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«Multi-objective algorithms for optimal product line design»

by

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Υπεύθυνη Δήλωση Συγγραφέα:

Δηλώνω ρητά ότι, σύμφωνα με το άρθρο 8 του Ν.1599/1986, η παρούσα εργασία αποτελεί αποκλειστικά προϊόν προσωπικής μου εργασίας, δεν προσβάλλει κάθε μορφής δικαιώματα διανοητικής ιδιοκτησίας, προσωπικότητας και προσωπικών δεδομένων τρίτων, δεν περιέχει έργα/εισφορές τρίτων για τα οποία απαιτείται άδεια των δημιουργών/δικαιούχων και δεν είναι προϊόν μερικής ή ολικής αντιγραφής, οι πηγές δε που χρησιμοποιήθηκαν περιορίζονται στις βιβλιογραφικές αναφορές και μόνον και πληρούν τους κανόνες της επιστημονικής παράθεσης.

Multi-objective algorithms for optimal product line design

ABSTRACT

Introducing new products has an important role in sustainability and profitability of a firm. Product Line Design (PLD) is a key decision area that product managers have to deal with in the early stages of product development, to estimate the potential success of a product. Even though several objectives may be simultaneously pursued during the product configuration process, most reported studies have focused on single-objective optimization. In this research, the multi-objective PLD (MOPLD) problem is addressed, by taking into account more than one objectives, to provide product managers with a better tradeoff among them, using 23 variants of seven state-of-the-art metaheuristics. The seven main multi-objective metaheuristics used in this research are Genetic Algorithms (GAs), Particle Swarm Optimization (PSO), Firefly Algorithm (FA), Differential Evolution (DE), Grey Wolf Optimizer (GWO), Teaching-Learning Based Optimization (TLBO) and Mayfly Algorithm (MA). Those seven multi-objective algorithms are fully adapted to the MOPLD problem, using popular diversity controlling operators, as well as using an extended to multi-objective optimization Fuzzy-Self-Tuning (FST) process. The purpose of the FST process, is to help the algorithms overcome specific difficulties when performing on different datasets, using the same parameter settings, by calculating the settings of parameters independently for each individual during the optimization process. The 23 variants are compared with each other, through popular performance metrics when it comes to multi-objective optimization, using two types of data sets, under five different MOPLD scenarios, without knowing the specific number of products. Moreover, factors affecting the performance of optimizers are investigated using statistical analysis. Finally, a multi-criteria decision analysis method is used to rank solutions according to the needs of product managers, and an attempt to estimate the possible moves of the competitors, is made.

Keywords: mathematical multi-objective optimization, multi-objective product line design, many-objective product line design, genetic algorithms, particle swarm optimization, firefly algorithm, differential evolution, grey wolf optimizer, teaching-learning based optimization, mayfly algorithm, multi-criteria decision analysis, statistically significant differences of algorithms

Αλγόριθμοι Πολυκριτήριας βελτιστοποίησης για το σχεδιασμό γραμμής προϊόντων

ΠΕΡΙΛΗΨΗ

Οι τακτικές που υιοθετούν οι παραγωγικές μονάδες για την εισαγωγή νέων και καινοτόμων προϊόντων στην αγορά, αποτελούν σημαντικό παράγοντα οικονομικής ανάπτυξης, κυριαρχίας, και εν γένει, βασικό συντελεστή βιωσιμότητας της επιχειρησιακής τους δραστηριότητας. Στην κατεύθυνση αυτή, ο σχεδιασμός μιας γραμμής προϊόντων αποτελεί μια καίρια επιχειρησιακή λειτουργία, για την a priori εκτίμηση της πιθανής της επιτυχίας, μιας και συνδέεται άμεσα τόσο με την κερδοφορία της επιχείρησης όσο και με τη διαμόρφωση του μεριδίου σε ανταγωνιστικές συνθήκες αγοράς. Παρόλο που τα προϊόντα αυτά μπορεί να κατασκευαστούν σύμφωνα με πολλά κριτήρια, οι περισσότερες προσεγγίσεις επικεντρώνονται στη βελτιστοποίηση ενός μονάχα κριτηρίου. Υπό την οπτική αυτή, η παρούσα διδακτορική μελέτη διαπραγματεύεται το πρόβλημα του βέλτιστου σχεδιασμού γραμμών προϊόντων, λαμβάνοντας υπόψη περισσότερα από ένα κριτήρια, χρησιμοποιώντας 23 παραλλαγές από επτά μεθευρετικούς πολυκριτήριους αλγορίθμους μαθηματικής βελτιστοποίησης. Οι επτά αυτοί πολυκριτήριοι αλγόριθμοι βελτιστοποίησης είναι οι Γενετικοί Αλγόριθμοι, ο Αλγόριθμος Βελτιστοποίησης Σμήνους Σωματιδίων, ο Αλγόριθμος Πυγολαμπίδας, ο Αλγόριθμος Διαφορικής Εξέλιξης, ο αλγόριθμος Γκρίζων Λύκων, η Βελτιστοποίηση βάσει Διδασκαλίας-Μάθησης και ο αλγόριθμος βελτιστοποίησης Εφημεροπτέρων. Αυτοί οι επτά πολυκριτήριοι αλγόριθμοι προσαρμόζονται πλήρως στο πολυκριτήριο πρόβλημα του βέλτιστου σχεδιασμού, χρησιμοποιώντας δημοφιλείς μηχανισμούς ελέγχου ποικιλομορφίας μη-κυριαρχούμενων λύσεων, και αυτόματης παραμετροποίησης. Σκοπός της αυτόματης παραμετροποίησης, είναι ο υπολογισμός των παραμέτρων ανεξάρτητα για κάθε λύση, κατά τη διάρκεια της βελτιστοποίησης, για να βοηθήσει τους αλγόριθμους να ξεπεράσουν συγκεκριμένες δυσκολίες κατά την εκτέλεση σε διαφορετικά σύνολα δεδομένων. Οι 23 παραλλαγές αυτές συγκρίνονται μεταξύ τους, μέσω δημοφιλών μετρικών απόδοσης πολυκριτήριων αλγορίθμων, χρησιμοποιώντας δύο διαφορετικά σύνολα δεδομένων μέσα από πέντε διαφορετικά σενάρια, χωρίς να είναι γνωστός ο ακριβής αριθμός των προϊόντων. Επιπλέον, οι παράγοντες που επηρεάζουν την απόδοση των αλγορίθμων διερευνώνται χρησιμοποιώντας στατιστική ανάλυση. Τέλος, χρησιμοποιείται μια μέθοδος πολυκριτήριας λήψης αποφάσεων με σκοπό την κατάταξη των λύσεων σύμφωνα με τις ανάγκες και γίνεται προσπάθεια εκτίμησης των πιθανών κινήσεων των ανταγωνιστών.

Λέξεις - κλειδιά: μαθηματική πολυκριτήρια βελτιστοποίηση, πολυκριτήριος σχεδιασμός σειράς προϊόντων, γενετικοί αλγόριθμοι, αλγόριθμος βελτιστοποίηση σμήνους σωματιδίων, αλγόριθμος πυγολαμπίδας, αλγόριθμος διαφορικής εξέλιξης, αλγόριθμος γκρίζων λύκων, βελτιστοποίηση βάσει διδασκαλίας-μάθησης, αλγόριθμος βελτιστοποίησης εφημεροπτέρων, πολυκριτήρια λήψη αποφάσεων, στατιστικά σημαντικές διαφορές αλγορίθμων

Στον Κώστα, την Άρτεμις, τον Φίλιππο και την Ελεάννα

ΕΥΧΑΡΙΣΤΙΕΣ

Αρχικά, θα ήθελα να ευχαριστήσω τον Αναπληρωτή Καθηγητή της Σχολής Μηχανικών Παραγωγής και Διοίκησης του Πολυτεχνείου Κρήτης, Δρ. Στέλιο Τσαφαράκη, για την επίβλεψη, την βοήθεια, την υποστήριξη σε όλα τα στάδια της πορείας αυτής της διατριβής, το ενδιαφέρον, τη διαρκή ενθάρρυνση, την εμπιστοσύνη και τις γνώσεις που μου προσέφερε, όλα αυτά τα χρόνια που συνεργάζομαι μαζί του. Θέλω επίσης να ευχαριστήσω τον Καθηγητή Μιχαήλ Δούμπο για τα πολύτιμα σχόλιά του.

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Chapter 1 Introduction

The introduction of novel products and the modification of existing ones play a key role in the sustainability, profitability, and success of a firm. By providing a larger product variety, the increasingly diversified customer needs are satisfied, resulting the maximization of the total firm's market share, income, or profit (Tseng & Jiao, 2001). Before the product production process begins, its potential success needs to be estimated by product managers. This can be accomplished by addressing the Product Line Design (PLD) problem, which has been studied for over 40 years by various researchers and is commonly formulated in the context of Conjoint Analysis (CA) or Multi-Dimensional Scaling (MDS) (Baier & Gaul, 1998; Kotler & Armstrong, 2012).

1.1 Designing product lines

Designing products was originally introduced by Zufryden (1977), and some years later, Green and Krieger (1985) addressed the PLD problem, to meet the diversity of the market. A product line consists of several related products, distinguished by similar structures, but different configurations. A set of attributes usually represents each product. Each attribute takes specific values (levels). A TV, for instance, consists of the attribute of *resolution*, which can take the three levels of *Full HD*, *4K Ultra HD* or *8K Ultra HD*, the attributes of *Smart TV*, *Media player*, *HDR* and *Curved Screen* which can take *Yes* or *No* values and the attribute of *screen* size which can take the values of *24"*, *32"*, *48"* and *55"*.

Since consumers purchase products according to their attributes and their levels that meet their needs, product managers need to be aware of their preferences. CA is a widely known method to measure consumer preferences (Luce & Tukey, 1964). Compared to MDS, not only CA, as an approach, has better analytical capabilities, but also it has had thousands of commercial applications (X. G. Luo & Kwong, 2010). Using CA, the perceived utility value of each level of a product's attributes (part-worths) for each consumer, is generated. Combining those values with choice models, the possible success of a product line can be estimated, using optimization techniques to search for the best configuration. To conduct the product search, the followings must be included (Sawtooth Software, 2003):

a) attribute levels to be considered,
b) part-worths estimates,
c) attribute cost information,
d) the number of products to search for,

e) any competing products, and
f) size of the market.

1.2 Motivation and objectives

The PLD problem is reported to be a nondeterministic-polynomial (NP)-hard class combinatorial optimization problem (Kohli & Krishnamurti, 1989). As a result, a complete enumeration of the problem space is not feasible (Papadimitriou & Steiglitz, 1982). In an attempt to address the PLD problem, researchers have used various optimization approaches in previous research, to provide (near) optimal (Belloni et al., 2008; Pantourakis et al., 2021; Tsafarakis et al., 2013, 2020, 2021; Tsafarakis & Matsatsinis, 2010) or guaranteed optimal solutions (Akçakuş & Misic, 2021; Belloni et al., 2008; Bertsimas & Mišić, 2019), in acceptable time.

However, most of the existing approaches address the problem from a deterministic perspective, optimizing only one criterion, such as profit, market share, income, or cost, even though conflicting business objectives should be taken into account, by product managers. For instance, profitability is provided no insight from market share. In a matter of fact, a market share increase could be achieved by lowering product prices. On the contrary, total profit could be increased by increasing product prices. However, this kind of price increase may have a negative impact on market share (Ferguson & Foster, 2013).

To investigate and demonstrate the difference between optimizing different objectives, like profit and market share, the data that Belloni et al., (2008) used in their research to optimize profit, were used. Belloni et al., (2008) implemented a Lagrangian relaxation with branch and bound with a computational time of one week, and detected the global optimum solution, which was a product line of five bags with a $12,226 of predicted earnings. According to the literature, Genetic Algorithm (GA), Simulated Annealing (SA) (Belloni et al., 2008), Particle Swarm Optimization (PSO) (Tsafarakis et al., 2011, 2013), Fuzzy Self-Tuning Differential Evolution (FSTDE) (Tsafarakis et al., 2020), Tabu Search (TS) (Tsafarakis et al., 2021) and Clonal Selection Algorithms (CSAs) (Pantourakis et al., 2021) are some of the most successful metaheuristic optimization approaches to reach the overall optimum of the problem, in acceptable time.

The predicted market share of the optimal solution provided by Belloni et al., (2008) was calculated and it was found that the optimal solution, as regards the profit, has a market share of 89.81%, as presented in Table 1.1. In Table 1.1 the configuration of the optimal solution, as regards profit is also demonstrated.

Table 1.1: Optimal product line when maximizing profit

Profit:			$12,226						
Market share:			89.81 %						
Product Configurations									
Price	Size	Color	School Logo	Handle	Gadget holder	Cell phone holder	Mesh pocket	Velcro flap	Reinforcing boot
$80	Normal	Black	No	No	Yes	Yes	Yes	Yes	Yes
$95	Normal	Black	No	Yes	No	No	No	Yes	Yes
$100	Normal	Black	Yes	Yes	Yes	Yes	Yes	No	No
$95	Large	Red	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$100	Large	Black	Yes	Yes	No	No	Yes	Yes	Yes

By implementing a FSTDE to optimize market share, as proposed by Tsafarakis et al., (2020) for the PLD problem, a predicted market share of 98.15% was detected. However, the predicted profit of the specific solution has a value of $7,922.5 ($4,303.5 less than the optimum solution as regards profit). Table 1.2 demonstrates the configuration of the optimal solution, as regards market share, as well as its differences (colored attribute levels) between the optimal solution as regards profit.

Table 1.2: Optimal product line when maximizing market share

Profit:			$7,922.5						
Market share:			98.15 %						
Product Configurations									
Price	Size	Color	School Logo	Handle	Gadget holder	Cell phone holder	Mesh pocket	Velcro flap	Reinforcing boot
$70	Normal	Black	No	No	No	No	No	Yes	No
$70	Normal	Black	No	Yes	No	No	No	No	No
$75	Normal	Black	No	No	Yes	Yes	Yes	Yes	Yes
$70	Large	Red	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$100	Large	Black	Yes	Yes	Yes	Yes	Yes	Yes	Yes

From Table 1.1 as well as Table 1.2, it is clear that in many cases profitability and market share can be two conflicting objectives, that is, to increase one, a sacrifice must occur in the other. Moreover, by running FSTDE for one more time, a solution with a market share of 98.15% and a profit of $7,506 was detected. Particularly, even though the two solutions obtained using FSTDE have the same market share (98.15%), the first solution presented in Table 1.2 with a profit of $7,922.5 clearly constitutes a better strategy for product managers, compared to the one with a profit of $7,506. It would therefore be ideal if the PLD problem could be modeled and solved as a multi-objective optimization problem (e.g., maximizing market share, maximizing income, minimizing development costs, etc.).

Furthermore, existing methods address the problem using a certain number of products. However, product managers, may intent to investigate the tradeoffs between dissimilar objectives for product lines with a different number of products. Particularly, using the same dataset as before, two market share values of 96.29 % and of 98.77 % were detected, when optimizing a product line of four and six products, respectively. However, the latter would have a cost of development of $17,652.50 which is bigger compared to the line of four products which have a predicted cost of development of $16,096.00. As a result, even though a product line of more products may result in a bigger percentage of market share, compared to one of less products (2.48% more), it may result in a bigger cost of development, compared to second one ($1,556.50 more). As a result, solving a Multi-objective Product Line Design (MOPLD) problem with the number of products not known a priori, becomes even more computationally difficult, since its complexity increases.

In this research, a total of 23 variants of seven multi-objective optimization approaches are used, for addressing the MOPLD problem using more than one criteria, under unknown number of products. These multi-objective optimization algorithms consist of multi-objective Evolutionary Algorithms (EAs) as well as multi-objective Swarm Intelligence (SI) algorithms whose performance is evaluated using popular performance metrics.

1.3 Multi-objective Optimization

Multi-objective optimization refers to the optimization process of optimizing more than one objective functions. It can be formulated as:

$$\min f(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_K(\mathbf{x})] \tag{1.1}$$

Subject to:

$$g_i(\mathbf{x}) \leq 0,\, i = 1,2, \dots, m \tag{1.2}$$

$$h_j(\mathbf{x}) = 0,\, j = 1,2, \dots, p \tag{1.3}$$

$$lb_l \leq x_l \leq ub_l,\, l = 1,2, \dots, n \tag{1.4}$$

where $f_s: A \rightarrow \mathbb{R}$ refer to the objective functions to be optimized. The domain A is a subset of the Euclidean space $\mathbb{R}^n$ and represents the search space, K is the number of the objectives to be optimized, x_s are the decision (problem) variables. n is the number of variables, m is the number of inequality constraints, p is the number of equality constraints, and lb_l and ub_l correspond to the lower the upper bound of the l^{th} variable (Zervoudakis & Tsafarakis, 2020; Zitzler et al., 2000).

According to literature, optimization problems that involve four or more objectives, are called as many-objective optimization problems (Deb & Jain, 2014; D. Guo et al., 2021).

1.3.1 Pareto Dominance

In a multi-objective optimization problem, the search space is generally ordered according to the relation between two different solutions. Those solutions can be related in the two following possible ways: either one dominates the other or neither dominates the other according to the following definitions (Zitzler et al., 2000):

Definition 1.1: Pareto Dominance

Let us consider, without the loss of generality, a multi-objective optimization problem as presented in Subsection 1.3 (equations 1.1 to 1.4). A decision vector **a**∈*A is said to* dominate *a decision vector* **b**∈*A (also written as* **a**≺**b***) if and only if:*

$$\forall \boldsymbol{k} \in \{\mathbf{1}, \dots, \boldsymbol{K}\}\text{:}\ f_k(\mathbf{a}) \leq f_k(\mathbf{b}) \wedge \exists \boldsymbol{z} \in \{\mathbf{1}, \dots, \boldsymbol{K}\}\text{:}\ f_z(\mathbf{a}) < f_z(\mathbf{b}) \quad (1.5)$$

Based on Definition 1.1 the Pareto-optimal solutions are now defined:

Definition 1.2: Pareto-optimal solutions

Let **a**∈*A be an arbitrary decision vector:*

1. **a** *is considered to be* non-dominated *regarding a set $A' \subseteq A$ if and only if there is no vector in A' which dominated* **a**:

$$\nexists a' \epsilon A' : a' \prec a \quad (1.6)$$

2. *The decision vector* **a** *is* Pareto-optimal *if and only if* **a** *is non-dominated regarding A.*

Pareto-optimal solutions are not possible to be improved in any objective at the same time. Similar to single-objective optimization problems, local optima points (a non-dominated set within a certain neighbor-hood) may exist (Zitzler et al., 2000):

Definition 1.3: Local and Global Pareto-optimal solutions

Let us now consider for once again the decision vectors $A' \subseteq A$:

1. *The set A' is considered to be a local Pareto-optimal one, if and only if:*

$$\forall \mathbf{a}' \in A' : \nexists \mathbf{a} \in A : \mathbf{a} \prec \mathbf{a}' \wedge \|\mathbf{a} - \mathbf{a}'\| < \varepsilon \wedge \|f(\mathbf{a}) - f(\mathbf{a}')\| < \delta \quad (1.7)$$

where $\|\cdot\|$ is a distance metric, $\varepsilon > 0$ and $\delta > 0$.

2. *The set A' is considered to be a global Pareto-optimal one, if and only if:*

$$\forall a' \epsilon A' : \nexists \mathbf{a} \in A : a \prec a' \tag{1.8}$$

Figure 1.1 demonstrates the Pareto front (red rhombus points) of the multi-objective optimization T_1, problem as presented by Zitzler et al., (2000), that minimizes two-objective functions. This figure, also graphically explains which solutions *dominate* (light blue area) a random solution (blue circle point), which ones *are dominated* by this random solution (grey area) and which neither *dominate* nor *are dominated* (green area).

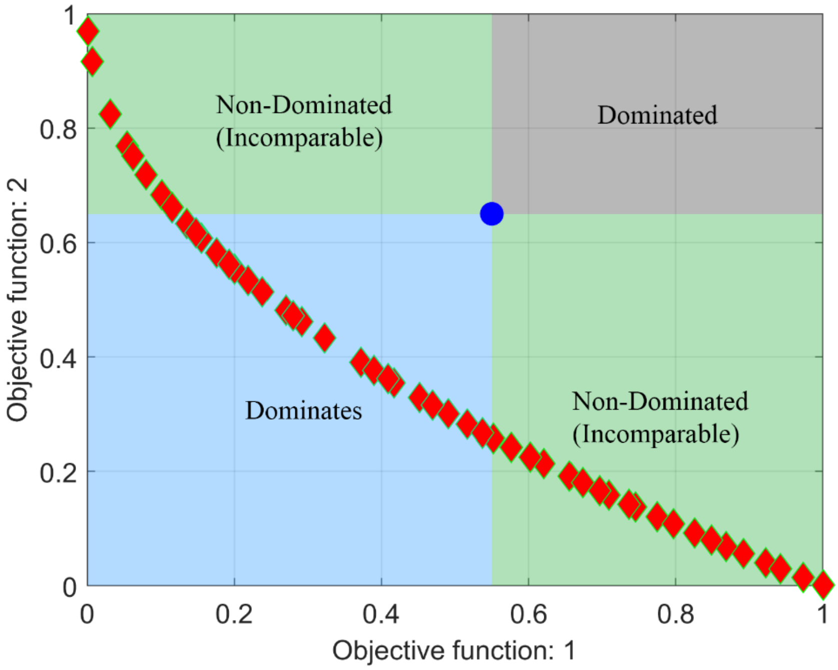


Figure 1.1: Example of Pareto Dominance

1.3.2 Aim of multi-objective optimization algorithms

Numerous algorithms have been proposed in the literature to address the multi-objective optimization problems. Their aim is to detect solutions that are as close to the optimal Pareto front as possible. However, finding an optimal Pareto front as well as proving that it is indeed optimal, especially when performing on complex real-world combinatorial optimization problems, is practically impossible. For this reason, the optimization process is performed using approximation approaches, capable of detecting an optimal set of non-dominated solutions, as close to the optimal Pareto front as possible, in a very short time (Konak et al., 2006).

According to Coello Coello (2006) from those approximation algorithms, EAs like GAs are expected to perform better in obtaining a Pareto front compared to heuristics like SA (Tsafarakis,

2016) or TS (Tsafarakis et al., 2021), since GAs (in general) search for solutions from a number of different points in the solution space, increasing the odds of finding near-optimal solutions.

According to Konak et al., (2006) the best-known Pareto front retrieved from multi-objective optimization approaches should be a) a subset of the Pareto front as possible, or at least it should be as close to it as possible, b) as uniformly distributed as possible, to give the decision maker a complete picture of his/her choices, and c) extended to the full range of the Pareto front which requires exploring the extreme solutions of the front (finding the optimal solutions of each objective function) as best as possible.

Some of the most successful multi-objective optimization approaches are the Non-dominated Sorting Genetic Algorithms (NSGA-II and NSGA-III) (Deb et al., 2002; Deb & Jain, 2014) as well as Multi-Objective Particle Swarm Optimization (MOPSO) (Coello Coello et al., 2004). To further investigate the most well-known multi-objective optimization algorithms presented in the literature to date, one can refer to the work of Zouache et al., (2018), Afshari et al., (2019), Monsef et al., (2019) and Liu et al., (2021).

1.4 Research innovation

The present study extends previous research in the following five important ways: First, 23 multi-objective variants of seven state-of-the-art metaheuristic optimizers, are applied for designing product lines without knowing the specific number of products for the first reported time in the literature. Second, the fuzzy self-tuning method proposed by Nobile et al., (2018) and Tsafarakis et al., (2020) for single objective optimization, is modified and extended to multi-objective optimization for helping the algorithms that suffer from initial parameter settings overcome problem-related difficulties. Third, five different scenarios are used in this research, each one using a different number of objective functions, in order to investigate how the number of objectives affects the results. For these experiments, two types of data sets were used. The first concerns existing data from previous research, while the second is used for the first time in the product line problem. Fourth, factors affecting the performance of optimizers are investigated using statistical analysis, in contrast to existing research which simply compares algorithms without being interested in their characteristics that make the difference in performance. Fifth and last, a multi-criteria decision analysis method is used to rank solutions according to the needs of product managers, and an attempt to estimate the possible moves of the competitors, is made.

1.5 Structure of doctoral dissertation

This doctoral dissertation is structured through seven chapters as follows: Chapter 2 provides a literature review regarding addressing the PLD problem using more than one objectives. In Chapter 3, the full research methodology is presented, while in Chapter 4 the multi-objective optimization algorithm to be used in this research as well as their adaptation to the problem, are described in detail. In Chapter 5, the effectiveness of each algorithm is evaluated through their comparison, using popular performance metrics, according to the literature. Furthermore, in Chapter 6, the process of ranking the non-dominated solutions according to the decision maker's preferences as well as an approach to estimate the competitors' next moves when it comes on product development, are demonstrated. Finally, Chapter 7 provides an overview of the main conclusions of the study and future research areas are suggested.

Chapter 2 Literature Review on Product Line Design

In this chapter, the literature review on PLD when optimizing more than one criteria either by optimizing a single compromised objective function or using multi-objective optimization, is presented.

By reviewing the literature, it was found that not only a few researchers have addressed the PLD problem when considering more than one criteria (either by optimizing each objective one by one or by using a weighted sum of two or more individual conflicting objectives to transform them into a compromised single objective function), but even fewer have addressed it using multi-objective optimization.

Research on addressing the PLD problem when considering more than one criteria in terms of both marketing and engineering perspectives, without using multi-objective optimization, can be found in various research publications (D'Souza & Simpson∗, 2010; Farrell & Simpson, 2003; Heese & Swaminathan, 2006). Some of the most important ones are the work of Besharati et al., (2004), who introduced an integrated approach considering variability (i.e., noise or uncertainty) in both domains of engineering design and consumer preferences in marketing, to decrease a set of design alternatives to a controllable size. In the same fashion, Michalek et al., (2006) coordinated decision models from design, business, and manufacturing with one another, using Analytical Target Cascading (ATC), to make tradeoffs with respect to a firm-level objective and thus detect a reliable solution that is considered "optimal" for the firm, in terms of both marketing and design perspectives. Furthermore, Michalek et al., (2011) proposed an extended to continuous search spaces ATC-based methodology for PLD, that avoids both the combinatorial complexity of binary or integer formulations and the need of assuming monotonic preferences, to coordinate attribute selection for each desired products on a heterogeneous market while ensuring that they can each be realized by a possible engineering design. Furthermore, L. Luo (2011) presented a product line optimization approach that enables product managers to simultaneously consider important factors from both marketing and engineering perspectives, taking into account the strategic reactions from the incumbent manufacturers and the retailer, in the search for a profit-maximizing product line.

Finally, in a more recently published research of Kwong et al., (2020) a nested bi-level GA combined with Stackelberg game theory approach was adopted to formulate joint optimization models that involve a manufacturer and a retailer considering various common types of contracts, while T. Wang and Gutierrez (2021) found that both designs of maximization of the value of the worst outcome (MaxMin) and minimization of the maximum relative ex-post regret (MinMaxRR) can decrease the downside risk, even though MaxMin may expose the firm to bigger upside ex-post opportunity losses on average, after exploring the tradeoffs between MaxMin and MinMaxRR on a PLD problem.

2.1 Previous research on multi-objective product line design

To detect previous research on multi-objective PLD as far as the use of multi-objective optimization is concerned, a systematic literature review was conducted, which included the definition of the criteria for inclusion or exclusion of a study, the research identification in bibliographic databases, their evaluation, the acceptance of those that met the inclusion criteria, and their findings recording.

The review was limited to international studies that were published in English language and focused on exploring previous research regarding the applications of multi-objective optimization on PLD using CA data sets. A search through numerous electronic databases provided 164 results by searching the phrases of "*product line design*", "*optimizing product line design*", "*optimization algorithms for product line design*", "*multi-objective optimization algorithms for product line design*", "*many-objective optimization algorithms for product line design*" and "*product optimization*".

The sum of the 164 publications collected during the identification phase was reduced to 154 after removing the duplicates, those that were not considered relevant and those that did not come from peer review journals, doctoral dissertations, book chapters, research reports and publications in conference proceedings. The full texts were then searched, retrieved, archived, and studied in order to clarify whether the predefined inclusion criteria were met. The findings revealed that research on approaching the PLD problem using multi-objective optimization is very limited.

Particularly, Besharati et al., (2006) used a Multi-Objective Genetic Algorithm (MOGA) (Narayanan & Azarm, 1999) with Kurpati et al.'s (2002) constraint handling technique as an optimizer, to obtain a set of Pareto designs, for a single product (motor), from both engineering design (output speed and mass removed), and marketing perspectives (market share variation and market share) to assess the performance and feasibility robustness of a design, when several uncontrollable parameters exist.

A few years later, X. G. Luo and Kwong, (2010) adopted Deb's NSGA-II (Deb et al., 2002) to optimize the total product demand in a multi-segment market as well as the total develop cost of a product line of digital cameras. Moreover, Kwong et al., (2011) proposed a one-step NSGA-II to address a PLD problem of digital cameras, using the three objectives of market share of a company's products, total product development cost of a product line and total product development cycle time. They also introduced a curve-fitting method into the part-worth utility models so that the optimization model can be applied to products with all kinds of attributes. Both studies mentioned above used the optimization process to also detect an optimal number of products.

Williams et al., (2011) examined the broad issue of how retail channel structures impact a manufacturer's optimal new Power Tool products, by using an NSGA-II to optimize manufacturer and retailer profits. The results showed that the multi-objective approach allows product managers to detect the attributes of the designs that are considered “optimal” under specific channel structure conditions, as well as the sensitivity of profits to different design variables. As a result, the multi-objective optimization approach is very useful to managers who are concerned about the acceptance of their new designs by the retail channel and getting shelf space.

Ferguson et al., (2012) synthesized advancements from market-based design and heuristic optimization research, to strategically construct targeted initial populations capable of reducing computational cost and improving the final solution quality of multi-objective GA, when performing on MOPLD considering a feature packaging problem for an automobile product line. The authors used profit and share of preference as objective functions. In this context, Foster and Ferguson (2013) extended the creation of targeted initial populations of NSGA-II by using the objectives of the problem, instead of product level utility, to identify candidate MP3 as well as automobile designs, by optimizing profit and market share. Furthermore, Ferguson and Foster (2013) demonstrated how multi-objective PLD can significantly influence and form a design strategy, using an NSGA-II (Deb et al., 2002) to locate a Pareto set between profit, market share and commonality associated with the design of an MP3 player product line.

