Structural Observability of Multi-Lane Traffic with Connected Vehicles

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***Abstract*— We establish sufficient and necessary conditions for the (weak) structural observability as well as the strong structural observability of lane-based highway traffic. Based on these results, we characterize the fixed detector configurations that guarantee the proper operation of a proposed traffic state estimation scheme. The structural observability analyses are based on a data-driven model, which is introduced for the per-lane traffic density dynamics. The proposed model is derived from the well-known conservation law equation via utilization of position and speed information from connected vehicle reports.**

I. Introduction

Despite the fact that numerous highway traffic state estimation methodologies exist, which are based on information stemming from connected vehicles, e.g., [4], [6], [8], [15], [18], [19], [24], [25], [26], [27], [28], those dealing with the *per-lane* traffic state estimation are scarce [30]. The development of per-lane traffic estimation techniques is of significant importance since the availability of traffic information at a lane level is necessary for implementation of lane-based traffic management strategies, e.g., [3], [20], [21], [22] [23], [29], which have great potential for traffic flow optimization, see, e.g., [7], [16]. We denote connected vehicles as vehicles that are capable of reporting information (i.e., position and speed) to an infrastructure-based system, which may be achieved via different technologies and communication paradigms. A basic scenario may simply consist of vehicles equipped with a GPS (Global Positioning System) and a system for mobile communication. However, more complex scenarios, for example, scenarios that incorporate communication among vehicles or between vehicles and roadside units, are also possible.

A prerequisite for the proper operation of a given model-based traffic state estimation scheme is the observability of the underlying model. In particular, studying the observability of traffic on a given highway stretch, one can derive the locations at which mainstream fixed-flow detectors should be placed, in order to guarantee that densities along the highway stretch may be reconstructed by measuring the flow only at those particular locations. The density estimation may be achieved in real time via the employment of a suitable estimation scheme, e.g., a Kalman filter. Yet, in existing model-based traffic estimation methods, observability is analytically studied, i) considering models that don't incorporate measurements from connected vehicles, ii) assuming availability of a fundamental diagram, and iii) only for specific examples of traffic networks, see, e.g., [1], [5], [12].

Generally speaking, observability of a system is usually studied employing certain algebraic conditions, see, e.g., [2]. However, for systems with a very large number of states or very large output matrices, or for time-varying systems (as in our case) it is difficult to formally check these conditions. For this reason, as an alternative, graph-theoretic approaches are may be adopted, which study the observability properties of a system by merely looking into its structure, see, e.g., [10], [11], [17]. In addition, the study of the *structural* observability properties of a system is useful in that one can determine under which measurement configurations a system is observable, by only investigating the structure of the zero and non-zero elements of the system's matrices.

In this paper, we study the observability properties of a lane-based highway traffic flow model. Specifically, we provide sufficient and necessary conditions for the (weak) structural observability as well as the strong structural observability of a dynamical model, which is introduced, for the per-lane traffic density dynamics. Based on the obtained results, we can characterize the fixed detector configurations that guarantee that densities along a highway stretch may be reconstructed in real time, employing a suitable estimation scheme, which utilizes the proposed model. The developed model may be viewed as a data-driven version of the conservation-of-vehicles equation (in its time- and space-discretized form) and it is largely based on position and speed information stemming from connected vehicle reports.

A. Notation and Definitions

a) Notation: We adopt the notation from [11], [17]:

- A structured matrix $\mathcal{A}$, i.e., a matrix with certain elements being either fixed zeros or free nonzero parameters, is called a pattern. We say that a matrix A is of pattern $\mathcal{A}$ if the structure of its zero and nonzero elements is the same with $\mathcal{A}$ for all times.
- The graph $\mathcal{G}(\mathcal{A},\mathcal{C})$, where $\mathcal{A}$ and $\mathcal{C}$ are of dimension $n\times n$ and $r\times n$, respectively, has vertices $\{1,\ldots,n+r\}$ and there is a (directed) edge from the vertex x to the vertex w if the element (w,x) of $(\mathcal{A},\mathcal{C})$ is a nonzero parameter. In this case, x is a predecessor of w, and w is a successor of x. For any set V of vertices, $\mathrm{Pre}(V)$ denotes the set of predecessors of V, namely, the set of all vertices with a directed edge to some vertex in V. The set $\mathrm{Post}(V)$ denotes the set of successors of V,

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namely, the set of all vertices with a directed edge from some vertex of V.

- Vertices $\{1, \ldots, n\}$ represent the states of the underlined dynamical system and are called "state" vertices, whereas vertices $\{n, \ldots, n+r\}$ represent those specific states for which noisy measurements are available, and are called "output" vertices.
- A vertex is called non-accessible if there exists no path (i.e., a sequence of directed edges connecting certain distinct vertices) from any output vertex to that vertex. A graph is said to contain a dilation if there exists a set V of state vertices such that the cardinality of the set $\text{Pre}\,(V)$ is strictly smaller than the cardinality of set V.

b) Definitions: Consider a system of the form

$$x(k+1) = A(k)x(k) + B(k)u(k) \quad (1)$$

$$y(k) = C(k)x(k) + D(k)u(k), \quad (2)$$

where $x \in \mathbb{R}^n$ is state, $u \in \mathbb{R}^M$ is input, $y \in \mathbb{R}^r$ is output, and $k = 0, 1, \ldots$ is the discrete time. We adopt the following definitions, see, e.g., [2], [11], [17]:

- The pair of matrices (A, C) is called observable on $[k_0, k_0 + M^*]$ if and only if for all initial conditions $x(k_0) \in \mathbb{R}^n$ and for all inputs $u : [k_0, k_0 + M^*] \to \mathbb{R}^r$, one can uniquely determine $x(k_0)$ from the information $\{(u(k), y(k)) \,| k \in [k_0, k_0 + M^*]\}$.

Let the matrices A and C be of pattern $\mathcal{A}$ and $\mathcal{C}$, respectively.

