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Traffic State Estimation Per Lane in Highways with Connected Vehicles

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Abstract

A model-based traffic state estimation approach is developed for per-lane density estimation as well as on-ramp and off-ramp flows estimation for highways in presence of connected vehicles, namely, vehicles that are capable of reporting information to an infrastructure-based system. Three are the basic ingredients of the developed estimation scheme: (1) a data-driven version of the conservation-of-vehicles equation (in its time- and space-discretized form); (2) the utilization of position and speed information from connected vehicles' reports, as well as total flow measurements obtained from a minimum number (sufficient for the observability of the model) of fixed detectors, such as, for example, at the main entry and exit of a given highway stretch; and (3) the employment of a standard Kalman filter. The performance of the estimation scheme is evaluated for various penetration rates of connected vehicles utilizing real microscopic traffic data collected within the Next Generation SIMulation (NGSIM) program. It is shown that the estimation performance is satisfactory, in terms of a suitable metric, even for low penetration rates of connected vehicles. The sensitivity of the estimation performance to variations of the model parameters (two in total) is also quantified, and it is shown that, overall, the estimation scheme is little sensitive to the model parameters.
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Keywords: Traffic state estimation; Connected vehicles

1. Introduction

Lane-specific highway traffic management has considerable potential in traffic flow optimization. As it is well-known since the early 1990s, the presence of connected and automated vehicles could play a key role in the exploitation of this potential, see, e.g., Varaiya (1993). More specifically, significant throughput improvements may be achieved via appropriate real-time lane assignment strategies. Prominent approaches are mandatory lane policies and lane advices, which can be achieved via lane-changing control strategies, such as, for example, Baskar et al. (2012), Roncoli et al. (2016b), Schakel & van Arem (2014), or lane-dependent variable speed limits Duret (2012), Zhang

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& Ioannou (2017). The operation of lane-based highway traffic control calls for real-time information reflecting the current traffic conditions per lane.

The effectiveness of lane-based traffic management strategies largely depends on the quality and accuracy of traffic monitoring at a lane level. Numerous traffic state estimation methodologies exist, amply based on the presence of connected vehicles, which, however, do not distinguish the density values among different lanes, such as, for example, Bekiaris-Liberis et al. (2016), Fountoulakis et al. (2016), Herrera & Bayen (2010), Qiu et al. (2010), Rempe et al. (2016), Roncoli et al. (2016a), Seo et al. (2015), Wang et al. (2017), Wright & Horowitz (2016), Yuan et al. (2012). The existing works dealing with the problem of traffic state estimation per lane are rare and, in addition, they employ almost exclusively measurements coming from fixed detectors Coifman (2003), Singh & Li (2012), Sheu (1999); with the notable exception of Zhou & Mirchandani (2015), where a somewhat heuristic, model-based approach is developed using Lagrangian coordinates, in which measurements stemming from connected vehicles' reports are utilized. It is worth highlighting that in Zhou & Mirchandani (2015) the knowledge of a fundamental diagram is required, which is not needed in the present paper.

In this paper, we address the problem of per-lane density as well as ramp flow estimation in highways, via the development of a model-based estimation approach, largely based on the presence of connected vehicles. We consider that connected vehicles are vehicles capable of reporting information (i.e., position and speed) to an infrastructure-based system, which may be achieved via different technologies and communication paradigms. A basic scenario may simply consist of vehicles equipped with a GPS (Global Positioning System) and a system for mobile communication. However, more complex scenarios, for example, scenarios that incorporate communication among vehicles or between vehicles and roadside units, are also possible.

The basic ingredients of the developed estimation scheme are: (1) a data-driven version of the conservation-of-vehicles equation (in its time- and space-discretized form); (2) the utilization of position and speed information from connected vehicles' reports, as well as total flow measurements obtained from a minimum number (sufficient for the observability of the model) of fixed detectors, such as, for example, at the entry and exit of a given highway stretch; and (3) the employment of a standard Kalman filter.

The performance of the estimation scheme is evaluated for various penetration rates of connected vehicles utilizing real microscopic traffic data collected within the Next Generation SIMulation (NGSIM) program, see, for example, Montanino & Punzo (2013a). It is shown that the estimation performance is satisfactory, in terms of a suitable metric, even for low penetration rates of connected vehicles. The sensitivity of the estimation performance to variations of the model parameters (two in total) is also quantified and it is shown that, overall, the estimation scheme is little sensitive to the model parameters.