Deng et al., (2014) adapted an NSGA-II (Deb et al., 2002) multi-objective optimization algorithm for addressing the MOPLD problem, to optimize profit, quality and performance, and cost of development of a product line of notebook. The authors also stated that companies could consider introducing techniques to help select an optimal solution. Moreover, Aydin et al., (2014) addressed the PLD problem with simultaneous consideration of remanufactured and new Tablet PCs using an NSGA-II (Deb et al., 2002) to detect an optimal Pareto front of product line alternatives, including specifications and prices of both remanufactured and new products, to estimate profit, market share as well as the launching time of remanufactured products and to investigate the tradeoffs among the objectives. Similarly, Aydin et al., (2015b) addressed various research issues, like the competition between remanufactured and novel products in markets; demand estimation of remanufactured products in markets; downgrading and upgrading their features and the launching time of remanufactured products in the markets. An NSGA-II (Deb et al., 2002) was adopted to determine the Pareto optimal solutions of the multi-objective optimization problem, when optimizing profit and market share for a PLD of tablet PCs. In the same context, Aydin et al., (2015a, 2016) investigated the coordination of a manufacturer and supply chain partners using a multi-objective optimization model based on Stackelberg game theory to determine the product line solutions, pricing decisions of supply chain partners, and product’s return rate for remanufacturing. An NSGA-II (Deb et al., 2002) was used to determine the Pareto optimal solutions of the multi-objective optimization problem, when optimizing profit and market share for a PLD of tablet PCs, with consideration of remanufactured tablet PCs, to illustrate the applicability and effectiveness of their approach.

Finally, Shin and Ferguson (2016) specified the reliability and robustness of a tablet PC product line, under uncertainty, when using discrete choice methods and integrating the objectives into a multi-objective optimization framework using an NSGA-II. The researchers attempted to maximize the average choice share of a product line and its robustness while minimizing the probability of failure. The authors also suggested that a multi-criteria decision analysis method could be adopted to help the decision maker choose the best design from the set of non-dominated solutions.

Interestingly enough, all researchers mentioned above, used GAs and more specifically the NSGA-II (Deb et al., 2002) to address the MOPLD problem. The reason behind this specific adaption, might be that not only NSGA-II has obtained very good results in solving complex optimization problems compared to the other approaches (Agarwal & Gupta, 2008), but also GAs (in general) are expected to perform well in the PLD problem, because they search for solutions from a number of different points in the solution space, increasing the odds of finding near-optimal solutions (Belloni et al., 2008). However, more multi-objective optimization techniques need to be tested when performing on the MOPLD problem.

Chapter 3 Research Methodology

In this chapter, the research methodology that was used in this research, is discussed in detail. It contains two main items, related to the research methodology, namely: Research questions and Research approach. The latter is further divided into seven items concerning the Data colection, the Data sets to be used, the Choice Models, the Objective Functions to be optimized, the Solution representation, the Performance metrics to be used and the Data analysis techniques used in this research. Each point is briefly elaborated in the following subsections:

3.1 Research questions

As mentioned in the Introduction, a key assumption of this research is that a multi-objective approach is more efficient than a single-objective one, to simultaneously address the subproblems of optimal design. Therefore, since multi-objective optimization is a more effective and more profitable approach for a company to address the PLD problem, the following questions arise:

1) How can the problem of optimal PLD be modeled as a multi-objective optimization problem?
2) Are the solutions to the problem (product lines) different depending on the optimized objective?
3) Is the relationship of tradeoffs between the criteria linear?
4) Can the PLD problem be addressed without knowing the number of products to be designed?
5) Which multi-objective optimization algorithms have the best performance in multi-objective PLD?
6) What factors affect the effectiveness of each algorithm?
7) Can the non-dominated solutions be evaluated or ranked (according to the decision maker)?
8) Can the future moves of competitors be predicted?

3.2 Research approach

3.2.1 Data colection

In this research, two data sets are to be used. The first one concerns an existing data set of an actual PLD problem faced by Timbuk2, a manufacturer of messenger bags. The company worked with a group of academic researchers to conduct a conjoint study that focused on price and nine binary product features. Additional details are reported in Toubia et al., (2003). The particular dataset has been used as benchmark in numerous popular studies (Belloni et al., 2008; Bertsimas & Mišić, 2019; Pantourakis et al., 2021; Tsafarakis et al., 2020, 2021; Zervoudakis et al., 2020) on evaluating the performance of optimization algorithms on the PLD problem and it is presented in detail in Subsection 3.2.2.1.

The second data set which is used for the first time on the PLD problem, concerns olive oil products. The Choice-Based Conjoint Analysis was used as a tool for interpreting and estimating consumers' purchase behavior.

3.2.1.1 Choice-Based Conjoint Analysis

Choice-Based Conjoint Analysis (CBCA) evaluates the preferences of consumers realistically, compared to other methods, by observing a quantity of choice decisions (Green et al., 2001). It can be described as a decomposition process where products are judged and part-worth utilities for each attribute or attribute level, are generated (Meyerding et al., 2018).

According to Meyerding and Merz (2018) CBCA is distinguished by a few major advantages over the rest of CA approaches. Initially, due to the involvement of a simulated purchase situation CBCA constitutes a more realistic approach to data acquisition. Secondly, the generated part-worth utilities of the different attributes reveal the choice of the consumers. Moreover, it allows the researcher to include the *None* option.

A major limitation of CBCA is that it does not provide the researcher the information whether a particular characteristic is not relevant for the consumer or if it does not catch his/her attention, like all CA methods.

The CBCA questionnaire used in this research, is demonstrated in Appendix E.

3.2.2 Data sets

In this subsection, the datasets used in this research are described.

3.2.2.1 Timbuk2 data set

Regarding the first data set, after a CA market survey, the partworths (preferences) of 324 consumers on the 10 attributes that comprise each bag, are generated. The first attribute concerns the *price* that can take seven different levels ($70, $75, $80, $85, $90, $95 or $100) while the remaining nine binary attributes, get yes/no values (exists / does not exist). According to Tsafarakis et al., (2021) by combining these attributes, 3,584 possible products (bags) can be designed, by which $4.9 \cdot 10^{15}$ different product lines consisting of five products can be formed. Table 3.1 presents the 10 attributes, along with their partworths and their marginal costs, while in Figure 3.1 the violin plots of each feature's partworths are demonstrated.

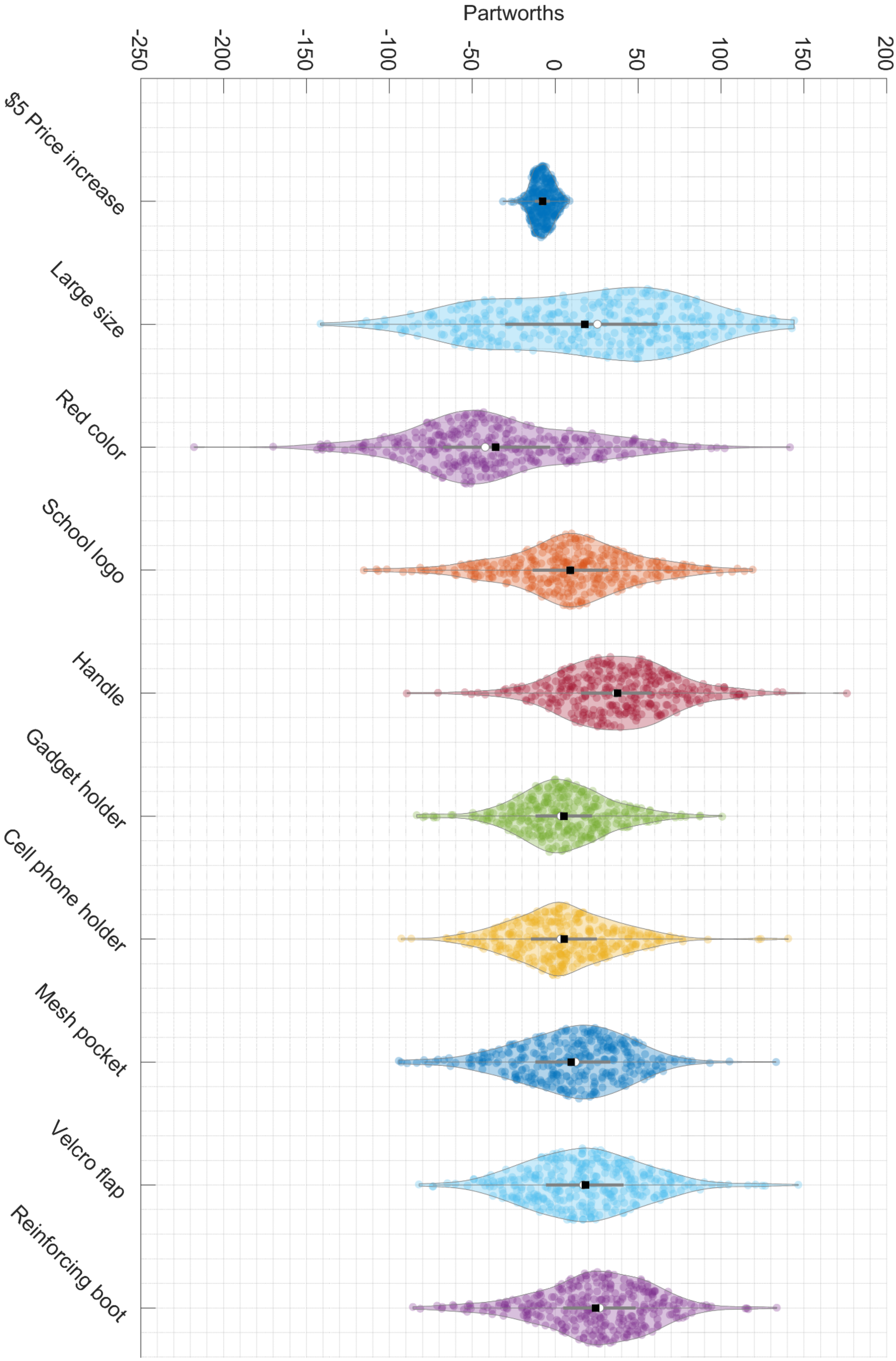


Figure 3.1: Violin plots of partworths on the Timbuk2 data set

Table 3.1: Partworths and Incremental marginal cost of each feature in the Timbuk2 data set

Feature	Part worth details					Incremental marginal cost ($)
	Max	Average	Median	Min	SD	
$5 Price increase	8.70	-7.58	-7.75	-31.50	6.03	-5.00
Large size	144.10	17.93	25.45	-141.50	57.71	3.50
Red color	141.60	-35.95	-42.20	-217.80	50.62	0.00
School logo	119.10	8.98	9.30	-115.40	39.57	2.00
Handle	175.90	37.67	36.70	-89.50	34.94	3.50
Gadget holder	100.70	5.15	3.50	-83.50	29.03	3.00
Cell phone holder	140.50	5.45	3.35	-92.80	32.32	3.00
Mesh pocket	133.20	9.67	12.15	-94.30	34.61	2.00
Velcro flap	146.60	18.21	17.00	-82.20	35.67	3.50
Reinforcing boot	133.60	24.36	26.90	-85.80	34.90	4.50

3.2.2.2 Olive oil data set

Regarding the second data set to be used, after a CBCA market survey on olive oil products, the partworths (preferences) of 417 consumers on the five attributes that comprise each product, are generated. Table 3.2 and Table 3.3 present the attributes and their levels, along with their partworths and their marginal costs, Table 3.4 presents the olive oil usage statistics in liters per month, while in Figure 3.2 the violin plots of each feature’s partworths, are demonstrated. By combining these attributes, 8,000 possible products can be designed.

Table 3.2: Partworths and Incremental marginal cost of each feature in the olive oil data set

Feature		Part worth details					Cost (€)
		Max	Average	Median	Min	SD	
Olive oil type	Plain	113.54	-63.56	-71.61	-170.58	41.17	3.30
	Extra Virgin	176.58	9.81	10.44	-133.14	28.75	4.51
	Organic extra virgin	216.37	29.11	26.47	-54.13	33.82	5.49
	POP Kolumpariou	162.52	14.29	16.04	-129.65	29.74	4.73
	Sustainable	113.62	10.35	14.33	-133.63	29.69	4.40
Package type	Plastic	104.89	-13.42	-10.03	-111.43	30.98	
	Glass	198.20	22.30	16.71	-127.60	37.01	
	Metal	112.54	0.89	1.41	-99.42	26.17	
	Pouch	101.40	-9.78	-7.26	-193.09	32.78	
Package size	0.75*lt*	109.10	-28.77	-35.62	-148.48	49.91	see: Table 3.3
	1*lt*	146.36	-2.19	-1.20	-162.00	47.46	
	2*lt*	130.89	7.78	6.33	-105.25	38.79	
	3*lt*	145.21	12.58	11.32	-114.67	43.12	
	5*lt*	211.21	10.59	3.59	-114.32	54.89	
Brand	Super Market	98.60	-78.28	-87.78	-195.99	42.69	0.05
	Altis	146.82	30.07	31.23	-82.13	30.39	0.15
	ABEA	232.30	33.26	33.90	-76.95	35.36	0.10

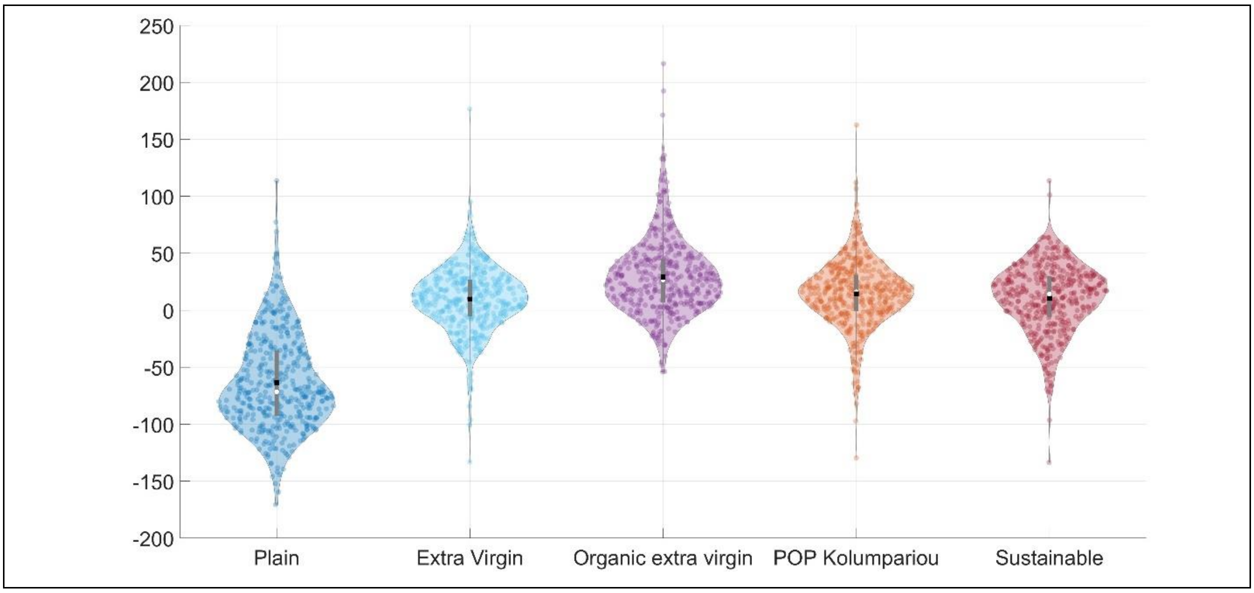
	MINERBA	221.37	14.95	19.09	-108.33	31.56	0.10
Price per liter	4.5€	175.32	30.13	24.24	0.60	26.31	-4.50
	5.3€	132.09	14.80	10.27	-29.58	18.20	-5.30
	6.2€	66.86	-0.70	-0.29	-84.71	15.45	-6.20
	7.0€	37.15	-13.39	-9.26	-100.50	19.53	-7.00
	7.9€	-0.51	-30.82	-25.69	-155.17	24.72	-7.90
None		292.55	46.78	48.52	-299.67	71.15	

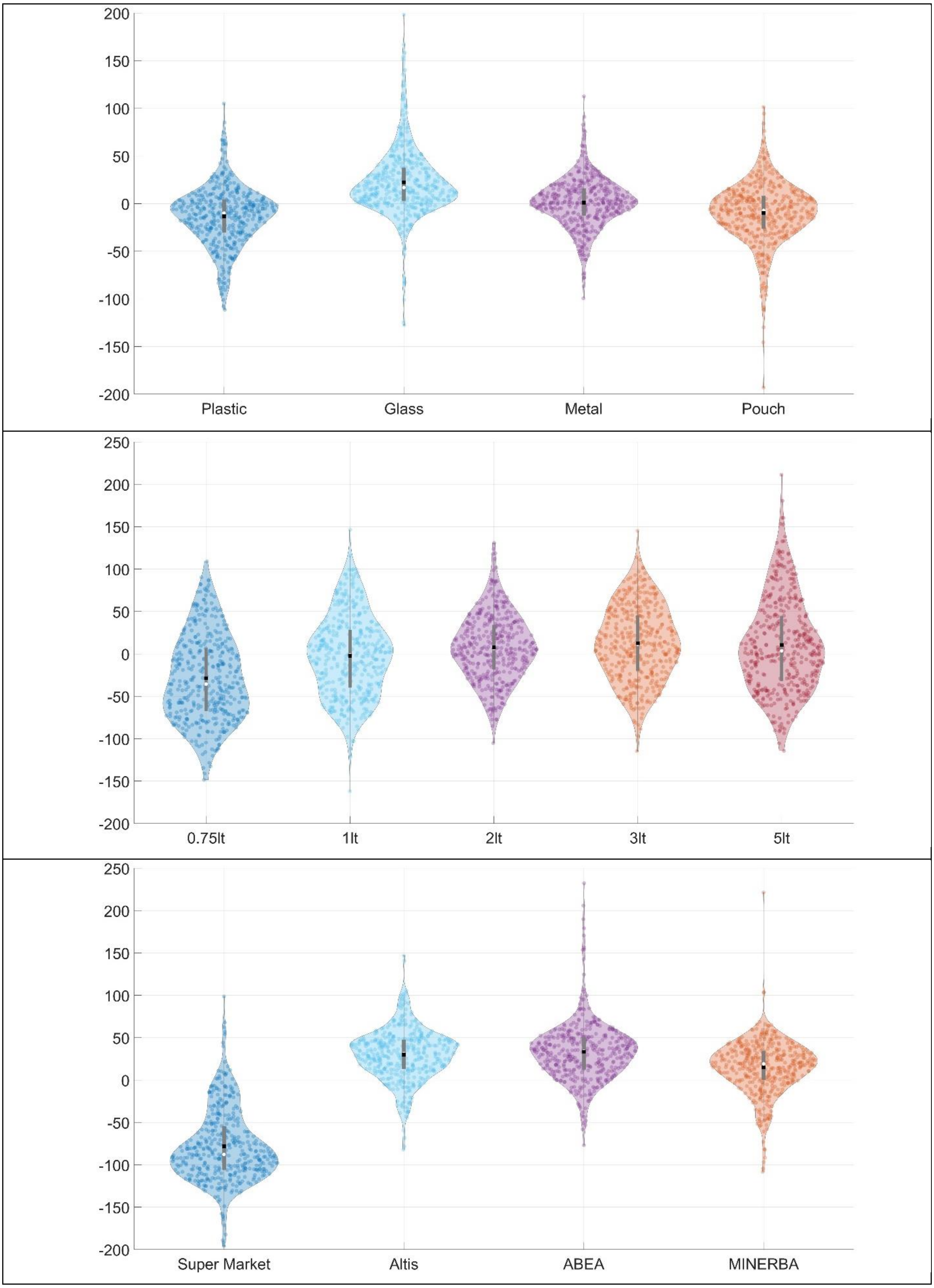
Table 3.3: Marginal cost of specific features in the olive oil data set

Feature/liter	0.75*lt*	1*lt*	2*lt*	3*lt*	5*lt*
Plastic	0.16	0.17	0.30	0.40	0.60
Glass	0.40	0.50	0.80	-	-
Metal	0.08	0.10	0.20	0.30	0.50
Pouch	-	-	0.75	0.80	0.85

Table 3.4: Olive oil usage statistics in liters per month

Statistics	Olive oil usage in liters
Maximum	7
Mean	1.97
Median	2
Minimum	1
Standard deviation	1.28




200
150
100
50
0
-50
-100
-150
-200
Plastic
Glass
Metal
Pouch
250
200
150
100
50
0
-50
-100
-150
-200
0.75lt
1lt
2lt
3lt
5lt
250
200
150
100
50
0
-50
-100
-150
-200
Super Market
Altis
ABEA
MINERBA

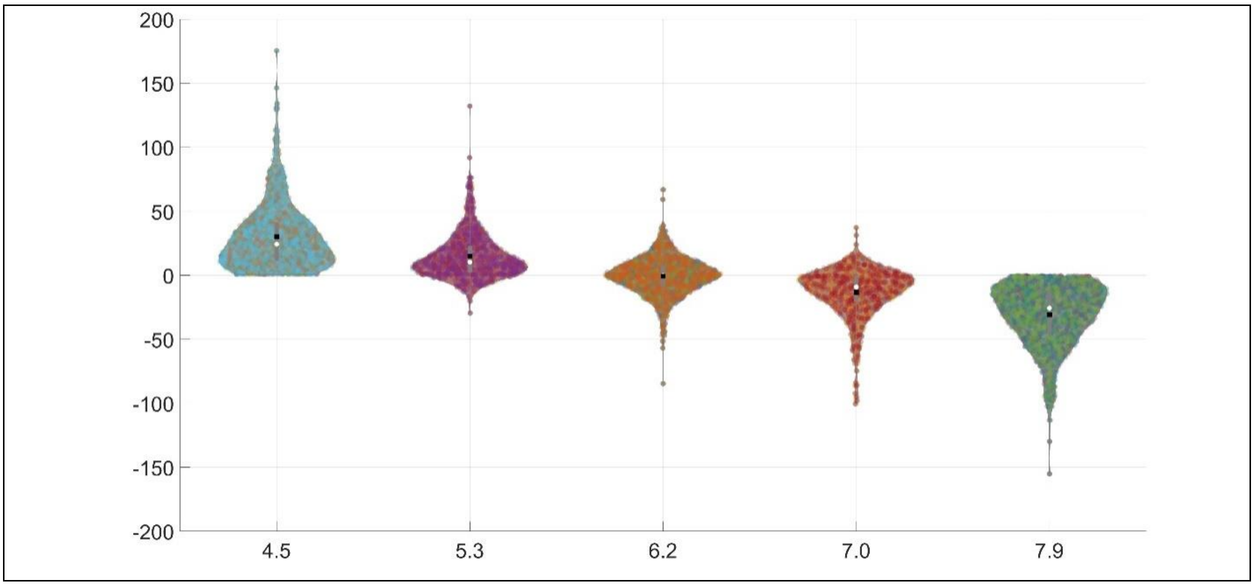


Figure 3.2: Violin plots of partworths on the Olive oil data set

During the data collection through CBCA, a few prohibitions were defined, as presented in Table 3.5. The same prohibitions were used during the optimization process. More specifically, when a prohibited product line is defined, it is automatically rejected.

Table 3.5: CBCA Prohibitions

Prohibition			
Prohibition 1	Glass	with	5*lt*
Prohibition 2	Glass	with	3*lt*
Prohibition 3	Pouch	with	0.75*lt*
Prohibition 4	Pouch	with	1*lt*
Prohibition 5	Super-market	with	7.9€
Prohibition 6	Sustainable	with	4.5€
Prohibition 7	POP Kolumpariou	with	4.5€

3.2.3 Choice Models

To simulate the consumer behavior, when it comes on product selection, a choice model must be in place, which refers to the process of which a consumer integrates information to buy a product, from a set of similar competing ones. Numerous choice models have been introduced with varying assumptions and purposes, which differ in the underlying logic structure that derives them (Manrai, 1995). Choice models, which can be either deterministic or probabilistic mathematical models converting the product utilities assigned to the set of alternatives under consideration to choice-probabilities, for each alternative per individual, represent the consumer's purchasing pattern by relating preference to choice.

The First Choice (or maximum utility) Model (FCM) is a deterministic one, which assumes that a consumer always purchases the product with the highest sum of utility values. In such a case, a probability of choice equal to *one* is assigned to the highest sum of utility values alternative, while *zero* probabilities are assigned to the rest. FCM's major weakness is that it provides information only for the maximum utility product, ignoring the rest of them. As a result, FCM may fail to successfully simulate the actual human choice behavior, since a consumer does not necessarily buy the highest utility alternative, due to buyer's confusion, out of stock occasions, high search costs, etc.

To overcome the FCM's difficulty to simulate the actual consumer choice behavior, probabilistic models, which are also known as *share of preference* models, may be used to assign choice probabilities among all products, according to their utility values. These models are further separated into constant utility models and random utility ones.

In contrast to models in the first category which assume that product utilities are constant and capture the stochastic nature of human behavior, by assuming a level of uncertainty in the decision model, models in the second category assume that the consumer always chooses the product with the highest utility (U), which consists of a deterministic component (V) specified as a function of the measured product attributes and individual preferences, and a stochastic term (e) representing possible measurement errors, computed as (Baltas & Doyle, 2001):

$$U = V + e \tag{3.1}$$

Bradley-Terry-Luce (BTL) (Bradley & Terry, 1952) is the most popular constant utility probabilistic model, which is calculated as:

$$p_{ij} = \frac{U_{ij}}{\sum_{j=1}^{n} U_{ij}} \tag{3.2}$$

where p_{ij} is the probability that consumer i selects product j, U_{ij} is the assigned by the consumer i utility to product j, and n is the number of competing products. To uniformly control the choice probabilities of BTL, BTL's extensions have been introduced in the literature (Pessemier et al., 1971).

The MNL (McFadden, 1974) is a widely used random utility model, which is calculated as:

$$p_{ij} = \frac{e^{U_{ij}}}{\sum_{j=1}^{n} e^{U_{ij}}} \tag{3.3}$$

According to Brice (1997) share of preference models like BTL and MNL, provide a better estimation of the actual market share, compared to the more extreme First Choice model.

Regarding the choice models used in this research, FCM was used when using the Timbuk2 data set, while BTL was used when using the olive oil data set.

3.2.4 Objective Functions

The MOPLD problem addressed in this research, can be formulated as:

Let Ξ={1,2,…N} be the set of products that compose the market, Ω = {1,2,…,K} denotes the set of K attributes that compromise the products, Φ_k={1, 2,..., J_k} is the set of J_k levels of attribute k, Ψ={1, 2,...,M} is the set of products to be designed ($\Psi \subset \Xi$), θ={1, 2,..., I} is the set of I customers and w_{ijk} is the part-worth that customer $i \in \theta$ assigns to level $j \in \Phi_k$ of attribute $k \in \Omega$. The x_{jkm} and U_{im} decision variables are also used, where:

$$x_{jkm} = \begin{cases} 1, & \text{if the level of product's } m \text{ attribute } k \text{ is } j \\ 0, & \text{otherwise} \end{cases}, \quad k \in \Omega, m \in \Psi \tag{3.4}$$

and U_{im} is the utility that customer i assigns to product m (sum of its part-worths), as:

$$U_{im} = \sum_{k \in \Omega} \sum_{j \in \Phi_k} w_{ijk} x_{jkm}, \quad m \in \Psi \tag{3.5}$$

In case the Timbik2 data set is used and therefore the FCM is used, the s_{im} decision variable is also used, which reveals whether customer i buys product m according to the highest utility value, as:

$$s_{im} = \begin{cases} 1, & \text{if } U_{im} = \max(U_{i\Xi}) \\ 0, & \text{otherwise} \end{cases}, \quad m \in \Psi \tag{3.6}$$

In this step, the Buyer's welfare can be estimated, as:

$$BW = \sum_{i \in \theta} s_{im} \cdot U_{im}, \quad m \in \Psi \tag{3.7}$$

The c_m decision variable which calculates the number of customers to buy product m is computed as:

$$c_m = \sum_{i \in \theta} s_{im}, \quad m \in \Psi \tag{3.8}$$

The predicted Market Share (%) of the line can be estimated, as:

$$MS = \frac{c_m}{I} \cdot 100 \tag{3.9}$$

The income of the product line to be designed can be calculated as:

$$Income = \sum_{i=1}^{m} inc_i \cdot c_i \tag{3.10}$$

where inc_i is the sale price of product i. The development cost of the product line to be designed can be also calculated as:

$$Cost = \sum_{i=1}^{m} cost_i \cdot c_i \tag{3.11}$$

where $cost_i$ is the development cost of product i.

In case the Olive oil data set is used and therefore the BTL is used, the probability $c_{\iota m}$ that customer *i* will choose product *m* is estimated as follows using equation (3.2).

In this step, the Buyer's welfare can be estimated, as:

$$BW = \sum_{i\in\theta} \sum_{m\in\Psi} c_{\iota m} \cdot U_{im} \tag{3.12}$$

and the predicted Market Share (%) of the line can be estimated, as:

$$MS = \sum_{i\in\theta} \sum_{m\in\Psi} c_{\iota m} \tag{3.13}$$

Because in this data set each consumer has its own needs and therefore consumers can consume a different amount of olive oil per month, their needs per liter have been used to calculate the final values of the objective functions. For instance, if a consumer may buy a one-liter product even though he consumes three liters per month, and therefore he/she needs two more products, then the product income for that particular customer will triple. As a result, the income of the product line to be designed can be calculated as:

$$Income = \sum_{i\in\theta} \sum_{m\in\Psi} cl_{im} \cdot inc_m \cdot c_{\iota m} \tag{3.14}$$

where inc_m is the sale price of product *m* and cl_{im} corresponds to the number of *m* products consumer *i* needs. The development cost of the product line to be designed can be also calculated as:

$$Cost = \sum_{i\in\theta} \sum_{m\in\Psi} cl_{im} \cdot cost_m \cdot c_{\iota m} \tag{3.15}$$

where $cost_m$ is the development cost of product *m* that is the cost of its attributes.

Furthermore, the profit of the particular product line on both data sets can be computed as:

$$Profit = Income - Cost \tag{3.16}$$

Finally, the Commonality Index (Martin & Ishii, 1996, 1997) was used a sixth objective function to measure unique parts, due to possible manufacturer interest as regards the commonality associated with the product line solution. It can be calculated as:

$$Commonality\ Index = 1 - \frac{u - max(k_i)}{\sum_{i=1}^{m} k_i - max(k_i)}, k \in \Omega \tag{3.17}$$

where *u* is the total number of distinct components, k_i represents the number of components used in product *i*. CI ranges from 0 to 1 where a smaller value indicates more unique parts.

Regarding the competitive products used in this research, when using the Timbuk2 data set, it is considered that the work of Belloni et al., (2008) represents the choices of the competitors and therefore, the optimal product line retrieved by Belloni et al., (2008), as presented in Table 1.1, is used as the competitive product line, in order to investigate the choices of the competitors of Belloni et al., (2008). As regards the competitive products used when using the olive oil data set, they are presented in Table 3.6.