- The system is strongly structurally observable if *for any* numerical realization of the structured matrices $\mathcal{A}$ and $\mathcal{C}$, the corresponding systems are observable in the sense of the formal definition above.
- The system is (weakly) structurally observable if *there exist* numerical realizations of the structured matrices $\mathcal{A}$ and $\mathcal{C}$, such that the corresponding systems are observable in the sense of the formal definition above.

The conditions that must be satisfied for strong or (weak) structural observability are provided in the Appendix for the convenience of the reader.

II. Model for Per Lane Traffic Density Dynamics

A. General Set-Up

We consider highway stretches consisting of M lanes, indexed by $j = 1, \ldots, M$, subdivided into N segments, indexed by $i = 1, \ldots, N$. We define a cell (i, j) to be the highway part that corresponds to lane j of segment i. The length of each segment is denoted by Δ_i, $i = 1, \ldots, N$.

The following variables are repeatedly used in the paper:

- Average speed $\left[\frac{\text{km}}{\text{h}}\right]$ of vehicles in cell (i, j), denoted by $v_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total traffic density $\left[\frac{\text{veh}}{\text{km}}\right]$ at cell (i, j), denoted by $\rho_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total longitudinal inflow $\left[\frac{\text{veh}}{\text{h}}\right]$ of cell $(i+1, j)$, denoted by $q_{i,j}$, for $i = 0, \ldots, N-1$ and $j = 1, \ldots, M$.
- Total on-ramp flow $\left[\frac{\text{veh}}{\text{h}}\right]$ entering at cell (i, j), denoted by $r_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Total off-ramp flow $\left[\frac{\text{veh}}{\text{h}}\right]$ exiting from cell (i, j), denoted by $s_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.

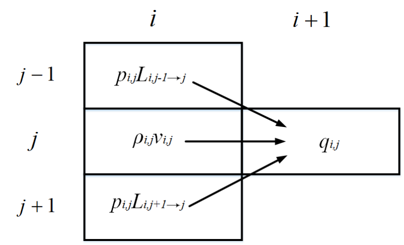

Fig. 1. Model of the exiting longitudinal flow from cell (i, j) as described in (4). It consists of a well-established term of the form $\rho_{i,j}v_{i,j}$, to the contribution of the cell density, and of additional terms, which are multiplied by a certain percentage p, due to the contribution of "diagonal" lateral flows.

- Total lateral flow $\left[\frac{\text{veh}}{\text{h}}\right]$ at segment i that enters lane j_2 from lane j_1, denoted by $L_{i,j_1 \to j_2}$, for $i = 1, \ldots, N$, $j_1 = 1, \ldots, M$, and $j_2 = j_1 \pm 1$.

B. Available Information from Connected Vehicles' Reports

The data-driven model presented in the next subsection, requires the availability of the following measurements:

- Average speed of connected vehicles at cell (i, j), denoted by $v^{\text{c}}_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Density of connected vehicles at cell (i, j), denoted by $\rho^{\text{c}}_{i,j}$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$.
- Lateral flow of connected vehicles at segment i that enters lane j_2 from lane j_1, denoted by $L^{\text{c}}_{i,j_1 \to j_2}$, for $i = 1, \ldots, N$, $j_1 = 1, \ldots, M$, and $j_2 = j_1 \pm 1$.

Average speeds, densities, and lateral flows of connected vehicles may be obtained via position and speed reports.

C. Model Description for the Density Dynamics

The conservation equation yields the following model for the density dynamics in each cell (i, j)

$$\begin{aligned}\rho_{i,j}(k+1) = {} & \rho_{i,j}(k) + \frac{T}{\Delta_i}\left(q_{i-1,j}(k) - q_{i,j}(k)\right. \\ & + L_{i,j-1\to j}(k) + L_{i,j+1\to j}(k) - L_{i,j\to j-1}(k) \\ & \left. - L_{i,j\to j+1}(k) + r_{i,j}(k) - s_{i,j}(k)\right), \end{aligned} \quad (3)$$

where T is time discretization step. For convenience, we assume $r_{i,j} \equiv s_{i,j} \equiv 0$, $\forall i$ and $1 \leq j \leq M-1$, where M denotes the right-most lane (assuming right-hand traffic); we have $L_{i,j_1\to j_2} \equiv 0$ if either j_1 or j_2 equals zero or $M+1$. We note that the inflows at the highway entry, namely, $q_{0,j}$, $j = 1, \ldots, M$, are treated as measured inputs to system (3).

The following relation is employed for total flows (Fig. 1)

$$\begin{aligned} q_{i,j}(k) = {} & v_{i,j}(k)\rho_{i,j}(k) + p_{i,j}L_{i,j-1\to j}(k) \\ & + p_{i,j}L_{i,j+1\to j}(k) + \bar{p}_{i,j}r_{i,j}(k), \end{aligned} \quad (4)$$

for $i = 1, \ldots, N$, $j = 1, \ldots, M$, where $p_{i,j}, \bar{p}_{i,j} \in [0, 1]$, $\forall (i, j)$, indicate the percentages of "diagonal" lateral movements, including lateral flows from an on-ramp "lane", for each specific cell. While the first term in (4) is well-known (see, e.g., [13]), the motivation for the rest of the terms is less obvious. Their choice is guided from the fact that at locations of strong lateral flows, e.g., at cells where an on-ramp is located or at segments that feature lane-drops, a significant amount of the lateral flow may appear close to the cell end (e.g., in the former case, at the acceleration lane end). As a result, the flow modeling may be more accurately described considering that a percentage of lateral or on-ramp flows actually acts as additional exiting longitudinal flow. This formulation is also employed in other works, e.g., [9].

