TECHNICAL UNIVERSITY OF CRETE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
AUTOMATION DEPARTEMENT

DIPLOMA THESIS

PREDICTOR-FEEDBACK ON-RAMP METERING CONTROL OF TRAFFIC FLOW AT DISTANT BOTTLENECKS

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Abstract

Increasing population in cities around the world creates congestion problems. Freeways,tunels, bridges are the most common bottlenecks where traffic jam occurs since drivers choose them as a quick path to their destination. Traffic conjestion can be avoided either by traffic police or by traffic lights. As far as the latter is concerned the use of an automatic control system for the coordination of the traffic controllers is necessary.

Although an automatic control system is an effective way for regulating traffic flow, a major drawback that inhibits its proper function is that of time delay (also known as dead time), this concerns the velocity in which a variable measure is received by the sensors of the control system, then to be processed, the distance to be transmitted and then to be evaluated.

Several techniques have been designed for dealing with time delay in contol systems such as Smith predictor, the Padé approximation, PID controller with a lead lag compensator, combinations of the aforementioned with observers or not.

A predictor-feedback law constitutes an alternative for delay compensation and it is employed here. More precisely there is a proposal for the use of a predictor-feedback law in the place of a PI controller under delay effect, as an efficient manner for the regulation of traffic flow in a distant bottleneck in a highway.

In first place the construction, under the state variable model, of a PI controller is taking place for the traffic flow management. We then study the delay effect for various parameters of the PI controller. Last but not least the implementation of a predictor-feedback law for the compensation of the delay is presented. Simulation results are also presented under each case and control system plus a comparison review for the aforementioned.

Περίληψη

Η αύξηση του πληθυσμού στις πολιτείες φέρνει με την σειρά της και μία αύξηση του πλήθους μηχανοκίνητων οχημάτων σε αυτές με αποτέλεσμα να παρουσιάζεται το φαινόμενο της κυκλοφοριακής συμφόρησης. Σημεία της πόλης όπως αυτοκινητόδρομοι,τούνελ, γέφυρες προσφέρουν μία γρήγορη διέξοδο την οποία θα χρησιμοποιήσει ο οδηγός ώστε να φτάσει γρήγορα στον προορισμό του, παράλληλα όμως αποτελούν και σημεία όπου παρουσιάζεται κυκλοφοριακό μποτιλιάρισμα. Το κυκλοφοριακό μποτιλιάρισμα μπορεί να αποφευχθεί είτε με την παρουσία τροχαίας είτε με φωτεινούς σηματοδότες. Οσον αφορά το τελευταίο, η χρήση ενός συστήματος αυτομάτου ελέγχου για τον συντονισμό του κυκλοφοριακού είναι απαραίτητη.

Αν και ένα σύστημα αυτομάτου ελέγχου είναι ένας αποδοτικός τρόπος για την ρύθμιση του κυκλοφοριακού στους δρόμους ένα σημαντικό μειονέκτημα που απειλεί την αποδοτικότητά του είναι αυτό της χρονοκαθυστέρης, αυτή αφορά την ταχύτητα που χρειάζεται μία πληροφορία για να διανύσει μία απόσταση ώστε να φτάσει έγκαιρα στο σύστημα ελέγχου και να την επεξεργαστεί.

Τεχνικές όπως ο προβλεπτής Smith, η μαθηματική προσέγγιση Padé, ο PID ελεγκτής με αντισταθμιστή καθυστέρησης καθώς και συνδυασμοί των προαναφερόμενων με ή χωρίς κάποιον παρατηρητή, είναι σύνηθες εφαρμοζόμενες.

Ο προβλεπτής-ανατροφοδότησης είναι επίσης μία εναλλακτική τεχνική για την εξάλειψη του παράγοντα καθυστέρηση και σε αυτή τη διπλωματική παρουσιάζεται η εφαρμογή ενός τέτοιου συστήματος ελέγχου. Πιο συγκεκριμένα προτείνεται η χρήση ενός κανόνα προβλεπτή-ανατροφοδότησης στη θέση ενός PI ελεγκτή υπό την επίδραση καθυστέρησης, ως ένας αποτελεσματικός τρόπος για τον έλεγχο του κυκλοφοριακού σε κάποιο απομακρυσμένο σημείο συμφόρησης ενός αυτοκινητόδρομου.

Σε πρώτη φάση γίνεται η κατασκευή, ακολουθώντας το μοντέλο μεταβλητών κατάστασης ,ενός PI ελεγκτή για τον έλεγχο της κυκλοφοριακής ροής. Επειτα εξετάζουμε την επίδραση της καθυστέρησης για μία σειρά παραμέτρων του PI ελεγκτή. Τέλος γίνεται η εφαρμογή ενός προβλεπτή για την αντιστάθμιση των περιπτώσεων χρονοκαθυστέρησης. Αποτελέσματα προσομοιώσεων επίσης παρουσιάζονται για κάθε περίπτωση καθυστέρησης, για το κάθε σύστημα, όπως και μία συγκριτική αξιολόγηση.

Acknowledgments

I would like to thank all those who I have colaborated with, all this years, coleagues, teachers and staff.

Specially, I would like to thank my advisor Assistant Professor Nikolaos Bekiaris - Limperis for his guidance throughout this diploma thesis. Furthermore I am greatful towards the other two committee members of my thesis, Associate Professor Eftychios Koutroulis and Edip Elefteria Sergaki, for their useful comments and their time to evaluate this work.

Last but not least, I would like to thank my family and my friends who supported me all this time so as to achive my goals.

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Chapter 1

Introduction

Traffic congestion in big cities is an important problem where human coordination is difficult to solve. Intersections equiped with traffic lights offer a solution to control the traffic flow but in order to maximize the efficiency the need of a coordinated system is necessary.

Communication processes are involved in traffic flow control presenting delay issues cause of measuring, evaluating,transfering information etc. leading in turn the control of the traffic flow in instability.

Different control strategies have been developed in the last decades for dealing input delay systems and these are devided in two categories, one is of frequency domain and the other is of time domain. Smith predictor along with its modified versions is considered a frequency-domain control technique, on the other side Lyapunov–Krasovskii, ALINEA [1] along with its evolved versions (PI ALINEA,FF-ALINEA) are considered time-domain control methods.

In this thesis an implementation of a predictor-feedback control is being presented aiming in the elimination of any delay effect in a control system that regulates traffic flow in a distant bottleneck of a specific length in a highway and render this system as stable as possible. The aforementioned implemantaton was held in matlab enviroment and in discrete time.

1.1 Thesis Outline

In **chapter 2** a methodology for the constuction of a nominal PI controller is being carried out. Calculations of traffic density in a distant bottleneck takes place along with the evaluation of the performance of the PI controller for five cases of gain factors. In **chapter 3** delay effect as a parameter of the PI controller is being evaluated in four scenarios. In **chapter 4** a predictor-feedback law is implemented as a compensator of the four delay scenarios. A performance comparison between predictor-feedback law and PI controller is conducted. Finally in **chapter 5** the main conclusions of this thesis are presented along with suggested potential directions for future work.

Chapter 2

Implemantation of a PI controller for traffic flow management in a distant bottleneck

A controller is a mechanism with main purpose to minimize the difference between the response of a system and the desired value that is set for the system (this difference is also known as steady-state error). Controllers are used in most automatic process control applications in industry to regulate flow, temperature, pressure, level, and many other process variables. The important uses of controllers include improvement of the steady-state accuracy (sensitivity) by decreasing the steady state error, improvement of system stability, reducing the unwanted offsets (sustained errors) produced by the system, control of the maximum overshoot of the system, reducing the noise signals produced by the system, speed up the slow response of an overdamped system, etc.

Controllers in first place are seperated according to their mode of control action. There are two modes of control action, one is continuous and the other is discontinuous as shown in figure 2.1

In the discontinuous mode of a controller the process variable changes between discrete values, the output signal generated by the controller shows a variation from one value to another. According to this mode of operation, controllers can be considered as On-Off/Two-position controllers and multiposition controllers. Examples of systems using two-position controllers are domestic heating systems, refrigeration, water tanks.

In the continuous mode of controller the process variable has an even variation over the entire range of operation. According to this mode of operation, a controller can be classified as a Proportional, Integral or Derivative. Practicaly there is a use of a combination of these modes to control

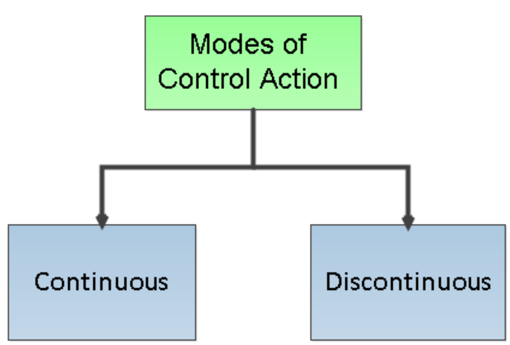


Figure 2.1: Controllers modes of operation

the system such that the process variable is equal or as close as it can to the setpoint. There are three combinations of controllers which are:

- Proportional and Integral controllers (PI Controllers)
- Proportional and Derivative controllers (PD Controllers)
- Proportional Integral Derivative controllers (PID Controllers)

A description for each seperate controller is presented below.

2.1 Control systems

2.1.1 Proportional controller

As the name suggests in a proportional controller the output is directly proportional to the error signal. Mathematicaly this can be described from the folowing equation:

$$u(t) = K_p e(t)$$

where K_p is the proportional gain and $e(t) = SetPoint - ProcessVariable$.

The proportional gain is the one that by increasing it tends to amplify the error thus making the system to respond faster. On the other hand increased proportional gain of this controller is a major cause for overshoot and oscillations as shown in figure 2.2.

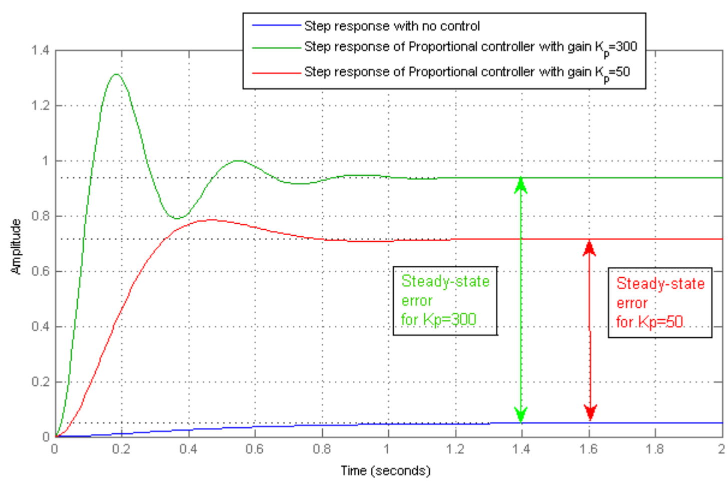


Figure 2.2: Proportional control response

Steady-state error can be reduced but not eliminated since proportional controller adjusts in the varying difference between the next steady state and the desired setpoint and not in the maintainance of the error.

2.1.2 Integral controller

As the name suggests in an integral controller the output is directly proportional to the integral of the error signal. Mathematicaly this can be described from the folowing equation:

$$u(t) = K_i \int_0^t e(t)dt$$

where K_i is the integral gain.

The integration of the error actualy stands as the summation of the error from zero time up to the current time t.

The I controller on its own has a very slow response because it needs the error to build up before it can start working (as shown in figure 2.3). This property helps to reduce the steady-state error, as long as there is no change in the error. On the other hand a change in the error causes an oscillatory behavior and makes the system unstable. To overcome this problem I controller is combined with a P controler.

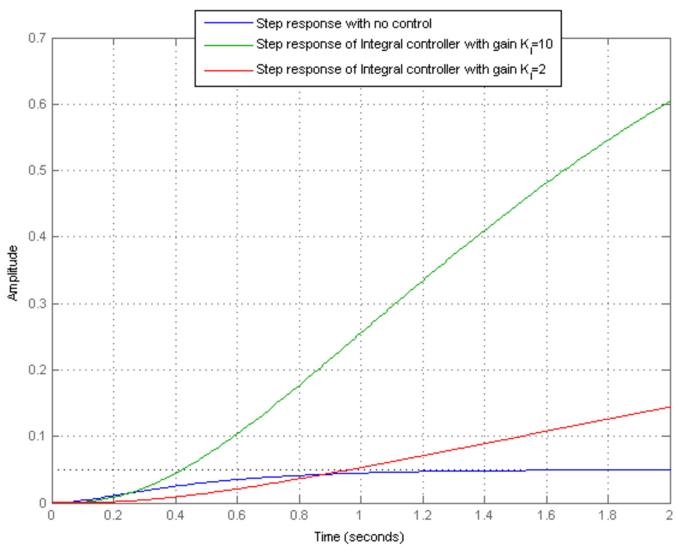


Figure 2.3: Integral control response

2.1.3 Derivative controller

As the name suggests in a derivative controller the output is directly proportional to the derivative of the error signal. Mathematicaly this can be described from the folowing equation:

$$u(t) = K_d \frac{de(t)}{dt}$$

where K_d is the derivative gain.

The derivative of the error actualy stands as the slope of the error signal at time t, where the slope implies that every rate of change of error signal provides a significantly different value of the output of the controller. Thus it never improves the steady-state error and also amplifies the noise signals produced in the system (as shown in figure 2.4).

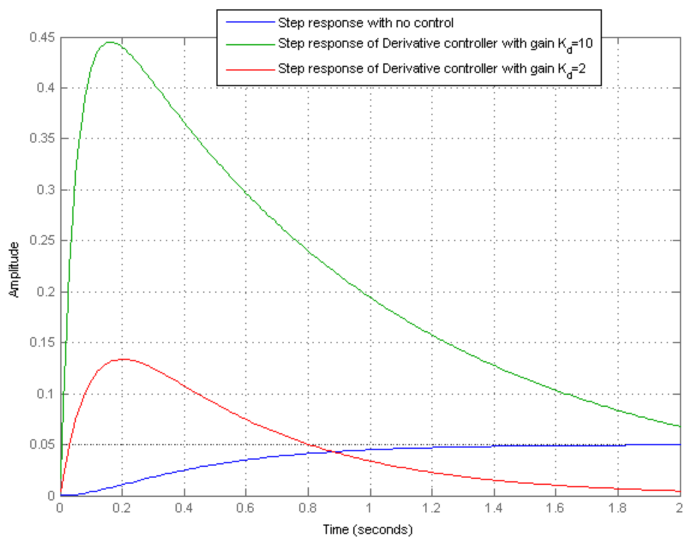


Figure 2.4: Derivative control response

The reason why this controller is used is that it can improve the transient response of the system and so derivative controller is never used alone.

Since in this thesis PI controller is the base element upon the implemented predictor-feedback law, an extensive description folows for it.

2.1.4 PI controller

PI controller is a combination of a proportional and an integral controller. As a consequence it combines the properties of proportional and integral action and thus eliminates the disadvantages associated with each one of them. Mathematical representation of proportional plus integral action is given from the folowing equation:

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt$$

It is therefore important to note that PI controllers are useful in systems where the response speed is not important. PI controllers have small effect on the rise time and cannot eliminate oscillations in a system because they are unable to predict future errors within the system.

PI controller is the most applied in automatic process control applications in industry [2]. It is a feedback closed loop mechanism (figure 2.5) that has the advantage to diminish the steady-state error of the system up to zero.