Table 3.6: Product line of competitors

Price per liter	Olive oil type	Package type	Package size	Brand
4.5€	Plain	Pouch	2*lt*	Super Market
5.3€	Extra Virgin	Plastic	3*lt*	MINERBA
6.2€	POP Kolumpariou	Metal	5*lt*	Altis
7.9€	Organic extra virgin	Glass	1*lt*	ABEA

3.2.5 Solution representation

Binary and integer representation are two ways of presenting a solution in a PLD problem. In the binary representation, a solution is presented as a table of $number\ of\ products \times total\ number\ of\ attribute\ levels$. A value of 1 is assigned to the level that corresponds to the specific attribute, while the rest of the elements take a value of zero. In the integer representation, a solution is presented as a table of $number\ of\ products \times total\ number\ of\ attributes$. The value of each attribute corresponds to its level. According to the literature, binary representation has shown better performance in similar problems (Tsafarakis et al., 2011), even though integer representation is reported to require smaller vector lengths and thus, less computational time.

Since in our approach the number of bags is not know in advance, one last attribute with a no/yes value (0 or 1), was added. This attribute shows if the specific product is considered within the line or not. A *maximum number of bags* as well as a *minimum number of bags* are given by the user. As a result, a potential binary solution to the problem is represented as a table of *maximum number of products* × *total number of attribute levels*. For instance, let a product line with a maximum number of five products be distinguished by three attributes, each one taking three levels. The potential solution would be presented as a table of 5×11. An example of a solution is presented in Figure 3.3.

Instead of binary representation of a solution, a discrete one can be used, which is represented as a table of *maximum number of products* × *total number of attributes*, as shown in Figure 3.4.

Attributes	A			B			C			Considered	
Levels	1	2	3	1	2	3	1	2	3	No	Yes
Product 1	1	0	0	0	1	0	1	0	0	0	1
Product 2	0	0	1	0	1	0	0	1	0	0	1
Product 3	1	0	0	0	0	1	0	0	1	1	0
Product 4	0	1	0	0	0	1	0	1	0	0	1
Product 5	0	1	0	0	1	0	1	0	0	0	1

Figure 3.3: Representation of a potential binary solution to the PLD problem

Attributes	A	B	C	Considered
Product 1	1	2	1	1
Product 2	3	2	2	1
Product 3	1	3	3	0
Product 4	2	3	2	1
Product 5	2	2	1	1

Figure 3.4: Representation of a potential discrete solution to the PLD problem

Since the third product has a *No* value in the *Considered* attribute, the final line consists of the rest four products. If the final line consists of less than the *minimum number of products* given by the user, then a randomly chosen product from the ones that has a *No* value is altered until the number of products within the line is equal to the *minimum number of products* given by the user. If the *minimum number of products* given by the user is equal to the *maximum number of products*, the line has a fixed number of products.

3.2.6 Performance metrics

In this subsection the performance metrics to be used are described. Those metrics are used because in the case of multi-objective optimization, the evaluation of quality is significantly more complex than for single-objective optimization problems (Zitzler et al., 2000).

The first metric to be used is the two-set coverage metric (*C*) proposed by Zitzler et al., (2000). It maps the ordered pair (X', X'') to the interval [0,1] using the following equation:

$$C(X', X'') := \frac{|\{a'' \in X''; \exists\, a' \in X' : a' \preccurlyeq a''\}|}{|X''|} \tag{3.18}$$

where X', X'' are two sets of decision vectors and $\|\cdot\|$ the Euclidean distance metric.

The value $C(X', X'') = 1$ means that all solutions in X'' are dominated by or equal to solutions in X'. The opposite, $C(X', X'') = 0$, represents the situation when none of the solutions in X'' are covered by the set X'. As Zitzler et al., (2000) mention in their research, both $C(X', X'')$ and $C(X'', X')$ have to be considered, since $C(X', X'')$ is not necessarily equal to 1- $C(X'', X')$.

The second metric is the M_1^* metric, proposed by Zitzler et al., (2000), which considers the average distance to the Pareto-optimal set $\bar{X} \subseteq X$ and is calculated as:

$$M_1^* (X') := \frac{1}{|X'|} \sum_{\mathbf{a}' \in X'} min\{\|\mathbf{a}' - \bar{\mathbf{a}}\|; \bar{\mathbf{a}} \in \bar{X}\} \tag{3.19}$$

where $\|\cdot\|$ the Euclidean distance metric. Since the PLD problem is proved to be an NP-hard class problem (Kohli & Krishnamurti, 1989) and therefore the Pareto-optimal front cannot be detected, to calculate the M_1^* metric, all the non-dominated solutions retrieved by all algorithms are considered as the Pareto optimal front.

The third metric Δ, measures the extent of spread achieved among the obtained solutions as:

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}} \tag{3.20}$$

where d_f and d_l are two are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained non-dominated set and $\bar{d}$ is the average of all distances $d_i, i = 1,2, \dots, (N-1)$, assuming that there are N solutions on the best non-dominated front (Deb et al., 2002). As Deb et al., (2002) mention in their research, for the most widely and uniformly spread out set of non-dominated solutions, the metric would be zero. For any other distribution, it would be greater than zero. More details can be found in the work of (Deb et al., 2002).

Finally, the fourth metric is the M_3^* metric proposed by Zitzler et al., (2000), which considers the extent of the set of objective vectors that correspond to X' and is calculated as:

$$M_3^* (X') := \sqrt{\sum_{i=1}^{n} max\{\|a_i' - b_i'\|; \mathbf{a}', \mathbf{b}' \in X'\}} \tag{3.21}$$

where $\|\cdot\|$ the Euclidean distance metric.

3.2.7 Data analysis

The collected results of the multi-objective algorithms were imported in the Jamovi statistical package (Şahin & Aybek, 2019) and analyzed quantitatively to assess the degree of effectiveness of each algorithm, as well as to investigate possible factors that affect it.

As previous research suggests, non-parametric tests were used (Carrasco et al., 2020; Derrac et al., 2011; García et al., 2008). Furthermore, Pearson's correlation coefficient *r* was used to measure possible statistically significant relationships between two continuous variables. The level of statistical significance (p) was set at 5% while the findings with a value of $p<0.05$ were considered statistically significant.

Since statistically significant differences are more likely to occur when large samples are utilized, practical significance was also calculated, since it addresses the magnitude of a treatment effect without the complication of sample size. As a result, it provides more meaningful information that has usefulness to researchers (Kirk, 2016). As far as the practical significance is concerned, the effect sizes were calculated. More specifically, a Point Biserial Correlation *r* (Kornbrot, 2014) is used when a Mann-Whitney U test is performed, which indicates a small effect for $r \approx 0.1$, a medium effect for $r \approx 0.3$ and a large effect for $r \approx 0.5$. Moreover, the Epsilon square (ε^2) (Kelley, 1935) is used when a Kruskal-Wallis test is performed, which indicates a negligible effect when $0.00 \le \varepsilon^2 \le 0.01$, a weak effect when $0.01 < \varepsilon^2 \le 0.04$, a moderate one when $0.04 < \varepsilon^2 \le 0.16$, a relatively strong when $0.16 < \varepsilon^2 \le 0.36$, a strong when $0.36 < \varepsilon^2 \le 0.64$ and a very strong when $0.64 < \varepsilon^2 \le 1.00$.

Chapter 4 Multi-objective optimization algorithms for product line design

In this Chapter, the description as well as the application of the multi-objective optimization algorithms to the MOPLD problem, is demonstrated.

4.1 Genetic Algorithms (GAs)

Genetic Algorithms (GAs) are powerful global search metaheuristics that imitate the natural evolution process, based on the survival and reproduction of the fittest (Goldberg, 1989). GAs are reported to have great performance in numerous real-world complex optimization problems and continues to be successfully applied and further improved by researchers to this day (Katoch et al., 2020). In GA, candidate solutions are mostly represented as binary or integer strings, known as chromosomes, which are randomly generated and evolve over the algorithm's iterations, through genetic operations known as crossover and mutation. The new chromosomes (offspring) are assessed using the predefined objective function. Should the offspring be better than the parents, they will replace them in the next iteration. The iterative process continues until a stopping criterion is met.

Various multi-objective GAs are reported in the literature for addressing complex multi-objective optimization problems. The most important ones are the Multi-objective EA based on Decomposition (MOEA/D) (Q. Zhang & Li, 2007), the Pareto Envelope region-based Selection (PESA-II) (Corne et al., 2001), the Strength Pareto Evolutionary Algorithm II (SPEA-II) (Zitzler et al., 2001), the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Deb et al., 2002) and the Non-dominated Sorting Genetic Algorithm III (NSGA-III) (Deb & Jain, 2014; Jain & Deb, 2014). The multi-objective GAs mentioned above are all used in this research, to address the multi-objective PLD problem.

Just like the single-objective GA, in multi-objective GAs, individuals perform genetic operators to produce offspring. The non-dominated solutions are then archived and a diversity controlling operator as presented in Appendix D, is applied, which constitutes the main difference between the multi-objective GAs mentioned above.

For instance, NSGA-II uses a fast non-dominated sorting operator combined with CD as described in Subsection D.1 of Appendix D, NSGA-III replaced the CD with the use of RP, while PESA-II use an Adaptive Grid Technique (GT) as briefly described in Subsection D.3 and D.2 of Appendix D, respectively. Furthermore, MOEA/D uses a decomposition process which means that it decomposes the addressed multi-objective optimization problems to several single-objective sub-problems, each one having its own best solution ever found. Additional details can be found in the work of Zhang et al., (2007). Finally, SPEA-II utilizes a process like *k*-Nearest Neighbor (Silverman, 1986) and a specialized ranking system to sort the population members and select the next iteration's generation. Additional details can be found in the work of Zitzler et al., (2001).

4.1.1 Multi-Objective GAs for PLD

Since GAs operate in discrete/binary spaces, each solution is represented as described in Subsection 3.2.5, which is the same GA formulation used in the research of Belloni et al., (2008), with the only difference being the unknown number of products.

During the iterative process, crossover and mutation operators, as described in Appendix B, are in place to produce new candidate solutions. The best non-dominated solutions are then selected for the next generation.

A general pseudo-code for multi-objective GAs used in this research, is presented in Algorithm 4.1.

Algorithm 4.1: Multi-objective GAs for MOPLD

```
Initialization
Select the number of chromosomes
Generate the initial population
Evaluate each chromosome according to the objective functions
Main loop
Do until a stopping criterion is met
    Produce new offspring through crossover operator
    Mutate chromosomes
    Evaluate new solutions according to the objective functions
    Apply a diversity controlling operator and keep the nPop chromosomes
Return non-dominated solutions
```

As literature suggests, the initial population size (chromosomes) should be approximately 7-10 times the number of design variables being considered (Ferguson et al., 2012). As a result, in this research all GAs use an initial population size of seven times the number of design variables to be considered. Regarding the mutation probability, it was set to 5%, as literature suggests for the PLD problem (Belloni et al., 2008; Ferguson & Foster, 2013; Kwong et al., 2011; X. G. Luo & Kwong, 2010).

4.2 Particle Swarm Optimization (PSO)

PSO is a swarm intelligence optimization algorithm, inspired from the social behavior of social organisms, such as birds and fish, and it was first introduced to address continuous optimization problems (Kennedy & Eberhart, 1995). It has already been successfully applied to the PLD problem, in previous research (Tsafarakis et al., 2011, 2013).

A potential solution to the problem is represented as the position of each particle in the search space. During the iterative process, the position of *i* particle (x_{ij}^{t}) is updated by adding a velocity (u_{ij}) as:

$$x_{ij}^{t+1} = x_{ij}^{t} + u_{ij}^{t+1} \tag{4.1}$$

where the velocity (u_{ij}) represents the changes that will be made to move a particle from one position to another and is updated so that each particle follows its previous best position ($pbest_{ij}$) and the best solution of the swarm ($gbest_{ij}$) (Tsafarakis et al., 2011; Zervoudakis et al., 2019). The velocity of a particle is updated using the following formula:

$$u_{ij}^{t+1} = u_{ij}^{t} + c_1 \cdot rand_1 \cdot \left(pbest_{ij} - x_{ij}^{t}\right) + c_2 \cdot rand_2 \cdot \left(gbest_j - x_{ij}^{t}\right) \tag{4.2}$$

where t is the iteration counter, $c_1,\ c_2$ are the acceleration coefficients and $rand_1,\ rand_2$ are two random numbers in [0,1].

To control the convergence behavior of PSO, Shi and Eberhart (1998) proposed the use of an inertia weight w to the updated formula that updates the particles' velocities, to prevent particles from developing great speeds. The velocities of particles are now updated using the following formula:

$$u_{ij}^{t+1} = w \cdot u_{ij}^{t} + c_1 \cdot rand_1 \cdot \left(pbest_{ij} - x_{ij}^{t}\right) + c_2 \cdot rand_2 \cdot \left(gbest_j - x_{ij}^{t}\right) \tag{4.3}$$

where $w = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \cdot iter$, w_{max}, w_{min} are the maximum and minimum values that the inertia weight can take, while $iter$ and $iter_{max}$ are the current iteration of the algorithm and the maximum number of iterations, respectively.

Regarding the application of PSO on multi-objective optimization problems, multi-objective Particle Swarm Optimization (MOPSO) was proposed by Coello Coello et al., (2004) to handle multi-objective optimization problems and continues to be successfully applied and further improved by researchers to this day (Feng et al., 2010; Gu et al., 2021; Habibollahzade, Houshfar, et al., 2021; Habibollahzade, Kazemi Mehrabadi, et al., 2021; Kalogiannis et al., 2021; Pan et al., 2018).

Just like PSO, particles in MOPSO are sharing information. As a result, they move towards global best particles and their own personal best memory. However, unlike single-objective PSO, there are more than one criteria to determine and define the best (global or personal). All non-dominated solutions are gathered into a repository, and every particle chooses its global best target, among members of this repository. Since the size of the repository is limited, whenever it gets full, a diversity controlling operator as presented in Appendix D, is applied, with those particles located in less populated areas of the search space are given priority over those lying in highly populated regions. For personal best, a domination-based rule combined with a probabilistic rule is utilized. A special mutation operator is also used to enrich the exploratory capabilities of MOPSO (Coello Coello et al., 2004).

4.2.1 Multi-Objective PSO for PLD

Since MOPSO operates in continuous spaces, the Smallest Position Value (SPV) rule, as presented in Appendix A, is in place. As Tsafarakis et al., (2011) report in their research, PSO combined with the SPV rule, has shown the best performance in the optimal PLD problem.

A secondary criterion for retention of the non-dominated solutions together with a continuous mutation operator as presented in Appendix B maintain the diversity of non-dominated solutions.

Even though PSO has already been applied to the PLD problem when optimizing one objective by Tsafarakis et al., (2011), who provide PSO's parameter settings in their research, the performance of PSO is not only highly dependent on its parameter settings, but also on the dataset and the objectives to be optimized. For this reason, a fuzzy self-tuning method, as presented in Appendix C is used to assist PSO in overcoming this particular difficulty.

To automatically determine the population size, MOPSO exploits the heuristic

$$N = \left\lceil 10 + 2\sqrt{maximum\ number\ of\ products\ \cdot\ total\ number\ of\ attribute\ levels} \right\rceil \quad (4.4)$$

which sets the value of population size according to the number of dimensions of the search space, as suggested by Nobile et al., (2018) and Tsafarakis et al., (2020) for PLD.

To automatically determine the rest of MOPSO's parameters a fuzzy self-tuning method, as presented in Appendix C, is used. During each iteration, each solution computes independently its own values for δ and φ, which are used to calculate the output variables according to the rules reported in Table 4.1.

Table 4.1: Fuzzy rules used by MOPSO

Rule no.	Parameter	Rule definition
1	w	if (φ is *Worse* or δ is *Same*) then (w is *Low)*
2		if (φ is *Same* or δ is *Near*) then (w is *Medium)*
3		if (φ is *Better* or δ is *Far*) then (w is *High)*
4	c_1	if (φ is *Worse* or δ is *Far*) then (c_1 is *Low)*
5		if (φ is *Same* or δ is *Same* or δ is *Near*) then (c_1 is *Medium)*
6		if (φ is *Better*) then (c_1 is *High)*
7	c_2	if (φ is *Better* or δ is *Near*) then (c_2 is *Low)*
8		if (φ is *Same* or δ is *Same*) then (c_2 is *Medium)*
9		if (φ is *Worse* or δ is *Far*) then (c_2 is *High)*
10	*mutation rate*	if (φ is *Better* or δ is *Near*) then (*mutation rate* is *Low)*
11		if (φ is *Same* or δ is *Same*) then (*mutation rate* is *Medium)*
12		if (φ is *Worse* or δ is *Far*) then (*mutation rate* is *High)*

In the consequent of these rules, the output variables are the MOPSO's parameters, which correspond to the respective settings of each solution. The final numerical value of this output variable is calculated using the Sugeno method (Sugeno, 1985) as presented in equation C.3 of Appendix C, according to Table 4.2.

Algorithm 4.2: Multi-objective PSO for MOPLD

1. Initialization
2. Select the number of particles
3. Generate the initial population and velocities of the particles
4. Evaluate each particle according to the objective functions
5. Keep the personal best of each particle
6. Store the non-dominated vectors found in an external Archive
7. Do until a stopping criterion is met
8.   For every particle:
9.   Select global best solution
10.   Compute parameters
11.   Compute the new velocity according to Eq. (4.3)
12.   Calculate the new position according to Eq. (4.1)
13.   Apply SPV rule
14.   Evaluate according to the objective functions
15.   If the new solution dominates its personal best
16.   Replace personal best with the new solution
17.   If no one dominates the other
18.   Replace personal best with a probability of 0.5
19.   Apply mutation
20.   Apply SPV rule
21.   Evaluate according to the objective functions
22.   Insert all non-dominated solutions found in the external Archive
23.   Apply a diversity controlling operator
24. Return non-dominated solutions

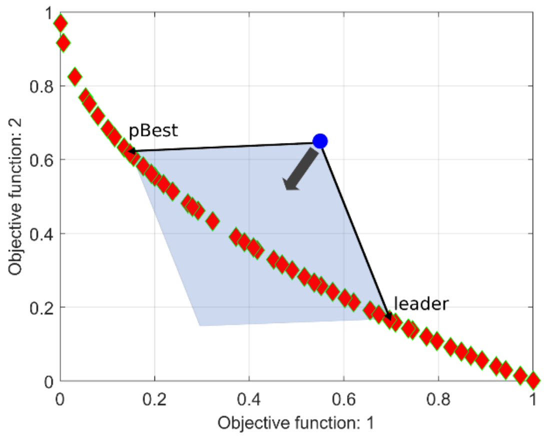
Table 4.2: Output variables and their defuzzification

Output Variable	Low	Medium	High
w	0.1	0.5	1
c_1	0.1	1.5	3
c_2	1	2	3
Mutation rate	0.01	0.05	0.1

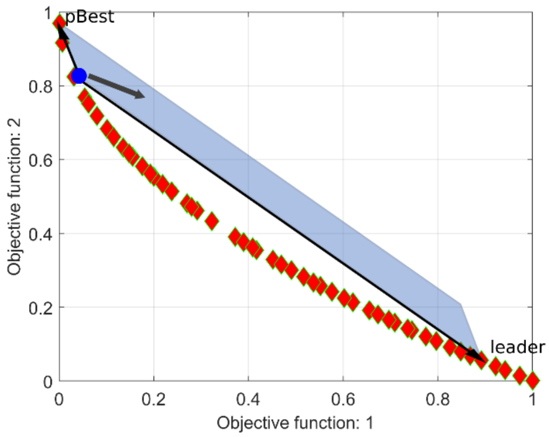
A general pseudo-code for the proposed MOPSO for MOPLD, is shown in Algorithm 4.2.

4.2.1.1 Leader selection

One of the most crucial factors that affect the performance of MOPSO is the global best selection process which is also referred as *Leader selection* in the literature of MOPSO (Coello Coello et al., 2004). As Pan et al., (2018) mention in their research, an inappropriate leader selection may result to opposite convergence directions. Two possible conditions of moving direction are demonstrated in Figure 4.1. The red rhombus points represent the non-dominated solutions, and the blue circle point represents a random particle. To make the influence of leader selection clear, the inertia direction of the particle is not considered in the following example. If the leaders are selected according to Figure 4.1a, the potential moving direction of the selected particle lies in the shadow area, which indicates an exploration in the uncultivated direction. Another possible movement is demonstrated in Figure 4.1b, where the parallelogram boxed by the particle, *pBest* and *leader* is in a reserve convergence direction resulting in slower convergence of the algorithm.



(a) Good leading



(b) Bad leading

Figure 4.1: Possibility analysis on particle's moving direction

According to Castro et al., (2018) and Castro et al., (2012) best results were obtained by using the Sigma (Mostaghim & Teich, 2003), the NWSum (Padhye et al., 2009) and the CD (Deb et al., 2002) methods, even though the objective search space still seems to have the greatest impact. In the first method, the Sigma vectors for all the solutions in the repository and all particles, are calculated. Each particle selects the solution with the smallest Sigma vector, considering the Euclidean distance, as its leader (Mostaghim & Teich, 2003). In the second method, particles choose as their leader, solutions with similar objective function values by maximizing a weighted sum of the objective vector. This particular technique is reported to introduce convergence in the algorithm (Padhye et al., 2009). In the third and last technique, for each individual, two different random solutions are selected and the one with the highest CD value is chosen as the final leader (Castro et al., 2018). More details about the selection leader methods mentioned, can be retrieved from the work of Castro et al., (2012).

Even though the methods mentioned above report promising results in the literature, a novel simple leader selection method is now introduced, in the context of this research. The objective of this method is to introduce convergence in the algorithm as well as to help the algorithm extend the non-dominated solutions in each objective.

To select a leader for each particle, a random number is generated in the range of (0,1). Should this random number be smaller than $\frac{1}{nob+1}$, where *nob* is the number of objective functions to be optimized, the particle choses its closest non-dominated solution as a leader, in terms of the Euclidean distance. In the opposite, the particle choses as its leader the closest best solution found so far, out of all the objective functions to be optimized. At this point, it should be mentioned that a particle cannot take as its leader a non-dominated solution from which it will have a zero distance. More details about the distance of two particles can be found in Subsection C.1 of Appendix C.

The Sigma method, the NWSum method, the CD method and the method proposed above (Selection Leader Spreading - SLSPR) are now compared, to adopt a proper leader selection strategy for the MOPLD problem. To make the comparisons, a MOPSO combined with CD was used to detect the pareto front of a product line of bags with a minimum number of two bags and a maximum number of 10 bags, using the Timbuk2 data set. All six objective functions mentioned in Subsection 3.2.4 are used in this comparison. Each method runs for 20 times until 100,000 function evaluations are reached. The leader selection methods were compared according to their average Performance metrics values, as presented in Table 4.3.

Table 4.3: Comparison of different leader selection methods

Leader selection method	M_1^*	Δ	M_3^* (%)
Sigma	62.66	0.70	98.97
NWSum	47.47	0.68	99.15
CD	35.05	**0.66**	99.01
SLSPR	**12.20**	**0.66**	**99.61**

Table 4.3 indicates that MOPSO combined with the proposed SLSPR method achieved the best average performance metric values. As a result, SLSPR is used for the rest of this research to select the leader of particles in MOPSO.

In this research, a fuzzy self-tuning MOPSO using the GT technique (FST-MOPSO-GT), a fuzzy self-tuning MOPSO using the CD technique (FST-MOPSO-CD-SLSPR) and a fuzzy self-tuning MOPSO using the RP technique (FST-MOPSO-RP), are applied. Previous research has already proven high performance results of MOPSO combined with GT, CD and RP (Allmendinger et al., 2008; Coello Coello et al., 2004; Feng et al., 2010; Figueiredo et al., 2016; X. Li, 2003; Raquel & Naval, 2005; Santana et al., 2009; W. Yang et al., 2019; J. Zhang & Li, 2014).

4.3 Firefly Algorithm (FA)

Firefly Algorithm (FA) is a nature-inspired optimization algorithm proposed by X.-S. Yang (2008) for optimizing continuous landscapes, based on the flashing patterns of fireflies. Even though FA is not reported in the literature to be applied to the PLD problem, the facts that it has already been used to advance the algorithmic structure of GA when applied to the PLD problem (Zervoudakis et al., 2020) as well as its reported advantages over PSO when it comes to objective landscape division and ability of dealing with multimodality, make it a promising swarm intelligence-based approach.

In FA algorithm, it is assumed that each firefly is unisex and will be attracted by other ones. The attractiveness is considered to be proportional to the brightness, which is determined according to the objective function landscape. As a result, for two random fireflies, the less bright one will move towards the brighter one, whilst the brighter one will move randomly. The light intensity *I(r)* varies according to the inverse square of a distance, formulated as:

$$I(r) = \frac{l_s}{r^2} \tag{4.5}$$

where l_s is the intensity of the light source. For a fixed light absorption coefficient γ, the light intensity I varies according to the distance r, as:

$$I = I_0 e^{-\gamma r^2} \tag{4.6}$$

where I_0 is the original light intensity determined from the objective landscape. Since the attractiveness of a firefly is proportional to the intensity of light seen by the others, the variation of attractiveness β with the distance r, is defined as:

$$\beta = \beta_0 e^{-\gamma r^2} \tag{4.7}$$

where β_0 is the attractiveness when $r=0$. More details about the distance of two particles can be found in Subsection C.1 of Appendix C.

Considering that firefly i is attracted by firefly j during iteration $t+1$, its position is determined as:

$$x_i^{t+1} = x_i^t + \beta_0 \cdot e^{-\gamma r_{ij}^2} \cdot \left(x_i^t - x_j^t\right) + a_t \cdot \varepsilon_i^t \tag{4.8}$$

where the second term is according to the attraction whilst the third one introduces a random movement. α_t is a randomization parameter and ε_i^t is a vector of random numbers generated by a Gaussian or uniform distribution. If $\beta_0 = 0$, a random walk is performed in contrast to the case of $\gamma = 0$, where it corresponds to a PSO variant (X.-S. Yang, 2008; X.-S. Yang & He, 2013).

Since α_t is used to introduce randomness and thus to control the solutions diversity, it can be reduced over the iterations as:

$$a_t = a_0 \cdot \delta^t, \quad 0 < \delta < 1 \tag{4.9}$$

where α_0 is the randomness scaling factor set by the user, and δ is a number in the range of (0,1), also set by the user. According to X.-S. Yang and He (2013) α_0 must be simulated to balance the local exploitation without jumping too far, in a single step. The variable α_0 should therefore be chosen wisely for each problem encountered, to help the algorithm perform as well as possible.

Regarding the application of FA in multi-objective optimization problems, multi-objective FA (MOFA) was proposed by X.-S. Yang (2013) and continues to be successfully applied and further improved by researchers to this day (Bajaj et al., 2021; G. Chen et al., 2018; Tsai et al., 2014; H. Wang et al., 2018; J. Zhang et al., 2021; L. Zhu et al., 2018).

Just like MOPSO and GAs, all non-dominated solutions are gathered into a repository, which whenever gets full, a diversity controlling operator, as demonstrated in Appendix D, is applied.

4.3.1 *Multi-Objective FA for PLD*

Like MOPSO, to convert the real values to discrete ones, MOFA uses the SPV rule. Moreover, to overcome possible problem-related difficulties, the fuzzy self-tuning method, as presented in Appendix C, is also used.

Similar to MOPSO, to automatically determine the population size, MOFA exploits the heuristic:

$$N = \left\lceil \frac{10+2\sqrt{maximum\ number\ of\ products \cdot total\ number\ of\ attribute\ levels}}{2} \right\rceil \quad (4.10)$$

which sets the value of population size according to the number of dimensions of the search space. The division of the heuristic by the number 2, resulted from the fact that for a relatively large population size, the algorithm complexity is reported to be increasing (X.-S. Yang & He, 2013).

To automatically determine the rest of MOFA's parameters, a fuzzy self-tuning method, as presented in Appendix C, is used. During each iteration, each solution computes its own values for δ and φ, independently, which are then used to calculate the output variables according to the rules reported in Table 4.4. Since the parameters are now calculated according to the distance between two fireflies, the movement of firefly *i* is now altered to:

$$x_i^{t+1} = x_i^t + \beta_i \cdot \left(x_i^t - x_j^t\right) + a_i \cdot \varepsilon_i^t \quad (4.11)$$

because it was considered that the use of γ is no longer needed.

Table 4.4: Fuzzy rules used by MOFA

Rule no.	Parameter	Rule definition
1	β	if (φ is *Better* or δ is *Near*) then (β is *Low)*
2		if (φ is *Same* or δ is *Same*) then (β is *Medium)*
3		if (φ is *Worse* or δ is *Far*) then (β is *High)*
4	*a*	if (φ is *Better* or δ is *Near*) then (*a* is *Low)*
5		if (φ is *Same* or δ is *Same*) then (*a* is *Medium)*
6		if (φ is *Worse* or δ is *Far*) then (*a* is *High)*

In the consequent of these rules, the output variables are the MOFA's parameters, which correspond to the respective settings of each solution. The final numerical value of this output variable is calculated using the Sugeno method (Sugeno, 1985), as presented in equation C.3 of Appendix C, according to Table 4.5.

Table 4.5: Output variables and their defuzzification for MOFA

Output Variable	Low	Medium	High
β	0.1	0.5	0.9
a	0.01	0.05	0.1

A general pseudo-code for the proposed MOFA, is demonstrated in Algorithm 4.3.

Regarding the reported sorting process at the end of each iteration (G. Chen et al., 2018), after the application of several experiments, it was found that MOFA performs better with the sorting process excluded. To make the comparisons, a MOFA combined with CD was used to detect the pareto front of a product line of bags with a minimum number of two bags and a maximum number of 10 bags, using the Timbuk2 data set. All six objective functions mentioned in Subsection 3.2.4 are used in this comparison. Each method runs for 20 times until 100,000 function evaluations are reached. The sorting methods were compared according to their average Performance metrics values, as presented in Table 4.6.