2. Per Lane Traffic Density Estimation Using a Data-Driven Model

2.1. General Set-Up

We consider highway stretches consisting of M lanes, indexed by $j = 1, \ldots, M$, subdivided into N segments, indexed by $i = 1, \ldots, N$. We define a cell (i, j) to be the highway part that corresponds to lane j of segment i. The length of each segment is denoted by Δ_i, $i = 1, \ldots, N$.

The following variables are repeatedly used in the paper:

- Average speed $\left[\frac{km}{h}\right]$ of vehicles in cell (i, j), denoted by $v_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total traffic density $\left[\frac{veh}{km}\right]$ at cell (i, j), denoted by $\rho_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total longitudinal inflow $\left[\frac{veh}{h}\right]$ of cell $(i + 1, j)$, denoted by $q_{i,j}$, for $i = 0, \ldots, N - 1$ and $j = 1, \ldots, M$.
- Total on-ramp flow $\left[\frac{veh}{h}\right]$ entering at cell (i, j), denoted by $r_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total off-ramp flow $\left[\frac{veh}{h}\right]$ exiting from cell (i, j), denoted by $s_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total lateral flow $\left[\frac{veh}{h}\right]$ at segment i that enters lane j_2 from lane j_1, denoted by $L_{i, j_1 \to j_2}$, for $i = 1, \ldots, N$, $j_1 = 1, \ldots, M$, and $j_2 = j_1 \pm 1$.
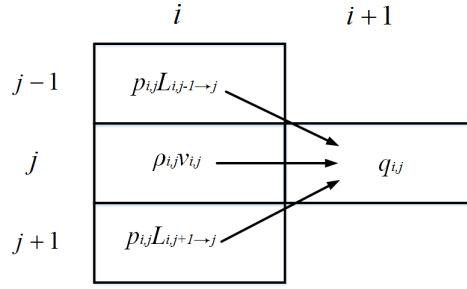
Fig. 1. Model of exiting longitudinal flow from cell (i, j) described in (2). It consists of a well-established term of the form $\rho_{i,j}v_{i,j}$, to the contribution of the cell density, and of additional terms, which are multiplied by a certain percentage p, due to the contribution of "diagonal" lateral flows.

2.2. Available Information from Connected Vehicles' Reports

The data-driven model presented in Section 2.3, requires the availability of the following measurements:

- Average speed of connected vehicles at cell (i, j), denoted by $v^c_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Density of connected vehicles at cell (i, j), denoted by $\rho^c_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Lateral flow of connected vehicles at segment i that enters lane j_2 from lane j_1, denoted by $L^c_{i,j_1 \to j_2}$, for $i = 1, \ldots, N$, $j_1 = 1, \ldots, M$, and $j_2 = j_1 \pm 1$.

Notice that average speeds, densities, and lateral flows of connected vehicles may be obtained based on position and speed reports (see also Section 3.1).

2.3. Model Description for the Density Dynamics

The conservation equation yields the following model for the density dynamics in each cell (i, j)

$$
\rho_{i,j}(k+1) = \rho_{i,j}(k) + \frac{T}{\Delta_i} \Big(q_{i-1,j}(k) - q_{i,j}(k) + L_{i,j-1 \to j}(k) + L_{i,j+1 \to j}(k)
$$
$$
- L_{i,j \to j-1}(k) - L_{i,j \to j+1}(k) + r_{i,j}(k) - s_{i,j}(k) \Big), \tag{1}
$$

where T is time discretization step. For convenience, we assume $r_{i,j} \equiv s_{i,j} \equiv 0$ $\forall i$ and $1 \leq j \leq M-1$, where M denotes the right-most lane (assuming right-hand traffic); also, we have $L_{i,j_1 \to j_2} = 0$ if either j_1 or j_2 equals zero or $M+1$. We note that inflows at the highway entry, namely, $q_{0,j}$, for all $j = 1, \ldots, M$, are treated as measured inputs to system (1).