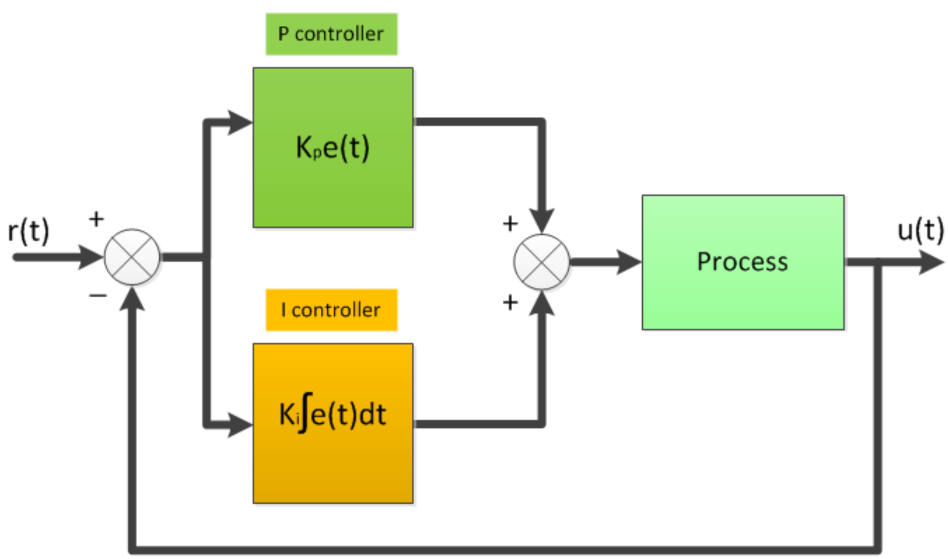


Figure 2.5: Closed loop control system with PI controller.

2.2 Description of the model for the traffic regulation in a distant bottleneck

In this section the description of a traffic flow management model is being presented. More precisely there is a given highway stretch with two controlled ramps U_1 and U_2 and a distant bottleneck of Δ length(see figure 2.6).

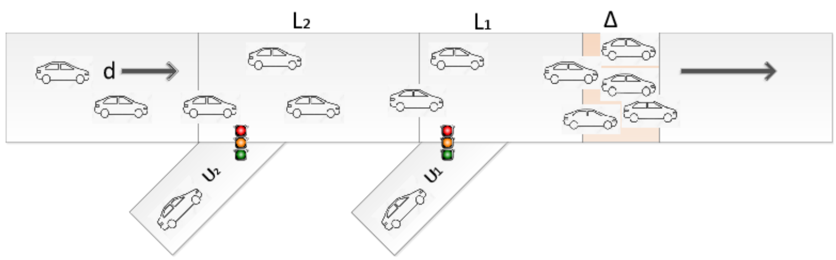


Figure 2.6: Traffic density regulation with two on ramp controllers U_1(t) and U_2(t).

The equation for the control of the traffic flow in the distant bottleneck is:

$$\frac{dY(t)}{dt} = \frac{1}{\Delta}[-avY(t) + U_1(t - D_1) + U_2(t - D_1 - D_2) + d] \tag{2.1}$$

where Y denotes the traffic density in the distant bottleneck, $D_1 = \frac{L_1}{v}$, $D_2 = \frac{L_2}{v}$ are time delays for road sections L1 and L2 respectively, $a \in [0.3, 0.9]$ is a slow factor (cause of the obstruction by other vehicles present in the same highway), υ>0 is the free flow velocity of the vehicles (that is, with no control), Δ>0 is the length of the distant bottleneck, d is the inflow of the vehicles before the main highway stretch and last U_1,U_2 are the corresponding ramps to be controlled by a PI controller respectively .

2.2.1 Traffic regulation under the state variable model

The state of a dynamic automatic control system can be expressed by a set of differential equations and thus by a set of variables. Being aware of the inputs of the system it is possible to predict the response of the system, that is the output for a future time moment t.

Since in this thesis only one PI controller is being considered then it is settled that $U_2 = 0$ and because in first case no delay is taking place the equation 2.1 is turned into:

$$\frac{dY(t)}{dt} = \frac{1}{\Delta}[-avY(t) + U_1(t) + d] \tag{2.2}$$

Additionaly the PI controller is expessed as:

$$U_1(t) = K_{11}(\Upsilon(t) - \Upsilon^*) + K_{12}\sigma(t) \tag{2.3}$$

where σ(t) is the integral operator from which:

$$\frac{d\sigma(t)}{dt} = \Upsilon(t) - \Upsilon^* \tag{2.4}$$

The state variable model which describes the traffic regulation system is :

$$\frac{dx(t)}{dt} = \begin{bmatrix} \frac{dY(t)}{dt} \\ \frac{d\sigma(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{(-av)}{\Delta} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Y(t) \\ \sigma(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{\Delta} \\ 0 \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \end{bmatrix} \begin{bmatrix} Y(t) \\ \sigma(t) \end{bmatrix} \tag{2.5}$$

the equation 2.5 can be briefly represented as:

$$\frac{x(t)}{dt} = Ax(t) + BKx(t) \tag{2.6}$$

where $A = \begin{bmatrix} \frac{(-av)}{\Delta} & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \frac{1}{\Delta} \\ 0 \end{bmatrix}$, $x(t) = \begin{bmatrix} Y(t) \\ \sigma(t) \end{bmatrix}$ and $K = \begin{bmatrix} K_{11} & K_{12} \end{bmatrix}$

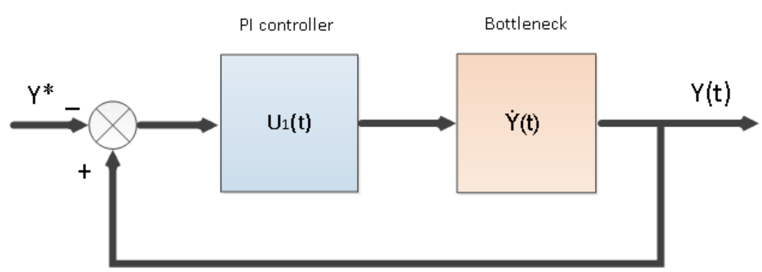


Figure 2.7: Block diagramm of the closed loop system for the traffic regulation at a distant bottleneck.

Equation 2.4 will be used for the deduction of the gain array K (that is, the determination of the gain coefficients K_{11},K_{12} of the PI controller).

2.2.2 Defining the gain matrix K of the PI controller

A system mathematically can be ordered by the degree of its highest derivative of its governing differential. The system depicted in equation 2.2 is a second order control system [1] of the following form:

$$Q(\lambda) = \lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 \tag{2.7}$$

[1]This can be prooved by evaluating the transfer function of the system

Second order systems are capable of an oscillatory response to a step input and their stability is determined by their characteristic equation. The latter is defined as:

$$det(\lambda I - (A - BK)) = 0 \tag{2.8}$$

As a consequence

$$det(\lambda I - (A - BK)) = det\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} \frac{-av-K_{11}}{\Delta} & \frac{-K_{12}}{\Delta} \\ 1 & 0 \end{bmatrix} \right| = \lambda^2 + \lambda(\frac{av + K_{11}}{\Delta}) + \frac{K_{12}}{\Delta} \tag{2.9}$$

Equalizing 2.9 with 2.7 it ends up that:

$$\frac{av + K_{11}}{\Delta} = 2\zeta\omega_n \tag{2.10}$$

$$\frac{K_{12}}{\Delta} = \omega_n^2 \tag{2.11}$$

So to evaluate the gains K_{11},K_{12} of the PI controller there must be a choise for values concerning the damping factor ζ [2] and the natural frequency ω_n [3] that will not give large overshoot and negative flow.

Before proceeding to the aforementioned evaluation there must be a testing for the system either being controllable or not.

2.2.3 System controllability

The definition of a system to be controllable or not[4] goes like this:

"A system is completely controllable if there exists an unconstrained control u(t) that can transfer any initial state x(t₀) to any other desired location x(t) in a finite time, $t_0 \leq t \leq T$. "

In order to ascertain for a system to be controllable or not it must be shown that rank $|A\ AB\ A^2B \cdots A^{n-1}B| = n$ or alternatively det $|A \quad AB \quad A^2B \cdots A^{n-1}B| \neq 0$. Indeed if P_c=[A AB] then:

$$detP_c = \begin{bmatrix} 1 & \frac{-av}{\Delta} \\ 0 & 1 \end{bmatrix} = 1 \cdot 1 - \frac{-av}{\Delta} \cdot 0 = 1 \neq 0 \tag{2.12}$$

Hence the system is controllable.

[2] Damping factor ζ is a dimensionless magnitude which express how fast an oscillation decays.
[3] Natural frequency ω_n(rad/sec) is the frequency in which the system oscillates for ζ=0.
[4] From Richard's C. Dorf and Robert's H Bishop "Modern Control Systems".

2.3 Simulation model and gain parameter evaluation

2.3.1 Simulation setup

The model of equation 2.2 represents the traffic density at a distant bottleneck of length Δ and the model of equation 2.3 represents the traffic flow on ramp U_1. For the former model, a parameter set up is presented in table 2.1.

Parameter	**value**
a	0.3
Δ(km)	1
v(km\h)	100
d(veh\h)	1000

Table 2.1: A set up of traffic density parameters

For simulating equation 2.2 in discrete time, derivative $\frac{dY(t)}{dt}$ was approximated with forward difference method [5], time is discretized with a model time step dt=0.00002 h, while 100000 samplings were evaluated in process time, all ending up in a 2 h represantation of traffic behavior (see appendix A).

2.3.2 Gain parameter evaluation

Before any traffic density and traffic flow calculation occurs, an optimal response must be considered for the control system, so there is the need to specify which damping factor ζ and natural frequency ω_n will be used so as to evaluate PI's controler gain factors K_{11},K_{12}. One diagram as displayed in figure 2.8 is used for choosing a value of natural frequency ω_n that corresponds to a certain gain factor K_{12}, and four diagrams as displayed in figure 2.9 are used for choosing which damping factor ζ is appropriate for gain factor K_{11}, under natural frequency ω_n.

[5] $\frac{dY(t)}{dt} = \frac{Y(t+dt)-Y(t)}{dt}$

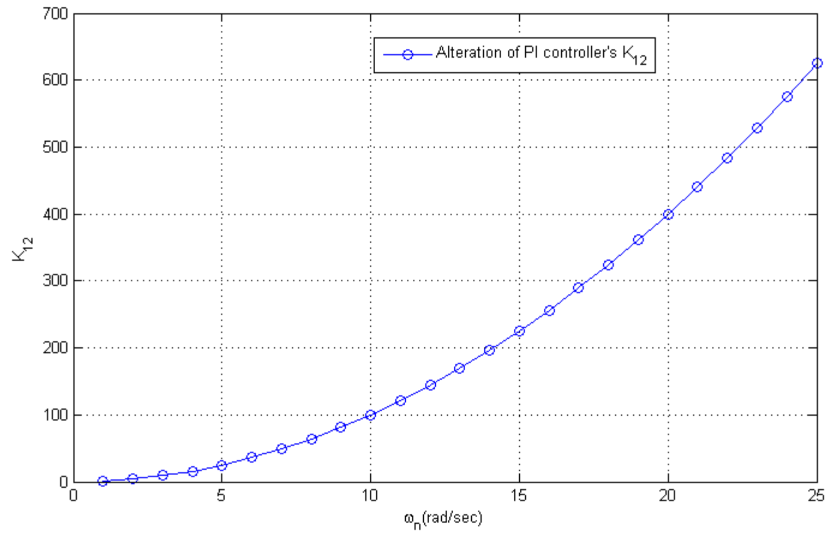


Figure 2.8: Integral gain variation related to natural frequency.

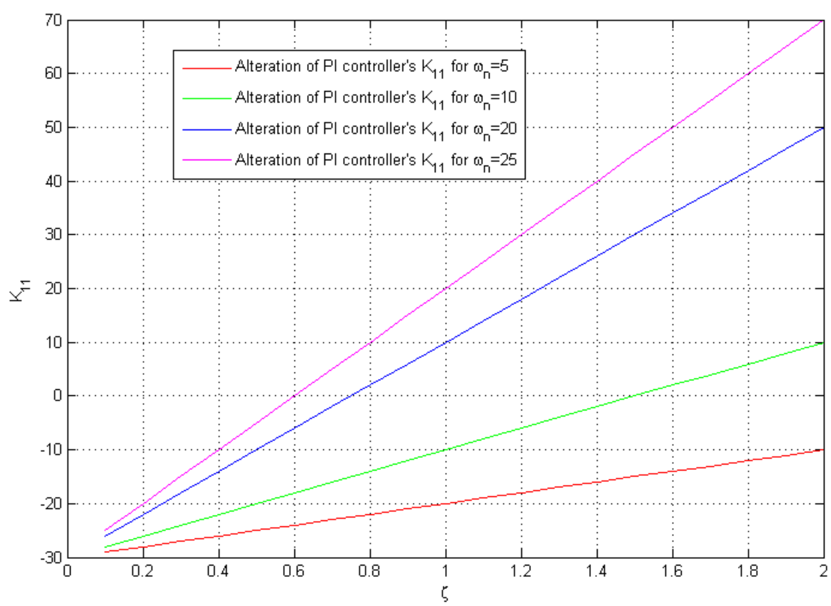


Figure 2.9: Proportional gain variation related to damping factor concerning ω_n=5, 10, 20 and 25 rad/sec (that is K_{12}=25, 100, 400 and 625 respectively).

We ascertain from figure 2.9 that gain factor K_{12} should be positive but for a small gain factor K_{11} because the latter exponentially increases. An adequent amplitude for the natural frequency and for the dambing factor can be specified yieldind a positive set of gain factors K_{11}, K_{12}, for the former that is $10 \leq \omega_n \leq 20$ while for the latter is $0.78 \leq \zeta \leq 2$.

2.3.3 Performance characteristics of a second order system

In order for the performance of a second order control system to be defined, a set of performance characteristics is being used. These are depicted in figure 2.10

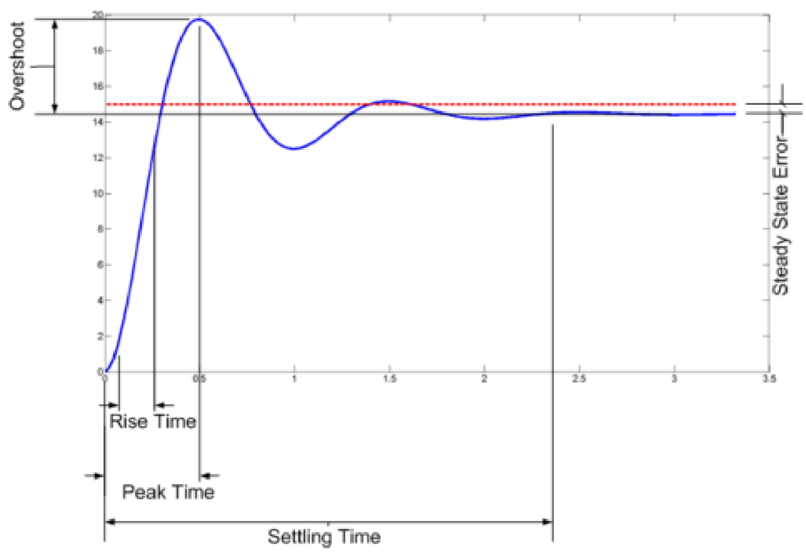


Figure 2.10: Performance characteristics of a response for a second order system.