Algorithm 4.3: Multi-objective FA for MOPLD

```
1.  Initialization
2.  Select the number of fireflies nPop
3.  Generate the initial population of fireflies
4.  Evaluate each firefly according to the objective functions
5.  Store the non-dominated vectors found in an external Archive
6.  Main loop
7.  Do until a stopping criterion is met
8.     For i=1:nPop do
9.        For j=1:nPop do
10.         Compute parameters
11.         if firefly j dominates firefly i
12.           move firefly i towards j according to equation 4.11
13.         else
14.           move firefly i randomly by using a continuous mutation operator (Appendix B)
              with a mutation rate of a.
15.         Apply SPV rule
16.         Evaluate according to the objective functions
17.         if the new position dominates the old one
18.           accept the new position
19.         elseif no one dominates the other
20.           if rand>0.5 (rand U(0,1))
21.             accept the new solution
22.    Insert the non-dominated solutions in the external Archive
23.    Apply a diversity controlling operator
24. Return non-dominated solutions
```

Table 4.6: Comparison of MOFA with and without the sorting process

MOFA variant	M_1^*	Δ	M_3^* (%)
With sort	31.54	0.72	96.97
Without sort	**12.35**	**0.71**	**98.31**

In this research, a fuzzy self-tuning MOFA using the GT technique (FST-MOFA-GT), a fuzzy self-tuning MOFA using the CD technique (FST-MOFA-CD) and a fuzzy self-tuning MOFA using the RP technique (FST-MOFA-RP), are applied to address the MOPLD problem. Previous research has already proven high performance results of MOFA combined with CD and RP (G. Chen et al., 2018; H. Wang et al., 2019; L. Zhu et al., 2018).

4.4 Differential Evolution (DE)

The Differential Evolution (DE) algorithm was first introduced by Storn and Price (1997) to optimize continuous landscapes. Similar to GA, it is classified as an EA. Even though DE is a continuous landscape optimizer, various attempts are reported in the literature, regarding its modification and application on binary and combinatorial optimization problems (Ali et al., 2018, 2019; Baioletti et al., 2018; Santucci et al., 2019; Tsafarakis et al., 2020).

Similar to most EAs, in DE, a population of individuals is generated (most of the times randomly to achieve a better diversity), each one representing a potential solution to the problem. Those solutions are then evaluated using the predefined objective function. During DE's iterative process, evolutionary operators like mutation and crossover, are applied.

According to the literature, DE's ability of handling complex objective functions as well as the fact that it is easy to code, since it requires a small amount of control parameters (population size, scale factor, and crossover probability), constitute its main advantages. According to Lampinen and Storn (2004) DE is characterized by good convergence capabilities, and it has a high probability of detecting optimal solutions. It has already been successfully applied to the PLD problem, in the recent research (Tsafarakis et al., 2020).

During DE's iterative process, a mutant vector is created, for each individual. The five most widely used mutation strategies, according to Das et al., (2016) are formulated as:

$$\text{DE/rand/1: } v_i^t = x_{R_1 i}^t + F\left(x_{R_2 i}^t - x_{R_3 i}^t\right) \tag{4.12}$$

$$\text{DE/best/1: } v_i^t = x_{best}^t + F\left(x_{R_1 i}^t - x_{R_2 i}^t\right) \tag{4.13}$$

$$\text{DE/current-to-best/1: } v_i^t = x_i^t + F(x_{best}^t - x_i^t) + F\left(x_{R_1 i}^t - x_{R_2 i}^t\right) \tag{4.14}$$

$$\text{DE/best/2: } v_i^t = x_{best}^t + F\left(x_{R_1 i}^t - x_{R_2 i}^t\right) + F\left(x_{R_3 i}^t - x_{R_4 i}^t\right) \tag{4.15}$$

$$\text{DE/rand/2: } v_i^t = x_{R_1 i}^t + F\left(x_{R_2 i}^t - x_{R_3 i}^t\right) + F\left(x_{R_4 i}^t - x_{R_5 i}^t\right) \tag{4.16}$$

where the vectors x_{R_1} to x_{R_5} are randomly chosen different solution vectors from the current population, which also differ from the current solution vector x_i. F is a positive mutation control parameter, used to scale the difference vectors and x_{best} is the best solution detected so far. It is worth noting that all mutation strategies mentioned above, differ from the genetic mutation operator, described in Subsection B.2.

The *dither* method constitutes one of the many attempts that have been made to improve DE's performance. In this method, F can be either randomized in each generation of the algorithm or for each target vector of the population in a specific generation. According to Price et al., (2005), in this method F is randomized as:

$$F = F_{low} + rand * \left(F_{high} - F_{low}\right) \tag{4.17}$$

where F_{low} and F_{high} are the highest and lowest values of F respectively, and $rand$ is a uniform random number in the range of [0, 1].

According to Tsafarakis et al., (2020) strategies containing the best solution detected so far, like DE/best/1, DE/best/2 and "DE/current-to-best/1," normally are distinguished by fast convergence speed. Consequently, even though they perform fine on unimodal objective functions, they are expected to get stuck at local optima points when performing on multimodal ones. In contrast, the DE/rand/1 strategy is mostly characterized by slow convergence speed and better exploration capability. As a result, it is expected to perform better when addressing multimodal problems compared to the strategies relying on the best solution found so far. The two-difference mutation strategies, like DE/best/2, DE/rand/2 and DE/current-to-best/1, are reported to have a higher performance, compared to the one-difference ones (Qin et al., 2009; Tsafarakis et al., 2020).

Moreover, in the work of Tsafarakis et al., (2020) for PLD, a novel promising mutation strategy was introduced, formulated as:

$$\text{FSTDE: } v_i^t = x_{R_{1}i}^t + F_1\left(x_{R_{2}i}^t - x_{R_{3}i}^t\right) + F_2\left(x_{best}^t - x_{R_{4}i}^t\right) \tag{4.18}$$

According to Tsafarakis et al., (2020) the purpose of this combination is to take advantage of the better exploration capability of DE/rand/1, along with the fast convergence speed of DE/current-to-best/1, depending on the distance of each solution from the best solution found so far, and the solution's improvement with respect to the previous iteration.

After generating the mutation vector, a crossover operator is then in place. Using this operator, which differs from the genetic crossover operator described in Subsection B.1, the mutant vector mixes its elements with the original vector x_i^t to form an offspring vector $u_i^t = \left(u_{i,1}^t, u_{i,2}^t, \dots, u_{i,d}^t\right)$ according to a predefined probability parameter *Cr*. As Das et al. (2016) mention in their research, binomial crossover can be described as:

$$u_{i,j}^t = \begin{cases} v_{i,j}^t, \ if\ j = k\ or\ rand_{i,j} \le Cr \\ x_{i,j}^t, \ otherwise \end{cases} \tag{4.19}$$

where k is randomly selected in $\{1,2,\dots,d\}$, $rand_{i,j}$ is a random number in the range $[0,1]$. The $j = k$ condition, guarantees that at least one component from v_i^t is chosen by u_i^t, to ensure that the new solution is not a duplicate of the original one.

A selection operator is then in place, which determines whether the original vector or the offspring survives for the following generation. This selection operation for minimization problems is described as:

$$x_i^{t+1} = \begin{cases} v_i^t, \ if\ f(v_i^t) \leq f(x_i^t) \\ x_i^t, \ otherwise \end{cases} \tag{4.20}$$

where $f(v_i^t)$ and $f(x_i^t)$ are the function values of v_i^t and x_i^t, respectively.

Regarding the application of DE on multi-objective optimization problems, Multi-Objective Differential Evolution (DEMO) was proposed by Xue et al., (2003) and continues to be successfully applied and further improved by researchers to this day (Cheng et al., 2016; Hancer, 2020; Kaur & Singh, 2020; Liang et al., 2019; Robič & Filipič, 2005; Sreedhar & Rajan M .R, 2013; Tasgetiren et al., 2015; X. Wang & Tang, 2016; K. Yu et al., 2021; Yue et al., 2021).

Just like DE, individuals in DEMO are sharing information. As a result, offspring are generated using the DE operators mentioned above. However, unlike single-objective DE, there is more than one criteria to determine whether the original vector or the offspring survives for the next generation. For that reason, equation 4.20 is altered as:

$$x_i^{t+1} = \begin{cases} v_i^t, \ if\ v_i^t\ dominates\ x_i^t \\ x_i^t, \ otherwise \end{cases} \tag{4.21}$$

All non-dominated solutions are gathered into a limited-sized repository, and a diversity controlling operator as presented in Appendix D, is applied, to ensure diversity.

4.4.1 Multi-Objective DE for PLD

Since DE is another optimization algorithm for continuous optimization, the SPV rule, as presented in Appendix A, is in place for once again, to convert the real values to discrete ones. Moreover, to overcome possible problem-related difficulties, the extended to multi-objective optimization fuzzy self-tuning method of Tsafarakis et al., (2020), as presented in Appendix C, is also used.

Since in this research more than one criteria are optimized, equation 4.18 is extended to multi-objective optimization, as follows:

$$\text{FSTDEMO: } v_i^t = x_{R_1 i}^t + F_1\left(x_{R_2 i}^t - x_{R_3 i}^t\right) + F_2\left(x_{R_{nd}}^t - x_{R_4 i}^t\right) \tag{4.22}$$

where $x_{R_{nd}}^{t}$ correspond to a random non-dominated solution.

To automatically determine the population size, MODE exploits the heuristic

$$N = \lceil 10 + 2\sqrt{maximum\ number\ of\ products\ \cdot\ total\ number\ of\ attribute\ levels} \rceil \quad (4.23)$$

which sets the value of population size according to the number of dimensions of the search space, as suggested by Tsafarakis et al., (2020).

To automatically determine the rest of DEMO's parameters, a fuzzy self-tuning method similar to the one suggested by Tsafarakis et al., (2020) for single-objective optimization, as presented in Appendix C, is used. During each iteration, each solution compute independently their own values for δ and φ, which are used to calculate the output variables, according to the rules reported in Table 4.7.

Table 4.7: Fuzzy rules used by DEMO

Rule no.	Parameter	Rule definition
1	$F_{low,1}$ and $F_{high,1}$	if (δ is *Far*) then ($F_{low,1}$ and $F_{high,1}$ are *Low)*
2		if (φ is *Same* or δ is *Same* or δ is *Near*) then ($F_{low,1}$ and $F_{high,1}$ are *Medium)*
3		if (φ is *Better*) then ($F_{low,1}$ and $F_{high,1}$ are *High)*
4	$F_{low,2}$ and $F_{high,2}$	if (φ is *Better* or δ is *Near*) then ($F_{low,2}$ and $F_{high,2}$ are *Low)*
5		if (φ is *Same* or δ is *Same*) then ($F_{low,2}$ and $F_{high,2}$ are *Medium)*
6		if (δ is *Far*) then ($F_{low,2}$ and $F_{high,2}$ are *High)*
7	*Cr*	if (φ is *Same* or φ is *Better*) then (Cr is *Low)*
8		if (δ is *Same* or δ is *Near*) then (Cr is *Medium)*
9		if (δ is *Far*) then (Cr is *High)*

In the consequent of these rules, the output variables are the DEMO's parameters, which correspond to the respective settings of each solution. The final numerical value of this output variable is calculated using the Sugeno method (Sugeno, 1985) as presented in equation C.3 of Appendix C, according to Table 4.8.

Table 4.8: Output variables and their defuzzification for MODE

Output Variable	Low	Medium	High
F_{low}	0.1	0.4	0.7
F_{high}	0.4	0.7	0.9
Cr	0.01	0.1	0.5

A general pseudo-code for the proposed DEMO, is demonstrated in Algorithm 4.4.

In this research, a fuzzy self-tuning DEMO using the GT technique (FST-DEMO-GT), a fuzzy self-tuning DEMO using the CD technique (FST- DEMO -CD) and a fuzzy self-tuning DEMO using the RP technique (FST- DEMO -RP), are applied. Previous research has already proven high performance results of DEMO combined with GT, CD and RP (Bao et al., 2017; Cheng et al., 2016; Reddy & Dulikravich, 2019; Yue et al., 2021).

Algorithm 4.4: Multi-objective DE for MOPLD

```
Initialization
Determine the number of individuals
Generate the initial population
Evaluate each one according to the objective functions
Store the non-dominated vectors found in an external Archive
Do until a stopping criterion is met
   For every individual:
      Compute parameters
      Compute the new mutant according to Eq. (4.22)
      Apply Crossover according to Eq. (4.19)
      Apply SPV rule
      Evaluate according to the objective functions
      Apply Selection Operator according to Eq. (4.21)
   Insert the non-dominated solutions found in the external Archive
   Apply a diversity controlling operator
Return non-dominated solutions
```

4.5 Grey Wolf Optimizer (GWO)

Grey Wolf Optimizer (GWO) is another popular swarm intelligence algorithm, for solving continuous optimization problems, introduced by Mirjalili et al., (2014). In contrast to various metaheuristics that in order to perform well, the right parameter setting must be provided, GWO is an optimization algorithm, with only a few parameters to adjust, that can be easily used for global optimization (Long et al., 2019). Even though a reported study on applying GWO to the PLD problem was not detected in the literature, its promising results have attracted researchers' attention over the past years (Gupta & Deep, 2020; J. Hu et al., 2021; P. Hu et al., 2020; Komaki & Kayvanfar, 2015; Nadimi-Shahraki et al., 2021).

It is inspired from the social behavior of grey wolf packs when it comes to social hierarchy of leadership and group hunting. Regarding the social hierarchy, the leader of a pack is referred to as the alpha wolf, which is supported by beta wolfs in decision-making and replacement, in case of death or illness. The rest of the hierarchy includes delta and omega wolfs. The best solution in a population corresponds to the alpha wolf (α), the following two best solutions are considered as the beta (β) and the delta (δ) wolfs, respectively. As a result, the remaining solutions in the population are considered as omega wolves (ω). As regards the group hunting process, tracking, chasing, pursuing, encircling, harassing, and attacking the prey, are included (Faris et al., 2017; Mirjalili et al., 2014). Wolves' hunting behavior is modeled by randomly generating a set of grey wolves. The position of the prey is predicted using the alpha, beta, and delta wolves. The potential solutions converge toward the prey, which assists in exploiting the search space, and diverge from the prey, in cases where values of A are between −1 and 1, and larger than 1 or smaller than−1, respectively. This can be modeled by the processes of encircling, hunting, attacking (exploitation) and searching (exploration).

Equations 4.24-4.27 simulate the encircling process. Equation 4.24 represents the vector calculated according to the position vector of the prey and the current position vector of the wolf, whilst using equation 4.25, the next position of a wolf is calculated. Equations 4.26 and 4.27 correspond to the *A* and *C* vectors, respectively.

$$D = \left| C \cdot X_p(t) - X(t) \right| \tag{4.24}$$

$$X(t+1) = X(t) - A \cdot D \tag{4.25}$$

$$C = 2 \cdot r_2 \tag{4.26}$$

$$A = 2a \cdot r_1 - a \tag{4.27}$$

where *t* is an iteration counter, *X* and X_p are the wolf and prey positions, respectively, r_1 and r_2 re generated randomly between 0 and 1, and finally, α is a linearly decreasing vector from two to zero.

By calculating the X_1, X_2 and X_3 values, the position *X* of a wolf at iteration *t+1* is estimated, as:

$$X_1 = X_a - A_1 \cdot |C_1 X_a - X| \tag{4.28}$$

$$X_2 = X_\beta - A_2 \cdot \left| C_2 X_\beta - X \right| \tag{4.29}$$

$$X_3 = X_\delta - A_3 \cdot |C_3 X_\delta - X| \tag{4.30}$$

$$X(t+1) = \frac{X_1 + X_2 + X_3}{3} \tag{4.31}$$

where the X_a, X_β and X_γ correspond to the position vectors of the *alpha*, *beta* and *delta* wolves. A_1 and C_1, A_2 and C_2 as well as A_3 and C_3 correspond to the alpha, beta and delta wolves coefficients, respectively.

According to Qaddoura et al., (2021) by decreasing the α value from two to zero results to a consequent decrease of *A* value from 1 to −1. As a result, wolves move toward the prey to attack it by the time its stops moving, which assists the exploitation capabilities of the algorithm. Finally, grey wolves diverge from the prey when *A* values are less than −1 or larger than 1. Additionally, the random values of *C* between 0 and 2, offer different moves to the wolves. Consequently, both techniques assist the algorithm on avoiding local optima as well as they enhance exploration capabilities of the algorithm.

When it comes on addressing multi-objective optimization problems using GWO, multi-objective GWO (MOGWO) was proposed by Mirjalili et al., (2016) and continues to be successfully applied and further improved by researchers to this day, in order to solve complex real-world multi-objective optimization problems (Al-Tashi et al., 2020; Lu et al., 2016; Y. Yang et al., 2020; Yousri et al., 2020; Zapotecas-Martínez et al., 2019; Z. Zhu & Zhou, 2020).

To apply GWO to multi-objective optimization, two new components must be integrated, similar to the ones used by Coello Coello et al., (2004) for MOPSO. The first one concerns a diversity controlling operator, as presented in Appendix D, for ensuring diversity. The second one concerns a leader selection strategy for choosing *alpha, beta,* and *delta* solutions as the leaders of the hunting process, from the non-dominated archive, to guide the other wolves towards promising regions of the search space. However, since in a multi-objective optimization problem, comparing solutions is not an easy task due to the Pareto optimality, a leader selection mechanism is designed to choose the least crowded regions of the objective landscape. Additional details can be retrieved from the work of Mirjalili et al., (2016).

4.5.1 Multi-Objective GWO for PLD

Since GWO is another metaheuristic for continuous landscapes, the SPV rule, as presented in Appendix A, is used to convert the real values to discrete ones.

Even though MOGWO has only population size, as a parameter to be adjusted, equation 4.4, as proposed to determine the population size of MOPSO in Subsection 4.2.1, is adopted for once again, to make the algorithm completely parameter-independent.

A general pseudo-code for the proposed MOGWO, is shown in Algorithm 4.5.

Regarding the X_a, X_β and X_γ selection, according to Mirjalili et al., (2016), MOGWO uses the same technique that MOPSO uses to select a leader, when combined with GT (Coello Coello et al., 2004), followed by a second criterion to ensure that alfa and beta wolves are selected from the least crowded areas, as described by Mirjalili et al., (2016). For that reason, MOGWO's performance using the technique as suggested by Mirjalili et al., (2016), was compared to MOGWO's performance using the most successful leader selection method according to Subsection 4.2.1.1. To make the comparisons, MOGWO combined with GT was used to detect the pareto front of a product line of bags with a minimum number of two bags and a maximum number of 10 bags, using the Timbuk2 data set. All six objective functions mentioned in

Subsection 3.2.4 were used in this comparison. Each method run for 20 times until 100,000 function evaluations are reached. The leader selection methods were compared according to their average Performance metrics values, as presented in Table 4.9.

Algorithm 4.5: Multi-objective GWO for MOPLD

```
Initialization
Select the number of grey wolves
Generate the initial population of grey wolves
Evaluate each grey wolf according to the objective functions
Store the non-dominated vectors found in an external Archive
Do until a stopping criterion is met
   For every grey wolf:
      Select Xα
      Exclude alpha from the archive temporarily to avoid selecting the same leader
      Select Xβ
      Exclude beta from the archive temporarily to avoid selecting the same leader
      Select Xδ
      Add back alpha and beta to the archive
      Update the position of the current search agent by equations 4.24-4.31
      Apply SPV rule
      Evaluate according to the objective functions
   Find the non-dominated solutions
   Insert the non-dominated solutions found in the external Archive
   Apply a diversity controlling operator
Return non-dominated solutions
```

The results of Table 4.9 demonstrate that MOGWO performs better when combined with SLSPR. For that reason, SLSPR is integrated in MOGWO for the rest of the experiments.

Table 4.9: Comparison of leader selection methods for MOGWO

Leader selection method	M_1^*	Δ	M_3^*
Default	12.98	**1.17**	98.97
SLSPR	**9.65**	1.19	**99.01**

In this research, a MOGWO using the GT technique (MOGWO-GT), a MOGWO using the CD technique (MOGWO-CD) and a MOGWO using the RP technique (MOGWO-RP), are applied. Previous research has already proven high performance results of MOGWO combined with GT and CD (Jangir & Jangir, 2018; Mirjalili et al., 2016).

4.6 Teaching Learning Based Optimization (TLBO)

Teaching Learning Based Optimization (TLBO) is another population-based algorithm, for global optimization problems (R. v. Rao et al., 2011). It is inspired by the influence of an educator on the level of the students in a class. Even though a reported study on applying TLBO to the PLD problem was not detected, the algorithm reports very promising results when addressing other complex optimization problems (Abdel-Basset et al., 2021; Z. Chen et al., 2021; Dong et al., 2021; S. Li et al., 2020; Wei et al., 2021).

According to Abdel-Basset et al., (2021) TLBO is based on two phases. The first phase concerns a class of N individuals, of which the fittest is considered as the teacher (x^*) who tries to enhance the mean value of students during the teaching process. Mathematically, the mean value of the learners in the class is computed as:

$$X_{mean} = \frac{1}{N}\sum_{i-1}^{N} X_i \tag{4.32}$$

where X_i correspond to each student.

During the teaching process, each student is updated as:

$$X_{i,new} = X_i + r \cdot (x^* - T_F \cdot X_{mean}) \tag{4.33}$$

where $X_{i,new}$ indicates the updated solution vector of i student, r is a random number in the range (0,1) and T_F is a 1 or 2 value, expressing the teaching factor. After computing $X_{i,new}$ its objective function value is computed. Should the new position be improved, the student accepts it.

The second phase of TLBO concerns the learner phase. In this phase students are improved by communicating with each other. Each one selects another from the class to discuss, to improve their knowledge, as:

$$X_{i,new} = \begin{cases} X_i + r \cdot \left(X_i - X_j\right), \ if\ f(X_i) < f(X_j) \\ X_i + r \cdot \left(X_j - X_i\right), \ otherwise \end{cases} \tag{4.34}$$

where i and j are two different students, and $f(X_i)$ and $f(X_j)$ correspond to their objective function values, respectively. For once again, after computing $X_{i,new}$ its objective function value is calculated. For once again, should the new position be improved, the student accepts it.

When it comes on addressing multi-objective optimization problems using TLBO, various reported studies on addressing numerous complex multi-objective optimization problems were found in the literature, using multi-objective TLBO (MOTLBO) (Chaves-González et al., 2015; Hajabdollahi et al., 2021; Natarajan et al., 2019; R. V. Rao, 2016; K. Yu et al., 2018; Zou et al., 2013).

To apply MOTLBO to multi-objective optimization problems, a diversity controlling operator, as presented in Appendix D, is integrated for ensuring diversity. Moreover, the centroid of the non-dominated solutions from current archive, is selected as the Mean of the learners, while the concept of dominance is used to decide when a candidate replaces the original solution. Finally, equation 4.34 is altered as:

$$X_{i,new} = \begin{cases} X_i + r \cdot \left(X_i - X_j\right), \ if \ X_i \ dominates \ X_j \\ X_i + r \cdot \left(X_j - X_i\right), \ otherwise \end{cases} \tag{4.35}$$

4.6.1 *Multi-Objective TBLO for PLD*

MOTBLO is another algorithm built to perform in continuous landscapes. As a result, the SPV rule, as presented in Appendix A, is used to convert its real values to discrete ones.

Similar to MOGWO, MOTBLO has only population size, as a parameter, to be adjusted. As a result, equation 4.4, as presented in Subsection 4.2.1, is adopted for once again, to make the algorithm completely parameter-independent.

A general pseudo-code for the proposed MOTBLO, similar to the one presented in the work of Zou et al., (2013), is presented in Algorithm 4.6.

Algorithm 4.6: Multi-objective TBLO for MOPLD

```
Initialization
Select the number of solutions
Generate the initial population
Evaluate each solution
Store the non-dominated vectors found in an external Archive
Do until a stopping criterion is met
    Select the centroid of the current on-dominated solutions as the Mean
    for i=1:N (Teaching Phase)
        Select randomly a non-dominated solution as Teacher
        Generate a trial solution using equation 4.33
        Apply SPV rule
        Evaluate according to the objective functions
        if the new solution dominates the old one
            Accept it
        elseif no one dominates the other
            Select randomly one of them with a probability of 0.5
    for i=1:N (Learning Phase)
        Select randomly an individual out of the population
        Generate a trial solution using equation 4.35
        Apply SPV rule
        Evaluate according to the objective functions
        if the new solution dominates the old one
            Accept it
        elseif no one dominates the other
            Select randomly one of them with a probability of 0.5
    Find the non-dominated solutions
    Insert the non-dominated solutions found in the external Archive
    Apply a diversity controlling operator
Return non-dominated solutions
```

In this research, a MOTLBO using the GT technique (MOTLBO -GT), a MOTLBO using the CD technique (MOTLBO -CD) and a MOTLBO using the RP technique (MOTLBO -RP), are applied. Previous reported research has already proven high performance results of MOTLBO combined with CD (K. Yu et al., 2018; Zou et al., 2013) and a grid-based technique (Patel & Savsani, 2016).

4.7 Mayfly Optimization Algorithm (MA)

Finally, the last optimization algorithm to be used in this research, is the Mayfly optimization Algorithm (MA) (Zervoudakis & Tsafarakis, 2020). MA is a recently developed optimization algorithm which has already been used by various researchers, to address their complex optimization problems (Abd Elaziz et al., 2021; Bhattacharyya et al., 2020; Gao et al., 2020; X. Guo et al., 2021; Liu et al., 2021; Majumdar et al., 2021; Ramasamy & Ravichandran, 2021; Yi et al., 2021; Zhao & Gao, 2020a, 2020b, 2020c).

In this algorithm, the position of each mayfly represents a potential solution to the problem. Initially, a set of female as well as a set of male mayflies, are randomly generated, as $\boldsymbol{x} = (x_1, \dots, x_d)$ for males, and $\boldsymbol{y} = (y_1, \dots, y_d)$ for females. Their performance is then assessed on the problem's objective function *f(***x***)*. Considering x_i^t as the present position of mayfly *i* in the objective space during step *t*, it is altered by summing it with a velocity v_i^{t+1}, similar to the equation 4.1 for PSO.

The velocity v of a male mayfly *i* is computed as:

$$v_{ij}^{t+1} = \begin{cases} g \cdot v_{ij}^t + a_1 e^{-\beta r_p^2}\left(pbest_{ij} - x_{ij}^t\right) + a_2 e^{-\beta r_g^2}\left(gbest_j - x_{ij}^t\right), \ if\ f(gbest) > f(x_i) \\ v_{ij}^t + d \cdot r, \ otherwise \end{cases} \quad (4.36)$$

while for a female mayfly *i*, the velocity is computed as:

$$v_{ij}^{t+1} = \begin{cases} g \cdot v_{ij}^t + a_2 e^{-\beta r_{mf}^2}\left(x_{ij}^t - y_{ij}^t\right), \ if\ f(y_i) > f(x_i) \\ v_{ij}^t + fl \cdot r, \ otherwise \end{cases} \quad (4.37)$$

where v_{ij}^t is the velocity of mayfly *i* in dimension *j = 1,...,n* at time step *t*, a_1 and a_2 are positive attraction constants. $pbest_{ij}$ is the best position mayfly *i* had ever gone and $gbest$ is the overall best solution, found so far. Moreover, β is a fixed visibility coefficient, r_p corresponds to the Cartesian distance between two individuals, which is calculated as presented in Subsection C.1 of Appendix C.

Finally, g is a constant in the range of $(0, 1]$, d is a positive nuptial dance coefficient, r is a random value in the range [-1, 1] and *fl* is a random number.

The mating process between a male and a female mayfly is formulated through the continuous (genetic) crossover operator as:

$$\begin{gathered} offspring1 = L \cdot male + (1 - L) \cdot female \\ offspring2 = L \cdot female + (1 - L) \cdot male \end{gathered} \tag{4.38}$$

where *male* is the male parent and *female* is the female one. L is a random value in the range (0,1). Initial velocities of offspring are set to zero.

Regarding MA's performance on multi-objective optimization problems, Multi-Objective MA (MOMA) was first introduced by Zervoudakis and Tsafarakis (2020), followed by Liu et al., (2021) who proposed a multi-objective version of MA for ensemble forecasting of short-term wind speed as well as Majumdar et al., (2021) who proposed a multi-objective chaotic MA for hydro-thermal-solar-wind scheduling based on available transfer capability problem.

According to Zervoudakis and Tsafarakis (2020), to apply MOMA to multi-objective optimization problems, a repository which maintains the best non-dominated solutions obtained so far, during the optimization process, is integrated. When optimizing more than one objectives, the movement of male mayflies works similarly to their movement when addressing single-objective optimization problems. Since there is no single best solution in multi-objective problems, the *gbest* solution is selected randomly from the repository of non-dominated solutions.

In MOMA, *i* male mayfly moves as:

$$v_{ij}^{t+1} = \begin{cases} g \cdot v_{ij}^t + a_1 e^{-\beta r_p^2} \left(pbest_{ij} - x_{ij}^t\right) + a_2 e^{-\beta r_g^2} \left(gbest_j - x_{ij}^t\right), \ if \ gbest \ dominates \ x_i \\ v_{ij}^t + d \cdot r, \ otherwise \end{cases} \tag{4.39}$$

while for a female mayfly *i*, the velocity is computed as:

$$v_{ij}^{t+1} = \begin{cases} g \cdot v_{ij}^t + a_2 e^{-\beta r_{mf}^2} \left(x_{ij}^t - y_{ij}^t\right), \ if \ male \ dominates \ female \\ v_{ij}^t + fl \cdot r, \ otherwise \end{cases} \tag{4.40}$$

Furthermore, equation (4.38) is then used for the mating process of MOMA, using the personal best position of each mayfly. Finally, a diversity controlling operator, as presented in Appendix D, for ensuring diversity, is applied.

4.7.1 Multi-Objective MA for PLD

MOMA is another optimization algorithm built to perform in continuous landscapes. As a result, the SPV rule, as presented in Appendix A, is used to convert its real values to discrete ones, for once again. Moreover, to overcome possible problem-related difficulties, the fuzzy self-tuning method, as presented in Appendix C, is also used. Finally, a_1 and a_2 were selected to have different values according to the dimension, using a method close to the *dither* method as presented in equation (4.17) of Subsection 4.4, as:

$$a = a_{low} + rand \cdot (a_{high} - a_{low}) \tag{4.41}$$

where a_{low} and a_{high} are the highest and lowest values of a_1 and a_2, and $rand$ is a uniform random number in the range of (0, 1).

Even though Zervoudakis and Tsafarakis (2020) provided MOMA's best parameter settings in their research, the performance of MOMA is not only highly dependent on its parameter settings, but also on the dataset and the objectives to be optimized like all parameter-dependent metaheuristics. For this reason, a fuzzy self-tuning method, as presented in Appendix C is used to overcome this particular difficulty.

Similar to MOFA, to automatically determine the population size, MOMA exploits the heuristic:

$$N = \left\lceil \frac{10+2\sqrt{maximum\ number\ of\ products \cdot total\ number\ of\ attribute\ levels}}{3} \right\rceil \tag{4.42}$$

which sets the value of population size of both males and females according to the number of dimensions of the search space.

Similar to MOPSO, the SLSPR method presented in Subsection 4.2.1.1 is used for selecting the leaders for each male mayfly, while the female mayflies follow their closest males with whom they mate.