For the lateral flows, we employ the following relation

$$L_{i,j_1\to j_2}(k)=\frac{L^{\mathrm{c}}_{i,j_1\to j_2}(k)}{\rho^{\mathrm{c}}_{i,j_1}(k)}\rho_{i,j_1}(k), \tag{5}$$

for $i=1,\ldots,N$, $j_1=1,\ldots,M$, and $j_2=j_1\pm 1$. Equation (5) is based on the reasonable assumption that the behavior of the population of connected vehicles in a given cell, with respect to lateral movements, is representative for the total vehicle population in that cell. This allows one to quantify the total lateral movements from a cell using (5), namely, by scaling the lateral movements of connected vehicles with the *inverse* of the percentage of connected vehicles in that cell. Note that, for analysis, the densities of connected vehicles are assumed to be strictly positive.

Plugging (4), (5) into (3), we get for all (i,j)

$$\begin{aligned}\rho_{i,j}(k+1)=&\left(1-\frac{T}{\Delta_i}v_{i,j}(k)-\frac{T}{\Delta_i}\frac{L^{\mathrm{c}}_{i,j\to j-1}(k)}{\rho^{\mathrm{c}}_{i,j}(k)}-\frac{T}{\Delta_i}\right.\\&\left.\times\frac{L^{\mathrm{c}}_{i,j\to j+1}(k)}{\rho^{\mathrm{c}}_{i,j}(k)}\right)\rho_{i,j}(k)+\frac{T}{\Delta_i}v_{i-1,j}(k)\\&\times\rho_{i-1,j}(k)+(1-p_{i,j})\frac{T}{\Delta_i}\left(\frac{L^{\mathrm{c}}_{i,j-1\to j}(k)}{\rho^{\mathrm{c}}_{i,j-1}(k)}\right.\\&\left.\times\rho_{i,j-1}(k)+\frac{L^{\mathrm{c}}_{i,j+1\to j}(k)}{\rho^{\mathrm{c}}_{i,j+1}(k)}\rho_{i,j+1}(k)\right)\\&+p_{i-1,j}\frac{T}{\Delta_i}\left(\frac{L^{\mathrm{c}}_{i-1,j-1\to j}(k)}{\rho^{\mathrm{c}}_{i-1,j-1}(k)}\rho_{i-1,j-1}(k)\right.\\&\left.+\frac{L^{\mathrm{c}}_{i-1,j+1\to j}(k)}{\rho^{\mathrm{c}}_{i-1,j+1}(k)}\rho_{i-1,j+1}(k)\right)+(1-\bar{p}_{i,j})\\&\times\frac{T}{\Delta_i}r_{i,j}(k)+\frac{T}{\Delta_i}\left(\bar{p}_{i-1,j}r_{i-1,j}(k)-s_{i,j}(k)\right).\end{aligned} \tag{6}$$

We adopt, as usual in absence of a descriptive dynamic model, a random walk to describe the dynamics of on/off-ramp flows. The deterministic parts of such models read

$$r_{i,M}(k+1) = r_{i,M}(k) \tag{7}$$

$$s_{i,M}(k+1) = s_{i,M}(k). \tag{8}$$

We write next compactly the overall system (6)–(8). For this, we define first the vector x as follows

$$\begin{aligned}x = &\left(\rho_{1,1},\cdots,\rho_{N,1},\cdots,\rho_{1,M}\cdots,\rho_{N,M},r_{1,M},\cdots,\right.\\&\left.r_{N,M},s_{1,M},\cdots,s_{N,M}\right)^{\mathrm{T}}.\end{aligned} \tag{9}$$

The average speed of connected vehicles is representative of the average cell speed, as motivated in [4] and justified with real data and in microscopic simulation in [19] and [6], respectively, even for connected-vehicle penetrations as low as 2%. Thus the unmeasured cell speeds $v_{i,j}$ may be replaced by the corresponding measured speeds $v^{\mathrm{c}}_{i,j}$; and, using (9), we re-write (6)–(8) in a compact form as

$$x(k+1) = A\left(v^{\mathrm{c}}(k),L^{\mathrm{c}}(k),\rho^{\mathrm{c}}(k)\right)x(k)+Bu(k), \tag{10}$$

where v^{c}, L^{c}, and ρ^{c} denote vectors that incorporate all average cell speeds of connected vehicles $v^{\mathrm{c}}_{i,j}$, lateral flows of connected vehicles $L^{\mathrm{c}}_{i,j_1\to j_2}$, and densities of connected vehicles $\rho^{\mathrm{c}}_{i,j}$, respectively, while u denotes the vector of inflows at the highway entrance, namely, $u=(q_{0,1},\cdots,q_{0,M})^{\mathrm{T}}$, $A\in\mathbb{R}^{(N\times M+2N)\times(N\times M+2N)}$, and $B\in\mathbb{R}^{(N\times M+2N)\times M}$.

Together with (10) we associate an output vector y, which holds all mainstream total flows that are measured by corresponding mainstream fixed detectors and, as follows from (4), (5), is given by

$$y(k)=C\left(v^{\mathrm{c}}(k),L^{\mathrm{c}}(k),\rho^{\mathrm{c}}(k)\right)x(k), \tag{11}$$

where $C\in\mathbb{R}^{(M+l_r+l_s-1)\times(N\times M+2N)}$, with l_r and l_s being the number of on-ramps and off-ramps, respectively. The minimum number of rows of C equals $M+l_r+l_s-1$ in order for system (10), (11) to be observable (see Section III).

III. Observability of the Model

A. Observability: Physical Implications

We provide next some physically oriented implications of the formal definitions of Section I-A (see, e.g., [2], [11]).

In less rigorous terms, the observability property of a system guarantees that the dynamic evolution of its internal states (i.e., the states that are not directly measured) may be extracted (observed) by measuring only some specific states (or, more generally, some outputs of the system). Thus, the study of observability of a system is useful since it provides the necessary sensor requirements, which guarantee that all internal sates may be reconstructed by measuring only certain outputs.

B. The Concept of Structural Observability

The motivation for the study of the structural observability properties of system (10), (11) has been already mentioned in the third paragraph of Section I. Depending on which specific notion is adopted, structural observability may be a sufficient or necessary condition for observability (see paragraphs *a)* and *b)* below and the definitions in Section I-A).