- Rise Time (T_r): the time taken for the output to go from 10% to 90% of the final value.
- Peak Time (T_p): the time taken for the output to reach its maximum value.
- Overshoot: (max value - final value)100/final value.
- Settling Time (T_s): The time taken for the signal to be bounded to within a tolerance of x% of the steady state value.
- Steady State Error (e_{ss}): The difference between the input set point (dashed line) and the final value.

If a control system is asked to follow certain criteria then a designing upon these parameters is useful. This can be done either by running a simulation and measure the parameters from the step response directly or to define expressions for the parameters in terms of the transfer function coefficients. Mathematical expressions regarding the latter are as follows:

$$Overshoot(\%) = 100exp(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}) \qquad for\ \ 0 < \zeta < 1 \tag{2.13}$$

$$T_r \approx \frac{1}{\omega_n}(2.3\zeta^2 - 0.078\zeta + 1.12) \qquad for\ \ 0 < \zeta < 1 \tag{2.14}$$

$$T_p \approx \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{2.15}$$

$$T_s \approx -\frac{ln(tolerance\ \ x\%)}{\zeta\omega_n} \qquad for\ \ \zeta << 1 \tag{2.16}$$

$$\tag{2.17}$$

In this thesis the evaluation of these parameters was done by running a simulation and measure the parameters from the step response directly.

2.3.4 Gain parameter results

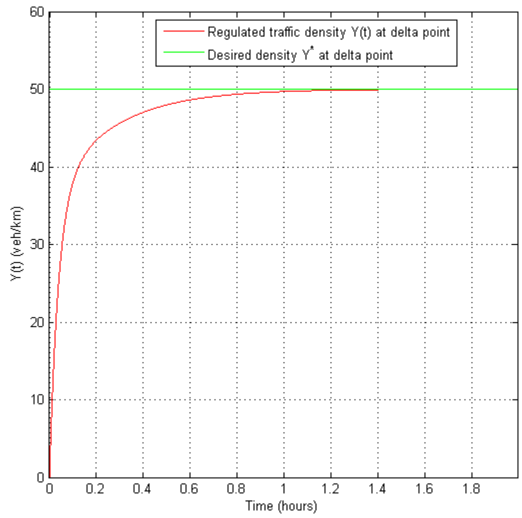
An applied combination of gain factors K_{11}, K_{12} for calculating U_1 and therefore Y(t), is presented in table 2.2.

ζ	**ω_n(rad\sec)**	**K_{11}**	**K_{12}**
1.5	10	0	100
1.5	15	15	225
1.8	15	24	225
1.8	20	42	400
2	20	50	400

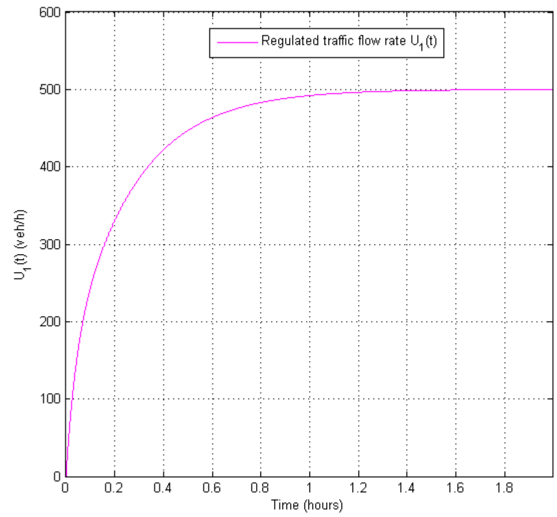
Table 2.2: A proposed combination of damping factor ζ with natural frequency ω_n

Of course other combinations can be used in terms of the response behaviour for the traffic control system as long as they are in the specified limits.

In the figures below (figures 2.11 - 2.15) we ascertain the set of gain factors K_{11}, K_{12} that yield an acceptable response (only positive) for traffic density Y(t) in accordace with flow U_1 on the corresponding ramp.

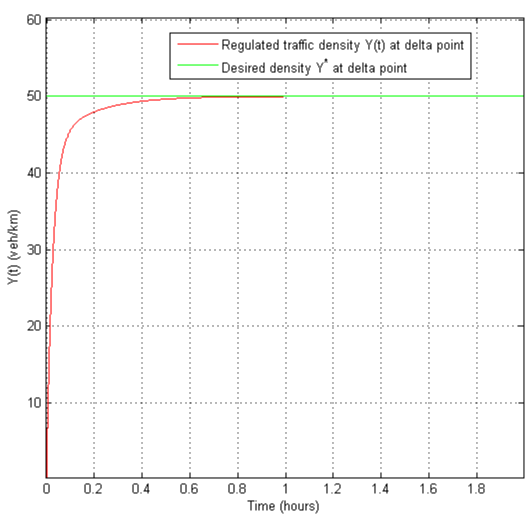


(i) Traffic density response for K_{11}=0 and K_{12}=100.

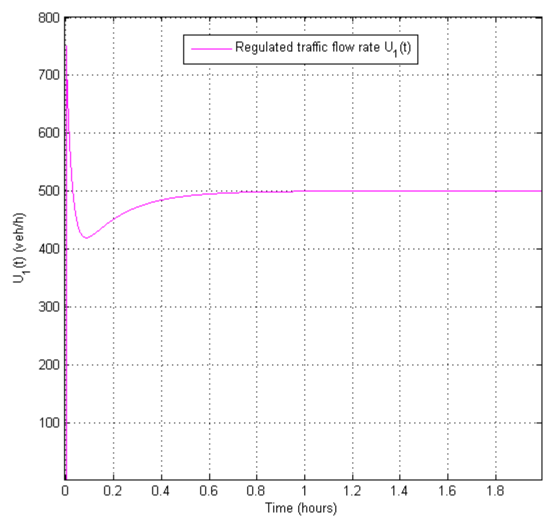


(ii) Flow response for K_{11}=0 and K_{12}=100.

Figure 2.11: Traffic density response Y(t) and flow response U_1(t) for ζ=1.5 and ω_n=10 (that is, K_{11}=0 and K_{12}=100 in PI controller).

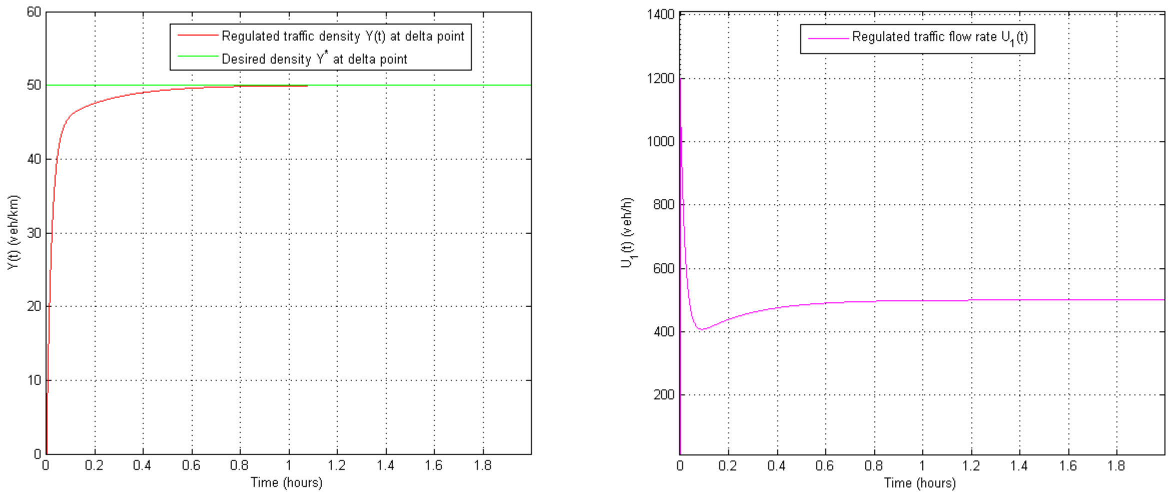


(i) Traffic density response for K_{11}=15 and K_{12}=225.



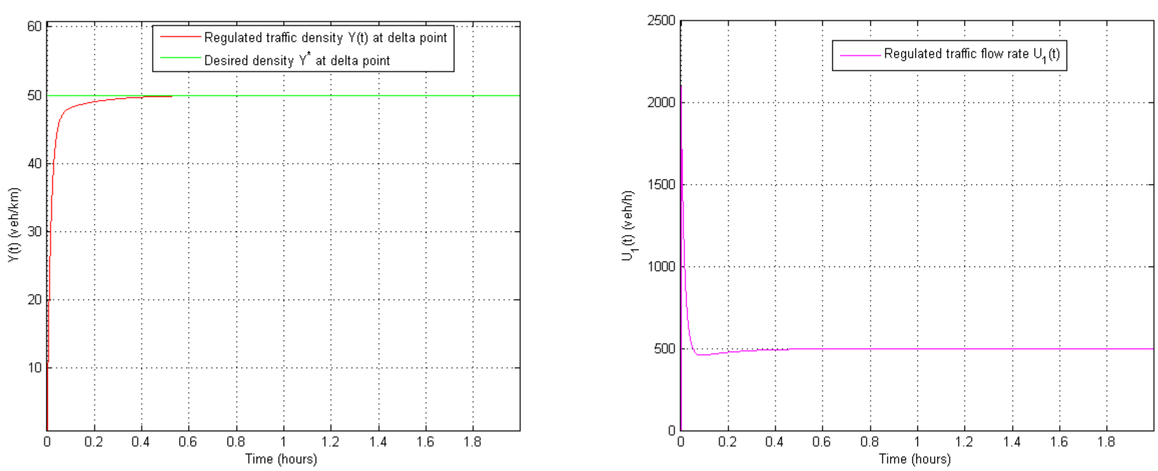
(ii) Flow response for K_{11}=15 and K_{12}=225.

Figure 2.12: Traffic density response Y(t) and flow response U_1(t) for ζ=1.5 and ω_n=15 (that is, K_{11}=15 and K_{12}=225 in PI controller).



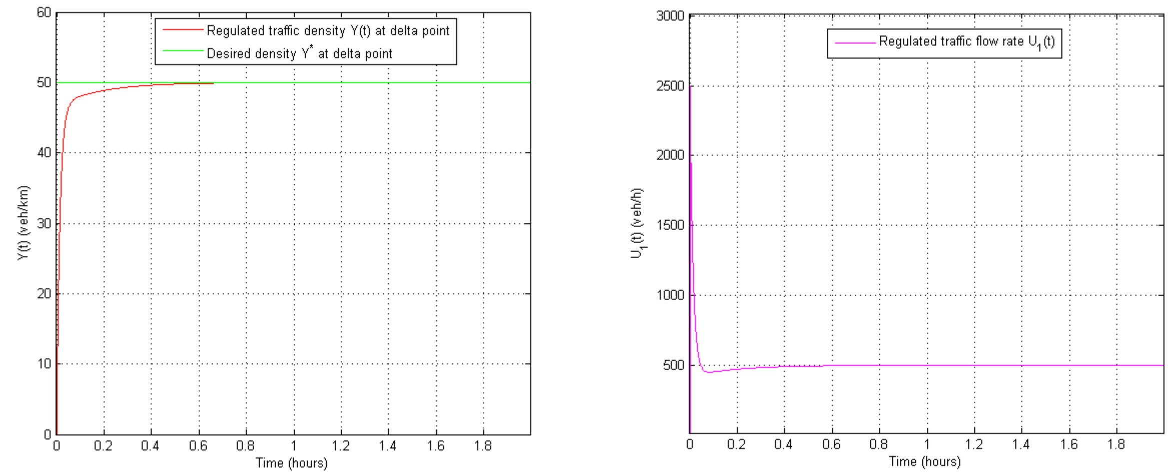
(i) Traffic density response for K_{11}=24 and K_{12}=225. (ii) Flow response for K_{11}=24 and K_{12}=225.

Figure 2.13: Traffic density response Y(t) and flow response U_1(t) for ζ=1.8 and ω_n=15 (that is, K_{11}=24 and K_{12}=225 in PI controller).



(i) Traffic density response for K_{11}=42 and K_{12}=400. (ii) Flow response for K_{11}=42 and K_{12}=400.

Figure 2.14: Traffic density response Y(t) and flow response U_1(t) for ζ=1.8 and ω_n=20 (that is, K_{11}=42 and K_{12}=400 in PI controller).

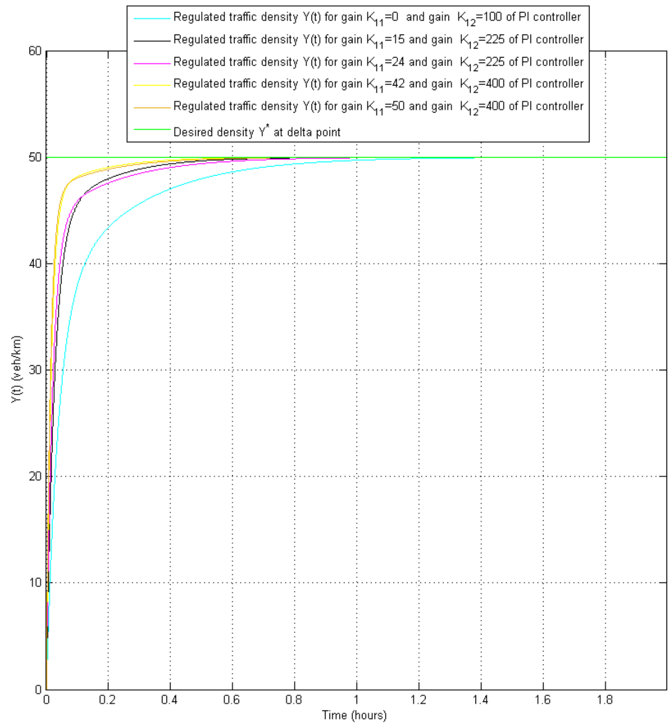


(i) Traffic density response for K_{11}=50 and K_{12}=400.

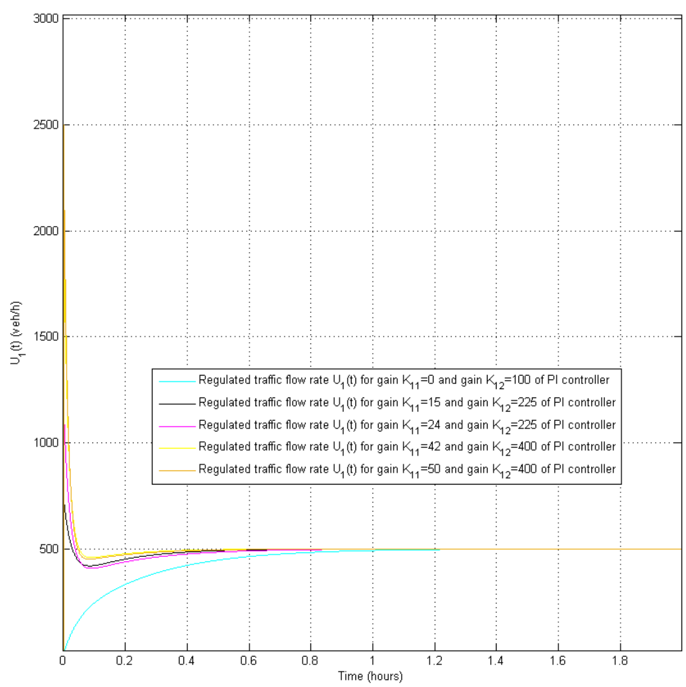
(ii) Flow response for K_{11}=50 and K_{12}=400.

Figure 2.15: Traffic density response Y(t) and flow response U_1(t) for ζ=2 and ω_n=20 (that is, K_{11}=50 and K_{12}=400 in PI controller).