To automatically determine the rest of MOMA's parameters a fuzzy self-tuning method, as presented in Appendix C, is used. During each iteration, each solution compute independently their own values for δ and φ, which are used to calculate the output variables according to the rules reported in Table 4.10.

In the consequent of these rules, the output variables are the MOMA's parameters, which correspond to the respective settings of each solution. The final numerical value of this output variable is calculated using the Sugeno method (Sugeno, 1985) as presented in equation C.3 of Appendix C, according to Table 4.11.

Table 4.10: Fuzzy rules used by MOMA

Rule no.	Parameter	Rule definition
1	g	if (φ is *Worse* or δ is *Same*) then (g is *Low)*
2		if (φ is *Same* or δ is *Near*) then (g is *Medium)*
3		if (φ is *Better* or δ is *Far*) then (g is *High)*
4	$a_{low,1}$ and $a_{high,1}$	if (φ is *Worse* or δ is *Far*) then ($a_{low,1}$ and $a_{high,1}$ is *Low)*
5		if (φ is *Same* or δ is *Same* or δ is *Near*) then ($a_{low,1}$ and $a_{high,1}$ is *Medium)*
6		if (φ is *Better*) then ($a_{low,1}$ and $a_{high,1}$ is *High)*
7	$a_{low,2}$ and $a_{high,2}$	if (φ is *Better* or δ is *Near*) then ($a_{low,2}$ and $a_{high,2}$ is *Low)*
8		if (φ is *Same* or δ is *Same*) then ($a_{low,2}$ and $a_{high,2}$ is *Medium)*
9		if (φ is *Worse* or δ is *Far*) then ($a_{low,2}$ and $a_{high,2}$ is *High)*
10	*mutation rate*	if (φ is *Better* or δ is *Near*) then (*mutation rate* is *Low)*
11		if (φ is *Same* or δ is *Same*) then (*mutation rate* is *Medium)*
12		if (φ is *Worse* or δ is *Far*) then (*mutation rate* is *High)*

Table 4.11: Output variables and their defuzzification for MOMA

Output Variable	Low	Medium	High
g	0.1	0.5	1
a_{low}	0.1	0.4	0.6
a_{high}	0.4	0.6	0.9
Mutation rate	0.01	0.05	0.09

Similar to MOFA, since the parameters are now calculated according to the distance between two mayflies, the movement of male mayfly *i* is now calculated as:

$$v_{ij}^{t+1} = \begin{cases} g \cdot v_{ij}^{t} + a_1\left(pbest_{ij} - x_{ij}^{t}\right) + a_2\left(gbest_j - x_{ij}^{t}\right), & if\ gbest\ dominates\ x_i \\ Apply\ mutation \text{ as described in Subsection B.2}, & otherwise \end{cases} \tag{4.43}$$

because it was considered that the use of the exponential term is no longer needed. The same holds for the movement of female mayfly *i* which is now calculated as:

$$v_{ij}^{t+1} = \begin{cases} g \cdot v_{ij}^{t} + a_2\left(x_{ij}^{t} - y_{ij}^{t}\right), & if\ male\ dominates\ female \\ Apply\ mutation \text{ as described in Subsection B.2}, & otherwise \end{cases} \tag{4.44}$$

A general pseudo-code for the proposed MOMA for PLD, similar to the one presented in the work of Zervoudakis and Tsafarakis (2020), is shown in Algorithm 4.7.

In this research, a MOMA using the GT technique (MOMA-GT), a MOMA using the CD technique (MOMA-CD) and a MOMA using the RP technique (MOMA-RP), are applied.

Algorithm 4.7: Multi-objective MA for MOPLD

```
Initialization
Select the number of male and female mayflies
Generate the initial population
Evaluate each solution
Store the non-dominated vectors found in an external Archive
Do until a stopping criterion is met
   For each male and female mayfly
      Calculate parameters
      Update velocities and positions of males and females (equations 4.43-4.44)
      Apply SPV rule
      Evaluate according to the objective functions
      if a new mayfly dominates its personal best
         Replace personal best with the new solution
      elseif no one dominates the other
         Select randomly one of them
   Mate the mayflies using equation (4.38)
   Apply SPV rule
   Evaluate offspring according to the objective functions
   Separate offspring to male and female randomly
   if an offspring dominates its same-sex parent
      Replace parent with the offspring
   elseif no one dominates the other
      Select randomly one of them
   Apply mutation as described in Subsection B.2
   Apply SPV rule
   Evaluate solutions according to the objective functions
   Find the non-dominated solutions
   Insert the non-dominated solutions found in the external Archive
   Apply a diversity controlling operator
Return non-dominated solutions
```

Chapter 5 Results

In this Chapter, the comparison of the multi-objective optimizers mentioned in Chapter 4, is demonstrated, when performing on the MOPLD problem. Five scenarios, as presented in Table 5.1, have been used to compare their performances, using the two datasets mentioned in Subsection 3.2.2.

Table 5.1: Five different scenarios used in this research

Scenarios	Optimization Types	Objectives to be used per scenario	Number of Products
First Scenario	Multi-objective	1. minimize Cost of development 2. maximize Market Share (%)	2 - 10
Second Scenario	Multi-objective	1. maximize Profit 2. minimize Cost of development 3. maximize Market Share (%)	2 - 10
Third Scenario	Many-objective	1. maximize Profit 2. minimize Cost of development 3. maximize Market Share (%) 4. maximize Income	2 - 10
Fourth Scenario	Many-objective	1. maximize Profit 2. minimize Cost of development 3. maximize Market Share (%) 4. maximize Income 5. maximize Buyer’s welfare	2 - 10
Fifth Scenario	Many-objective	1. maximize Profit 2. minimize Cost of development 3. maximize Market Share (%) 4. maximize Income 5. maximize Buyer’s welfare 6. maximize Commonality Index	2 - 10

5.1 First Scenario: Using two objective functions

In this Subsection the performance of the algorithms when using two objective functions is assessed. The two objective functions involved in this comparison, are presented in Table 5.1.

5.1.1 Results on the Timbuk2 data set when using two objectives

Regarding the performance of the comparing algorithms while performing on the Timbuk2 data set when using two objectives, in Figure 5.1, the sets of non-dominated solutions, provided from each algorithm in a random run, are presented.

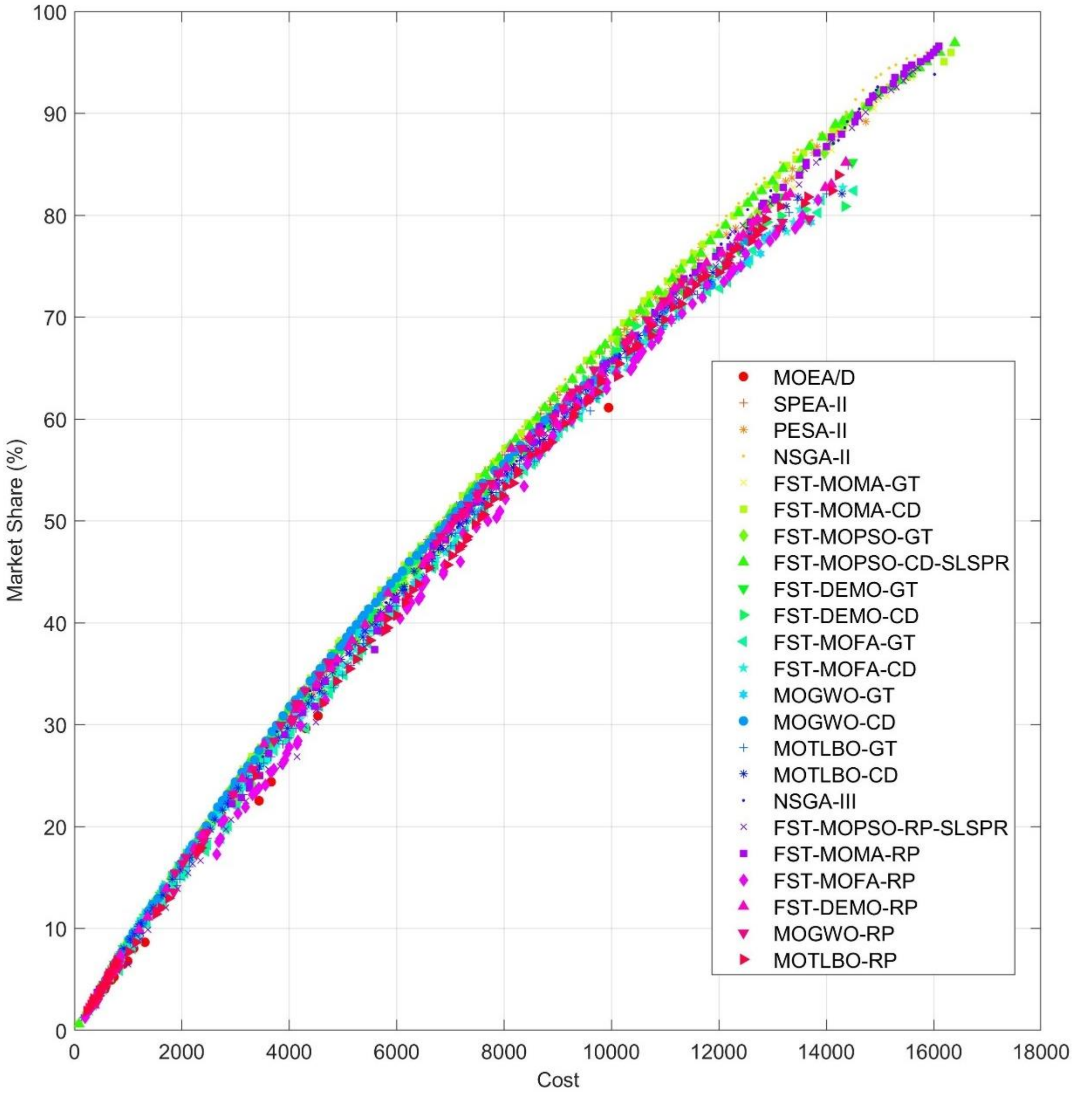


Figure 5.1: Detected non-dominated sets when using two objectives on the Timbuk2 data

Figure 5.1 illustrates that some optimizers like NSGA-II, FST-MOMA-CD and FST-MOPSO-CD appear to converge faster towards the optimal Pareto front.

Tables 5.2 and 5.3 show the results of the average *C* metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U Test.

Table 5.2: Comparison of methods according their average C metric values when using two objectives on the Timbuk2 data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.00	0.01	0.01	0.04	0.02	0.06	0.02	0.01	0.02	0.25	0.10
SPEA-II	0.08	1.00	0.65	0.05	0.77	0.24	0.88	0.41	0.84	0.90	1.00	0.99
PESA-II	0.11	0.07	1.00	0.02	0.49	0.05	0.65	0.09	0.54	0.46	0.93	0.87

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
NSGA-II	0.12	0.37	0.64	1.00	0.77	0.51	0.82	0.63	0.68	0.73	0.86	0.84
FST-MOMA-GT	0.13	0.03	0.15	0.02	1.00	0.02	0.43	0.03	0.37	0.23	0.84	0.76
FST-MOMA-CD	0.12	0.21	0.54	0.07	0.70	1.00	0.77	0.37	0.65	0.69	0.86	0.84
FST-MOPSO-GT	0.10	0.02	0.09	0.01	0.20	0.02	1.00	0.03	0.26	0.17	0.80	0.68
FST-MOPSO-CD-SLSPR	0.12	0.12	0.45	0.06	0.65	0.14	0.73	1.00	0.62	0.63	0.84	0.81
FST-DEMO-GT	0.16	0.01	0.10	0.02	0.17	0.02	0.37	0.02	1.00	0.09	0.92	0.83
FST-DEMO-CD	0.15	0.02	0.20	0.03	0.34	0.04	0.55	0.04	0.47	1.00	0.94	0.92
FST-MOFA-GT	0.06	0.00	0.00	0.00	0.01	0.00	0.02	0.00	0.01	0.00	1.00	0.12
FST-MOFA-CD	0.11	0.00	0.02	0.00	0.03	0.00	0.08	0.01	0.02	0.01	0.60	1.00
MOGWO-GT	0.12	0.01	0.11	0.01	0.16	0.01	0.33	0.02	0.22	0.08	0.85	0.73
MOGWO-CD	0.16	0.07	0.37	0.06	0.45	0.10	0.62	0.15	0.51	0.44	0.92	0.88
MOTLBO-GT	0.11	0.00	0.02	0.01	0.04	0.01	0.09	0.01	0.06	0.02	0.66	0.42
MOTLBO-CD	0.12	0.01	0.05	0.01	0.08	0.02	0.19	0.02	0.12	0.03	0.85	0.73
NSGA-III	0.11	0.02	0.11	0.03	0.27	0.10	0.39	0.11	0.29	0.19	0.76	0.66
FST-MOPSO-RP-SLSPR	0.08	0.01	0.03	0.01	0.12	0.03	0.15	0.04	0.16	0.12	0.58	0.47
FST-MOMA-RP	0.10	0.02	0.07	0.03	0.22	0.06	0.27	0.05	0.27	0.20	0.68	0.57
FST-MOFA-RP	0.11	0.00	0.01	0.01	0.02	0.01	0.06	0.01	0.01	0.01	0.36	0.15
FST-DEMO-RP	0.16	0.01	0.08	0.03	0.15	0.03	0.31	0.02	0.25	0.10	0.90	0.81
MOGWO-RP	0.15	0.01	0.12	0.02	0.19	0.03	0.33	0.02	0.22	0.10	0.80	0.67
MOTLBO-RP	0.11	0.00	0.02	0.02	0.03	0.02	0.07	0.02	0.04	0.03	0.56	0.38

Table 5.3: Comparison of methods according their average C metric values when using two objectives on the Timbuk2 data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.08	0.05	0.14	0.03	0.11	0.11	0.04	0.19	0.03	0.12	0.09
SPEA-II	0.71	0.73	0.98	0.98	0.79	0.93	0.74	0.78	0.82	0.64	0.93
PESA-II	0.46	0.35	0.86	0.76	0.53	0.84	0.56	0.77	0.54	0.42	0.83
NSGA-II	0.58	0.61	0.84	0.81	0.75	0.89	0.74	0.74	0.69	0.53	0.81
FST-MOMA-GT	0.35	0.25	0.75	0.63	0.34	0.67	0.37	0.69	0.36	0.32	0.74
FST-MOMA-CD	0.54	0.53	0.84	0.81	0.68	0.86	0.69	0.73	0.66	0.50	0.80
FST-MOPSO-GT	0.27	0.20	0.68	0.49	0.24	0.59	0.31	0.66	0.26	0.26	0.68
FST-MOPSO-CD-SLSPR	0.51	0.45	0.82	0.78	0.63	0.85	0.66	0.72	0.62	0.46	0.78
FST-DEMO-GT	0.30	0.17	0.74	0.56	0.29	0.61	0.30	0.75	0.28	0.27	0.74
FST-DEMO-CD	0.44	0.29	0.87	0.81	0.47	0.74	0.43	0.79	0.46	0.39	0.82
FST-MOFA-GT	0.04	0.02	0.11	0.01	0.03	0.19	0.04	0.32	0.01	0.05	0.18
FST-MOFA-CD	0.08	0.04	0.26	0.03	0.08	0.32	0.10	0.51	0.03	0.08	0.34
MOGWO-GT	1.00	0.10	0.65	0.46	0.26	0.55	0.28	0.63	0.23	0.20	0.62
MOGWO-CD	0.44	1.00	0.82	0.74	0.54	0.74	0.47	0.74	0.49	0.39	0.77
MOTLBO-GT	0.12	0.07	1.00	0.12	0.09	0.30	0.10	0.56	0.05	0.12	0.40
MOTLBO-CD	0.18	0.12	0.58	1.00	0.16	0.44	0.18	0.71	0.11	0.19	0.62
NSGA-III	0.29	0.22	0.68	0.52	1.00	0.66	0.39	0.64	0.28	0.27	0.67
FST-MOPSO-RP-SLSPR	0.21	0.18	0.48	0.35	0.09	1.00	0.15	0.55	0.17	0.21	0.50
FST-MOMA-RP	0.29	0.25	0.60	0.47	0.19	0.53	1.00	0.64	0.28	0.28	0.63
FST-MOFA-RP	0.07	0.05	0.14	0.02	0.06	0.16	0.05	1.00	0.02	0.08	0.16
FST-DEMO-RP	0.30	0.22	0.75	0.55	0.27	0.56	0.28	0.76	1.00	0.32	0.73
MOGWO-RP	0.22	0.14	0.62	0.43	0.29	0.56	0.28	0.66	0.21	1.00	0.61
MOTLBO-RP	0.13	0.12	0.29	0.12	0.08	0.20	0.07	0.58	0.05	0.15	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.2 demonstrates the violin plots which show the distribution of the time in seconds, each method needed. White circles as well as black squares within these plots, correspond to the median and mean values, respectively.

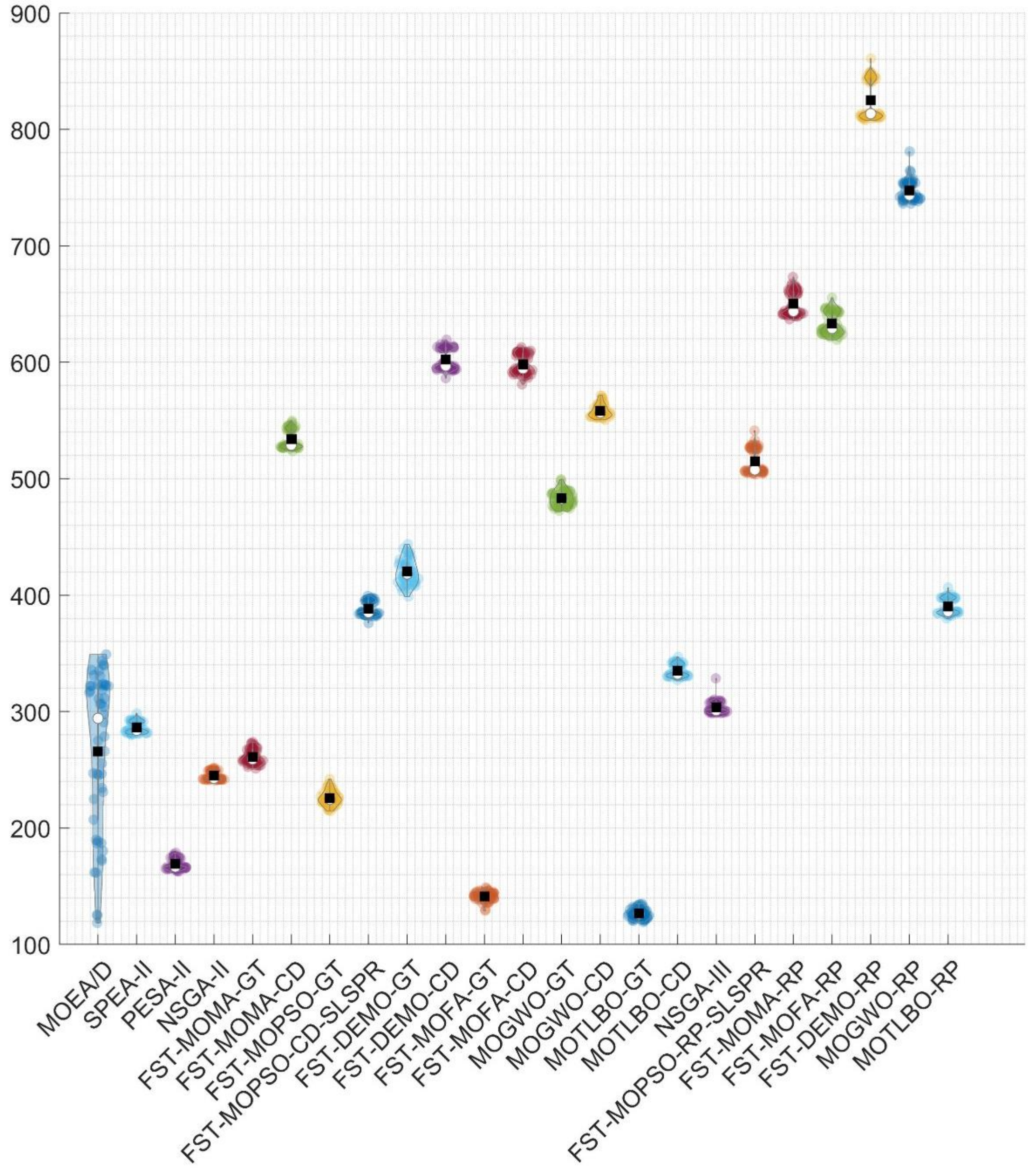


Figure 5.2: Violin plots of *Time* values when using two objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,136, p<0.001, ε^2=0.99. Their Median values along with the rest of statistics are demonstrated in Table 5.4.

Table 5.4: Statistics of *Time* when using two objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	349	266	294	**119**	**230**	**68.10**
SPEA-II	299	286	284	280	19	4.97
PESA-II	179	169	167	163	16	4.69
NSGA-II	252	245	243	241	**11**	**3.69**

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	274	261	259	251	23	6.16
FST-MOMA-CD	550	534	529	524	26	8.24
FST-MOPSO-GT	242	226	225	215	27	6.42
FST-MOPSO-CD-SLSPR	399	388	385	376	23	6.23
FST-DEMO-GT	444	420	418	399	45	10.80
FST-DEMO-CD	619	602	597	586	33	9.19
FST-MOFA-GT	149	141	141	129	20	4.10
FST-MOFA-CD	613	598	595	581	32	8.44
MOGWO-GT	499	483	483	472	27	6.64
MOGWO-CD	572	558	557	551	21	5.47
MOTLBO-GT	**135**	**127**	**126**	**119**	16	4.25
MOTLBO-CD	347	335	332	327	20	5.38
NSGA-III	329	304	301	298	31	5.33
FST-MOPSO-RP-SLSPR	541	515	508	504	37	10.90
FST-MOMA-RP	673	650	644	637	36	10.40
FST-MOFA-RP	655	634	629	619	36	9.65
FST-DEMO-RP	**861**	**825**	**813**	**808**	53	17.20
MOGWO-RP	781	747	744	736	45	8.98
MOTLBO-RP	407	390	386	380	27	6.94

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.5, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.5: Pairwise comparisons of *Time* when using two objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
SPEA-II	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	0	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	0	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.2 and Table 5.4 illustrate that MOTLBO-GT needs the least time, while FST-DEMO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.3 demonstrates the violin plots of the M_1^* values of each method.

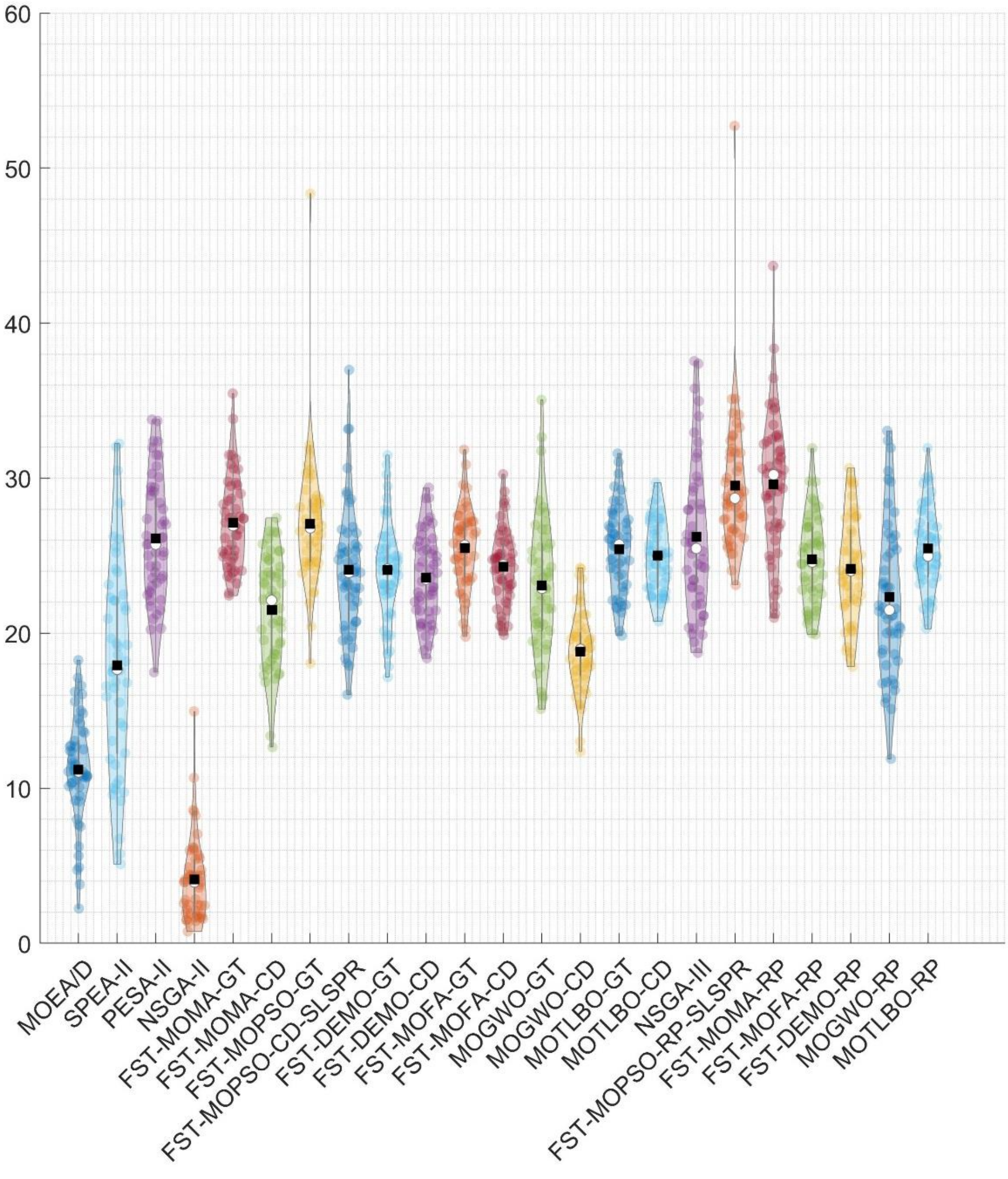


Figure 5.3: Violin plots of M_1^* metric values when using two objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=562$, $p<0.001$, $\varepsilon^2=0.49$. Their Median values along with the rest of statistics are demonstrated in Table 5.6.

Table 5.6: Statistics of M_1^* values when using two objectives on Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	18.3	11.2	11.1	2.2	16.1	3.5
SPEA-II	32.2	17.9	17.7	5.1	27.1	**6.7**
PESA-II	33.8	26.1	25.7	17.5	16.3	3.9
NSGA-II	**15.0**	**4.1**	**4.0**	**0.8**	14.2	2.7

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	35.5	27.1	26.9	22.4	13.1	2.9
FST-MOMA-CD	27.4	21.5	22.1	12.7	14.7	3.5
FST-MOPSO-GT	48.3	27.1	26.8	18.1	**30.2**	4.2
FST-MOPSO-CD-SLSPR	37.0	24.1	23.9	16.0	21.0	4.2
FST-DEMO-GT	31.5	24.1	24.1	17.2	14.3	3.2
FST-DEMO-CD	29.4	23.6	23.5	18.4	11.0	2.8
FST-MOFA-GT	31.8	25.5	25.7	19.8	12.0	2.6
FST-MOFA-CD	30.3	24.3	24.4	19.9	10.4	2.5
MOGWO-GT	35.0	23.1	22.9	15.1	19.9	4.3
MOGWO-CD	24.2	18.8	19.0	12.4	11.8	2.6
MOTLBO-GT	31.6	25.4	25.7	19.8	11.8	2.8
MOTLBO-CD	29.7	25.0	25.0	20.8	**8.9**	**2.2**
NSGA-III	37.5	26.2	25.5	18.7	18.8	4.8
FST-MOPSO-RP-SLSPR	**52.7**	29.5	28.7	**23.1**	29.6	4.5
FST-MOMA-RP	43.7	**29.6**	**30.2**	21.0	22.7	4.5
FST-MOFA-RP	31.9	24.8	24.6	19.9	12.0	2.8
FST-DEMO-RP	30.6	24.1	24.0	17.8	12.8	3.4
MOGWO-RP	33.1	22.3	21.5	11.9	21.2	4.9
MOTLBO-RP	31.9	25.5	25.0	20.3	11.6	2.7

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.7, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.7: Pairwise comparisons of M_1^* when using two objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1
PESA-II	1	1	-	1	0	1	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	1	1	1	0	1	1	1	0	0	0	0	0	1	1	1	0
FST-MOMA-CD	1	0	1	1	1	-	1	0	0	0	1	1	0	1	1	1	1	1	1	1	0	0	1
FST-MOPSO-GT	1	1	0	1	0	1	-	1	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0
FST-MOPSO-CD-SLSPR	1	1	0	1	1	0	1	-	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0
FST-DEMO-GT	1	1	0	1	1	0	1	0	-	0	0	0	0	1	0	0	0	1	1	0	0	0	0
FST-DEMO-CD	1	1	0	1	1	0	1	0	0	-	0	0	0	1	0	0	0	1	1	0	0	0	0
FST-MOFA-GT	1	1	0	1	0	1	0	0	0	0	-	0	0	1	0	0	0	1	1	0	0	1	0
FST-MOFA-CD	1	1	0	1	1	1	1	0	0	0	0	-	0	1	0	0	0	1	1	0	0	0	0
MOGWO-GT	1	1	0	1	1	0	1	0	0	0	0	0	-	1	0	0	0	1	1	0	0	0	0
MOGWO-CD	1	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	0	1	0	1	0	0	0	0	0	0	0	1	-	0	0	1	1	0	0	1	0
MOTLBO-CD	1	1	0	1	0	1	0	0	0	0	0	0	0	1	0	-	0	1	1	0	0	0	0
NSGA-III	1	1	0	1	0	1	0	0	0	0	0	0	0	1	0	0	-	1	0	0	0	1	0
FST-MOPSO-RP-SLSPR	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	0	-	1	1	1	1
FST-MOFA-RP	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	1	-	0	0	0
FST-DEMO-RP	1	1	0	1	1	0	1	0	0	0	0	0	0	1	0	0	0	1	1	0	-	0	0
MOGWO-RP	1	0	1	1	1	0	1	0	0	0	1	0	0	1	1	0	1	1	1	0	0	-	1
MOTLBO-RP	1	1	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	-

Figure 5.3 and Table 5.6 illustrate that NSGA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOMA-RP provides the farthest. Table 5.7 reveals that FST-

MOMA-RP does not have statistically significant differences with FST-MOMA-GT, FST-MOPSO-GT, NSGA-III and FST-MOPSO-RP-SLSPR.