Here we focus on two different notions of structural observability, namely the *strong* structural observability and the (*weak*)[1] structural observability whose (informal) definitions can be found in Section I-A. We provide next some of the implications of the two structural observability properties on observability, and consequently, on traffic state estimation.

a) Strong Structural Observability: The strong structural observability property guarantees that "no matter what" values the non-zero system matrix elements may take, the system remains observable. Thus, clearly, strong structural observability is sufficient for observability.

[1] Note that, in the literature, the weak structural observability property comes usually under the name "structural observability". However, here, we use the name "(weak) structural observability" so no confusion arises.

b) (Weak) Structural Observability: Is a necessary condition for observability. The (weak) structural observability concept, in fact, provides a physically intuitive way to the study of observability which, in *practice*, typically implies indeed system observability. The reason is that the loss of observability of a (weakly) structurally observable system may happen only in some "pathological" cases when the elements of the A and C matrices accidentally happen to satisfy some specific conditions, see, e.g., [11], [17]. Thus, in practice, (weak) structural observability implies in the rule the proper operation of a given estimation scheme. It should be also noted that, since (weak) structural observability is a weaker property than strong structural observability, one may expect that a smaller number of sensors may be required to guarantee (weak) structural observability than strong structural observability.

C. Structural Assumptions for the Considered Traffic Models

We consider systems of the form (10), (11) with the patterns $\mathcal{A}$ and $\mathcal{C}$ under the assumptions that for all (i, j)

$$0 \neq v^{\mathrm{c}}_{i,j}(k) \tag{12}$$

$$1 \neq \frac{T}{\Delta_i}\left(v^{\mathrm{c}}_{i,j}(k) + \frac{L^{\mathrm{c}}_{i,j\to j-1}(k)}{\rho^{\mathrm{c}}_{i,j}(k)} + \frac{L^{\mathrm{c}}_{i,j\to j+1}(k)}{\rho^{\mathrm{c}}_{i,j}(k)}\right), \tag{13}$$

for all $k \geq 0$. Both (12), (13) are satisfied when for all (i, j)

$$0 < v^{\mathrm{c}}_{i,j}(k) < \frac{\Delta_i}{T} - \frac{L^{\mathrm{c}}_{i,j\to j-1}(k)}{\rho^{\mathrm{c}}_{i,j}(k)} - \frac{L^{\mathrm{c}}_{i,j\to j+1}(k)}{\rho^{\mathrm{c}}_{i,j}(k)}, \tag{14}$$

for all $k \geq 0$. The right-hand side of (14), in the case of zero lateral flows of connected vehicles (e.g., in the case of one-lane highway), is satisfied when the traffic model respects the so-called Courant-Friedrichs-Lewy condition (which must anyhow hold for stable discrete traffic flow models). The left-hand side of (14) is satisfied when at least one connected vehicle is not completely stopped within a cell.

D. Traffic Models Under Study

In the following structural observability study we consider subsequently the following four basic models derived from the general model (10), (11); each sub-model turns out to have different structural observability properties:

I. One-lane highway stretch with N segments ($\equiv$ cells), where all percentage values $\bar{p}_{i,j}$ for diagonal on-ramp[2] flows are set to zero (which is equivalent to the non-lane-based estimation scheme for a highway stretch considered in [4]).
II. General three-lane highway stretch[3] where all percentage values $p_{i,j}$ and $\bar{p}_{i,j}$ are set equal to zero, for both diagonal lateral and on-ramp flows, whereas the lateral flows of connected vehicles may or may not be zero.
III. General three-lane highway stretch.
IV. One-lane highway stretch where the percentage values $\bar{p}_{i,j}$ for diagonal on-ramp flows may be non-zero.

[2]Since off-ramp and on-ramp flow dynamics are identical, for simplicity of presentation we consider the case in which all ramps are on-ramps.

[3]For simplicity of presentation we consider a *three*-lane highway stretch.

In the study of (weak) structural observability we consider in addition the following two models:

V. Model I where matrix A is replaced by $\frac{1}{T}(A - I)$, which can be viewed as an approximation of the continuous-time version of Model I (the time derivative is approximated by a forward finite difference formula).
VI. Model III where A is replaced by $\frac{1}{T}(A - I)$, which can be viewed as an approximation of the continuous-time version of Model III (where the time derivative is approximated by a forward finite difference formula).

The motivation for studying these additional models will be explained later (see Section III-F).

Note that, as it is already mentioned in Section II, the inflows at the main entry of each lane of the considered highway stretch are modeled as arbitrary, but measured, inputs. Yet, all the specific models and the following observability analysis could be adapted to the case where some inflows are not directly measured. For simplicity of presentation we consider the case in which none of the on-ramp flows is directly measured.

E. Strong Structural Observability of Lane-Based Traffic

We establish the following claims for each of the models I, II, III, and IV:

1. Model I is strongly structurally observable if and only if: i) total flow fixed detectors are placed at the main exit of the considered highway stretch; and ii) total flow fixed detectors are placed at every segment immediately upstream of a segment with an on-ramp[4].
2. Model II is strongly structurally observable if and only if: i) total flow fixed detectors are placed at the main exit in each lane of the considered highway stretch; and ii) total flow fixed detectors are placed at every segment immediately upstream of a segment with an on-ramp.
3. Model III is not strongly structurally observable under the measurement configuration i) and ii) of Claim 2.
4. Model IV is not strongly structurally observable under the measurement configuration i) and ii) of Claim 1.

In order to establish Claims 1–4 we employ the results from [17] under the structural assumptions of Section III-C.

a) Claim 1: We start by constructing the graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$, shown in Fig. 2, for Model I. Employing Corollary IV.2 from [17], it follows that the highway

[4]This may also be proved as in [4] employing an algebraic approach and utilizing the classical definition of observability.