We employ the following relation for the total flows (see Fig. 1)

$$
q_{i,j}(k) = v_{i,j}(k)\rho_{i,j}(k) + p_{i,j}L_{i,j-1 \to j}(k) + p_{i,j}L_{i,j+1 \to j}(k) + \bar{p}_{i,j}r_{i,j}(k), \quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, M, \tag{2}
$$

where $p_{i,j} \in [0, 1]$, $\bar{p}_{i,j} \in [0, 1]$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, are real numbers that indicate the percentages of "diagonal" lateral movements, including lateral flows from an on-ramp "lane", for each specific cell. While the choice of the first term in (2) is well-known (see, e.g., Papageorgiou & Messmer (1990)), the motivation for the choice of the rest three terms is less obvious. This choice is guided from the fact that at locations where strong lateral flows are observed, such as, for example, at cells with an on-ramp or at segments that feature lane-drops, a significant amount of the lateral flow may appear close to the end of those cells (e.g., in the former case, where the acceleration lane ends). As a result, the flow modeling may be more accurately described when considering that a percentage of lateral and on-ramp flows actually acts as additional exiting longitudinal flow. This conclusion is confirmed by the estimation results obtained using NGSIM data, in which we observe that an estimation bias may appear if all $p_{i,j}$ and $\bar{p}_{i,j}$ values are set to zero. In fact, this formulation is also employed in other works, see, e.g., Laval & Daganzo (2006).

The percentages $p_{i,j}$ and $\bar{p}_{i,j}$ are viewed as tuning parameters. In the most general case, these percentages may be different to each other. For example, one may expect that two percentages $\bar{p}_{i,j}$ that correspond to two respective on-ramps with different acceleration lane lengths are different to each other. However, it was observed in the testing

with NGSIM data in Section 3, that the estimation performance is not really affected if the percentage values are chosen equal to zero for all segments, except those featuring strong lateral flows, such as segments with on-ramps or lane drops. This significantly reduces the number of the parameters that need to be specified[1].

For the lateral flows, we employ the following relation

$$L_{i,j_1 \to j_2}(k) = \frac{L^c_{i,j_1 \to j_2}(k)}{\rho^c_{i,j_1}(k)} \rho_{i,j_1}(k), \quad \text{for } i = 1, \ldots, N, \ j_1 = 1, \ldots, M, \text{ and } j_2 = j_1 \pm 1. \tag{3}$$

Equation (3) is based on the reasonable assumption that the behavior of the population of connected vehicles in a given cell, with respect to lateral movements, is representative for the total vehicle population in that cell. This allows one to quantify the total lateral movements from a cell using (3), namely, by scaling the lateral movements of connected vehicles with the *inverse* of the percentage of connected vehicles in that cell. The accuracy of formulation (3) is tested in the estimation results of Section 3. Note that, for analysis, the densities of connected vehicles are assumed to be strictly positive, whereas, in practice, the problem of the possible division, in relation (3) with zero at some time instants, may be resolved by employing a heuristic procedure (see Section 3.1).

Plugging (2), (3) into (1), we re-write the total density dynamics for all (i, j) as

$$\begin{aligned}
\rho_{i,j}(k+1) = {}& \left(1 - \frac{T}{\Delta_i} v_{i,j}(k) - \frac{T}{\Delta_i} \frac{L^c_{i,j \to j-1}(k)}{\rho^c_{i,j}(k)} - \frac{T}{\Delta_i} \frac{L^c_{i,j \to j+1}(k)}{\rho^c_{i,j}(k)}\right) \rho_{i,j}(k) + \frac{T}{\Delta_i} v_{i-1,j}(k) \rho_{i-1,j}(k) \\
& + (1 - p_{i,j}) \frac{T}{\Delta_i} \frac{L^c_{i,j-1 \to j}(k)}{\rho^c_{i,j-1}(k)} \rho_{i,j-1}(k) + (1 - p_{i,j}) \frac{T}{\Delta_i} \frac{L^c_{i,j+1 \to j}(k)}{\rho^c_{i,j+1}(k)} \rho_{i,j+1}(k) + p_{i-1,j} \frac{T}{\Delta_i} \frac{L^c_{i-1,j-1 \to j}(k)}{\rho^c_{i-1,j-1}(k)} \\
& \times \rho_{i-1,j-1}(k) + p_{i-1,j} \frac{T}{\Delta_i} \frac{L^c_{i-1,j+1 \to j}(k)}{\rho^c_{i-1,j+1}(k)} \rho_{i-1,j+1}(k) + (1 - \bar{p}_{i,j}) \frac{T}{\Delta_i} r_{i,j}(k) + \bar{p}_{i-1,j} \frac{T}{\Delta_i} r_{i-1,j}(k) - \frac{T}{\Delta_i} s_{i,j}(k). \tag{4}
\end{aligned}$$