It is obvious from subfigures (i) and (ii) of figure 2.11 that only the set K_{11}=0, K_{12}=100 (that is, ζ=1.5, ω_n=10) presents a smooth behavior for both traffic density Y(t) and flow U_1(t). The rest of gain K sets are also accepted depending on how fast the traffic density response Y(t) must respond in the distant bottleneck and as a consequence which overshoot is tolerated on the ramp U_1 so as the corresponding PI controller to be implemented.



(i) Traffic density response for all sets of K_{11}, K_{12}.



(ii) Flow response for all sets of K_{11}, K_{12}.

Figure 2.16: Traffic density response Y(t) and flow response $U_1(t)$ for all sets of K_{11}, K_{12}.

For all K set cases except the first one, flow response $U_1(t)$ exposes significant increased rise time T_r and heightened overshoot. On the contrary settling time T_s is proportionally decreased as matrix K increases. As far as traffic density response is concerned, this exposes increased rise time (T_r), zero overshoot and decreased settling time (T_s) as matrix K increases.

An other point that subfigures of on ramp flow control exhibit is that the desired traffic density of 50 veh/km in the distant bottleneck corresponds to an on ramp flow of 500 veh/h. Flow is related with density from the following equation:

$$Q(veh/h) = Density(veh/km) \cdot Velocity(km/h) \tag{2.18}$$

So for an on ramp velocity of 100 km/h the on ramp traffic density would be 5 veh/km and for a bottleneck velocity of 100 km/h the flow through bootleneck would be 5000 veh/h, since the bootleneck length is 1 km this means that 50 vehicles cross the bottleneck.

As a conclusion in order for a ramp metering to be effective, the sum of road sections L_2, L_1 demand and on ramp demand should be higher than the bottleneck capacity.

Chapter 3

Traffic flow control in a distant bottleneck under delay effect

Delay effect is a phenomenon that is caused mainly due to the transmission of information, that is the distance in which the information travells in a specific velocity. Communication networks, chemical processes, teleoperation systems, biosystems, processes industries and so on undergo delay phenomenons. Time delay may cause performance decline, even instability of any system [3].

In this section delay effect scenarios, as an input parameter for the PI controller, are being evaluated for equation 2.2.

3.1 Delay scenarios

Four cases of the delay effect D_1 are considered for all gain sets of table 2.2. These are shown in table 3.1

$\mathbf{D_1}$
0.0084 h (30 sec)
0.017 h (1 min)
0.083 h (5 min)
0.16 h (10 min)

Table 3.1: Delay scenarios for U_1 controller.

Under delay effect equation 2.2 turns into:

$$\frac{dY(t)}{dt} = \frac{1}{\Delta}[-avY(t) + U_1(t - D_1) + d] \tag{3.1}$$

Why there is no control in time space $[0, D_1)$ a vehicle free flow speed is 100 km/h, thus the flow for the corresponding bottleneck length (that is, 1 km) will be 100 veh/h, so equation 3.1 becomes:

$$\frac{dY(t)}{dt} = \begin{cases} \frac{1}{\Delta}[-avY(t) + 100 + d] & \text{for } 0 \leq t < D_1 \\ \\ \frac{1}{\Delta}[-avY(t) + U_1(t - D_1) + d] & \text{for } t \geq D_1 \end{cases} \tag{3.2}$$

now equation 3.2 shows altogether how traffic density is evaluated before and after delay effect.

From relation $D_1 = \frac{L_1}{v}$, the distance L_1between the exit of the ramp and the entrace of the bottleneck can be estimated. Table 3.2 presents the corresponding distances for the proposed delay cases.

D₁	**L₁**
0.0084 h (30 sec)	0.84 km
0.017 h (1 min)	1.7 km
0.083 h (5 min)	8.3 km
0.16 h (10 min)	16 km

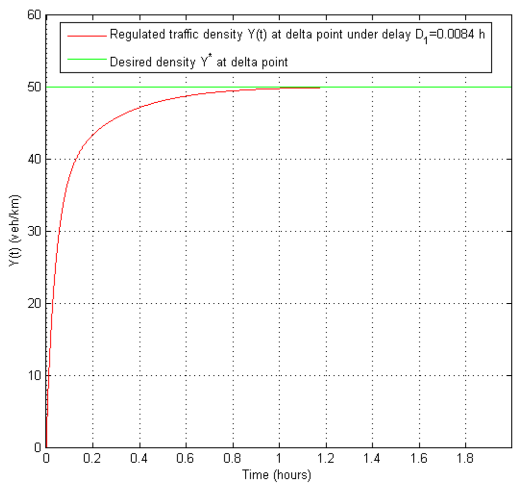
Table 3.2: Corresponding distance between ramp U_1 exit and entrance of bottleneck for the delay scenarios.

Time is discretized with a model time step Δt=0.00002 h, while 100000 samplings were evaluated in process time, all ending up in a 2 h represantation of traffic behavior (see appendix B).

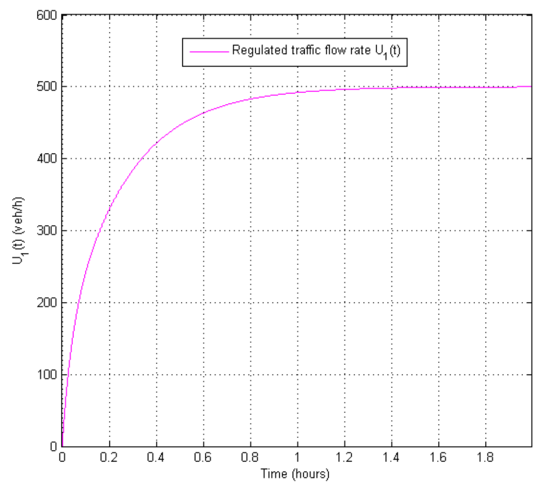
3.2 Simulation results

The following results appeard after implemanting equations 3.2, 2.2 for each delay case in order.

- **case K_{11}=0, K_{12}=100 (ζ=1.5, ω_n=10)**

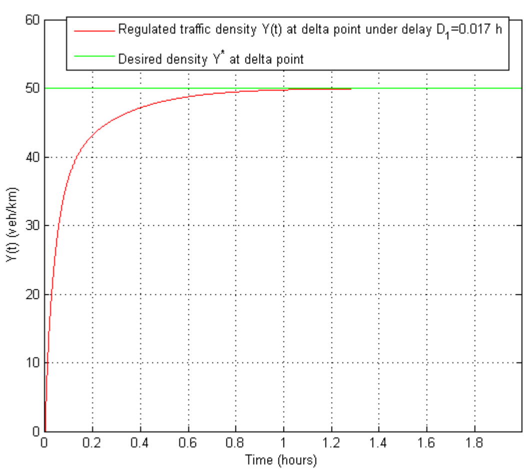


(i) Traffic density response for D_1=0.0084 h.

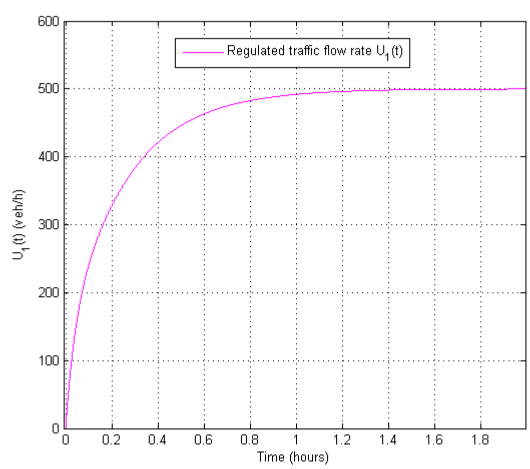


(ii) On ramp U_1 flow response.

Figure 3.1: Traffic density response Y(t) for delay effect D_1=0.0084 h and flow response U_1(t).

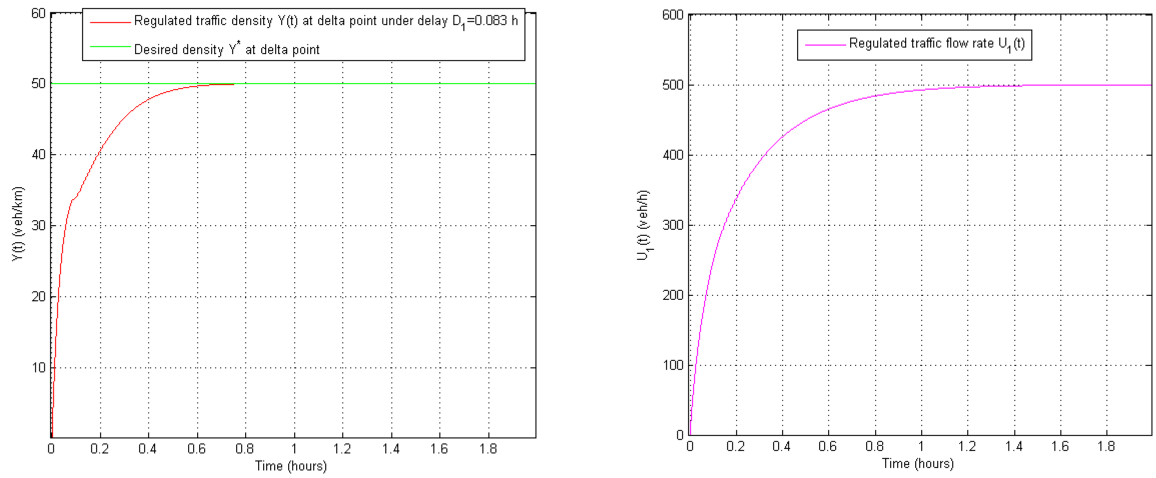


(i) Traffic density response for D_1=0.017 h.



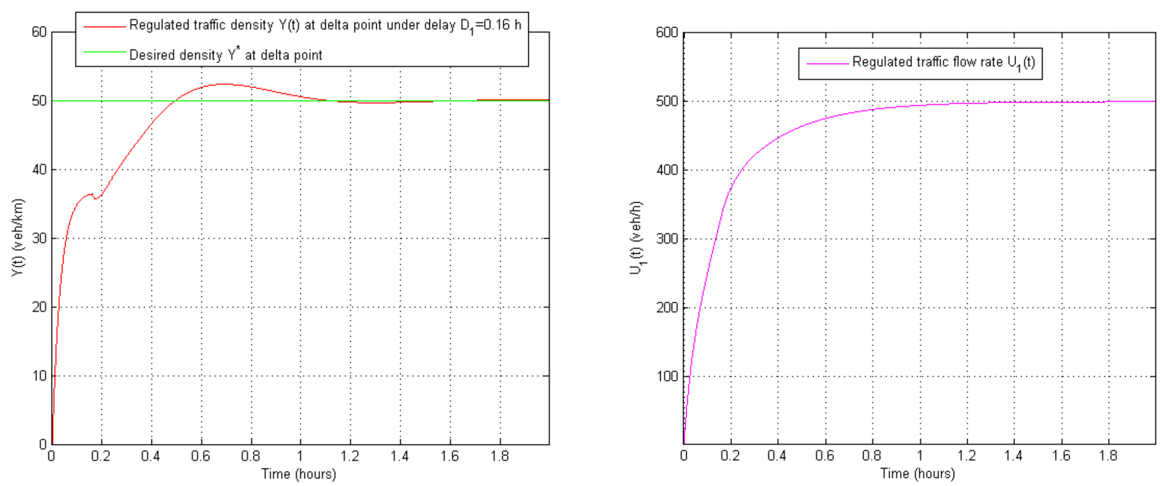
(ii) On ramp U_1 flow response.

Figure 3.2: Traffic density response Y(t) for delay effect D_1=0.017 h and flow response U_1(t).



(i) Traffic density response for D_1=0.083 h. (ii) On ramp U_1 flow response.

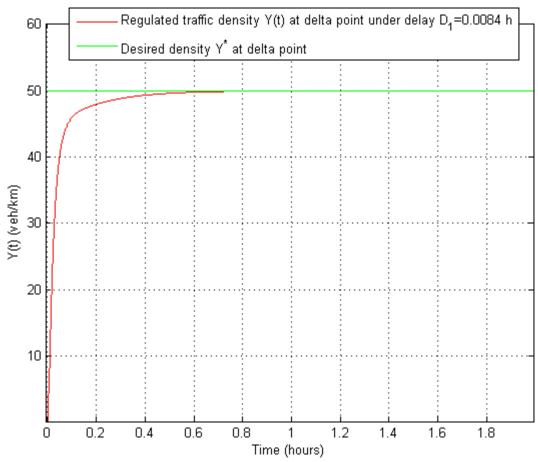
Figure 3.3: Traffic density response Y(t) for delay effect D_1=0.083 h and flow response $U_1(t)$.



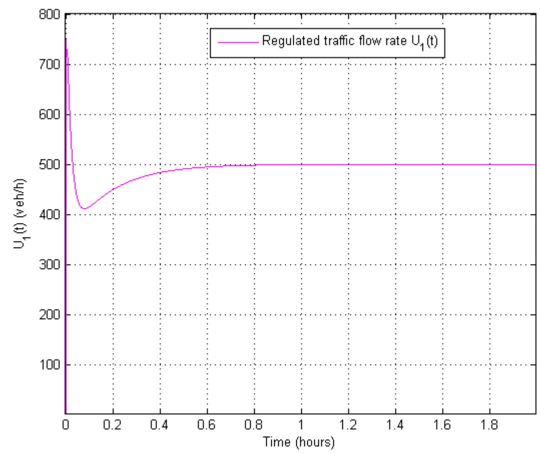
(i) Traffic density response for D_1=0.16 h. (ii) On ramp U_1 flow response.

Figure 3.4: Traffic density response Y(t) for delay effect D_1=0.16 h and flow response $U_1(t)$.

- **case K_{11}=15, K_{12}=225 (ζ=1.5, ω_n=15)**

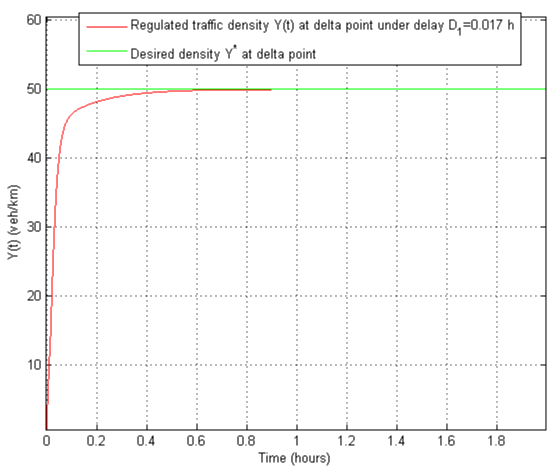


(i) Traffic density response for D_1=0.0084 h.

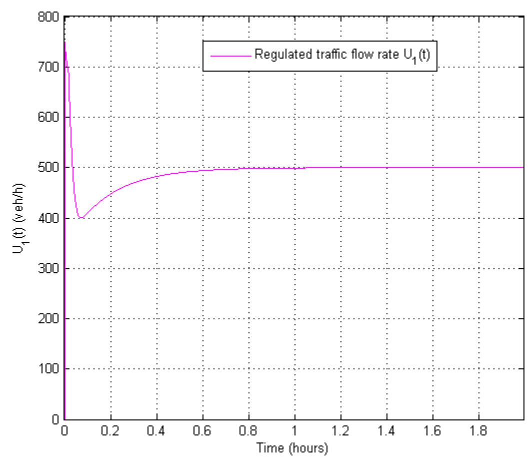


(ii) On ramp U_1 flow response.