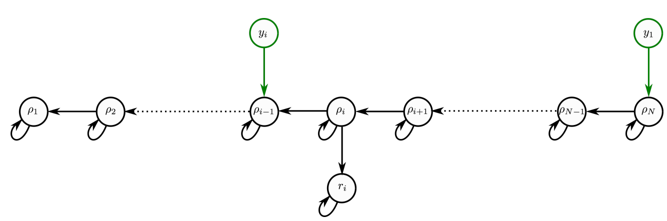

Fig. 2. The graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ for patterns $\mathcal{A}$ and $\mathcal{C}$ that include matrices A and C, respectively, of system (10), (11), for the case of a simple one-lane highway stretch with N segments (Model I).

stretch with one lane is strongly structurally observable on $[k_0, k_0 + N + m]$, for any $k_0 \geq 0$, where m denotes the total number of on-ramps (hence, that every system of pattern $(\mathcal{A}, \mathcal{C})$ is observable on $[k_0, k_0 + N + m]$) if and only if condition $G_0 \cap G_1$ is satisfied. For the reader's convenience, conditions G_0 and G_1 can be found in the Appendix.

A sufficient requirement for condition G_0 to be satisfied is that the total flow at the exit of the considered highway stretch is measured, or, more generally, that there is an output vertex y_1 as shown in Fig. 2. To see this, first observe from Fig. 2 that for every subset $S \subseteq \{\rho_1, \ldots, \rho_N\}$ it holds that $S \cap \text{Post}(\{\rho_{j+1}\}) = \{\rho_j\}$, where ρ_j corresponds to the vertex with the maximum index that belongs to S with $\rho_j \equiv y_1$ when $j = N + 1$. Moreover, for every such subset S that also includes vertices of the form r_i, it holds that $S \cap \text{Post}(\{r_i\}) = \{r_i\}$.

We next show that a sufficient requirement for condition G_1 to hold is that, simultaneously, mainstream, total flow fixed detectors are placed at every segment immediately upstream of a segment with an on-ramp as well as at the exit of the stretch. We first note that the requirement of condition G_1, that $V \subseteq \text{Pre}(V)$, is indeed satisfied for any subset $V \subseteq \{\rho_1, \ldots, \rho_N, r_1, \ldots, r_m\}$, since, as it is observed from Fig. 2, for every vertex there is a directed edge from that vertex to itself. We employ next the same argument to the proof of satisfaction of condition G_0, namely that for every subset $S \subseteq \{\rho_1, \ldots, \rho_N\}$ it holds that $S \cap \text{Post}(\{\rho_{j+1}\}) = \{\rho_j\}$, where ρ_j corresponds to the vertex with the maximum index that belongs to S with $\rho_j \equiv y_1$ when $j = N + 1$. Since ρ_{j+1} doesn't belong to S, it follows that condition G_1 is satisfied for every subset V that contains only density vertices. Thus, in order to guarantee that condition G_1 is satisfied it is sufficient to show that for every subset $V \subseteq \{\rho_1, \ldots, \rho_N, r_1, \ldots, r_m\}$ (i.e., which contains vertices of the form r_i) there exists a vertex x that doesn't belong to V and, moreover, it is such that $V \cap \text{Post}(\{x\})$ is a singleton. From Fig. 2 it is evident that the only cases the latter requirement may not hold are the following. The subset V contains pairs of vertices of the form $\{r_i, \rho_{i-1}\}$ (namely an on-ramp vertex and a vertex that corresponds to the density of the segment immediately upstream of the segment with the on-ramp, respectively) and, potentially, every other vertex on the left of ρ_{i-1} (irrespectively of being a density or an on-ramp vertex), but it doesn't contain vertex ρ_i (or any other vertex, density or on-ramp, on the right of ρ_i). To see this, note that, otherwise, $V \cap \text{Post}(\{\rho_{i+1}\}) = \{\rho_i\}$ or, in the case where V contains only on-ramp vertices, $V \cap \text{Post}(\{\rho_i\}) = \{r_i\}$. Thus, one can conclude that condition G_1 is satisfied when a mainstream total flow fixed detector is placed at segment $i - 1$, namely at the segment immediately upstream of the segment with the on-ramp, since then $V \cap \text{Post}(\{y_i\}) = \{\rho_{i-1}\}$. In other words, condition G_1 holds when an output vertex y_i exists as shown in Fig. 2.

The fact that the existence of the output vertex y_i is also necessary for the satisfaction of condition G_1 (and thus, necessary for strong structural observability [17]) follows from the previous discussion by taking $V = \{r_i, \rho_{i-1}\}$ and noting that the only vertex that has a successor in V (but, it doesn't belong to V) is ρ_i which, however, has two successors in V. Finally, the fact that the existence of the output vertex y_1 is a necessary condition for G_1 to hold, can be shown by taking $V = \{\rho_N\}$ and noting that there exists no vertex which doesn't belong to V with a successor in V.

b) Claim 2: The corresponding graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ for Model II is shown in Fig. 3. It is evident from Fig. 3 that strong structural observability is preserved under the same mainstream, fixed total flow measurement requirements as in the single-lane case (i.e., as in Claim 1). Note that a red edge may not exist, if the corresponding lateral flow of connected vehicles is zero, without affecting the strong structural observability of the system.

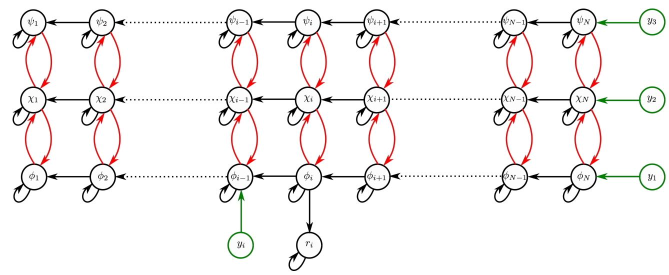

Fig. 3. The graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ for patterns $\mathcal{A}$ and $\mathcal{C}$ that include matrices A and C, respectively, of system (10), (11), for the case of a three-lane highway stretch with $\bar{p}_{i,j} = p_{i,j} = 0$, for all i and j, when the lateral flows of connected vehicles may be nonzero (Model II).