To enable also the estimation of unmeasured on-ramp and off-ramp flows within the highway stretch, we will adopt, as usual in absence of a descriptive dynamic model, a random walk for the quantities to be estimated. The deterministic parts of such models (to be eventually augmented with additive zero-mean, white Gaussian noise) for on-ramp and off-ramp flow estimation, respectively, read

$$r_{i,M}(k+1) = r_{i,M}(k), \quad s_{i,M}(k+1) = s_{i,M}(k). \tag{5}$$

We write next compactly the overall system (4), (5). For this, we define first the vector x as follows

$$x = (\rho_{1,1}, \cdots, \rho_{N,1}, \cdots, \rho_{1,M} \cdots, \rho_{N,M}, r_{1,M}, \cdots, r_{N,M}, s_{1,M}, \cdots, s_{N,M})^T. \tag{6}$$

The average speed of connected vehicles is representative of the average cell speed, as motivated in Bekiaris-Liberis et al. (2016) and justified with real data and in microscopic simulation in Roncoli et al. (2016a) and Fountoulakis et al. (2016), respectively, even for connected-vehicle penetrations as low as 2%. Thus the unmeasured cell speeds $v_{i,j}$ may be replaced by the corresponding measured speeds $v^c_{i,j}$; and, using (6), we re-write (4), (5) in a compact form as

$$x(k+1) = A\left(v^c(k), L^c(k), \rho^c(k)\right) x(k) + Bu(k), \tag{7}$$

where v^c, L^c, and ρ^c denote vectors that incorporate all average cell speeds of connected vehicles $v^c_{i,j}$, lateral flows of connected vehicles $L^c_{i,j_1 \to j_2}$, and densities of connected vehicles $\rho^c_{i,j}$, respectively, while u denotes the vector of inflows at the highway entrance, namely, $u = (q_{0,1}, \cdots, q_{0,M})^T$, $A \in \mathbb{R}^{(N \times M + 2N) \times (N \times M + 2N)}$, and $B \in \mathbb{R}^{(N \times M + 2N) \times M}$.

Together with (7) we associate an output vector y, which holds all mainstream total flows that are measured by corresponding mainstream fixed detectors and, as follows from (2), (3), is given by

$$y(k) = C\left(v^c(k), L^c(k), \rho^c(k)\right) x(k), \tag{8}$$

where $C \in \mathbb{R}^{(M + l_r + l_s - 1) \times (N \times M + 2N)}$, with l_r and l_s being the number of on-ramps and off-ramps, respectively. The minimum number of rows of C equals $M + l_r + l_s - 1$ in order for system (7), (8) to be observable, see Bekiaris-Liberis et al. (2016), which is true when the total flows at the last segment of every lane as well as the total flows of lane M at every segment that is anywhere between two consecutive unmeasured ramps, are measured.

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[1] For instance, in the testing with NGSIM data, only one percentage value is needed to be non-zero (and thus, to be tuned), namely, the value that corresponds to the cell where an on-ramp is located.
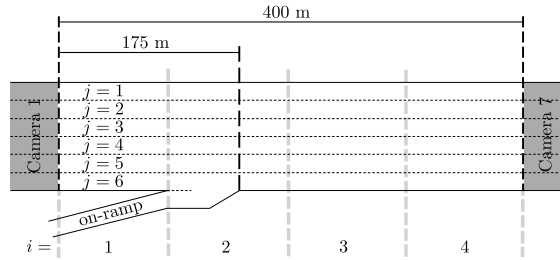
Fig. 2. Graphical representation of the considered stretch of highway I-80 in Emeryville, California, employed in the evaluation of the estimation scheme with NGSIM data. The left-most lane is a so-called high occupancy-vehicle (HOV) lane.