Figure 3.5: Traffic density response Y(t) for delay effect D_1=0.0084 h and flow response $U_1(t)$.

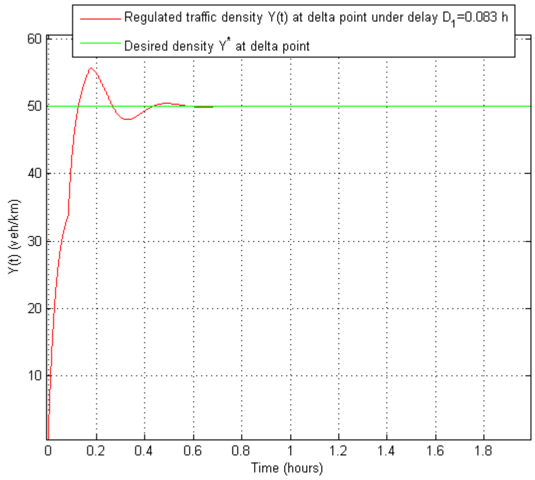


(i) Traffic density response for D_1=0.017 h.

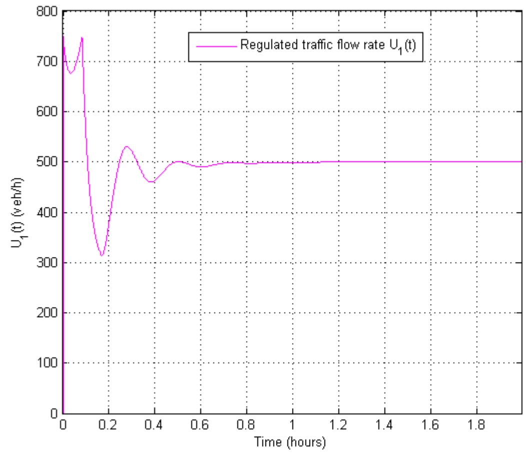


(ii) On ramp U_1 flow response.

Figure 3.6: Traffic density response Y(t) for delay effect D_1=0.017 h and flow response $U_1(t)$.

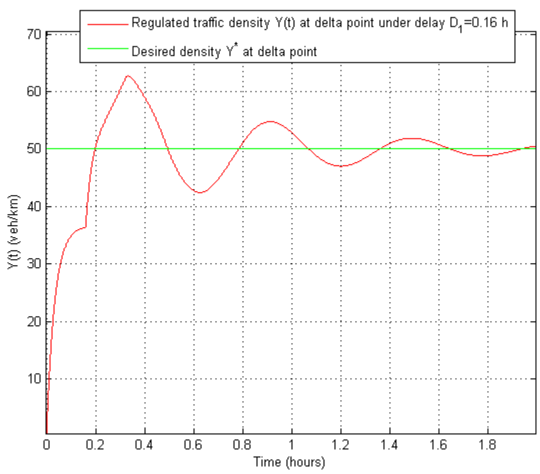


(i) Traffic density response for D_1=0.083 h.

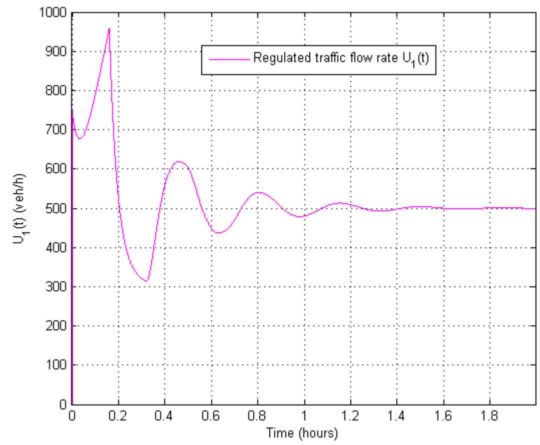


(ii) On ramp U_1 flow response..

Figure 3.7: Traffic density response Y(t) for delay effect D_1=0.083 h and flow response U_1(t).



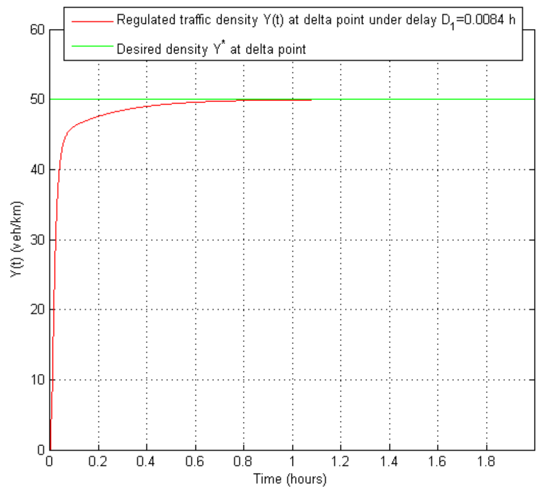
(i) Traffic density response for D_1=0.16 h.



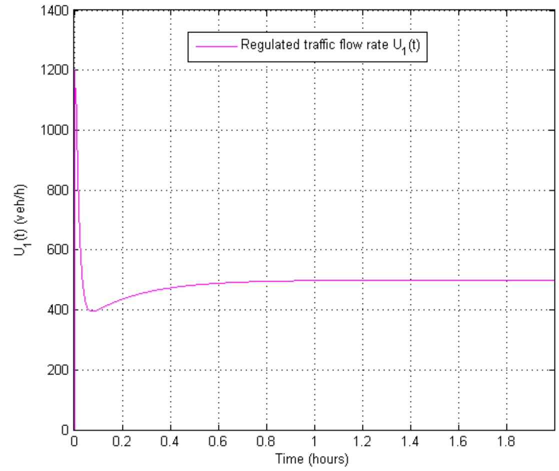
(ii) On ramp U_1 flow response.

Figure 3.8: Traffic density response Y(t) for delay effect D_1=0.16 h and flow response U_1(t).

- **case K_{11}=24, K_{12}=225 (ζ=1.8, ω_n=15)**

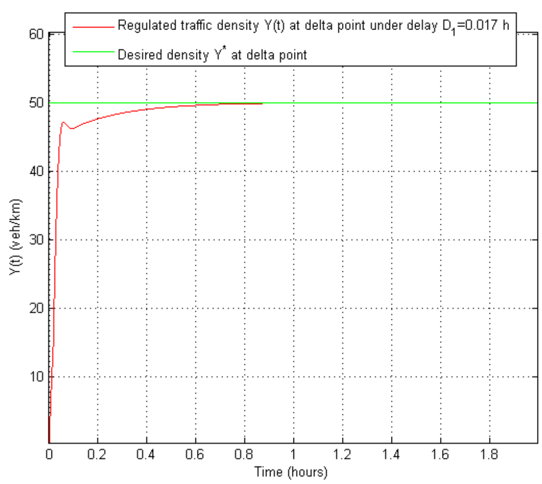


(i) Traffic density response for D_1=0.0084 h.

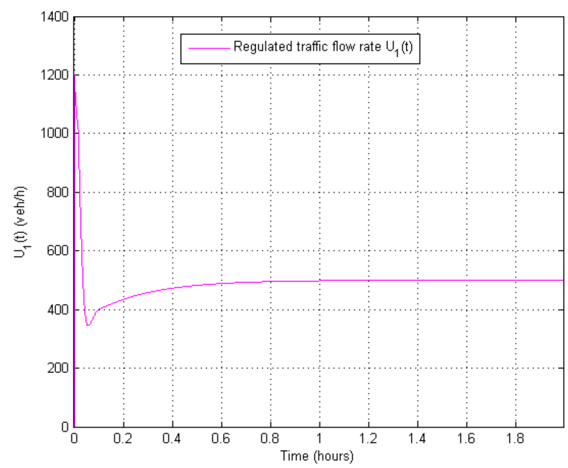


(ii) On ramp U_1 flow response.

Figure 3.9: Traffic density response Y(t) for delay effect D_1=0.0084 h and flow response U_1(t).

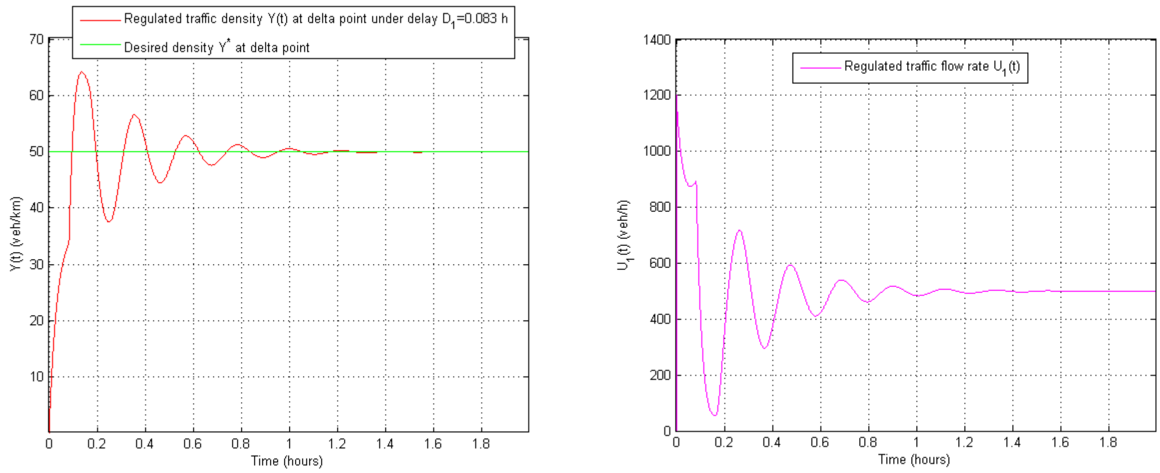


(i) Traffic density response for D_1=0.017 h.



(ii) On ramp U_1 flow response.

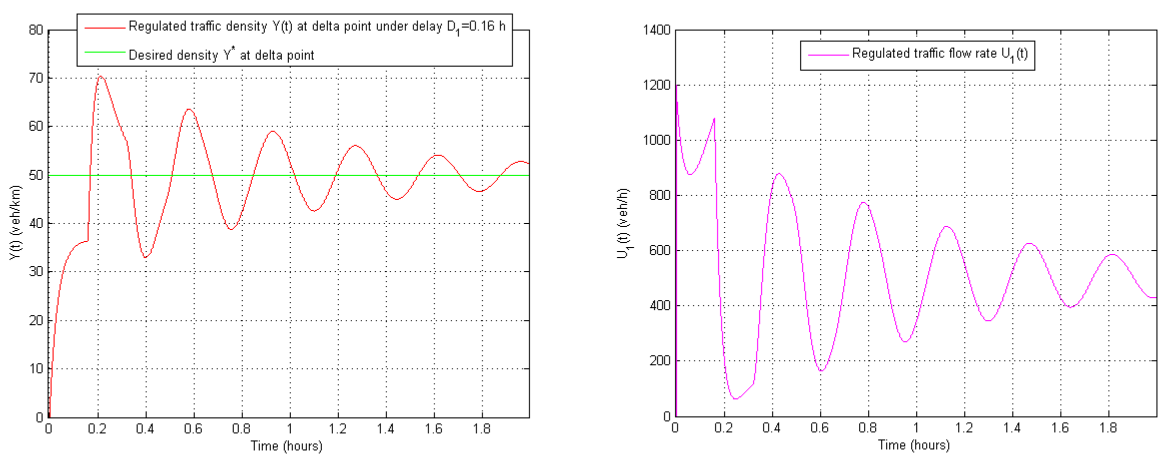
Figure 3.10: Traffic density response Y(t) for delay effect D_1=0.017 h and flow response U_1(t).



(i) Traffic density response for D_1=0.083 h.

(ii) On ramp U_1 flow response.

Figure 3.11: Traffic density response Y(t) for delay effect D_1=0.083 h and flow response U_1(t).

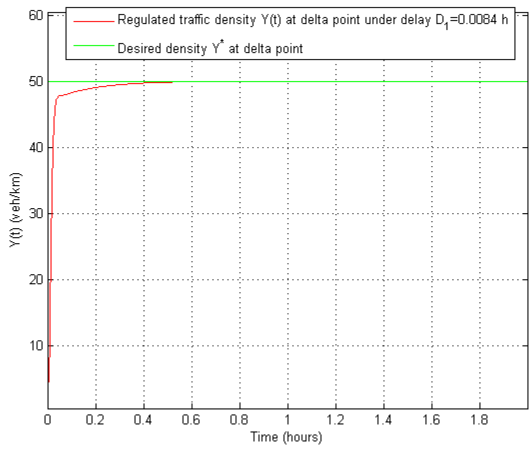


(i) Traffic density response for D_1=0.16 h.

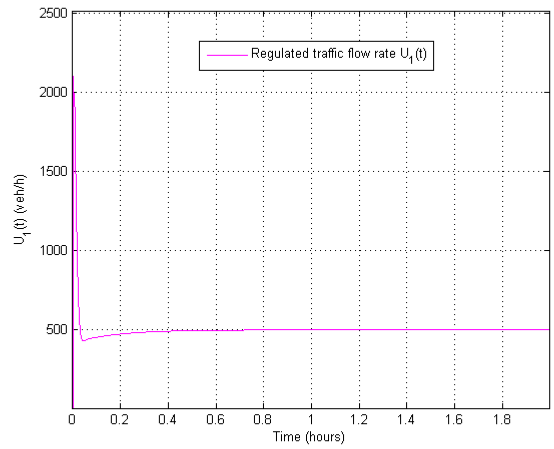
(ii) On ramp U_1 flow response.

Figure 3.12: Traffic density response Y(t) for delay effect D_1=0.16 h and flow response U_1(t).

- **case $K_{11}=42$, $K_{12}=400$ ($\zeta=1.8$, $\omega_n=20$)**

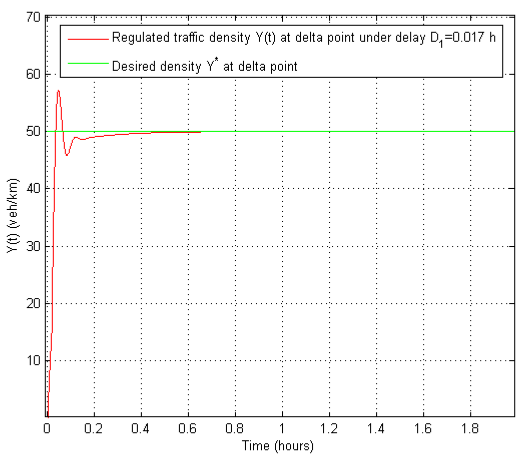


(i) Traffic density response for $D_1=0.0084$ h.

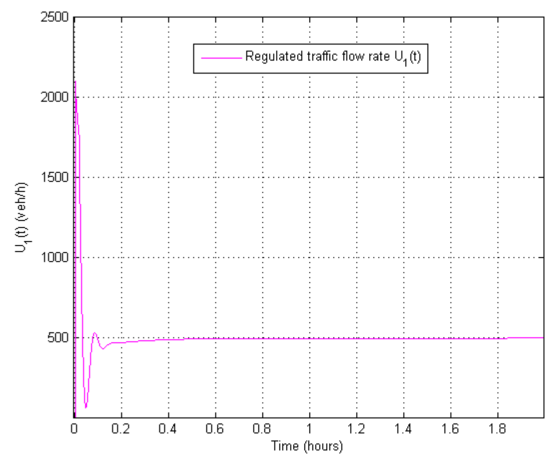


(ii) On ramp U_1 flow response.