The fact that the measurement configuration i) and ii) of Claim 2 is also necessary for strong structural observability can be shown as follows. Consider the case where, e.g., the output vertex y_1 doesn't exist. Then, in the case in which the red edge from χ_N to ϕ_N doesn't exist, condition G_1 obviously cannot hold for $V = \{\phi_N\}$. The necessity of the measurement configuration ii) follows analogously with Claim 1, by choosing the set $V = \{\phi_{i-1}, r_i\}$ and considering a case in which there is no red edge from χ_{i-1} to ϕ_{i-1}. The only vertex that has a successor in V (but, doesn't belong to V) is ϕ_i which, however, has two successors in V.

c) Claim 3: We next turn our attention to Model III, i.e., to the most general case of highway stretches modeled by system (10), (11), where the percentage values of diagonal on-ramp or lateral flows may be nonzero. The corresponding graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ is shown in Fig. 4. Unfortunately, condition G_0 cannot be satisfied for this general highway stretch with the measurement configuration shown in Fig. 4. This can be seen, for example, by choosing the set $\{\phi_1, \chi_1, \psi_1\}$ since there exists no vertex x such that the set $\{\phi_1, \chi_1, \psi_1\} \cap \text{Post}(\{x\})$ contains only one element[5].

d) Claim 4: For Model IV, condition G_1 cannot be satisfied for subsets V of the form $\{\phi_i, r_i\}$. This can be seen from Fig. 4, for the special case of one lane, as follows. The

[5]Note that for not making the corresponding graph shown in Fig. 4 more complex than needed, we consider the case where the total flows measured at the segments with mainstream fixed detectors are modeled by (4), with the percentage values of diagonal flows equal to zero. The conclusion drawn is not changed when some of the corresponding percentage values are nonzero.

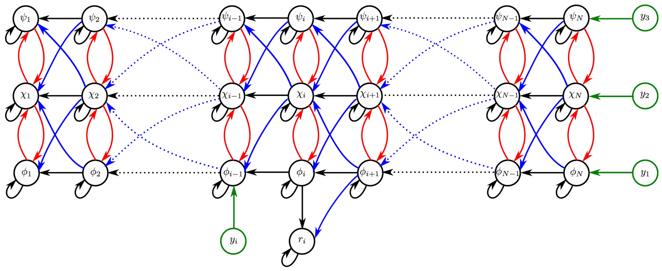

Fig. 4. The graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ for patterns $\mathcal{A}$ and $\mathcal{C}$ that include matrices A and C, respectively, of system (10), (11), for the case of a three-lane highway stretch where the percentage values of diagonal on-ramp or lateral flows may be nonzero (Model III).

only vertex that doesn't belong to V and has a successor in V is ϕ_{i+1}, which, however, has two successors in V.

F. (Weak) Structural Observability of Lane-Based Traffic

We establish the following claims for models I–VI:

5. Model V is (weakly) structurally observable if and only if: i) total flow fixed detectors are placed at the main exit of the considered highway stretch; and ii) for each pair of on-ramps, an additional fixed flow sensor is placed anywhere between two consecutive on-ramps.
6. Model I is (weakly) structurally observable under the measurement configuration i) and ii) of Claim 5.
7. Model I is (weakly) structurally observable if and only if total flow fixed detectors are placed at the main exit of the considered highway stretch.
8. Model VI is (weakly) structurally observable if and only if: i) for each pair of on-ramps, an additional fixed flow sensor is placed anywhere between two consecutive on-ramps; and either iia) total flow fixed detectors are placed at the main exit of every lane of the considered highway stretch, when some lateral flows of connected vehicles may be zero; or iib) total flow fixed detectors are placed at the main exit of at least one of the lanes of the considered highway stretch, when all lateral flows of connected vehicles are always nonzero.
9. Model III is (weakly) structurally observable under the measurement configurations i) and iia) or i) and iib) of Claim 8.
10. Model III is (weakly) structurally observable if and only if: either ia) total flow fixed detectors are placed at the main exit of every lane of the considered highway stretch, when some lateral flows of connected vehicles may be zero; or ib) total flow fixed detectors are placed at the main exit of at least one of the lanes of the considered highway stretch, when all lateral flows of connected vehicles are always nonzero.

Note that Claims 9, 10 and Claims 6, 7 trivially extend to Model II and Model IV, respectively, and thus, they are not presented here. One can see this by observing that the proofs of Claims 6, 7, 9, and 10 are not affected by the existence or not of blue edges, i.e., of edges due to non-zero percentages.

a) Claim 5: We start by constructing the graph $\mathcal{G}\left(\bar{\mathcal{A}}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$, shown in Fig. 5, where $\bar{\mathcal{A}} = \frac{1}{T}\left(\mathcal{A} - \mathcal{I}\right)$. Utilizing the results, e.g., from [11], the one-lane highway stretch is (weakly) structurally observable if the graph $\mathcal{G}\left(\bar{\mathcal{A}}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$, shown in Fig. 5, contains no non-accessible vertex and no dilation. A necessary and sufficient condition for the graph to contain no non-accessible node is that a fixed flow sensor is placed at the exit of the considered highway stretch, or, in other words, that an output vertex y_1 is placed as shown in Fig. 5. Moreover, from Fig. 5 one can observe that every density vertex contains a self-edge, and hence, every density vertex has at least two predecessors. Therefore, the only possibility for a dilation to exist is when one considers subsets of vertices like the ones indicated in blue circle in Fig. 5 (i.e., when one considers subsets of vertices that include two consecutive on-ramp vertices together with the density vertices at and between the cells with the on-ramps). Thus, no dilation exists if and only if an additional fixed flow sensor is placed anywhere between two consecutive on-ramps, as it is indicated in Fig. 5 with the output vertex y_{i^*}. With the same reasoning, it is easy to conclude that, in case there is only one on-ramp, one sensor only, namely a sensor at the exit of the considered highway stretch, is sufficient and necessary for (weak) structural observability.