3. Performance Evaluation Using NGSIM Data

We employ a standard Kalman filter, utilizing model (7), (8), along the lines of Bekiaris-Liberis et al. (2016), Fountoulakis et al. (2016), Roncoli et al. (2016a), for per lane total density estimation. The highway stretch considered in the evaluation of the proposed estimation scheme is shown in Fig. 2. Free-flow conditions prevail in the HOV lave over the whole simulation period, whereas a massive congestion is present within the stretch in the rest of the lanes, where congestion waves are coming from downstream, crossing the entire stretch, for details see Roncoli et al. (2016a).

3.1. Data Computation Description

We employ a processed version of NGSIM vehicle trajectory data, recorded from 4:00 PM to 4:15 PM on April 13, 2005, from Montanino & Punzo (2013a), Montanino & Punzo (2013b). Despite the fact that NGSIM data extend over relatively short time period and highway stretch, they have the advantage of including very detailed information, obtained from high-resolution real vehicles trajectories. In the investigations of Fountoulakis et al. (2016), for the cross-lane case, more extensive data are employed, which, however, are obtained from microscopic simulation. All the required measurements utilized by the estimation scheme as well as those used for computing the ground truth traffic state are produced from the available trajectory data as follows.

Data employed by the estimator:

- Vehicles entering the highway stretch are randomly marked as connected, according to the considered penetration rate, based on a uniform distribution. The measured average cell speeds, namely, v^c, employed by the estimator, are computed as the arithmetic average of speeds of connected vehicles that are present within a cell at a time instant kT, where $T = 5$ seconds is the estimation scheme's execution period. If there is no connected vehicle at a time instant kT then the measured average speed of the specific cell is set equal to the previous one.
- The measured lateral flows of connected vehicles, namely, L^c, are produced by counting the number of connected vehicles that moved from one lane to another during a time interval $(kT, (k + 1)T]$. Moreover, the cell densities of connected vehicles, namely, ρ^c, are computed by counting the number of connected vehicles that are present within a cell at time instant kT divided by the cell length. Because terms of the form $\frac{L^c}{\rho^c}$ may contain several spiky values, especially at low penetration rates, mainly due to the rare appearance of connected vehicles' at a time instant kT (and thus, also the rare appearance of lateral movements), we employ in the estimation scheme an exponentially smoothed version of the originally computed terms of the form $\frac{L^c}{\rho^c}$, where the smoothing factor $\alpha \in [0, 1]$ is yet to be chosen.
- Total flow measurements at the entry and exit of the stretch, namely, y and u, respectively, are produced via two respective series of virtual spot detectors, one placed in all lanes at the entrance of the considered highway stretch and another placed in all lanes at the exit of the highway stretch, where the corresponding flow values are produced by counting the number of vehicles (irrespectively of being connected or not) that cross the virtual detector locations during the corresponding periods.
- Total speed measurements at the exit of the considered stretch, i.e., at each of the last cell of every lane, say, $v^m_{4,j}$, $j = 1, \ldots, 6$, are computed as the arithmetic average at time instant kT of the speeds of all vehicles that are present within each cell.

Note that since we assume anyway the availability of flow detectors at the exit of the stretch, it is reasonable to assume that speed measurements from detectors may be available as well. For this reason, we replace the corresponding speeds v^c in the output equation (8) by the speeds v^m. Alternatively, when occupancy detectors are available instead, the densities at the exit of the stretch may be directly extracted. In this case, all elements of the C matrix in (8) would simply be equal to one. In general, we expect small variations in the estimation performance among these cases, so we present only one of them. Finally, note that since there is only one unmeasured on-ramp within the considered highway stretch only the total flow measurements at the entry and exit of the stretch are needed for observability, see Bekiaris-Liberis et al. (2016), and thus, for the proper operation of the estimation scheme.

Data for ground truth generation:

- The total cell densities are computed by counting the number of all vehicles that are present within a cell at a time instant kT divided by the segment length.
- The on-ramp flow is computed similarly to the lateral flows, i.e., by counting the number of vehicles leaving the on-ramp "lane" and entering the mainstream during time interval $(kT, (k + 1)T]$.