Figure 3.13: Traffic density response Y(t) for delay effect $D_1=0.0084$ h and flow response $U_1(t)$.



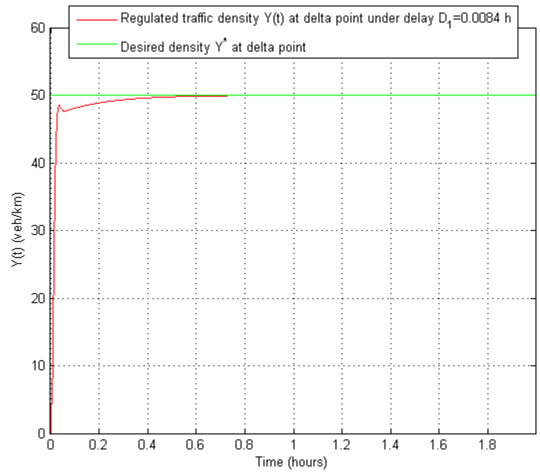
(i) Traffic density response for $D_1=0.017$ h.



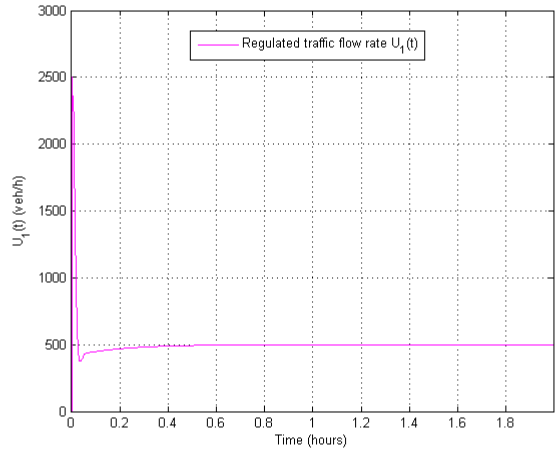
(ii) On ramp U_1 flow response.

Figure 3.14: Traffic density response Y(t) for delay effect $D_1=0.017$ h and flow response $U_1(t)$.

- **case K_{11}=50, K_{12}=400 (ζ=2, ω_n=20)**



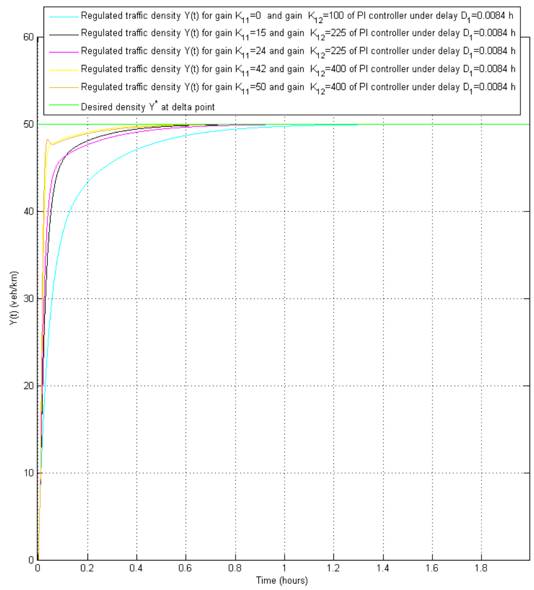
(i) Traffic density response for D_1=0.0084 h.



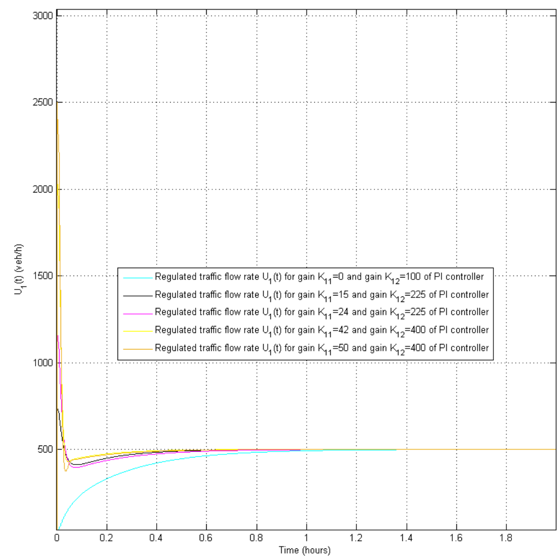
(ii) On ramp U_1 flow response.

Figure 3.15: Traffic density response Y(t) for delay effect D_1=0.0084 h and flow response $U_1(t)$.

- **Overall cases under delay D_1**



(i) Traffic density response for D_1=0.0084 h.

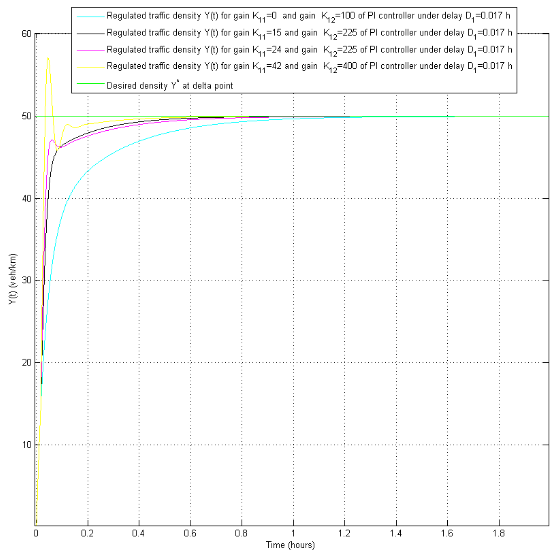


(ii) On ramp U_1 flow response.

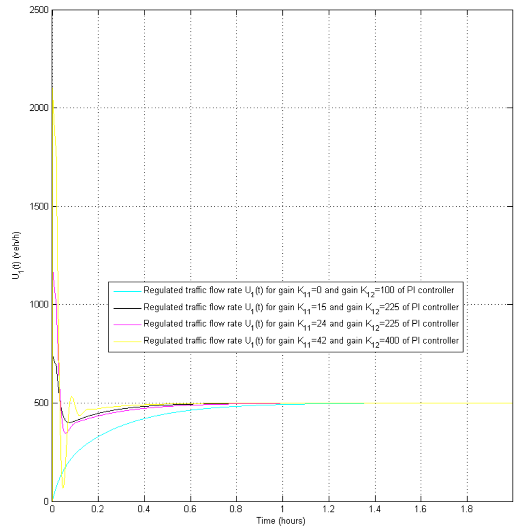
Figure 3.16: Traffic density response Y(t) for delay effect D_1=0.0084 h and flow response $U_1(t)$ for all sets of K_{11}, K_{12}.

Examining the overall progress of flow response $U_1(t)$ in subfigure (ii) of figure 3.16 there is the conclusion that the overshoot is increasing as gains K_{11}, K_{12} are increasing respectively. Furthermore T_r exhibits a rapid increase and T_s decreases. As a consequence a more fast traffic density response Y(t) is being produced with no oscillatory behavior.

Generall the delay effect D_1=0.0084 h has a small impact in the system response, that is obvious by comparing it with the equivallent graphs of figure 2.16.



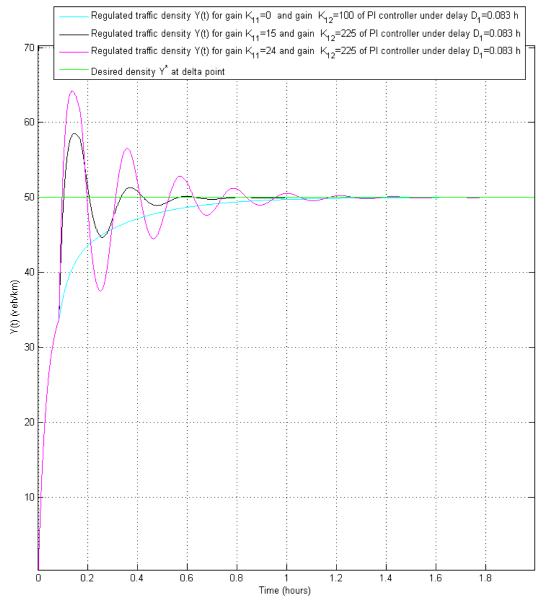
(i) Traffic density response for D_1=0.017 h.



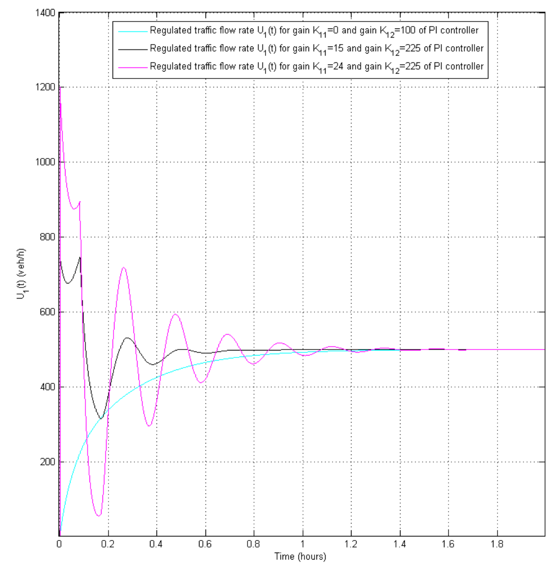
(ii) On ramp U_1 flow response.

Figure 3.17: Traffic density response Y(t) for delay effect D_1=0.017 h and flow response U_1(t) for all sets of K_{11}, K_{12}.

Comparing subfigure (ii) of figures 3.17, 3.16 it can be seen that delay effect D_1=0.017 h has caused a decreasing overshoot, regarding flow response U_1(t), for all sets of K_{11}, K_{12} except the first one. Moreover, delay effect D_1=0.017 h has caused a deceleration in T_r and an increasing behaviour in T_s. Oscillatory behavior is presented under the case K_{11}=42, K_{12}=400 both for traffic density and flow rate. For case K_{11}=50, K_{12}=400 flow response U_1(t) exhibited negative values so it is not recommended for a control system to be applied.



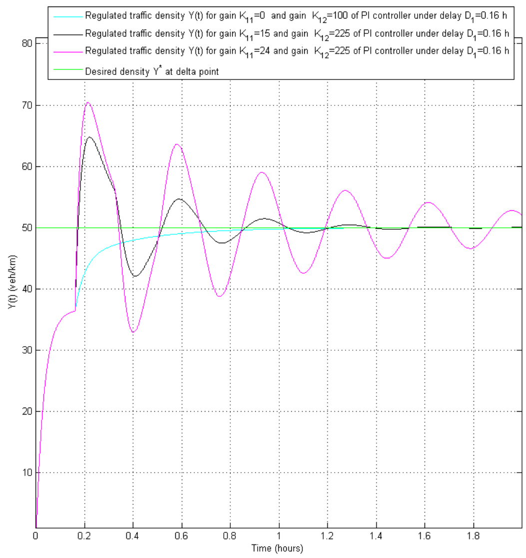
(i) Traffic density response for D_1=0.083 h.



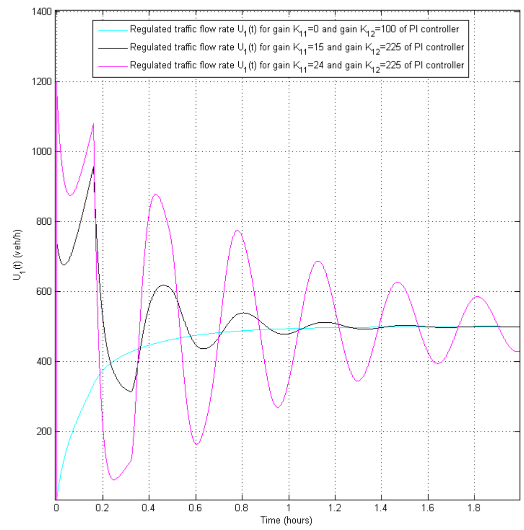
(ii) On ramp U_1 flow response.

Figure 3.18: Traffic density response Y(t) for delay effect D_1=0.083 h and flow response $U_1(t)$ for all sets of K_{11}, K_{12}.

From figure 3.18, comparing it with the previous figures 3.17-3.16 it is ascertained that delay effect D_1=0.083 h has caused a decreased overshoot, as far as flow response $U_1(t)$ is concerned, except the first set of K_{11}, K_{12}. Rise time (T_r) is decreased and settling time (T_s) is getting higher ending in an oscillatory behavior and all this because PI controller responds late. The last two cases for K_{11}, K_{12} yield negative values in their response so they were excluded.



(i) Traffic density response for D1=0.16 h.



(ii) On ramp U1 flow response.

Figure 3.19: Traffic density response Y(t) for delay effect D_1=0.16 h and flow response U_1(t) for all sets of K_{11}, K_{12}.

From figure 3.19 it is verified that as delay increses oscillatory behavior becomes more intense. The last two cases for K_{11}, K_{12} yield negative values in their response so they were excluded. Only the set K_{11}=0, K_{12}=100 presented a smooth response reaching the desired set point with a zero steady state error.

Chapter 4

Predictor-feedback implementation

Predictors as their name suggests are predicting the future values of a state variable for the control system. This property is used mainly for the compensation of the input delay in a control system thus render it stable and even with no oscillations [4].

The predictor feedback law is the product of a feedback gain matrix with an exponential term that represents the phase lag, the predicted state of the controlled system and a finite summation of past values of the predictor in a defined time window, altogether.

In this section an evaluation of a predictor-feedback law, for the traffic flow control system of equation 2.2, is being presented as a time delay compensator. The implemented predictor-feedback control system is given by the following equation:

$$U_1(t) = K\left(e^{AD_1}\begin{bmatrix} Y(t) \\ \sigma(t) \end{bmatrix} + \int_{t-D_1}^{t} e^{A(t-\theta)} B U_1(\theta) d\theta\right) \tag{4.1}$$

where K is the gain matrix, $B = \begin{bmatrix} \frac{1}{\Delta} \\ 0 \end{bmatrix}$, $U_1(\vartheta)$ represents past values of the predictor-feedback law in the time window [t-D_1, t] and e^{AD1}, $e^{A(t-\theta)}$ are the delay factors to be compensated. Figure 4.1 presents the block diagramm of the implemented predictor-feedback controller for the traffic flow management in a distant bottleneck.

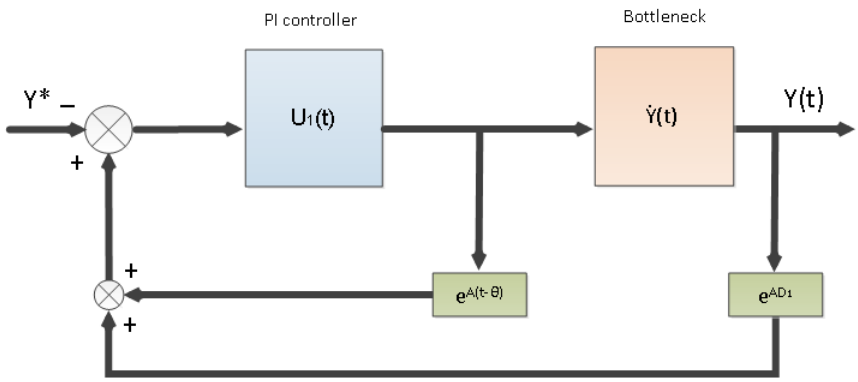


Figure 4.1: Block diagramm of an implemented predictor-feedback law for traffic flow management in a distant bottleneck.