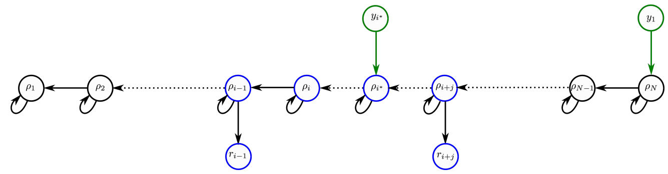

Fig. 5. The graph $\mathcal{G}\left(\bar{\mathcal{A}}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ for patterns $\bar{\mathcal{A}} = \frac{1}{T}\left(\mathcal{A} - \mathcal{I}\right)$ and $\mathcal{C}$ that include matrices A and C, respectively, of system (10), (11), for the case of a simple one-lane highway stretch with N segments (Model V).

b) Claim 6: The total flow measurement configurations in Claim 5 above are sufficient for the (weak) structural observability of the original system of pattern $\mathcal{A}$ since the graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ is similar to the graph $\mathcal{G}\left(\bar{\mathcal{A}}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$, with the only addition that there are self-edges at all on-ramp vertices. In fact, it is not difficult to see in Fig. 5 that with the addition of self-edges at the on-ramp vertices, there is still no non-accessible vertex, and, in addition, there exists no dilation since every state vertex has a self-edge.

c) Claim 7: For graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ the existence of only one output vertex, namely of y_1 (that corresponds to a fixed flow sensor at the exit of the considered highway stretch), is sufficient for (weak) structural observability. This can be seen by observing from Fig. 2 that all state vertices have a self-edge, thus no dilation occurs, and, all state vertices are accessible from the output vertex y_1. Necessity is not difficult to establish by observing that when the output vertex y_1 is not located at the position shown in Fig. 2, the vertex ρ_N cannot be accessible.

We now explain the motivation for studying Models V, VI.

Remark 1: For physical systems, (weak) structural observability is usually sufficient for observability since "a possible loss of observability of a (weakly) structurally observable system can occur only in pathological cases when there are accidental constraints of the system parameters", as it is stated in [10]. Yet, the random walk dynamics introduced, which correspond to diagonal elements of matrix A that are equal to each other (in fact, they are all exactly equal to one), impose some rather non-physical interconnections in model (10). In principle, such fictitious dependencies/symmetries among specific elements of matrix A may cause a (weak) structural observability test to eventually underestimate the number of sensors needed for observability, see, e.g., [11][6]. The following example illustrates this fact.

Example 1: Consider the case of an one-lane highway stretch with two segments and two on-ramps, where free-flow conditions prevail (assume zero percentages of on-ramp diagonal flows). Matrices A and C of (10), (11) reduce to $A = \begin{bmatrix} 1-\frac{T}{\Delta}v_f & 0 & \frac{T}{\Delta} & 0 \\ \frac{T}{\Delta}v_f & 1-\frac{T}{\Delta}v_f & 0 & \frac{T}{\Delta} \\ 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & v_f & 0 & 0 \end{bmatrix}$, respectively, where the elements of A that correspond to the random walk dynamics have been replaced by some arbitrary values g_1 and g_2 (instead of one). It can be shown that the determinant of the observability matrix (see paragraph *f)* in the Appendix) is given by $\det(O) = \frac{T^4 v_f^6}{\Delta^5}(g_1-g_2)(\Delta g_2 - \Delta + T v_f)$. From the last relation it is clear that, irrespectively of the values for v_f, T, and Δ, the system is not observable (i.e., the determinant of the observability matrix is zero), when the on-ramp flow dynamics are identical to each other (which is the case when both on-ramp flow dynamics are modeled by random walk equations), i.e., when $g_1 = g_2$[7], although from Claim 7 Model I is (weakly) structurally observable.

Models V and VI could be viewed as approximations of the continuous-time version of Models I and III, respectively (see Section III-D for details). As such, the dynamics of the deterministic part of a random walk equation (modeling the on-ramp flows) are zero. Replacing in the random walk equations the corresponding ones with fixed zeros, breaks potential fictitious symmetries in the structure of the system, which may appear. Thus, compared to the discrete-time case, it is more likely that the resulting conditions for (weak) structural observability are more physically oriented as well as sufficient. Indeed, when Models V and VI are (weakly) structurally observable, then Models I and III are so, since the addition of edges never weakens the (weak) structural observability of a system (see, e.g., [11]).

d) Claim 8: The graph for Model VI is identical to the graph of Fig. 4 with the only difference that on-ramp vertices have no self-edges. Observe next that the graph of Model VI is derived by the graph of Model V, shown in Fig. 5, by adding extra edges that correspond to diagonal flow percentages or lateral flows of connected vehicles, as well as extra vertices that correspond to densities of additional lanes. One can show that these additional edges and vertices don't affect the (weak) structural observability of the system under the measurement configuration of Claim 5 as follows. Since every additional state vertex has a self-edge, still no dilation occurs. Furthermore, all state vertices are accessible by an output vertex when there are measurements in all lanes at the exit of the considered highway stretch, no matter if lateral flows of connected vehicles are zero or not. Finally, when there is a measurement at at least one lane at the stretch's exit, it can be seen that all state vertices are still accessible by an output vertex as long as the lateral flows of connected vehicles are always nonzero (i.e., the red edges always exist).