As regards the Δ values of each algorithm, Figure 5.4 demonstrates the violin plots of the Δ metric values, of each method.

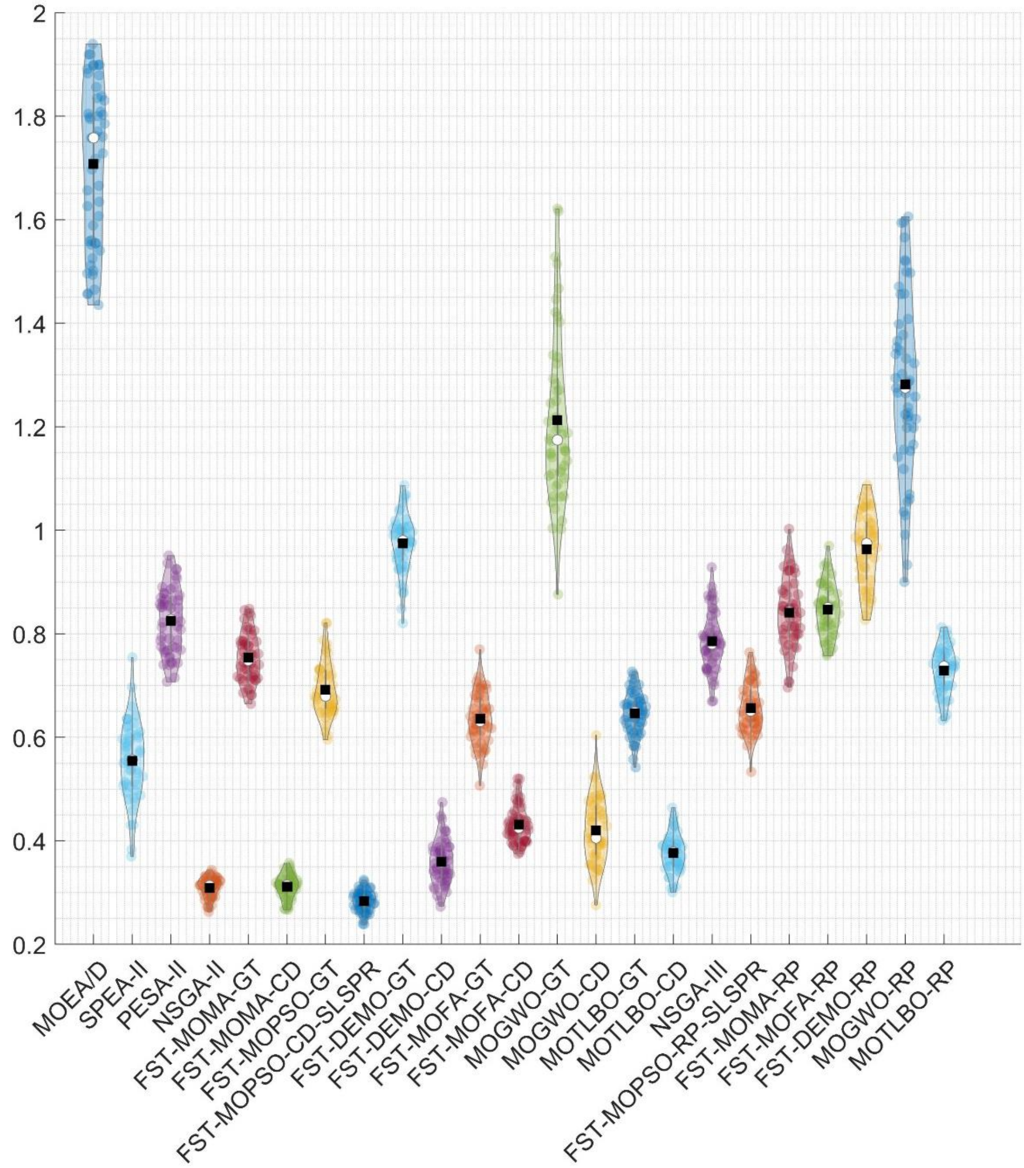


Figure 5.4: Violin plots of Δ metric values when using two objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)$=1,103, p<0.001, ε^2=0.96. Their Median values along with the rest of statistics are demonstrated in Table 5.8.

Table 5.8: Statistics of Δ metric values when using two objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	**1.94**	**1.71**	**1.76**	**1.44**	0.50	0.16
SPEA-II	0.75	0.55	0.56	0.37	0.38	0.08
PESA-II	0.95	0.83	0.83	0.71	0.24	0.06
NSGA-II	0.34	0.31	0.31	0.26	**0.08**	**0.02**

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	0.85	0.75	0.75	0.67	0.18	0.05
FST-MOMA-CD	0.36	0.31	0.32	0.27	0.09	0.02
FST-MOPSO-GT	0.82	0.69	0.68	0.60	0.23	0.05
FST-MOPSO-CD-SLSPR	**0.32**	**0.28**	**0.28**	**0.24**	0.09	0.02
FST-DEMO-GT	1.09	0.97	0.98	0.82	0.27	0.06
FST-DEMO-CD	0.48	0.36	0.36	0.27	0.20	0.04
FST-MOFA-GT	0.77	0.64	0.63	0.51	0.26	0.05
FST-MOFA-CD	0.52	0.43	0.43	0.38	0.15	0.04
MOGWO-GT	1.62	1.21	1.17	0.88	**0.75**	0.16
MOGWO-CD	0.60	0.42	0.41	0.28	0.33	0.06
MOTLBO-GT	0.73	0.65	0.65	0.54	0.19	0.04
MOTLBO-CD	0.46	0.38	0.38	0.30	0.16	0.04
NSGA-III	0.93	0.79	0.78	0.67	0.26	0.06
FST-MOPSO-RP-SLSPR	0.76	0.66	0.65	0.53	0.23	0.05
FST-MOMA-RP	1.00	0.84	0.84	0.70	0.31	0.07
FST-MOFA-RP	0.97	0.85	0.85	0.76	0.21	0.05
FST-DEMO-RP	1.09	0.96	0.98	0.83	0.26	0.07
MOGWO-RP	1.61	1.28	1.28	0.90	0.71	**0.18**
MOTLBO-RP	0.81	0.73	0.74	0.63	0.18	0.04

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.9, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.9: Pairwise comparisons of *Δ* metric when using two objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1
NSGA-II	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0
FST-MOMA-CD	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	0	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	1	0	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1
FST-MOFA-RP	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.4 and Table 5.8 illustrate that FST-MOPSO-CD-SLSPR provides the best distribution among the non-dominated solutions, while MOEA/D provides the worst.

Finally, as regards the M_3^* values of each algorithm, Figure 5.5 demonstrates the violin plots of the M_3^* values of each method.

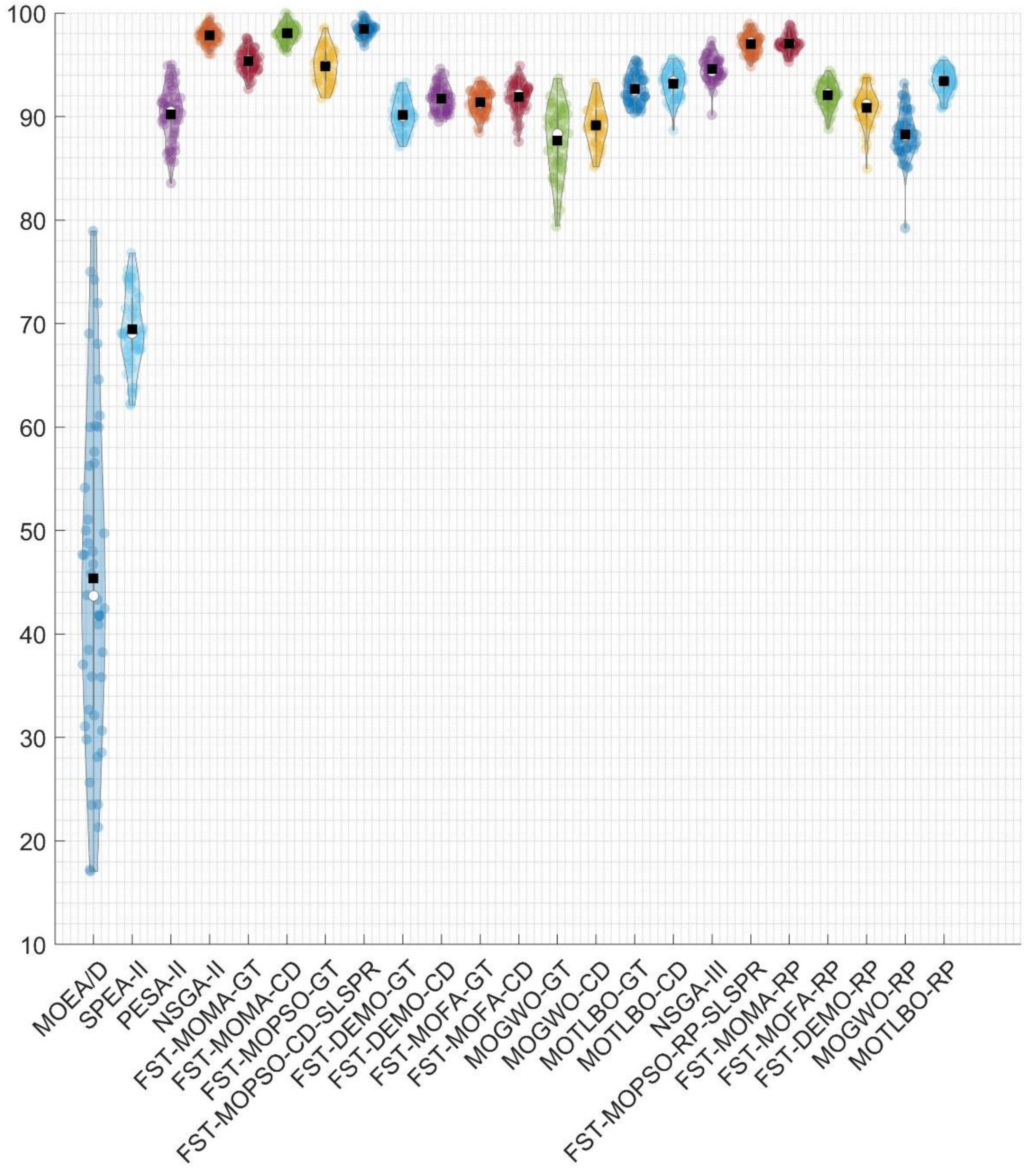


Figure 5.5: Violin plots of M_3^* metric values when using two objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=994.69, p<0.001, ε^2=0.87. Their Median values along with the rest of statistics are demonstrated in Table 5.10.

Table 5.10: Statistics of M_3^* metric values when using two objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	78.92	**45.38**	**43.69**	**17.05**	**61.87**	**15.63**
SPEA-II	**76.80**	69.42	69.07	62.12	14.69	3.70
PESA-II	95.00	90.21	90.46	83.56	11.43	2.68
NSGA-II	99.59	97.85	97.85	96.03	3.56	0.78

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	97.60	95.35	95.41	92.66	4.93	1.12
FST-MOMA-CD	**100.00**	98.05	98.04	96.24	3.76	0.86
FST-MOPSO-GT	98.57	94.88	94.93	91.77	6.80	1.68
FST-MOPSO-CD-SLSPR	99.78	**98.44**	**98.48**	**96.83**	**2.95**	**0.62**
FST-DEMO-GT	93.27	90.16	90.01	87.12	6.15	1.57
FST-DEMO-CD	94.61	91.72	91.73	89.49	5.12	1.24
FST-MOFA-GT	93.45	91.39	91.31	88.47	4.98	1.15
FST-MOFA-CD	94.89	91.88	92.12	87.57	7.32	1.47
MOGWO-GT	93.69	87.66	88.33	79.39	14.30	3.42
MOGWO-CD	93.24	89.18	89.13	85.15	8.09	1.99
MOTLBO-GT	95.45	92.66	92.51	90.33	5.12	1.46
MOTLBO-CD	95.59	93.17	93.40	88.65	6.94	1.47
NSGA-III	97.30	94.58	94.40	90.16	7.14	1.24
FST-MOPSO-RP-SLSPR	98.96	97.01	97.16	94.85	4.10	0.93
FST-MOMA-RP	98.87	97.04	97.04	95.29	3.57	0.80
FST-MOFA-RP	94.42	92.04	92.20	88.80	5.62	1.23
FST-DEMO-RP	93.77	90.86	91.18	84.96	8.81	1.78
MOGWO-RP	93.18	88.27	88.17	79.22	13.96	2.36
MOTLBO-RP	95.43	93.41	93.46	90.81	4.62	1.17

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.11, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.11: Pairwise comparisons of M_3^* metric when using two objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1	1	0	0	1
NSGA-II	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	0	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	0	1	1	1	1	1	-	1	1	1	1	0	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	1	0	1	1	1	1	1	1	-	0	0	1	1	0	1	1	1	1	0	0	1	1
FST-MOFA-GT	1	1	0	1	1	1	1	1	1	0	-	0	1	1	1	1	1	1	1	0	0	1	1
FST-MOFA-CD	1	1	0	1	1	1	1	1	1	0	0	-	1	1	0	1	1	1	1	0	0	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	0	1	1	1	1	1	0	1	1	1	0	-	1	1	1	1	1	1	1	0	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	0	1	0	1	1	-	0	1	1	1	0	1	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	0
NSGA-III	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	-

Figure 5.5 and Table 5.10 illustrate that FST-MOPSO-CD-SLSPR provides the best extend of the non-dominated solutions, while MOEA/D provides the worst. Table 5.11 reveals that FST-MOPSO-CD-SLSPR does not have statistically significant differences with FST-MOMA-CD.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.6.

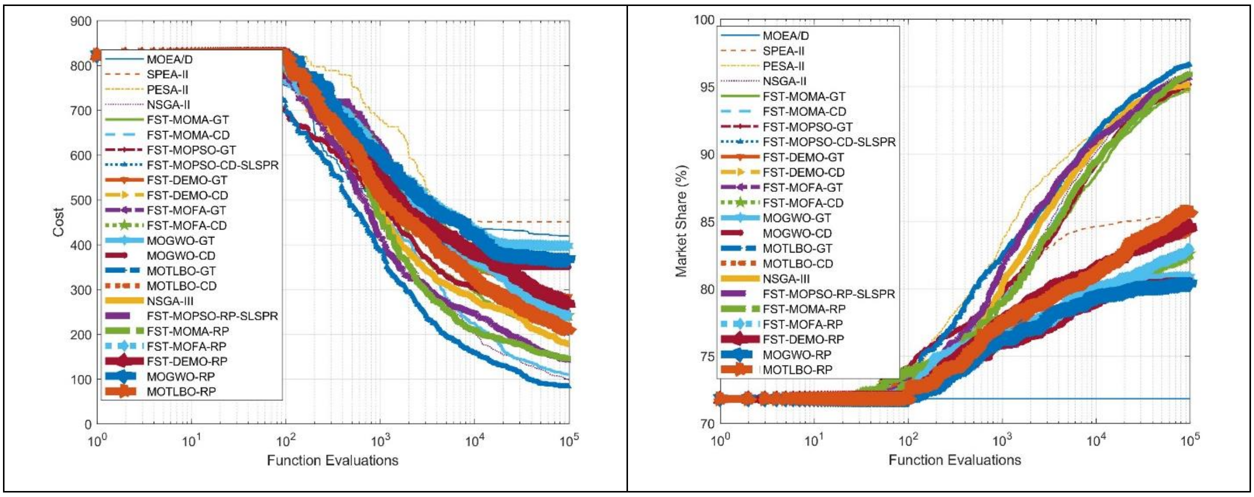


Figure 5.6: Average convergence characteristic curves when using two objectives on the Timbuk2 data

5.1.2 Results on the Olive oil data set when using two objectives

Regarding the performance of the comparing algorithms while performing on the olive oil data set when using two objectives, in Figure 5.7, the sets of non-dominated solutions, provided from each algorithm in a random run, are presented.

Figure 5.7 illustrates that some optimizers like NSGA-II, FST-MOMA-CD, FST-MOMA-RP and FST-MOPSO-CD appear to converge faster towards the optimal Pareto front.

Tables 5.12 and 5.13 show the results of the average C metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between C(A,B) and C(B,A), according to a Mann Whitney U Test.

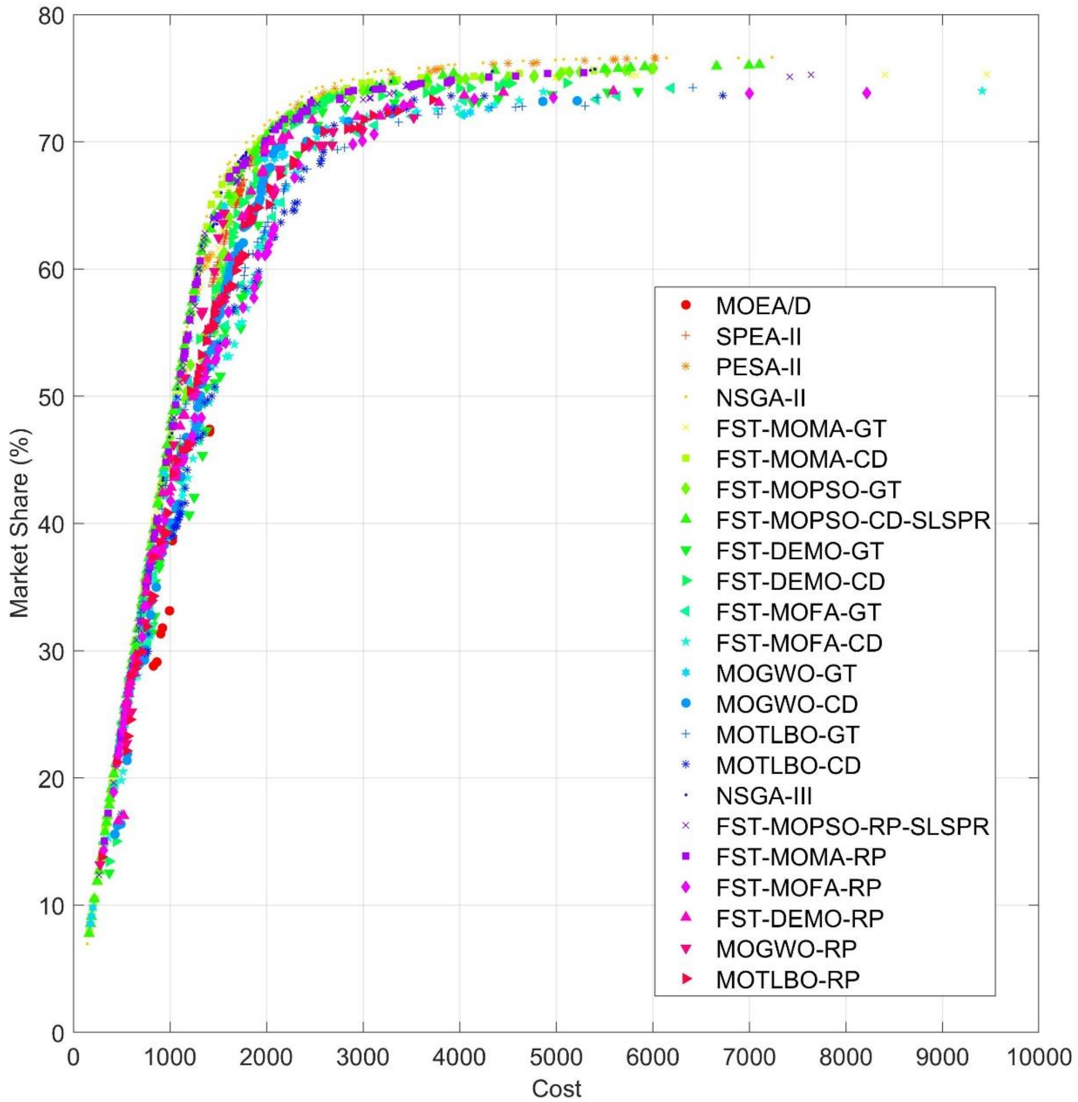


Figure 5.7: Detected non-dominated sets when using two objectives on the Olive oil data

Table 5.12: Comparison of methods according their average C metric values when using two objectives on the Olive oil data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.04	0.05
SPEA-II	0.68	1.00	0.17	0.00	0.11	0.01	0.23	0.01	0.86	0.94	1.00	1.00
PESA-II	0.33	0.14	1.00	0.08	0.45	0.38	0.48	0.37	0.68	0.71	0.99	0.99
NSGA-II	0.38	0.26	0.54	1.00	0.61	0.53	0.64	0.57	0.75	0.77	1.00	1.00
FST-MOMA-GT	0.39	0.19	0.15	0.02	1.00	0.12	0.34	0.08	0.71	0.72	1.00	1.00
FST-MOMA-CD	0.44	0.28	0.32	0.09	0.46	1.00	0.53	0.28	0.78	0.79	1.00	1.00
FST-MOPSO-GT	0.37	0.17	0.11	0.02	0.20	0.10	1.00	0.06	0.71	0.71	1.00	1.00
FST-MOPSO-CD-SLSPR	0.42	0.26	0.29	0.07	0.47	0.29	0.55	1.00	0.77	0.78	1.00	1.00
FST-DEMO-GT	0.37	0.03	0.01	0.00	0.02	0.00	0.04	0.00	1.00	0.32	0.93	0.93
FST-DEMO-CD	0.41	0.04	0.01	0.00	0.03	0.01	0.04	0.01	0.41	1.00	0.96	0.94
FST-MOFA-GT	0.25	0.00	0.01	0.00	0.01	0.00	0.02	0.00	0.11	0.10	1.00	0.31
FST-MOFA-CD	0.25	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.11	0.09	0.31	1.00
MOGWO-GT	0.44	0.14	0.08	0.01	0.13	0.08	0.17	0.07	0.57	0.57	0.95	0.95

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOGWO-CD	0.51	0.12	0.10	0.02	0.11	0.05	0.14	0.04	0.62	0.62	0.98	0.97
MOTLBO-GT	0.33	0.02	0.01	0.00	0.01	0.00	0.01	0.00	0.18	0.15	0.61	0.58
MOTLBO-CD	0.30	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.18	0.16	0.56	0.55
NSGA-III	0.48	0.36	0.46	0.02	0.57	0.43	0.68	0.46	0.88	0.89	1.00	1.00
FST-MOPSO-RP-SLSPR	0.45	0.30	0.26	0.01	0.37	0.13	0.47	0.14	0.81	0.83	1.00	1.00
FST-MOMA-RP	0.36	0.22	0.22	0.01	0.37	0.17	0.46	0.16	0.79	0.78	1.00	1.00
FST-MOFA-RP	0.26	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.12	0.10	0.32	0.33
FST-DEMO-RP	0.39	0.04	0.00	0.00	0.03	0.00	0.02	0.00	0.41	0.37	0.95	0.97
MOGWO-RP	0.49	0.17	0.09	0.00	0.15	0.09	0.20	0.07	0.65	0.68	0.99	0.99
MOTLBO-RP	0.36	0.02	0.00	0.00	0.01	0.00	0.01	0.00	0.19	0.18	0.64	0.64

Table 5.13: Comparison of methods according their average C metric values when using two objectives on the Olive oil data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.04	0.00	0.03	0.02	0.01	0.00	0.00	0.06	0.02	0.03	0.02
SPEA-II	0.52	0.72	0.97	0.98	0.04	0.08	0.10	1.00	0.88	0.47	0.98
PESA-II	0.50	0.59	0.92	0.96	0.11	0.40	0.41	0.99	0.67	0.50	0.93
NSGA-II	0.58	0.72	0.95	0.96	0.38	0.58	0.58	1.00	0.73	0.59	0.95
FST-MOMA-GT	0.49	0.59	0.94	0.96	0.06	0.20	0.22	0.99	0.70	0.51	0.94
FST-MOMA-CD	0.54	0.72	0.95	0.96	0.18	0.40	0.41	0.99	0.75	0.55	0.95
FST-MOPSO-GT	0.49	0.58	0.92	0.96	0.05	0.18	0.18	0.99	0.69	0.49	0.94
FST-MOPSO-CD-SLSPR	0.55	0.70	0.95	0.96	0.16	0.40	0.41	1.00	0.76	0.55	0.95
FST-DEMO-GT	0.32	0.38	0.73	0.75	0.01	0.02	0.02	0.93	0.33	0.33	0.73
FST-DEMO-CD	0.29	0.36	0.78	0.75	0.01	0.03	0.03	0.93	0.38	0.29	0.73
FST-MOFA-GT	0.09	0.10	0.21	0.22	0.01	0.01	0.01	0.29	0.11	0.09	0.20
FST-MOFA-CD	0.10	0.10	0.21	0.22	0.01	0.01	0.01	0.29	0.12	0.10	0.21
MOGWO-GT	1.00	0.40	0.84	0.85	0.05	0.12	0.12	0.94	0.56	0.36	0.83
MOGWO-CD	0.28	1.00	0.87	0.87	0.06	0.09	0.13	0.97	0.61	0.26	0.86
MOTLBO-GT	0.13	0.17	1.00	0.41	0.01	0.01	0.02	0.57	0.17	0.13	0.35
MOTLBO-CD	0.14	0.16	0.38	1.00	0.01	0.01	0.01	0.56	0.18	0.11	0.36
NSGA-III	0.72	0.79	0.98	0.98	1.00	0.57	0.57	1.00	0.88	0.73	0.98
FST-MOPSO-RP-SLSPR	0.65	0.73	0.97	0.99	0.08	1.00	0.28	1.00	0.81	0.65	0.98
FST-MOMA-RP	0.61	0.70	0.96	0.97	0.06	0.33	1.00	0.99	0.76	0.60	0.96
FST-MOFA-RP	0.10	0.11	0.21	0.22	0.01	0.01	0.02	1.00	0.11	0.11	0.21
FST-DEMO-RP	0.33	0.39	0.80	0.78	0.00	0.02	0.02	0.95	1.00	0.30	0.77
MOGWO-RP	0.44	0.54	0.91	0.93	0.02	0.11	0.11	0.99	0.66	1.00	0.90
MOTLBO-RP	0.13	0.16	0.45	0.45	0.00	0.01	0.01	0.64	0.19	0.13	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.8 demonstrates the violin plots which show the distribution of time each method needed.

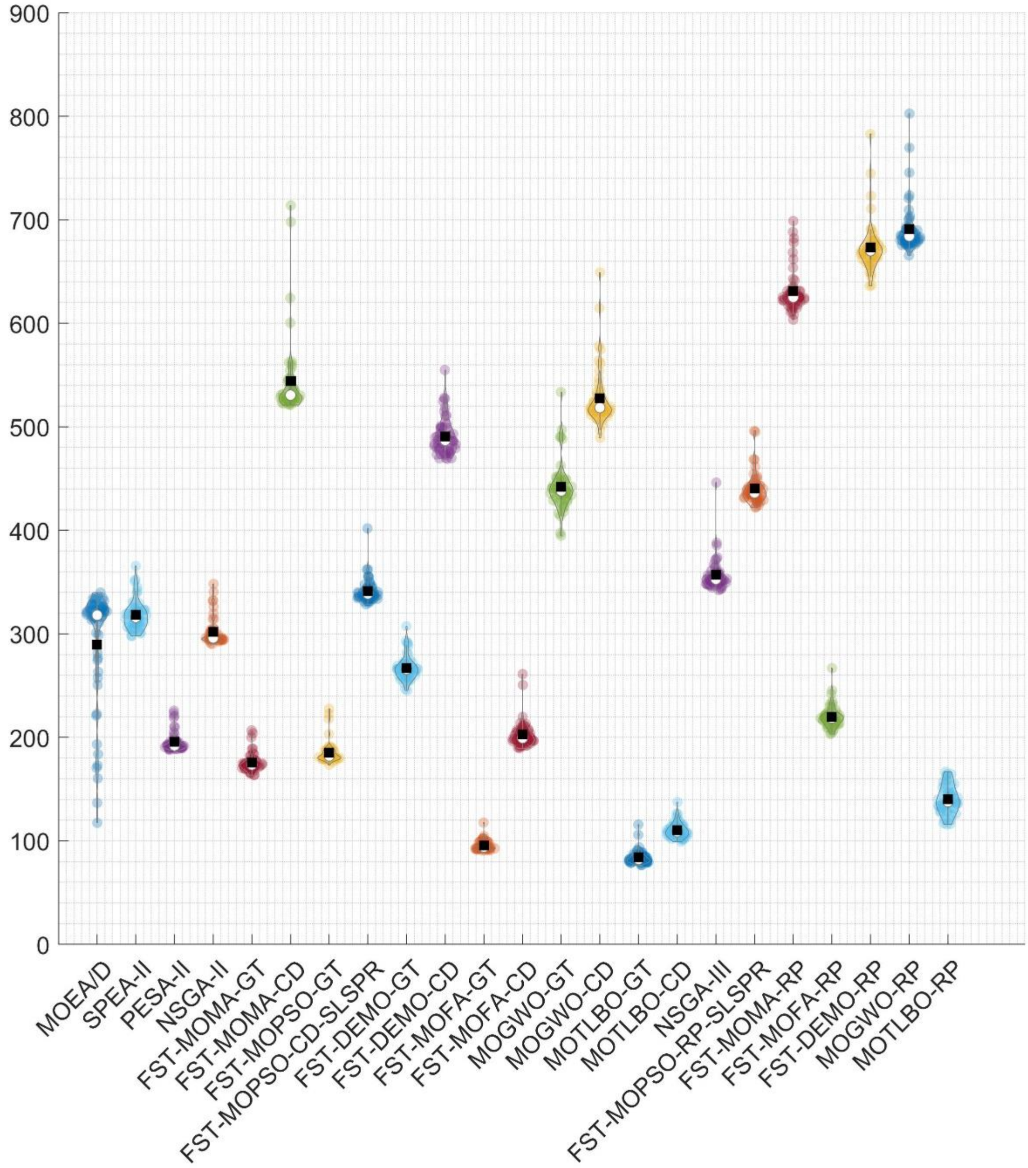


Figure 5.8: Violin plots of *Time* values when using two objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the time needed, $H(22)$=1,129.97, p<0.001, ε^2=0.98. Their Median values along with the rest of statistics are demonstrated in Table 5.14.