3.2. Performance Evaluation for Various Penetration Rates

For details on the selection of the estimation scheme's parameters see Bekiaris-Liberis et al. (2016), Fountoulakis et al. (2016), Roncoli et al. (2016a). For a fair comparison of the performance of the estimation scheme in quantitative terms, the filter is always initialized with the real density and on-ramp flow values, whereas the initial conditions for the filtered versions of the terms of the form $\frac{L^c}{\rho^c}$ are always set equal to zero. Moreover, only the percentage $\bar{p}$, modeling the diagonal lateral movement due to the on-ramp's acceleration lane, where the effect of diagonal lateral movements is expected to be quite strong, has a non-zero value. All of the rest percentage values are set equal to zero.

Quantitative Performance Measure. The performance index is chosen as the average, over 10 replications, where in each replication a different set of vehicles is tagged as connected (since, for a given penetration rate, every vehicle that enters the stretch is randomly selected, from a uniform distribution, to be connected), of the Coefficient of Variation (CV) of the root mean square error of the (30 second-averaged) estimated densities and on-ramp flow defined by

$$CV_\rho = \frac{\sqrt{\frac{1}{MNK} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K} \left(\rho_{i,j}(k) - \hat{\rho}_{i,j}(k)\right)^2}}{\frac{1}{MNK} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K} \rho_{i,j}(k)}, \quad CV_r = \frac{\sqrt{\frac{1}{K} \sum_{k=1}^{K} (r(k) - \hat{r}(k))^2}}{\frac{1}{K} \sum_{k=1}^{K} r(k)}, \tag{9}$$

respectively, where a quantity $\hat{w}$ indicates the estimate of w and $M = 6$, $N = 4$, $K = 23$.

Estimation Performance for a Baseline Case. In Fig. 3 we show the estimation results for 20% penetration rate of connected vehicles (for one random replication, which is closest to the average performance). The estimation results are obtained with a percentage $\bar{p} = 0.3$ and a smoothing factor $\alpha = 0.05$, resulting in $CV_\rho = 18\%$ and $CV_r = 41\%$. From Fig. 3 it is evident that density estimation is quite accurate both during transient periods and at steady-state, considering also the high time and space resolution of the considered stretch and the available data. The on-ramp flow estimation is somewhat sluggish and it suffers from some delay, due to the distance of the total flow detector at the exit of the considered stretch from the on-ramp location. Yet, one could make it faster with a different choice of the filter's parameters (see Bekiaris-Liberis et al. (2016), Fountoulakis et al. (2016), Roncoli et al. (2016a) for details).

Sensitivity of the Estimation Performance to Variations of the Percentage of On-Ramp Diagonal Flow and the Smoothing Factor. We show in the left plot of Fig. 4 the performance index CV_ρ (9) obtained for various percentages $\bar{p}$ of diagonal on-ramp flow and $\alpha = 0.05$, for various penetration rates. In the right plot of Fig. 4 we show CV_ρ (9) obtained for $\bar{p} = 0.3$ and for various values of the smoothing factor α, for various penetration rates. From Fig. 4 we observe that the overall estimation performance is quite insensitive to changes of the parameters $\bar{p}$ and α.