4.1 Simulation setup

For the evaluation of the predictor feedback law of equation 4.1 in discrete time there was a use of a trapezoidal rule for the integral summation. The general form of this rule is as follows.

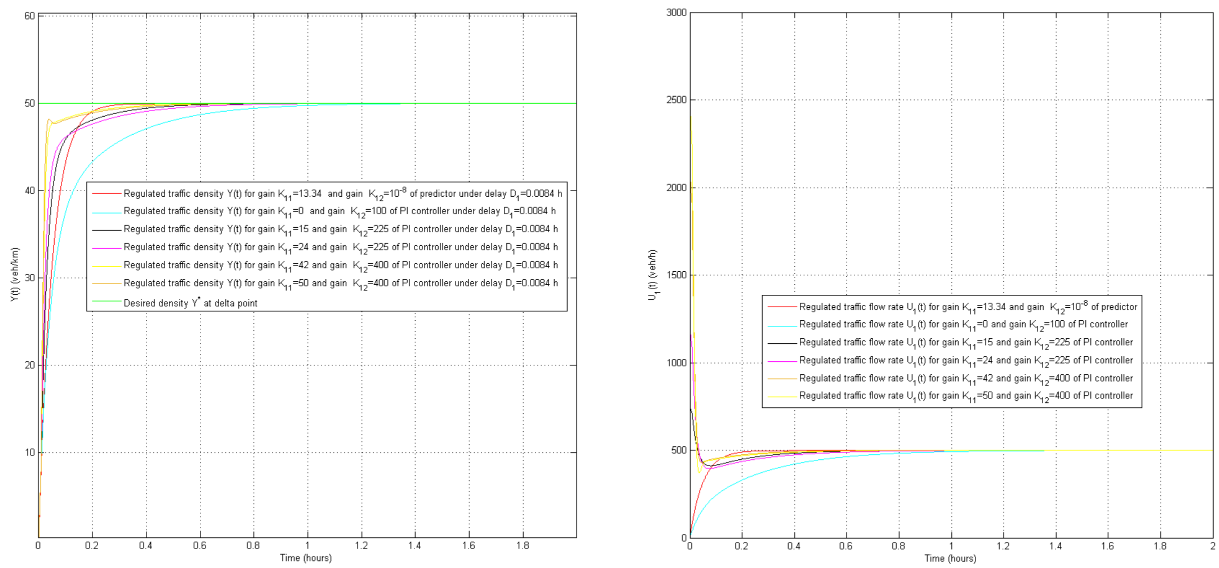
$$\int_a^b f(x)dx \approx \sum_{k=1}^{Q} \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_\kappa \tag{4.2}$$

where x_0=a, x_Q=b and $\Delta x_k = x_k - x_{k-1}$. The approximation increases as Q intervals increase too, which means also that Δx_k decreases.

A number of samplings N=100000 where estimated with an interval dt=$\Delta x_k = 2 * 10^{-5}$ ending in a 2 h simulation process while index Q of equation 4.2 was seted for 1000 (see appendix c). Delay values to be compansated are these of table 3.1.

4.2 Simulation results

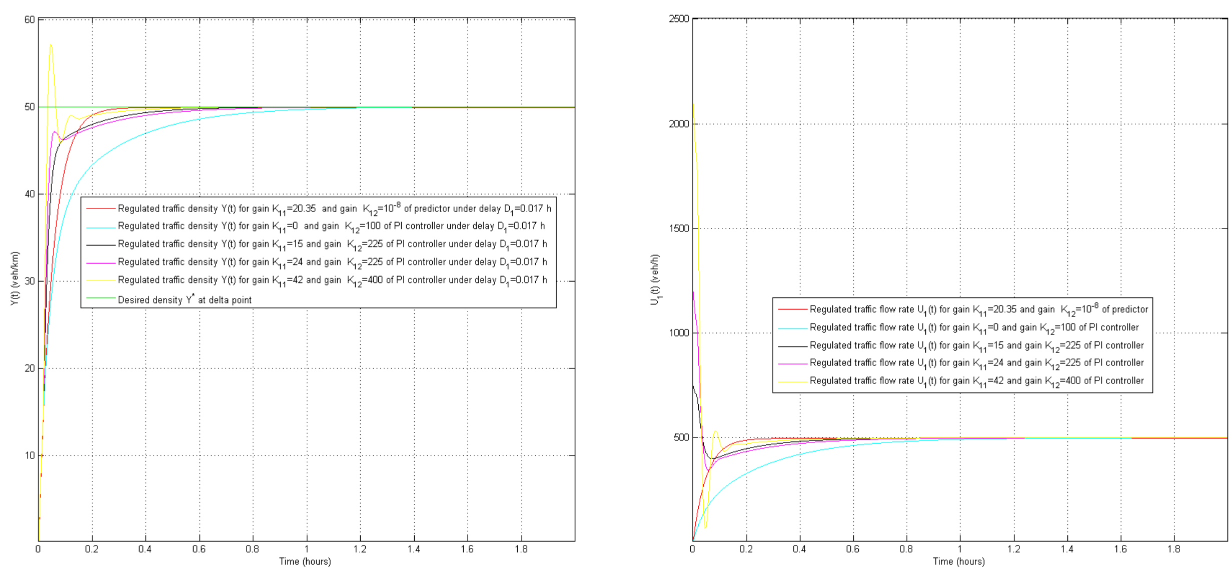
- **Predictor Vs PI controller under delay D_1=0.0084 h**



(i) Traffic density response for D_1=0.0084 h. (ii) On ramp U_1 flow response.

Figure 4.2: Comparing predictor response with that of PI controller under delay D_1=0.0084 h.

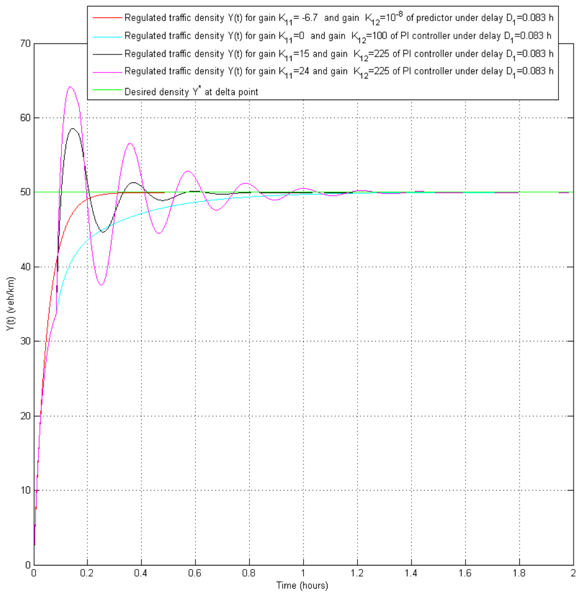
- **Predictor Vs PI controller under delay D_1=0.017 h**



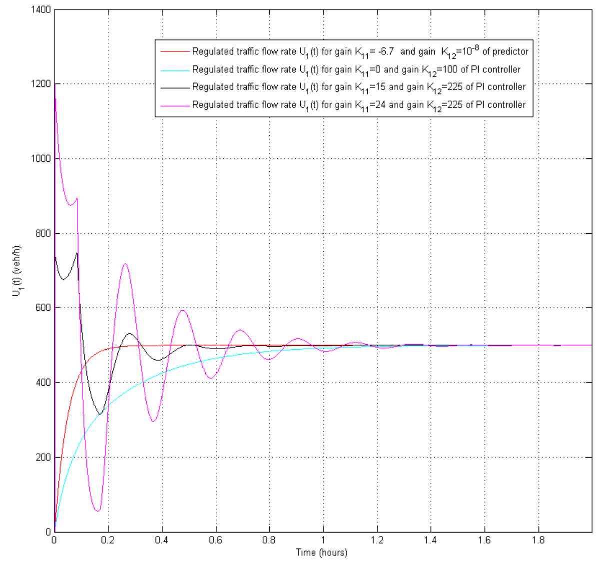
(i) Traffic density response for D_1=0.017 h. (ii) On ramp U_1 flow response.

Figure 4.3: Comparing predictor response with that of PI controller under delay D_1=0.017 h.

- **Predictor Vs PI controller under delay D_1=0.083 h**



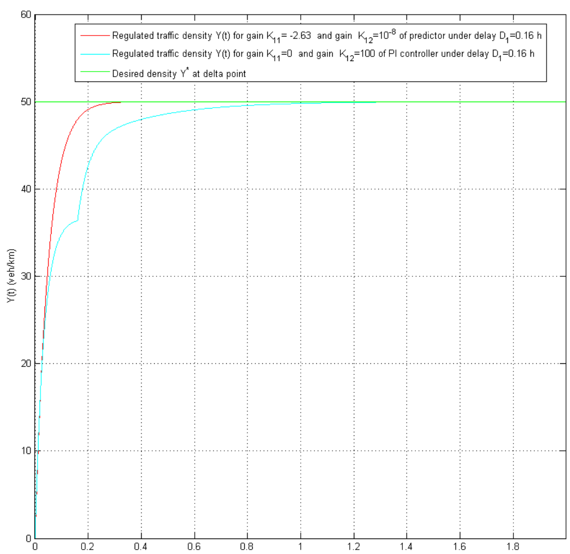
(i) Traffic density response for D_1=0.083 h.



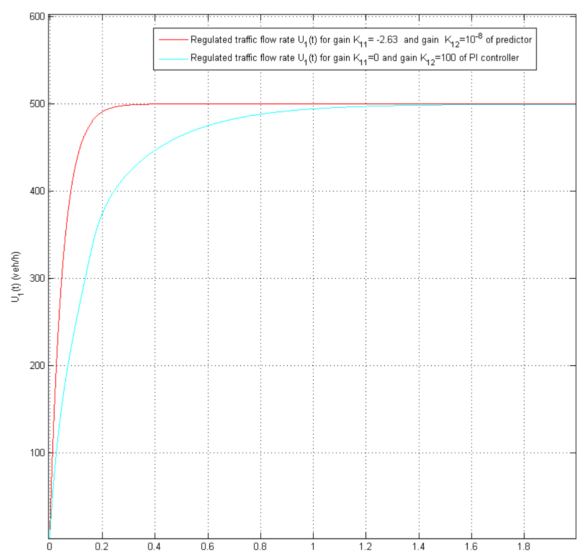
(ii) On ramp U_1 flow response.

Figure 4.4: Comparing predictor response with that of PI controller under delay D_1=0.083 h.

- **Predictor Vs PI controller under delay D_1=0.16 h**



(i) Traffic density response for D_1=0.16 h.



(ii) On ramp U_1 flow response.

Figure 4.5: Comparing predictor response with that of PI controller under delay D_1=0.16 h.

- **Overall evaluation**

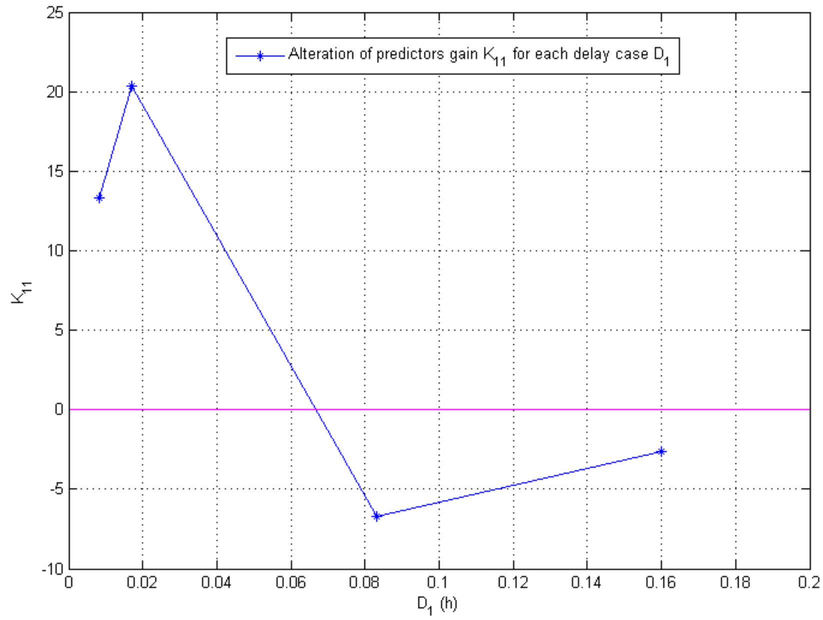


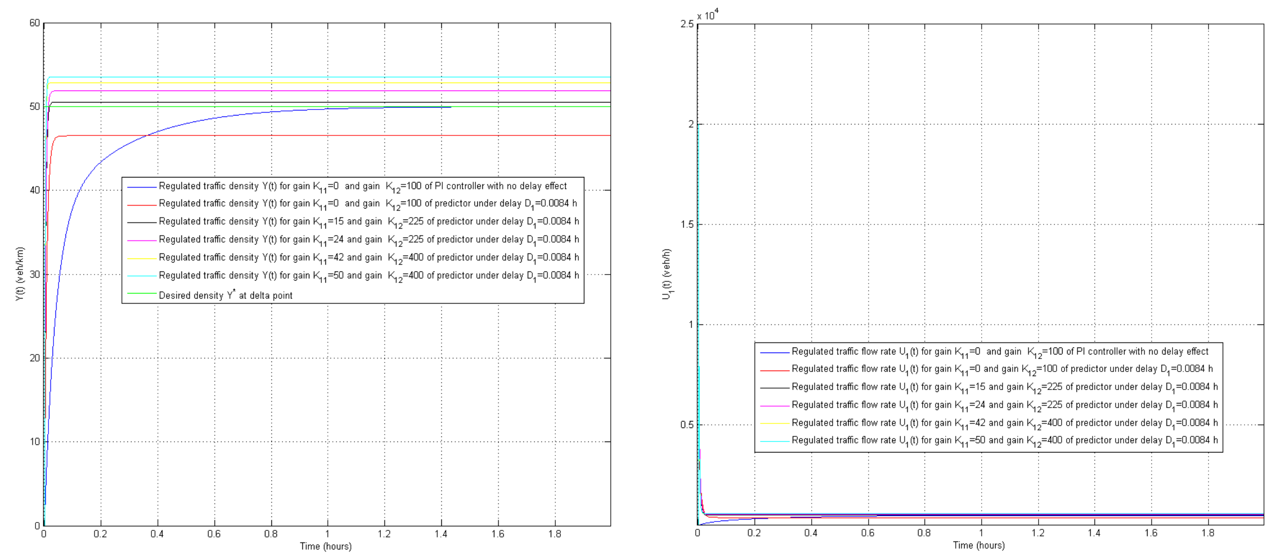
Figure 4.6: Alteration of predictors gain K_{11} through delay D_1 and for steady gain $K_{12}= 10^{-8}$.

In all the above cases predictor exhibits the same rise time T_r, same settling time T_s, zero overshoot and no oscillations both for traffic density and flow. Furthermore it yields a smooth transient response that rests in the desired set point thus producing a zero steady state error (e_{ss}=0).

What changes in each delay case for the predictor is the gain factor K_{11}, it acquires positive values for the first two delay cases and negative for the last two (figure 4.6). On the contrary gain factor K_{12} remains the same for all delay cases (of course other combinations of gains K_{11},K_{12} can be used proportionally to yield the same response for the predictor).

Now, what was presented in figures 4.2-4.5 is how gain K of predictor was altered from the view of delay effect D_1. In the following figures traffic density and flow response of the predictor is presented from the view of the proposed set of gain factors K_{11}, K_{12} of table 2.2 compared with an optimal PI controller (that is, PI controller with K_{11}=0, K_{12}=100, with no delay effect, which yielded a smooth response both for traffic density and flow rate).