Table 5.14: Statistics of *Time* when using two objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	339.9	289.3	318.0	117.5	**222.4**	**59.1**
SPEA-II	365.5	318.5	315.9	298.0	67.5	14.4
PESA-II	225.9	195.8	192.1	187.9	38.0	9.3
NSGA-II	348.2	302.1	296.1	290.2	58.0	13.0
FST-MOMA-GT	206.7	175.6	173.6	163.4	43.3	9.1
FST-MOMA-CD	713.9	544.0	530.6	521.0	192.9	38.6
FST-MOPSO-GT	227.7	185.4	182.1	173.4	54.2	10.8
FST-MOPSO-CD-SLSPR	401.9	341.5	338.7	328.6	73.4	11.7
FST-DEMO-GT	307.3	267.0	265.7	245.4	62.0	11.9
FST-DEMO-CD	554.9	490.8	486.9	469.0	85.8	17.8
FST-MOFA-GT	117.7	95.8	94.1	90.6	**27.1**	**4.8**

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-CD	261.2	203.0	199.4	189.5	71.7	13.0
MOGWO-GT	533.6	441.9	438.2	394.8	138.8	23.9
MOGWO-CD	649.1	527.8	518.5	489.4	159.7	28.2
MOTLBO-GT	**115.8**	**84.1**	**82.2**	**76.5**	39.3	6.8
MOTLBO-CD	137.5	110.3	108.9	99.2	38.3	7.5
NSGA-III	446.1	357.1	352.9	342.2	103.8	16.1
FST-MOPSO-RP-SLSPR	496.1	440.5	436.2	422.0	74.2	15.3
FST-MOMA-RP	698.8	631.2	625.2	603.6	95.3	20.4
FST-MOFA-RP	266.9	220.1	218.7	203.0	63.9	10.9
FST-DEMO-RP	782.8	673.3	670.1	636.1	146.7	24.5
MOGWO-RP	**802.6**	**691.2**	**684.3**	**665.5**	137.1	24.3
MOTLBO-RP	166.7	140.0	137.7	115.9	50.8	13.7

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.15, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.15: Pairwise comparisons of *Time* when using two objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	0	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	0	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.8 and Table 5.14 illustrate that MOTLBO-GT needs the least time, while MOGWO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.9 demonstrates the violin plots of the M_1^* values of each method.

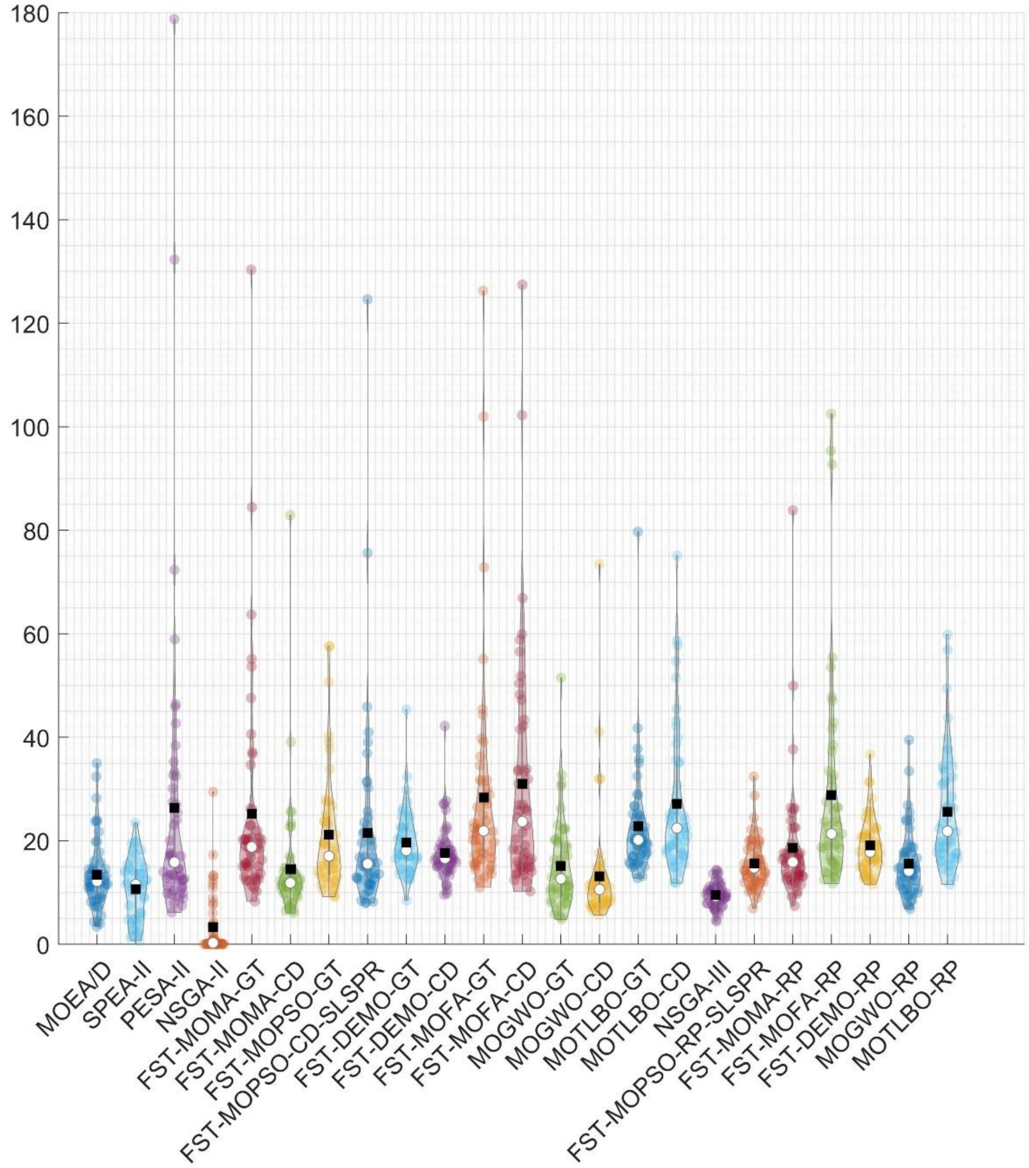


Figure 5.9: Violin plots of M_1^* metric values when using two objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=469.58$, $p<0.001$, $\varepsilon^2=0.41$. Their Median values along with the rest of statistics are demonstrated in Table 5.16.

Table 5.16: Statistics of M_1^* values when using two objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	35.0	13.5	12.2	3.4	31.6	6.8
SPEA-II	23.5	10.7	11.6	0.7	22.8	5.8
PESA-II	**178.8**	26.3	15.9	6.2	**172.6**	**30.4**
NSGA-II	29.5	**3.3**	**0.3**	**0.0**	29.5	5.9
FST-MOMA-GT	130.3	25.2	18.8	8.3	122.0	21.3
FST-MOMA-CD	82.9	14.5	11.9	6.1	76.9	11.5
FST-MOPSO-GT	57.6	21.2	17.1	9.2	48.4	11.5
FST-MOPSO-CD-SLSPR	124.6	21.5	15.6	8.0	116.6	19.1
FST-DEMO-GT	45.4	19.6	18.2	8.5	36.9	6.1
FST-DEMO-CD	42.2	17.7	16.6	9.6	32.6	5.4
FST-MOFA-GT	126.3	28.4	21.9	11.0	115.3	21.3

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-CD	127.4	**31.0**	**23.7**	10.3	117.2	22.9
MOGWO-GT	51.5	15.2	12.7	4.8	46.7	8.6
MOGWO-CD	73.5	13.1	10.6	5.7	67.8	10.9
MOTLBO-GT	79.7	22.8	20.2	**12.8**	66.9	10.4
MOTLBO-CD	75.1	27.2	22.5	11.8	63.3	13.9
NSGA-III	**14.4**	9.5	9.3	4.4	**10.0**	**2.3**
FST-MOPSO-RP-SLSPR	32.4	15.7	14.8	7.0	25.5	4.8
FST-MOMA-RP	83.9	18.7	15.9	7.4	76.4	11.8
FST-MOFA-RP	102.5	28.9	21.4	11.8	90.7	20.5
FST-DEMO-RP	36.7	19.1	17.9	11.6	25.1	5.4
MOGWO-RP	39.5	15.6	14.2	6.8	32.7	6.5
MOTLBO-RP	59.8	25.6	21.9	11.6	48.3	11.1

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.17, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.17: Pairwise comparisons of M_1^* when using two objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	0	1	1	0	1	0	1	1	1	1	0	0	1	1	0	0	0	1	1	0	1
SPEA-II	0	-	1	1	1	0	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	0	1
PESA-II	0	1	-	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0
FST-MOMA-CD	0	0	0	1	1	-	1	1	1	1	1	1	0	0	1	1	1	0	1	1	1	0	1
FST-MOPSO-GT	1	1	0	1	0	1	-	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
FST-MOPSO-CD-SLSPR	0	1	0	1	0	1	0	-	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1
FST-DEMO-GT	1	1	0	1	0	1	0	0	-	0	0	0	1	1	0	0	1	1	0	0	0	1	0
FST-DEMO-CD	1	1	0	1	0	1	0	0	0	-	1	0	0	1	1	1	1	0	0	1	0	0	1
FST-MOFA-GT	1	1	0	1	0	1	0	0	0	1	-	0	1	1	0	0	1	1	1	0	0	1	0
FST-MOFA-CD	1	1	0	1	0	1	0	0	0	0	0	-	1	1	0	0	1	1	1	0	0	1	0
MOGWO-GT	0	0	0	1	1	0	0	0	1	0	1	1	-	0	1	1	1	0	0	1	1	0	1
MOGWO-CD	0	0	1	1	1	0	1	1	1	1	1	1	0	-	1	1	0	1	1	1	1	1	1
MOTLBO-GT	1	1	0	1	0	1	0	0	0	1	0	0	1	1	-	0	1	1	1	0	0	1	0
MOTLBO-CD	1	1	0	1	0	1	0	1	0	1	0	0	1	1	0	-	1	1	1	0	0	1	0
NSGA-III	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	0	1	0	1	0	0	0	0	1	0	1	1	0	1	1	1	1	-	0	1	0	0	1
FST-MOMA-RP	0	1	0	1	0	1	0	0	0	0	1	1	0	1	1	1	1	0	-	1	0	0	1
FST-MOFA-RP	1	1	0	1	0	1	0	0	0	1	0	0	1	1	0	0	1	1	1	-	0	1	0
FST-DEMO-RP	1	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	-	1	0
MOGWO-RP	0	0	0	1	0	0	0	0	1	0	1	1	0	1	1	1	1	0	0	1	1	-	1
MOTLBO-RP	1	1	0	1	0	1	0	1	0	1	0	0	1	1	0	0	1	1	1	0	0	1	-

Figure 5.9 and Table 5.16 illustrate that NSGA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-CD provides the farthest ones. Table 5.17 reveals that FST-MOFA-CD does not have statistically significant differences with PESA-II, FST-MOMA-GT, FST-MOPSO-GT, FST-MOPSO-CD-SLSPR, FST-DEMO-GT, FST-DEMO-CD,

FST-MOFA-GT, MOTLBO-GT, MOTLBO-CD, FST-MOFA-RP, FST-DEMO-RP, MOTLBO-RP.

As regards the *Δ* values of each algorithm, Figure 5.10 demonstrates the violin plots of the *Δ* metric values, of each method.

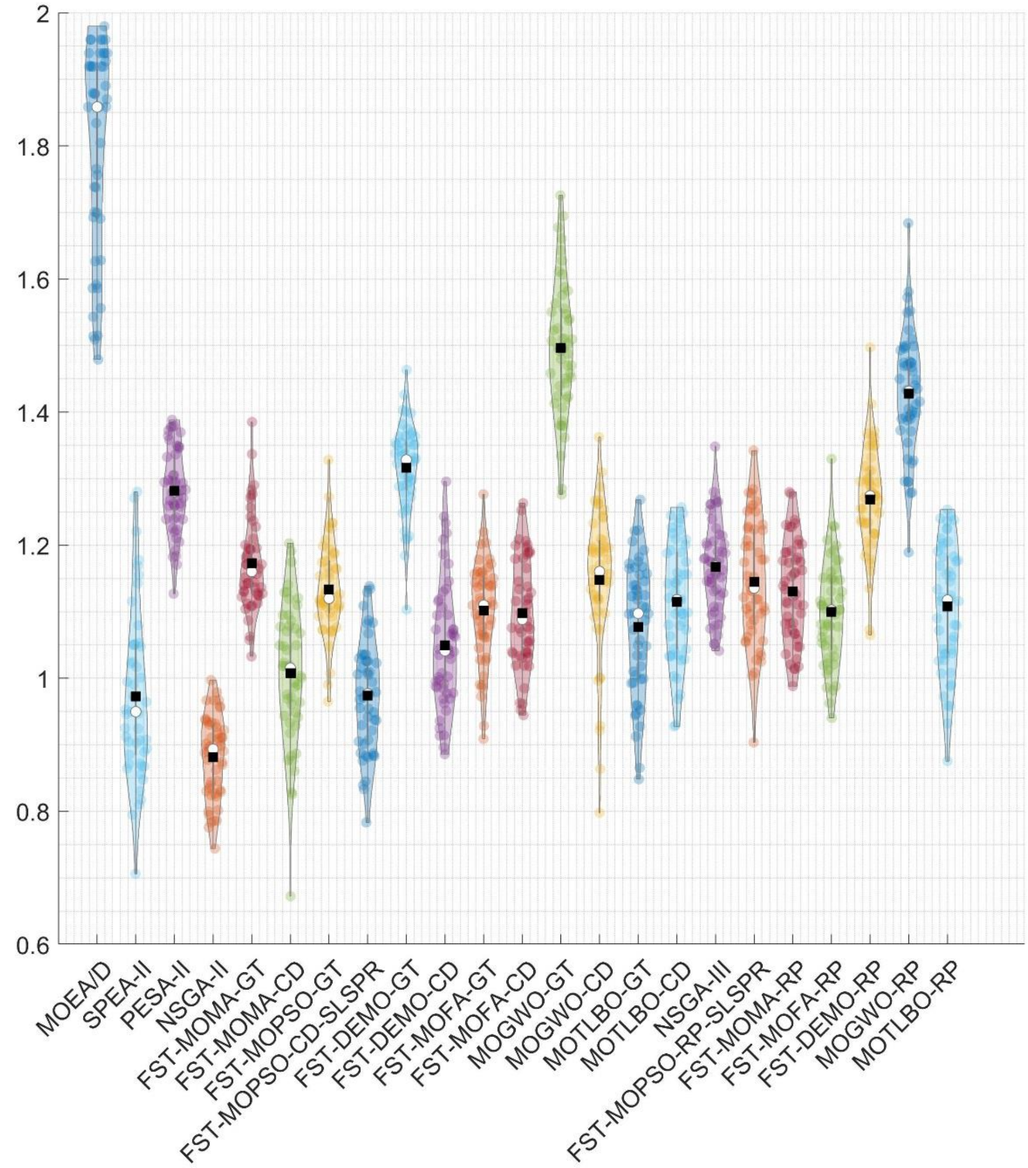


Figure 5.10: Violin plots of *Δ* metric values when using two objectives on the Olive oil data

A Kruskal-Wallis test revealed that the optimizer selection significantly affects the distribution of the solutions, $H(22)=809.37$, $p<0.001$, $\varepsilon^2=0.71$. Their Median values along with the rest of statistics are demonstrated in Table 5.18.

Table 5.18: Statistics of *Δ* metric values when using two objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	**1.98**	**1.80**	**1.86**	**1.48**	0.50	**0.16**
SPEA-II	1.28	0.97	0.95	0.71	**0.57**	0.13
PESA-II	1.39	1.28	1.28	1.13	0.26	**0.06**
NSGA-II	**1.00**	**0.88**	**0.89**	0.74	**0.25**	**0.06**

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	1.39	1.17	1.16	1.03	0.35	0.07
FST-MOMA-CD	1.20	1.01	1.02	**0.67**	0.53	0.10
FST-MOPSO-GT	1.33	1.13	1.12	0.96	0.36	0.07
FST-MOPSO-CD-SLSPR	1.14	0.97	0.98	0.78	0.36	0.08
FST-DEMO-GT	1.46	1.32	1.33	1.10	0.36	0.07
FST-DEMO-CD	1.30	1.05	1.04	0.89	0.41	0.10
FST-MOFA-GT	1.28	1.10	1.11	0.91	0.37	0.07
FST-MOFA-CD	1.26	1.10	1.09	0.94	0.32	0.08
MOGWO-GT	1.73	1.50	1.50	1.28	0.45	0.10
MOGWO-CD	1.36	1.15	1.16	0.80	0.56	0.11
MOTLBO-GT	1.27	1.08	1.10	0.85	0.42	0.10
MOTLBO-CD	1.26	1.11	1.12	0.93	0.33	0.09
NSGA-III	1.35	1.17	1.17	1.04	0.31	0.07
FST-MOPSO-RP-SLSPR	1.34	1.15	1.14	0.90	0.44	0.09
FST-MOMA-RP	1.28	1.13	1.13	0.99	0.29	0.07
FST-MOFA-RP	1.33	1.10	1.10	0.94	0.39	0.08
FST-DEMO-RP	1.50	1.27	1.27	1.06	0.43	0.08
MOGWO-RP	1.68	1.43	1.43	1.19	0.49	0.09
MOTLBO-RP	1.25	1.11	1.12	0.88	0.38	0.09

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.19, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.19: Pairwise comparisons of *Δ* metric when using two objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	0	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	0
FST-MOMA-CD	1	0	1	1	1	-	1	0	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	0	1	-	1	1	1	0	0	1	0	0	0	0	0	0	0	1	1	0
FST-MOPSO-CD-SLSPR	1	0	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	0	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	1	1	1	1	0	1	1	1	-	0	0	1	1	0	0	1	1	1	0	1	1	0
FST-MOFA-GT	1	1	1	1	1	1	0	1	1	0	-	0	1	0	0	0	1	0	0	0	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	0	1	1	0	0	-	1	0	0	0	1	0	0	0	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	0	1	0	1	1	1	0	0	1	-	0	0	0	0	0	0	1	1	0
MOTLBO-GT	1	1	1	1	1	0	0	1	1	0	0	0	1	0	-	0	1	0	0	0	1	1	0
MOTLBO-CD	1	1	1	1	0	1	0	1	1	0	0	0	1	0	0	-	0	0	0	0	1	1	0
NSGA-III	1	1	1	1	0	1	0	1	1	1	1	1	1	0	1	0	-	0	0	1	1	1	0
FST-MOPSO-RP-SLSPR	1	1	1	1	0	1	0	1	1	1	0	0	1	0	0	0	0	-	0	0	1	1	0
FST-MOMA-RP	1	1	1	1	0	1	0	1	1	1	0	0	1	0	0	0	0	0	-	0	1	1	0
FST-MOFA-RP	1	1	1	1	1	1	0	1	1	0	0	0	1	0	0	0	1	0	0	-	1	1	0
FST-DEMO-RP	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	1	-

Figure 5.10 and Table 5.18 illustrate that NSGA-II provides the best distribution among the non-dominated solutions, while MOEA/D provides the worst.

Finally, as regards the M_3^* values of each algorithm, Figure 5.11 demonstrates the violin plots of the M_3^* values of each method.

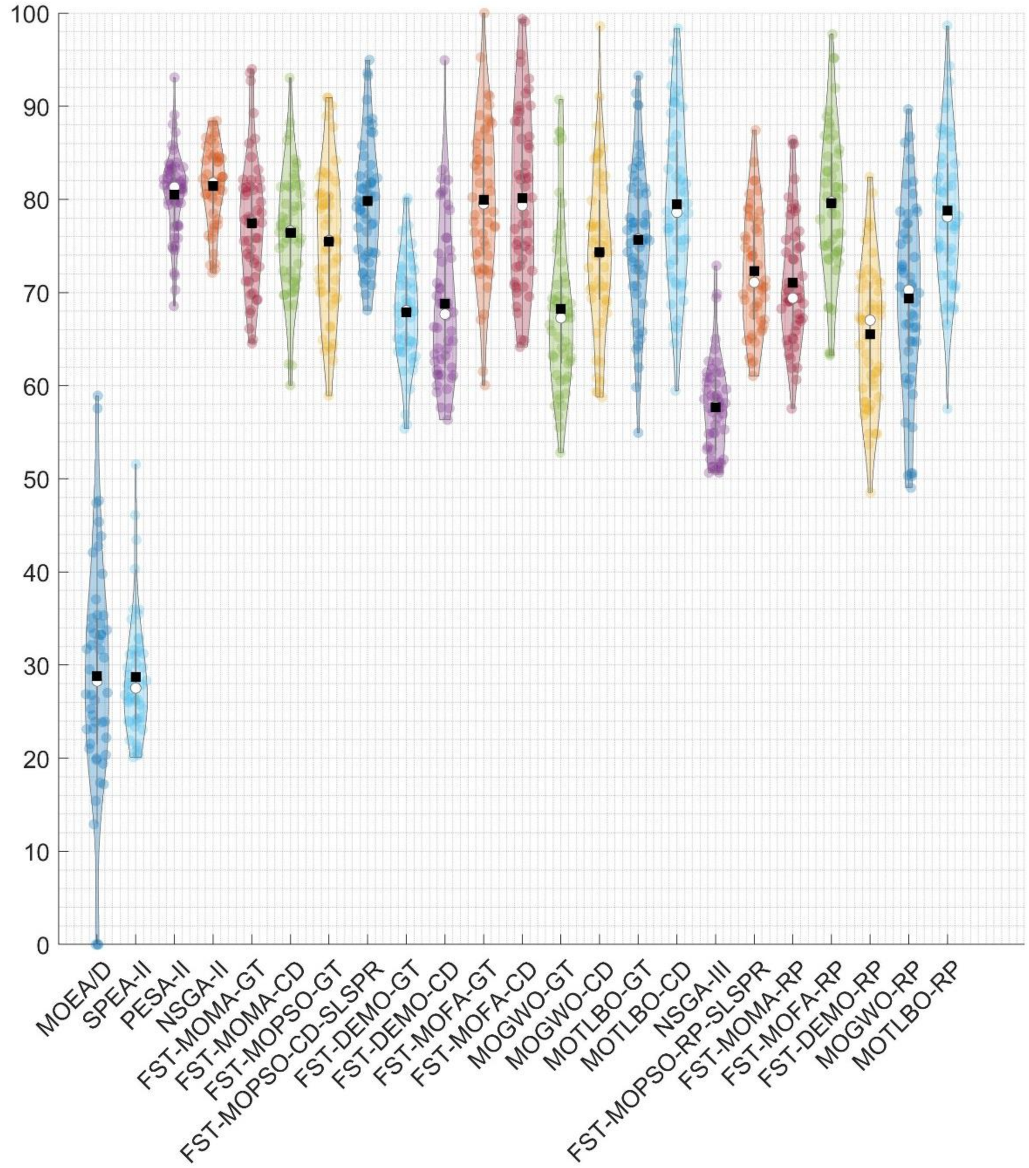


Figure 5.11: Violin plots of M_3^* metric values when using two objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=606.62, p<0.001, ε^2=0.53. Their Median values along with the rest of statistics, are demonstrated in Table 5.20.

Table 5.20: Statistics of M_3^* metric values when using two objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	58.90	28.82	28.27	**0.00**	**58.90**	**12.50**
SPEA-II	**51.55**	**28.74**	**27.53**	20.10	31.45	6.59
PESA-II	93.09	80.47	81.32	68.55	24.54	4.74
NSGA-II	88.41	**81.43**	**81.79**	**72.13**	**16.28**	**4.30**

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	93.97	77.45	77.50	64.55	29.41	7.01
FST-MOMA-CD	93.05	76.40	76.59	60.04	33.01	6.70
FST-MOPSO-GT	90.92	75.48	75.57	58.93	31.99	7.93
FST-MOPSO-CD-SLSPR	94.96	79.81	79.82	68.09	26.88	6.52
FST-DEMO-GT	80.12	67.89	68.06	55.38	24.74	5.65
FST-DEMO-CD	94.93	68.77	67.67	56.32	38.60	8.32
FST-MOFA-GT	**100.00**	79.94	79.51	60.04	39.96	8.25
FST-MOFA-CD	99.37	80.14	79.34	64.16	35.21	9.34
MOGWO-GT	90.70	68.23	67.27	52.81	37.89	8.96
MOGWO-CD	98.58	74.30	74.29	58.76	39.82	8.62
MOTLBO-GT	93.27	75.64	75.73	54.93	38.34	8.20
MOTLBO-CD	98.35	79.47	78.60	59.45	38.90	9.00
NSGA-III	72.88	57.70	57.78	50.65	22.23	5.16
FST-MOPSO-RP-SLSPR	87.42	72.30	71.09	61.03	26.39	6.31
FST-MOMA-RP	86.41	71.08	69.40	57.54	28.87	6.79
FST-MOFA-RP	97.72	79.60	79.75	63.24	34.48	8.00
FST-DEMO-RP	82.41	65.51	67.02	48.52	33.88	7.67
MOGWO-RP	89.66	69.36	70.27	49.03	40.63	10.17
MOTLBO-RP	98.64	78.78	78.13	57.52	41.12	8.03

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.21, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.21: Pairwise comparisons of M_3^* metric when using two objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	0	0	0	1	0	1	1	0	0	1	1	0	0	1	1	1	0	1	1	0
NSGA-II	1	1	0	-	0	1	1	0	1	1	0	0	1	1	1	0	1	1	1	0	1	1	0
FST-MOMA-GT	1	1	0	0	-	0	0	0	1	1	0	0	1	0	0	0	1	0	1	0	1	1	0
FST-MOMA-CD	1	1	0	1	0	-	0	0	1	1	0	0	1	0	0	0	1	0	1	0	1	0	0
FST-MOPSO-GT	1	1	1	1	0	0	-	0	1	1	0	0	1	0	0	0	1	0	0	0	1	0	0
FST-MOPSO-CD-SLSPR	1	1	0	0	0	0	0	-	1	1	0	0	1	0	0	0	1	1	1	0	1	1	0
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	0	1	1	0	1	1	1	1	0	0	1	0	0	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	0	-	1	1	0	0	1	1	1	0	0	1	0	0	1
FST-MOFA-GT	1	1	0	0	0	0	0	0	1	1	-	0	1	0	0	0	1	1	1	0	1	1	0
FST-MOFA-CD	1	1	0	0	0	0	0	0	1	1	0	-	1	0	0	0	1	1	1	0	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	0	0	1	1	-	0	1	1	1	0	0	1	0	0	1
MOGWO-CD	1	1	1	1	0	0	0	0	1	0	0	0	0	-	0	0	1	0	0	0	1	0	0
MOTLBO-GT	1	1	0	1	0	0	0	0	1	1	0	0	1	0	-	0	1	0	0	0	1	0	0
MOTLBO-CD	1	1	0	0	0	0	0	0	1	1	0	0	1	0	0	-	1	1	1	0	1	1	0
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	1	1	-	0	1	1	0	1
FST-MOMA-RP	1	1	1	1	1	1	0	1	0	0	1	1	0	0	0	1	1	0	-	1	0	0	1
FST-MOFA-RP	1	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	-	0	1
MOGWO-RP	1	1	1	1	1	0	0	1	0	0	1	1	0	0	0	1	1	0	0	1	0	-	1
MOTLBO-RP	1	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	1	1	0	1	1	-

Figure 5.11 and Table 5.20 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while SPEA-II provides the worst. Table 5.21 reveals that SPEA-II does not have statistically significant differences with MOEA/D, as well as that NSGA-II does not have statistically significant differences with PESA-II, FST-MOMA-GT, FST-MOPSO-CD-SLSPR, FST-MOFA-GT, FST-MOFA-CD, MOTLBO-CD, FST-MOFA-RP and MOTLBO-RP.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.12.

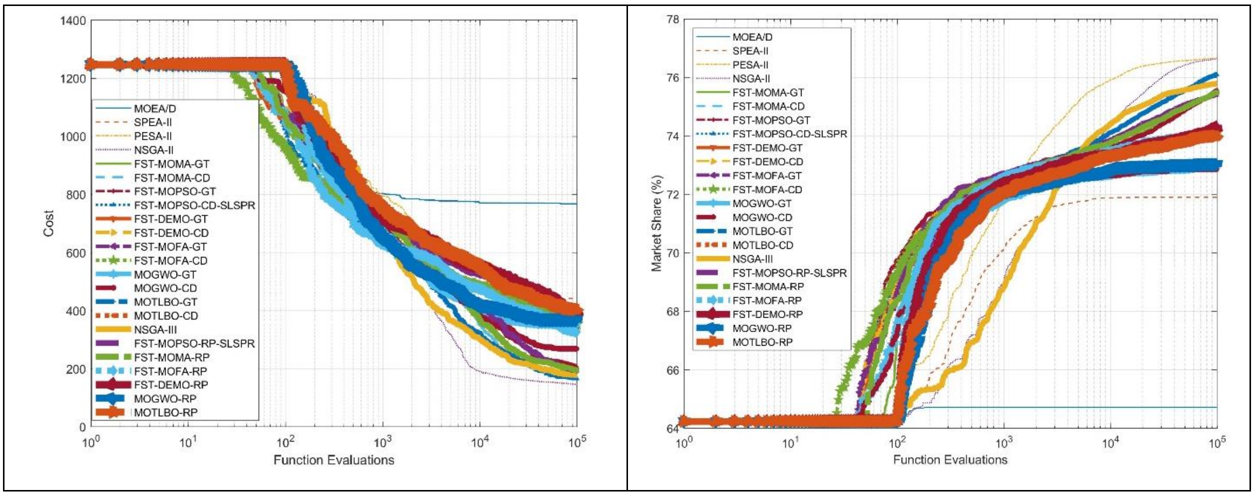


Figure 5.12: Average convergence characteristic curves when using two objectives on the Olive oil data

5.2 Second Scenario: Using three objective functions

In this Subsection the performance of the algorithms when using three objective functions is assessed. Three objective functions are involved in the first comparison, as presented in Table 5.1.

5.2.1 Results on the Timbuk2 data set when using three objectives

Regarding the performance of the comparing algorithms while performing on the Timbuk2 data set when using three objectives, in Figure 5.13, the sets of non-dominated solutions, provided from each algorithm in a random run, are presented.

Tables 5.22-5.23 show the results of the average *C* metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U test.