4. Conclusions

We presented a model-based traffic state estimation approach for per-lane density estimation and on-ramp flow estimation in highway sections. The developed approach is largely based on the presence of connected vehicles
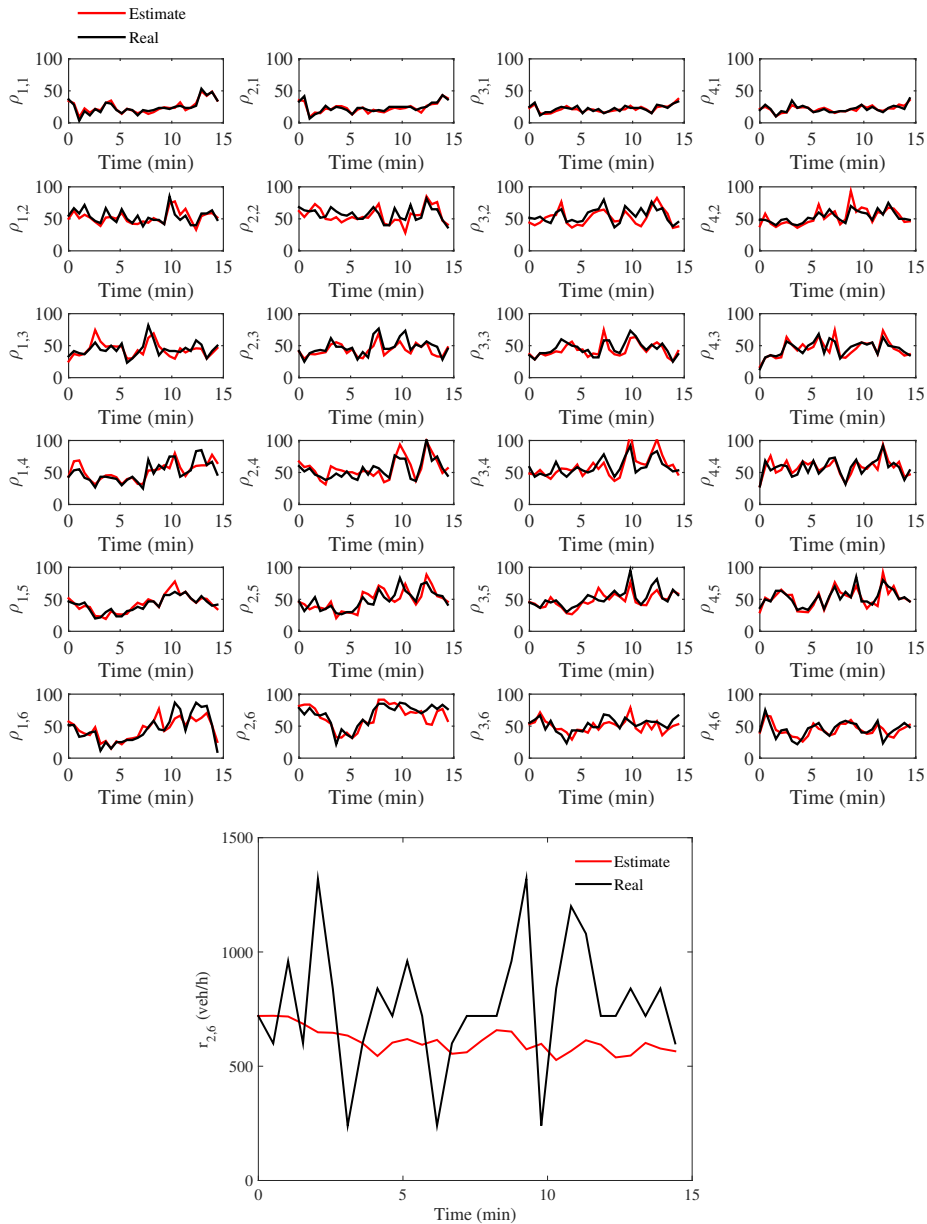
Fig. 3. Top: Estimated densities (red line) produced by the estimation scheme versus real densities (black line) obtained from NGSIM data. Bottom: Estimated on-ramp flow (red line) produced by the estimation scheme versus real on-ramp flow (black line) obtained from NGSIM data.

since it mainly employs position, speed, and lane change measurements from connected vehicles. It was shown, via the utilization of NGSIM data, that the performance of the proposed estimation scheme is satisfactory even for low penetration rates of connected vehicles. It was demonstrated that the scheme is insensitive to the model parameters.
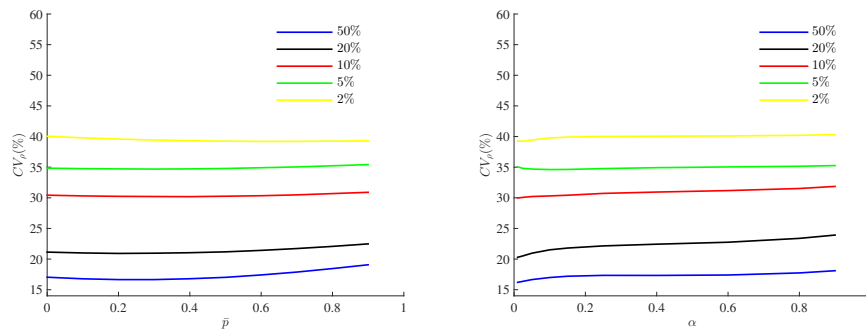
Fig. 4. Left: Performance index CV_ρ (9) versus percentage of on-ramp diagonal flow for various penetration rates of connected vehicles when $\alpha = 0.05$. Right: Performance index CV_ρ (9) versus smoothing factor for various penetration rates of connected vehicles when $\bar{p} = 0.3$.

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