The fact that the measurement configuration i) of Claim 8 is also a necessary condition for (weak) structural observability can be shown as follows. Consider the set of blue vertices in Fig. 5 and assume that there is no incoming edge to that particular set, which could potentially exist due to nonzero percentages of diagonal flows or due to nonzero lateral flows to the adjacent lane. Then this set contains a dilation when the output vertex y_i^* doesn't exist, and hence, the system is not (weakly) structurally observable. Moreover, the measurement configuration iia) is a necessary condition for (weak) structural observability, which can be seen from Fig. 4 as follows. Consider, for example, the case in which there is no output vertex y_3, and, in addition, the red edge from χ_N to ψ_N doesn't exist. Then the vertex ψ_N is non-accessible, and thus, the system is not (weakly) structurally observable. Finally, it is trivial to show that the measurement configuration iib) is a necessary condition for (weak) structural observability since when condition iib) doesn't hold there is no output vertex at the exit of any lane. Hence, all vertices of every segment on the right of some segment with an output vertex y_i are non-accessible.

e) Claims 9, 10: The proofs of Claims 9 and 10 follow from Claim 8 similarly to Claims 6 and 7, respectively.

The (weak) structural observability results of this section concern the time-invariant version of A and C with the structure $\mathcal{A}$ and $\mathcal{C}$, respectively. Yet, adapting the results from [14] (see also, [11]) to the case of the observability matrix we conclude that the original time-varying system (10), (11) is also (weakly) structurally observable. This is shown noting that the observability matrix that corresponds to the time-varying version of A and C, with structure $\mathcal{A}$ and $\mathcal{C}$ for all times, has a generic rank $MN+m$, on any interval $[k_0, k_0+MN+m], \forall k_0 \geq 0$. The latter conclusion follows combining [14] with the (weak) structural observability results of this section, and noting that the elements of matrices A and C are well-defined, explicit functions of speeds, lateral flows, and densities of connected vehicles (which are all assumed to be uniformly bounded from above and, in addition, speeds and densities are assumed to be positive and uniformly bounded from below, see Sections II-C and III-E).

Example 2: See Fig. 6.

[6]There may be additional symmetries due to, e.g., in the case of one-lane highway, terms of the form $1-\frac{T}{\Delta}v_{i,j}$ and $\frac{T}{\Delta}v_{i,j}$ that appear in the main diagonal and the diagonal immediately below of matrix A, respectively. However, such dependencies don't seem to cause an underestimation of the measurement requirements needed for observability since such symmetries/dependencies are inherent to the actual physical system.

[7]One can observe from the expression for $\det(O)$ that the system is not observable when $g_2 = \frac{\Delta - T v_f}{\Delta}$ as well. However, the latter condition is just an accidental condition without any meaningful physical interpretation.

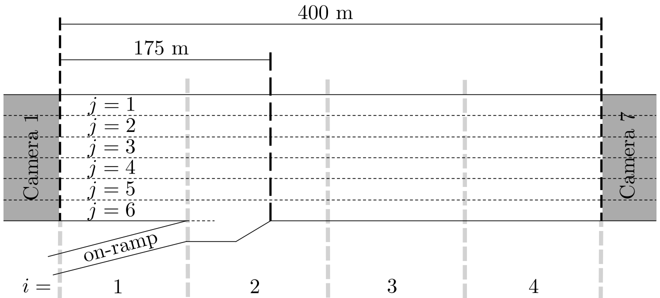

Fig. 6. Stretch of highway I-80 in Emeryville, California. Assume per-lane density dynamics modeled by (10), (11), where, for simplicity, all diagonal flow percentages are zero. From Claim 2, the system is strongly structurally observable when flow detectors are placed at the exit of cells $(1, 6)$, $(4, j)$, j=1, . . . , 6, and at the entry of cells $(1, j)$, j=1, . . . , 6. From Claim 10, the system is (weakly) structurally observable when flow detectors are placed at the exit and entry of cells $(4, j)$ and $(1, j)$, j = 1, . . . , 6, respectively.

Appendix

a) Condition G_0 from [17]: For every non-empty subset $V \subseteq \{1, \ldots, n\}$ of (state) vertices of $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ there exists a vertex $v \in \{1, \ldots, n+r\}$ such that $V \cap \mathrm{Post}\left(\{v\}\right)$ is a singleton.

b) Condition G_1 from [17]: For every non-empty subset $V \subseteq \{1, \ldots, n\}$ of (state) vertices of $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ that satisfies $V \subseteq \mathrm{Pre}\left(V\right)$ there exists a vertex $v \in \{1, \ldots, n+r\} \setminus V$ such that $V \cap \mathrm{Post}\left(\{v\}\right)$ is a singleton.

c) Conditions for (Weak) Structural Observability (see, e.g., [11]): A linear system (A, C) is (weakly) structurally observable if and only if: i) The graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ contains no non-accessible vertex; and ii) the graph $\mathcal{G}\left(\mathcal{A}^{\mathrm{T}}, \mathcal{C}^{\mathrm{T}}\right)$ contains no dilation.

d) Observability Gramian: Observability Gramian for (1), (2) is matrix $G\left(k_0, k_0 + N^*\right) = \sum_{k=k_0}^{k_0+N^*-1} \theta^{\mathrm{T}}\left(k, k_0\right) C^{\mathrm{T}}(k) C(k) \theta\left(k, k_0\right)$, where θ is as $\theta\left(k, k_0\right) = A\left(k-1\right) A\left(k-2\right) \cdots A\left(k_0\right)$, $\forall k > k_0$, and it satisfies $\theta\left(k, k\right) = I$, with I denoting the identity matrix.

e) Condition for Observability of Linear Time-Varying Systems (see, e.g., [2]): The pair (A, C) is observable on $[k_0, k_0 + N^*]$ if and only if: $\det\left(G\left(k_0, k_0 + N^*\right)\right) \neq 0$.

f) Specialization to Linear Time-Invariant Systems [2]: The pair (A, C) is observable if and only if: $\mathrm{rank}\left(O\right) = n$, where n is the dimension of matrix A and O is the observability matrix $O^{\mathrm{T}} = \left[\begin{array}{cccc} C^{\mathrm{T}} & A^{\mathrm{T}} C^{\mathrm{T}} & \cdots & {A^{n-1}}^{\mathrm{T}} C^{\mathrm{T}} \end{array}\right]$.

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