- **Predictor under delay D_1=0.0084 h for the proposed set of gain factors K_{11}, K_{12} Vs an optimal PI controller**



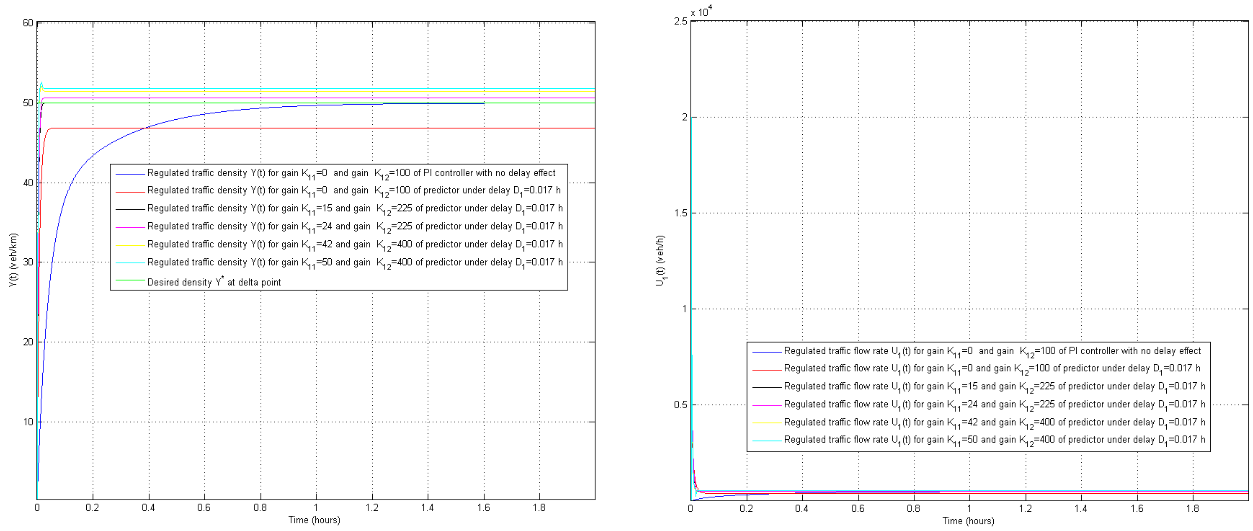
(i) Traffic density response for D_1=0.0084 h.

(ii) On ramp U_1 flow response.

Figure 4.7: Comparing predictor response for the proposed gain factors K_{11}, K_{12} under D_1=0.0084 h with that of a PI controller with no delay effect.

What can be verified from subfigure (i) of figure 4.7 is that no set of the proposed gain factors K_{11}, K_{12} is appropriate for the predictor to meet the desired value. If there is a small tolerance in the desired value to be met (or else a tolerance in the steady-state error) then the most appropriate set would be that of K_{11}=15, K_{12}=225, but again it deviates from the gain set of the optimal PI controller.

- **Predictor under delay D_1=0.017 h for the proposed set of gain factors K_{11}, K_{12} Vs an optimal PI controller**



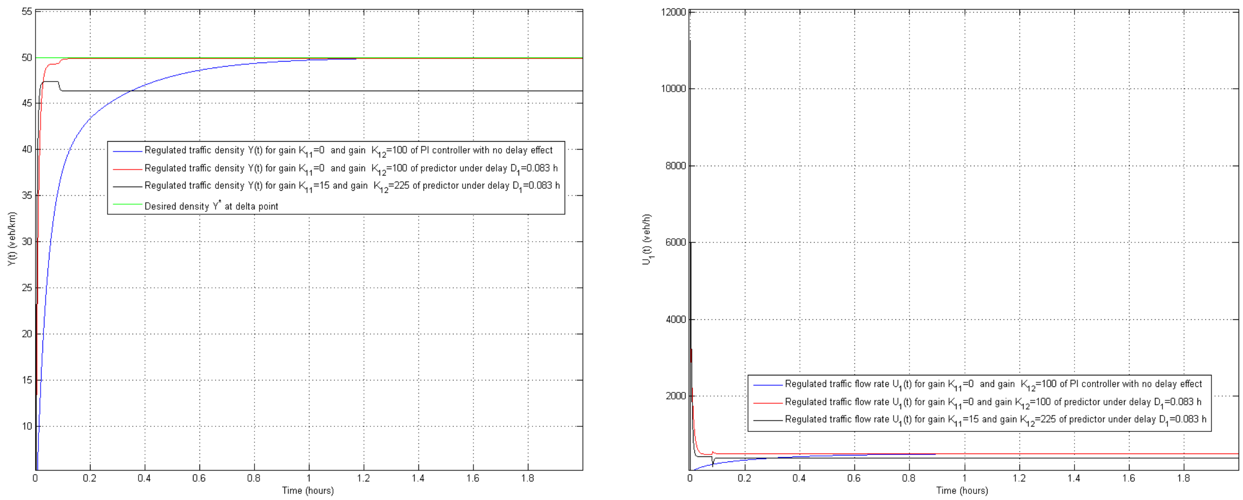
(i) Traffic density response for D_1=0.017 h.

(ii) On ramp U_1 flow response.

Figure 4.8: Comparing predictor response for the proposed gain factors K_{11}, K_{12} under D_1=0.017 h with that of a PI controller with no delay effect.

From subfigure (i) of figure 4.8 it is ascertain that only the set K_{11}=15, K_{12}=225 of the predictor meets the desired value with zero steady-state error. The only issue for the set K_{11}=15, K_{12}=225 of the predictor is that it deviates from the gain set of the optimal PI controller.

- **Predictor under delay D_1=0.083 h for the proposed set of gain factors K_{11}, K_{12} Vs an optimal PI controller**



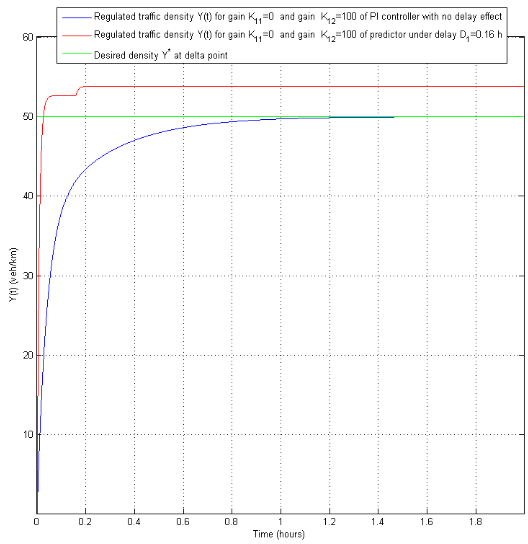
(i) Traffic density response for D_1=0.083 h.

(ii) On ramp U_1 flow response.

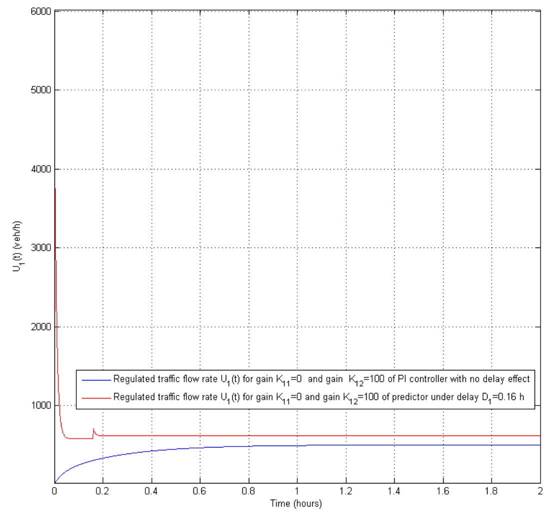
Figure 4.9: Comparing predictor response for the proposed gain factors K_{11}, K_{12} under D_1=0.083 h with that of a PI controller with no delay effect.

In subfigure (i) of figure 4.9 it is verified that the set K_{11}=0, K_{12}=100 of the predictor meets the desired value with zero steady-state error. Moreover an overshoot 90% is presented in the flow rate of the predictor (subfigure (ii) of figure 4.9).

- **Predictor under delay D_1=0.16 h for the proposed set of gain factors K_{11}, K_{12} Vs an optimal PI controller**



(i) Traffic density response for D_1=0.16 h.



(ii) On ramp U_1 flow response.

Figure 4.10: Comparing predictor response for the proposed gain factors K_{11}, K_{12} under D_1=0.16 h with that of a PI controller with no delay effect.

What can be verified from subfigure (i) of figure 4.10 is that the set K_{11}=0, K_{12}=100 of the predictor fails to meets the desired value for delay case D_1=0.16 h. So it is appropriate here for the optimal PI controller to be used in place of the predictor.

Chapter 5

Conclusions

In this thesis, an evaluation of a predictor-feedback law was presented for the traffic flow regulation in a distant bottleneck in a highway. Furthermore there was a performance comparison of the implemented predictor-feedback law with a PI controller under scenarios concerning delay effect.

First of all, a traffic regulation model with a PI controller was introduced applying the state variable model. There was an estimation for the gain matrix of the PI controller and for the controllability of the system proving that the traffic regulation system is controllable. Simulations were carried out for five gain sets of matrix K and for four cases of delay effect for the PI controller seperately and together. Finaly a predictor-feedback law was implemented as a delay compensator for the traffic control system under the same delay scenarios and the same gain sets of matrix K.

As far as the PI controller is concerned, free of delay effect, simulation results proved that it responded fast as gain matrix K increases. Overshoot increased proportionally ending in the desired value with no significant oscillations. A question that emerges is what overshoot can be tolerated for the ramp to be implemented the PI controller when there is a need of a fast response.

For the PI controller, under delay effect, simulation results proved that increased gain K combined with increased delay make the system unstable and thus not suitable for traffic regulation. On the other hand gain set K_{11}=0, K_{12}=100 was the one that presented a smooth response both for traffic density in the distant bottleneck and flow rate on ramp U_1, thus this gain set was considered optimal for the PI controller and as a reference point for comparing it with the predictor-feedback law.

About predictor-feedback law, comparing estimations from view of delay effect and the proposed gain K matrices it is concluded that there is a need of calibration for the two first delay cases and the last one. As far as the third delay case is concerned, predictor-feedback law reached the desired value with fast response while the corresponding flow rate exhibited a high overshoot, so there is a choise between fast response for traffic density and ramp vehicle tolerance. So, what can be declared here is that the implemented predictor-feedback law is suitable after being calibrated.

In future work ramp U_2 may be added for further analysis. In addition, an other mathematical approach for discetizing the predictor-feedback law may be implemented for facing the presented calibration issues. An other option is that of implementing a different predictor-feedback law(e.g a discrete-time Smith predictor) for controlling traffic flow in the proposed distant bottleneck.

Appendix A

%variables
Delta=1; %bottleneck length in Kilometers.
a=0.3; %slow factor for velocity of cars.
d=1000; %the inflow (#veh/h) at the mainstream of the considered stretch lane
v=100; %free flow $car_velocitykm/h$
dt=0.00002; %sim step-2h output response
N=100000; %100000 number of samplings in time
q=1000; %index of summation of trapezoidal rule

Desired_density=50; %The desired traffic density (number of vehicles per km)
traffic_dens=zeros(1,N);
traffic_dens1=zeros(1,N);
pi_controller=traffic_dens;
predictor=pi_controller;
bottleneck=traffic_dens;
var4=zeros(1,N);
error=var4;

%PI controllers gains k_{11},k_{12}
%k12
wn=0.0001; %natural frequency
$ki = w_n^2$;
%k11
$z = 20.3 * 10^4$;%damping factor
$kp = ((2 * z * wn) * Delta) - a * v$;

%main equation. Use this for control with no delay effect

```
par=(1+((a*v*dt)/Delta));
var4=zeros(1,N);
error=var4;
for   n=1:N
 if   n<2
     traffic_dens(1,n)=0;
     error(1,n)=0;
 else
     error(1,n)=(Desired_density-traffic_dens(1,n-1));
     traffic_dens(1,n)=((d+ kp*( error(1,n))+dt*ki*sum( error) )*dt/Delta)+traffic_dens(1,n-1);
     traffic_dens(1,n)=traffic_dens(1,n)/par;
     pi_controller(1,n)=( kp*( error(1,n))+dt*ki*sum( error) )/Delta;
 end
end
```

Appendix B

```
%pi controller with delay effect.
delay=420;%in discrete time 420,850,4150,8000
par=(1+((a*v*dt)/Delta));
var4=zeros(1,N);
error=var4;
for  n=1:N
   if   n<2
     traffic_dens(1,n)=0;
     error(1,n)=0;
   else
     error(1,n)=(Desired_density-traffic_dens(1,n-1));
     pi_controller(1,n)=( kp*( error(1,n))+dt*ki*sum( error) )/Delta;
     if n-delay>1
       traffic_dens(1,n)=((d+ pi_controller(1,n-delay))*dt/Delta)+traffic_dens(1,n-1);
       traffic_dens(1,n)=traffic_dens(1,n)/par;
     else
       traffic_dens(1,n)=((d+100)*dt/Delta)+traffic_dens(1,n-1);
       traffic_dens(1,n)=traffic_dens(1,n)/par;
       pi_controller(1,n)=100/Delta;
     end
   end
end

% PI's plots
figure(1)
plot(time,traffic_dens,'r');
hold on
line([0 N*dt],[Desired_density Desired_density],'Color','green');
ylim([0 60])
legend('Regulated traffic density Y(t) at delta point ','Desired density Y^* at delta point')xlabel('Time (hours)');
ylabel('Y(t) (veh/km)');
grid on
figure(2)
plot(time,pi_controller,'m');
%ylim([0 600])
legend('Regulated traffic flow rate U_1(t) ')%under delay D_1=0.0084 h
xlabel('Time (hours)');
ylabel('U_1(t) (veh/h)');
grid on
```

Appendix C

```
delay=420;%in discrete time 420,850,4150,8000
%predictor's Kp,Ki
%k12
wn=0.0001;%natural frequency
ki = wn^2;
%k11
z = 20 * 10^4;%damping factor (0.0084h) kp=((2*z*wn)*Delta)-a*v;

%feedback-predictor. A=[-a*v 0;1 0];
B=[1;0];
D1=delay;
par=(1+((a*v*dt)/Delta));
var4=zeros(1,N);
error=var4;

smt=(eye(2)+A*D1*dt);%approximate value of exp(AD1)
for  n=1:N
  if   n<2
    predictor(1,n)=0;
    traffic_dens1(1,n)=0;
    error(1,n)=0;
  else
    error(1,n)=(Desired_density-traffic_dens1(1,n-1));
    K=[kp ki];
    Y=[ traffic_dens1(1,n-1);error(1,n-1)];
    if n-delay>1
      for i=2:q %Integral to summation
        AD11=(eye(2)+A*D1*dt*(i-1))*B*predictor(1,i-1);%dt is in hours
        AD12=(eye(2)+A*D1*dt*(i))*B*predictor(1,i);
        AD1=(AD12+AD11)*dt/2;
      end
      predictor(1,n)=K*( smt*Y+(AD1));
    end
    if n-delay<=1
      predictor(1,n)=K*(smt*Y);
    end
    traffic_dens1(1,n)=((d+predictor(1,n) )*dt/Delta)+traffic_dens1(1,n-1);
    traffic_dens1(1,n)=traffic_dens1(1,n)/par;
  end
end
```

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