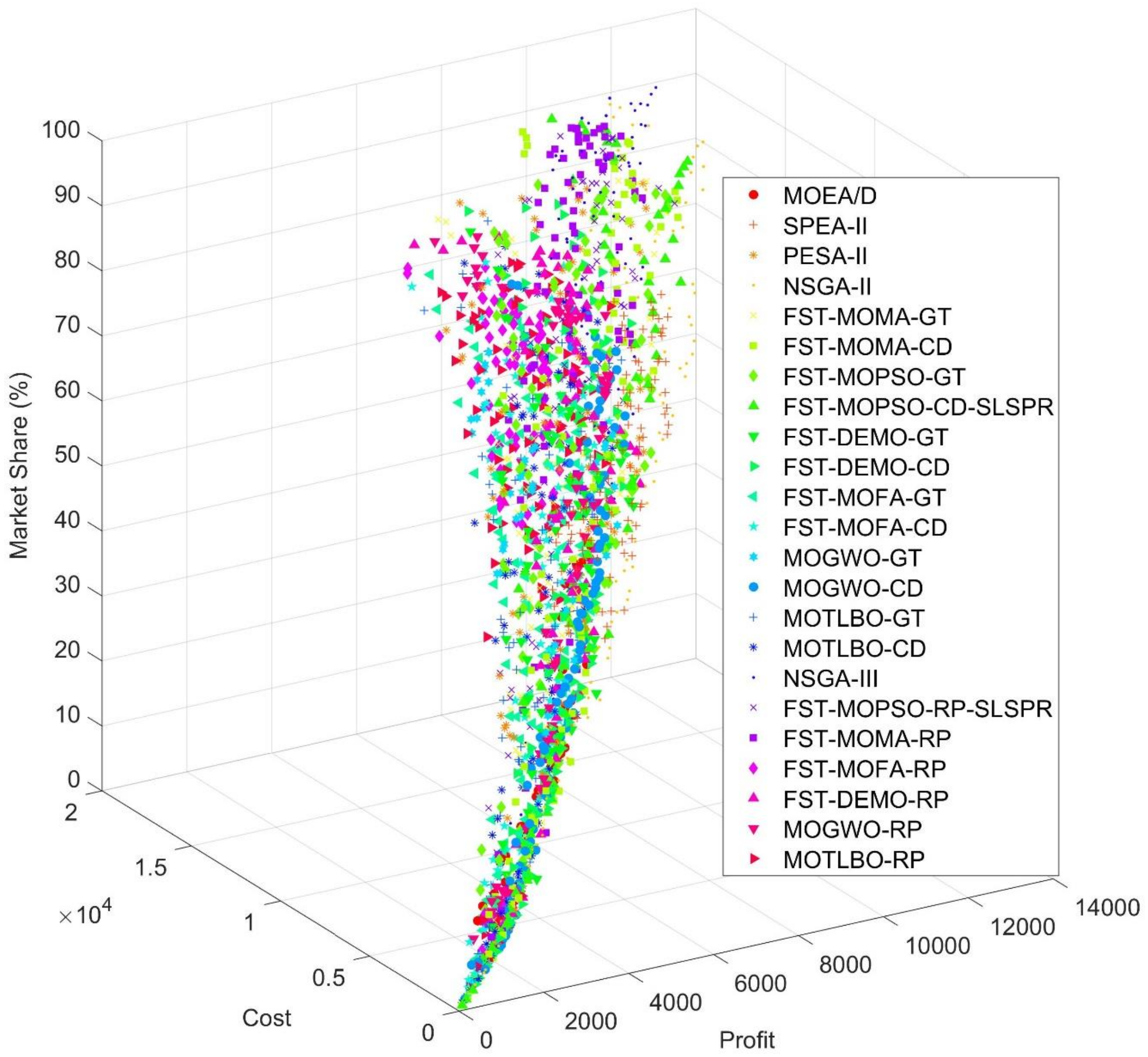


Figure 5.13: Detected non-dominated sets when using three objectives on the Timbuk2 data

Table 5.22: Comparison of methods according their average C metric values when using three objectives on the Timbuk2 data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.01	0.14	0.03	0.11	0.06	0.24	0.09	0.02	0.03	0.36	0.30
SPEA-II	0.17	1.00	0.55	0.12	0.66	0.56	0.81	0.62	0.53	0.55	0.94	0.93
PESA-II	0.06	0.01	1.00	0.02	0.24	0.12	0.35	0.16	0.15	0.16	0.53	0.50
NSGA-II	0.20	0.08	0.37	1.00	0.46	0.44	0.58	0.53	0.27	0.32	0.65	0.69
FST-MOMA-GT	0.06	0.00	0.08	0.01	1.00	0.05	0.23	0.10	0.09	0.08	0.50	0.46
FST-MOMA-CD	0.09	0.00	0.16	0.04	0.22	1.00	0.34	0.22	0.12	0.14	0.53	0.54
FST-MOPSO-GT	0.04	0.00	0.05	0.00	0.10	0.04	1.00	0.07	0.07	0.07	0.41	0.37
FST-MOPSO-CD-SLSPR	0.09	0.00	0.13	0.04	0.19	0.14	0.28	1.00	0.10	0.11	0.48	0.49
FST-DEMO-GT	0.13	0.00	0.20	0.03	0.25	0.15	0.42	0.22	1.00	0.12	0.70	0.67
FST-DEMO-CD	0.15	0.01	0.21	0.05	0.27	0.17	0.42	0.24	0.16	1.00	0.71	0.71
FST-MOFA-GT	0.02	0.00	0.02	0.00	0.02	0.01	0.07	0.02	0.00	0.00	1.00	0.13
FST-MOFA-CD	0.03	0.00	0.03	0.01	0.02	0.01	0.09	0.03	0.01	0.01	0.25	1.00
MOGWO-GT	0.11	0.02	0.27	0.04	0.34	0.22	0.51	0.30	0.21	0.20	0.75	0.70
MOGWO-CD	0.18	0.03	0.30	0.07	0.38	0.30	0.53	0.35	0.25	0.27	0.76	0.75

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOTLBO-GT	0.04	0.00	0.04	0.00	0.04	0.01	0.10	0.04	0.02	0.01	0.29	0.23
MOTLBO-CD	0.05	0.00	0.06	0.01	0.06	0.03	0.17	0.07	0.04	0.03	0.40	0.36
NSGA-III	0.04	0.02	0.17	0.06	0.28	0.29	0.37	0.32	0.15	0.18	0.44	0.45
FST-MOPSO-RP-SLSPR	0.02	0.00	0.05	0.01	0.13	0.07	0.17	0.10	0.09	0.11	0.33	0.33
FST-MOMA-RP	0.03	0.01	0.10	0.03	0.21	0.15	0.27	0.17	0.11	0.15	0.42	0.43
FST-MOFA-RP	0.02	0.00	0.02	0.01	0.02	0.01	0.04	0.02	0.01	0.01	0.26	0.17
FST-DEMO-RP	0.11	0.01	0.17	0.03	0.25	0.13	0.39	0.20	0.18	0.14	0.72	0.70
MOGWO-RP	0.09	0.01	0.18	0.02	0.24	0.14	0.41	0.18	0.18	0.17	0.67	0.64
MOTLBO-RP	0.02	0.00	0.03	0.00	0.03	0.01	0.08	0.03	0.04	0.02	0.36	0.28

Table 5.23: Comparison of methods according their average C metric values when using three objectives on the Timbuk2 data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.05	0.02	0.24	0.20	0.18	0.31	0.04	0.08	0.04	0.04	0.14
SPEA-II	0.35	0.31	0.89	0.84	0.49	0.77	0.52	0.61	0.49	0.29	0.73
PESA-II	0.09	0.10	0.46	0.38	0.12	0.33	0.16	0.42	0.15	0.09	0.43
NSGA-II	0.17	0.24	0.61	0.60	0.36	0.60	0.35	0.43	0.27	0.17	0.50
FST-MOMA-GT	0.05	0.06	0.40	0.30	0.09	0.27	0.08	0.38	0.07	0.06	0.36
FST-MOMA-CD	0.07	0.10	0.46	0.42	0.16	0.39	0.16	0.38	0.12	0.09	0.40
FST-MOPSO-GT	0.04	0.05	0.31	0.24	0.05	0.20	0.06	0.33	0.06	0.04	0.31
FST-MOPSO-CD-SLSPR	0.07	0.09	0.40	0.37	0.15	0.35	0.13	0.34	0.10	0.07	0.35
FST-DEMO-GT	0.10	0.08	0.58	0.47	0.20	0.45	0.17	0.42	0.16	0.11	0.45
FST-DEMO-CD	0.11	0.13	0.59	0.51	0.22	0.46	0.19	0.44	0.16	0.12	0.48
FST-MOFA-GT	0.01	0.00	0.09	0.05	0.03	0.12	0.02	0.10	0.01	0.01	0.09
FST-MOFA-CD	0.01	0.02	0.14	0.09	0.05	0.15	0.02	0.18	0.01	0.02	0.14
MOGWO-GT	1.00	0.09	0.64	0.53	0.26	0.52	0.23	0.38	0.18	0.12	0.48
MOGWO-CD	0.16	1.00	0.68	0.62	0.34	0.56	0.26	0.42	0.23	0.17	0.50
MOTLBO-GT	0.02	0.02	1.00	0.11	0.05	0.18	0.03	0.20	0.02	0.02	0.17
MOTLBO-CD	0.03	0.05	0.27	1.00	0.08	0.24	0.05	0.28	0.04	0.03	0.26
NSGA-III	0.09	0.12	0.40	0.38	1.00	0.53	0.36	0.39	0.18	0.11	0.41
FST-MOPSO-RP-SLSPR	0.05	0.09	0.29	0.26	0.03	1.00	0.08	0.36	0.11	0.06	0.33
FST-MOMA-RP	0.06	0.11	0.38	0.33	0.06	0.36	1.00	0.42	0.15	0.09	0.41
FST-MOFA-RP	0.02	0.03	0.14	0.07	0.03	0.07	0.02	1.00	0.01	0.02	0.14
FST-DEMO-RP	0.11	0.10	0.60	0.48	0.17	0.41	0.20	0.56	1.00	0.12	0.55
MOGWO-RP	0.11	0.11	0.53	0.48	0.19	0.42	0.19	0.50	0.19	1.00	0.49
MOTLBO-RP	0.03	0.04	0.23	0.11	0.03	0.11	0.03	0.36	0.03	0.03	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.14 demonstrates the violin plots which show the distribution of time each method needed. A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)=1{,}144.26$, $p<0.001$, $\varepsilon^2=1.00$. Their Median values along with the rest of statistics are demonstrated in Table 5.24.

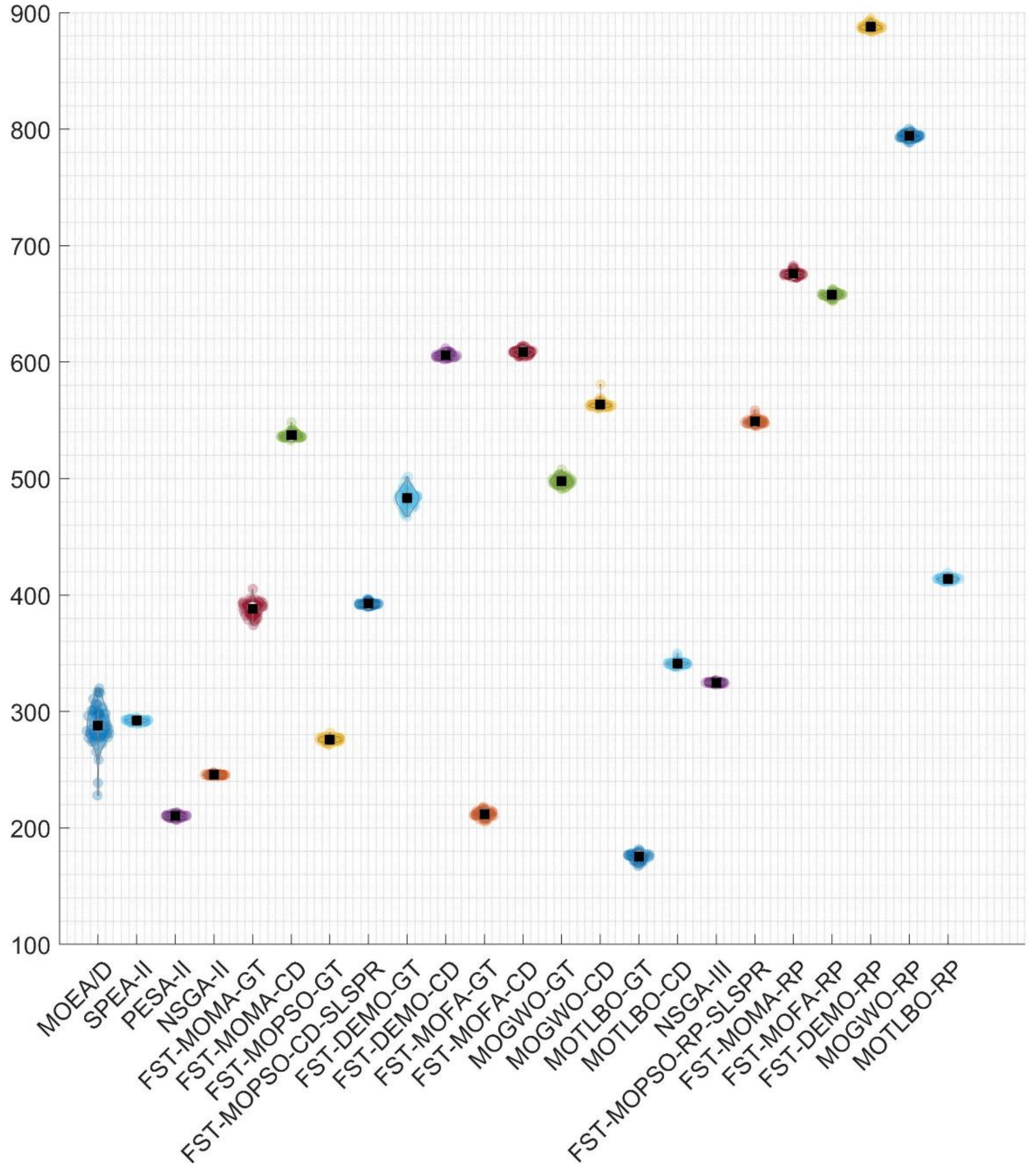


Figure 5.14: Violin plots of *Time* values when using three objectives on the Timbuk2 data

Table 5.24: Statistics of *Time* when using three objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	319.78	288.00	287.87	227.94	**91.85**	**17.70**
SPEA-II	295.06	292.07	292.17	289.55	5.52	1.18
PESA-II	213.29	210.18	210.20	206.86	6.42	1.43
NSGA-II	247.43	245.72	245.60	244.73	**2.71**	**0.63**
FST-MOMA-GT	405.16	388.09	389.53	374.11	31.05	6.02
FST-MOMA-CD	548.22	537.27	536.38	532.93	15.30	2.58
FST-MOPSO-GT	281.79	275.87	275.81	271.49	10.30	1.99
FST-MOPSO-CD-SLSPR	396.45	392.54	392.29	389.86	6.59	1.45
FST-DEMO-GT	501.66	483.26	483.35	467.49	34.17	7.27
FST-DEMO-CD	611.62	606.01	605.80	602.61	9.01	1.95
FST-MOFA-GT	217.93	211.73	211.97	205.86	12.07	2.80
FST-MOFA-CD	613.96	608.61	608.61	604.72	9.24	2.35
MOGWO-GT	507.74	497.92	497.76	491.02	16.72	3.50
MOGWO-CD	580.89	563.65	563.10	560.05	20.84	3.12
MOTLBO-GT	**181.59**	**175.12**	**175.77**	**167.43**	14.16	3.07
MOTLBO-CD	349.55	341.19	340.99	338.19	11.37	1.95
NSGA-III	327.04	324.81	324.78	323.37	3.67	0.74

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	558.54	548.93	548.52	545.27	13.27	2.30
FST-MOMA-RP	682.76	675.83	675.37	672.43	10.33	2.18
FST-MOFA-RP	662.70	657.67	657.76	652.28	10.42	2.08
FST-DEMO-RP	**895.16**	**888.01**	**887.59**	**883.41**	11.75	2.43
MOGWO-RP	800.29	794.22	794.44	788.70	11.59	2.27
MOTLBO-RP	418.94	413.83	413.99	411.42	7.52	1.44

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.25, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.25: Pairwise comparisons of *Time* when using three objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	0	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.14 and Table 5.24 illustrate that MOTLBO-GT needs the least time, while FST-DEMO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.15 demonstrates the violin plots of the M_1^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=909.31$, $p<0.001$, $\varepsilon^2=0.79$. Their Median values along with the rest of statistics are demonstrated in Table 5.26.

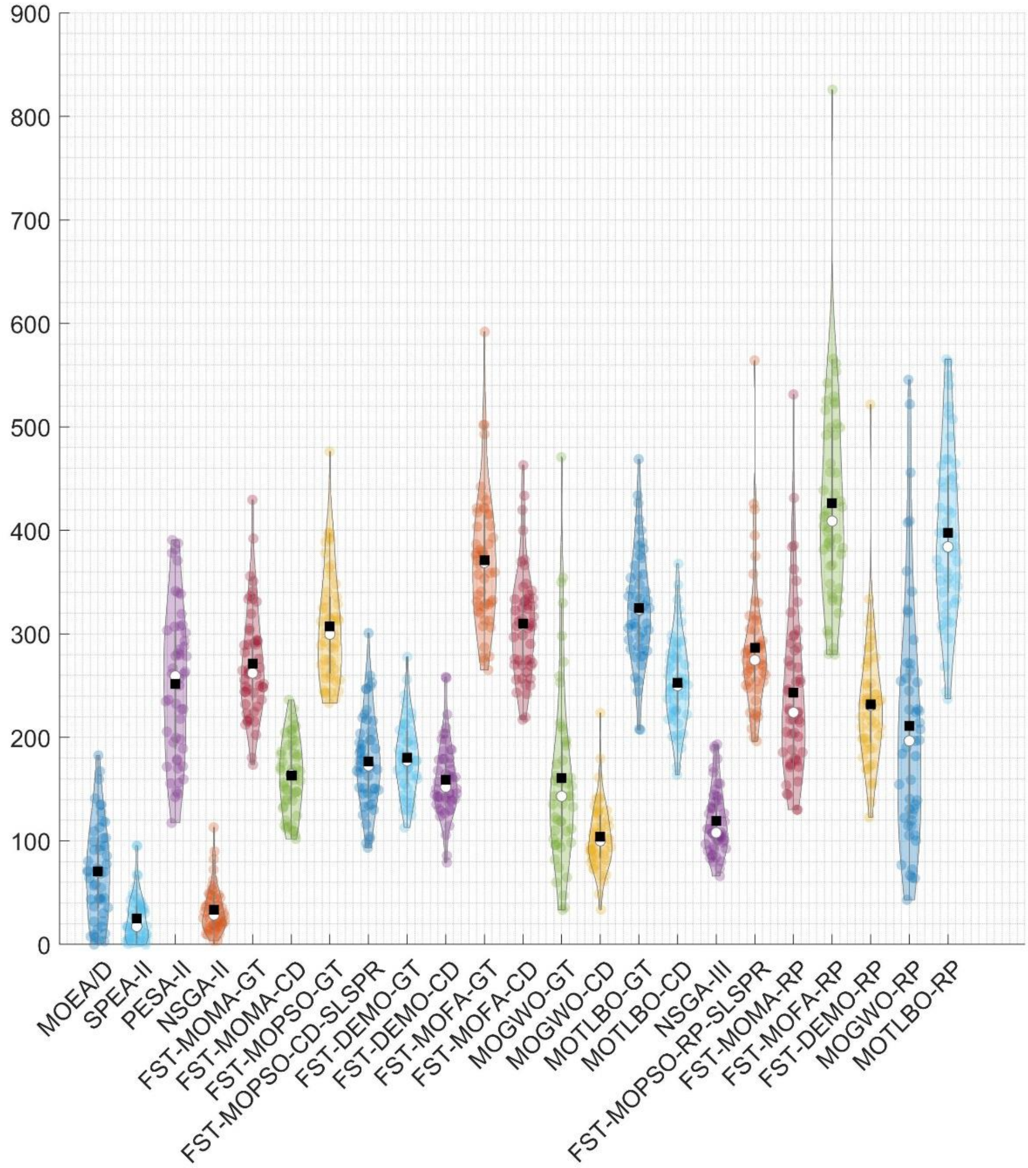


Figure 5.15: Violin plots of M_1^* metric values when using three objectives on the Timbuk2 data

Table 5.26: Statistics of M_1^* values when using three objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	182.56	70.57	70.10	**0.00**	182.56	46.92
SPEA-II	**95.44**	**25.13**	**17.43**	**0.00**	**95.44**	23.04
PESA-II	390.61	251.58	259.29	117.36	273.24	73.16
NSGA-II	113.08	33.62	28.58	3.57	109.52	**22.34**
FST-MOMA-GT	429.56	271.46	262.15	173.64	255.92	52.31
FST-MOMA-CD	236.36	163.06	164.20	101.97	134.39	35.56
FST-MOPSO-GT	476.17	307.13	299.73	233.15	243.02	53.52
FST-MOPSO-CD-SLSPR	300.75	176.94	172.74	93.42	207.32	45.47
FST-DEMO-GT	277.84	180.35	177.46	112.88	164.95	35.25
FST-DEMO-CD	258.34	159.32	152.03	79.16	179.19	36.15
FST-MOFA-GT	591.82	371.26	368.97	265.12	326.70	65.07
FST-MOFA-CD	462.99	309.87	309.41	217.16	245.83	52.19
MOGWO-GT	470.72	160.68	143.17	33.35	437.37	87.89
MOGWO-CD	223.57	104.00	100.19	33.64	189.93	32.85
MOTLBO-GT	468.67	324.90	323.53	207.32	261.35	53.81
MOTLBO-CD	367.69	253.00	249.96	163.90	203.79	42.85
NSGA-III	193.14	119.20	107.97	66.05	127.09	31.99

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	564.10	286.64	274.76	196.32	367.78	64.55
FST-MOMA-RP	531.39	243.09	224.21	130.14	401.25	81.95
FST-MOFA-RP	**825.75**	**426.45**	**408.99**	**280.02**	**545.73**	98.64
FST-DEMO-RP	521.58	231.86	230.90	122.87	398.71	62.80
MOGWO-RP	545.43	211.14	196.70	43.09	502.33	**117.66**
MOTLBO-RP	565.26	397.59	384.18	237.21	328.05	81.67

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.27, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.27: Pairwise comparisons of M_1^* when using three objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	1
NSGA-II	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	0	1
FST-MOPSO-GT	1	1	1	1	0	1	-	1	1	1	1	0	1	1	0	1	1	0	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	0	1	-	0	0	1	1	0	1	1	1	1	1	1	1	1	0	1
FST-DEMO-GT	1	1	1	1	1	0	1	0	-	0	1	1	0	1	1	1	1	1	1	1	1	0	1
FST-DEMO-CD	1	1	1	1	1	0	1	0	0	-	1	1	0	1	1	1	1	1	1	1	1	0	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	1	0	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	0	1	1	1	1	-	1	1	0	1	1	0	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	0	1	0	0	0	1	1	-	1	1	1	0	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	0	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	0	1	1	1	0	0	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	-	1	0	0	1	0	0	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	0	1	0	1	0	1	1	1	1	0	1	1	1	0	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	-	1	0	0	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	-	0	1
MOGWO-RP	1	1	0	1	1	0	1	0	0	0	1	1	0	1	1	0	1	1	0	1	0	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	-

Figure 5.15 and Table 5.26 illustrate that SPEA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-RP provides the farthest ones. Table 5.27 reveals that SPEA-II does not have statistically significant differences with NSGA-II, while FST-MOFA-RP does not have statistically significant differences with FST-MOFA-GT and MOTLBO-RP.

As regards the *Δ* values of each algorithm, Figure 5.16 demonstrates the violin plots of the *Δ* metric values, of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)$=901.23, p<0.001, ε^2=0.78. Their Median values along with the rest of statistics are demonstrated in Table 5.28.

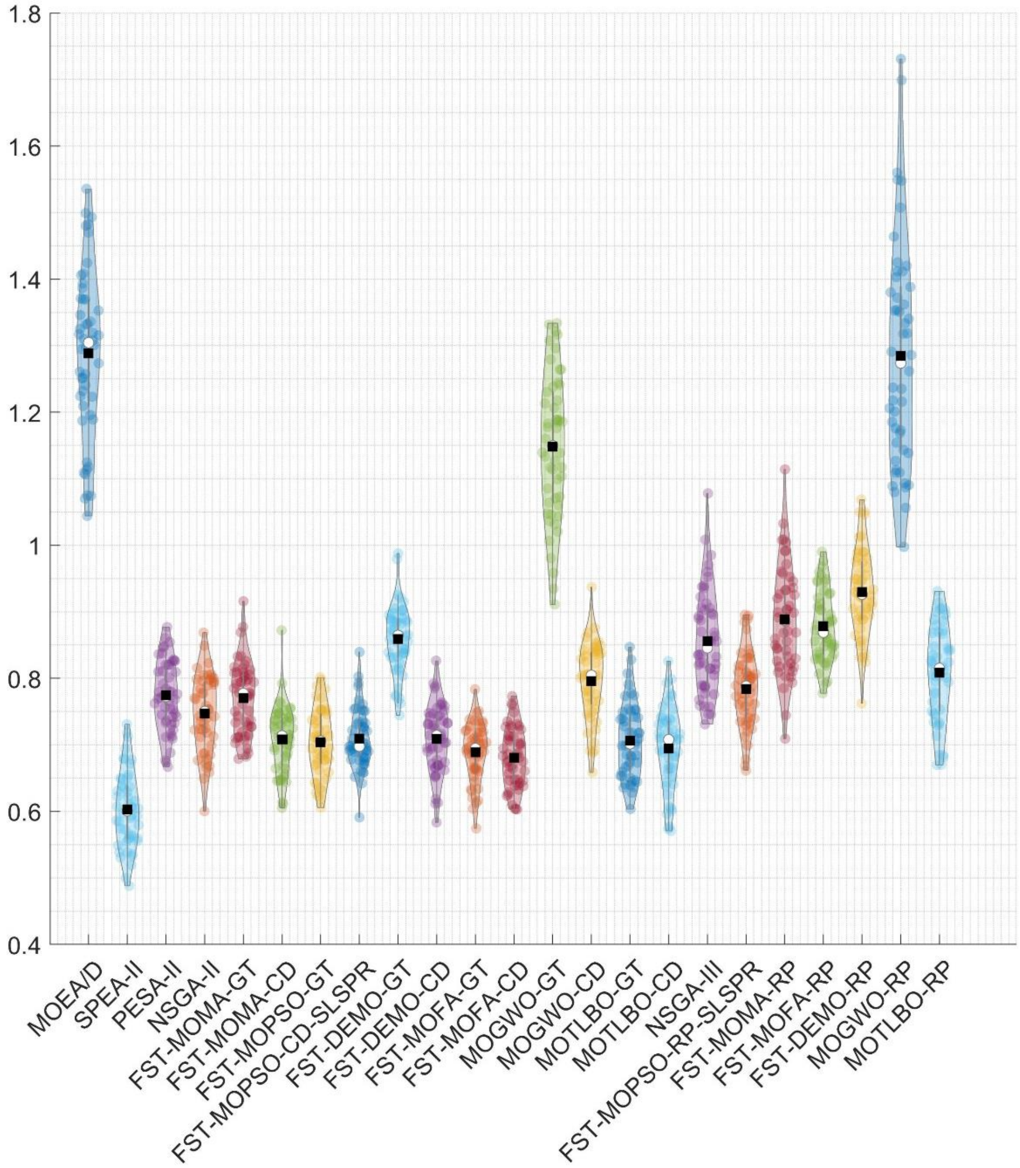


Figure 5.16: Violin plots of Δ metric values when using three objectives on the Timbuk2 data

Table 5.28: Statistics of Δ metric values when using three objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.54	**1.29**	**1.30**	**1.04**	0.49	0.12
SPEA-II	**0.73**	**0.60**	**0.60**	**0.49**	0.24	0.06
PESA-II	0.88	0.77	0.77	0.67	0.21	0.05
NSGA-II	0.87	0.75	0.75	0.60	0.27	0.06
FST-MOMA-GT	0.92	0.77	0.78	0.68	0.24	0.05
FST-MOMA-CD	0.87	0.71	0.71	0.61	0.27	0.05
FST-MOPSO-GT	0.80	0.70	0.71	0.61	0.20	0.05
FST-MOPSO-CD-SLSPR	0.84	0.71	0.70	0.59	0.25	**0.04**
FST-DEMO-GT	0.99	0.86	0.86	0.74	0.24	0.05
FST-DEMO-CD	0.83	0.71	0.71	0.58	0.24	0.05
FST-MOFA-GT	0.78	0.69	0.69	0.57	0.21	**0.04**
FST-MOFA-CD	0.77	0.68	0.68	0.60	**0.17**	**0.04**
MOGWO-GT	1.33	1.15	1.15	0.91	0.42	0.11
MOGWO-CD	0.94	0.80	0.80	0.66	0.28	0.06
MOTLBO-GT	0.85	0.71	0.70	0.60	0.24	0.05
MOTLBO-CD	0.83	0.69	0.71	0.57	0.26	0.06
NSGA-III	1.08	0.86	0.85	0.73	0.35	0.08

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	0.90	0.78	0.79	0.66	0.23	0.06
FST-MOMA-RP	1.11	0.89	0.89	0.71	0.40	0.08
FST-MOFA-RP	0.99	0.88	0.87	0.78	0.21	0.05
FST-DEMO-RP	1.07	0.93	0.93	0.76	0.31	0.06
MOGWO-RP	**1.73**	1.28	1.27	1.00	**0.73**	**0.17**
MOTLBO-RP	0.93	0.81	0.82	0.67	0.26	0.07

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.29, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.29: Pairwise comparisons of *Δ* metric when using three objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	0	0	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0
NSGA-II	1	1	0	-	0	0	1	0	1	0	1	1	1	1	0	1	1	0	1	1	1	1	1
FST-MOMA-GT	1	1	0	0	-	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0
FST-MOMA-CD	1	1	1	0	1	-	0	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	0	-	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	0	1	0	0	-	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	0	1	0	0	1	1	0
FST-DEMO-CD	1	1	1	0	1	0	0	0	1	-	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	0	0	0	1	0	-	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	0	0	0	1	0	0	-	1	1	0	0	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	0	1	0	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	1	0
MOTLBO-GT	1	1	1	0	1	0	0	0	1	0	0	0	1	1	-	0	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	0	0	0	1	0	0	0	1	1	0	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	-	1	0	0	1	1	0
FST-MOPSO-RP-SLSPR	1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1	-	1	1	1	1	0
FST-MOMA-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	-	0	0	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	0	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	-	1	1
MOGWO-RP	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	0	1	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	1	1	1	-

Figure 5.16 and Table 5.28 illustrate that SPEA-II provides the best distribution among the non-dominated solutions, while MOEA/D provides the worst. According to Table 5.29, MOEA/D does not have statistically significant differences with MOGWO-RP.

Finally, as regards the M_3^* values of each algorithm, Figure 5.17 demonstrates the violin plots of the M_3^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=1,023.45, p<0.001, ε^2=0.89. Their Median values along with the rest of statistics, are demonstrated in Table 5.30.

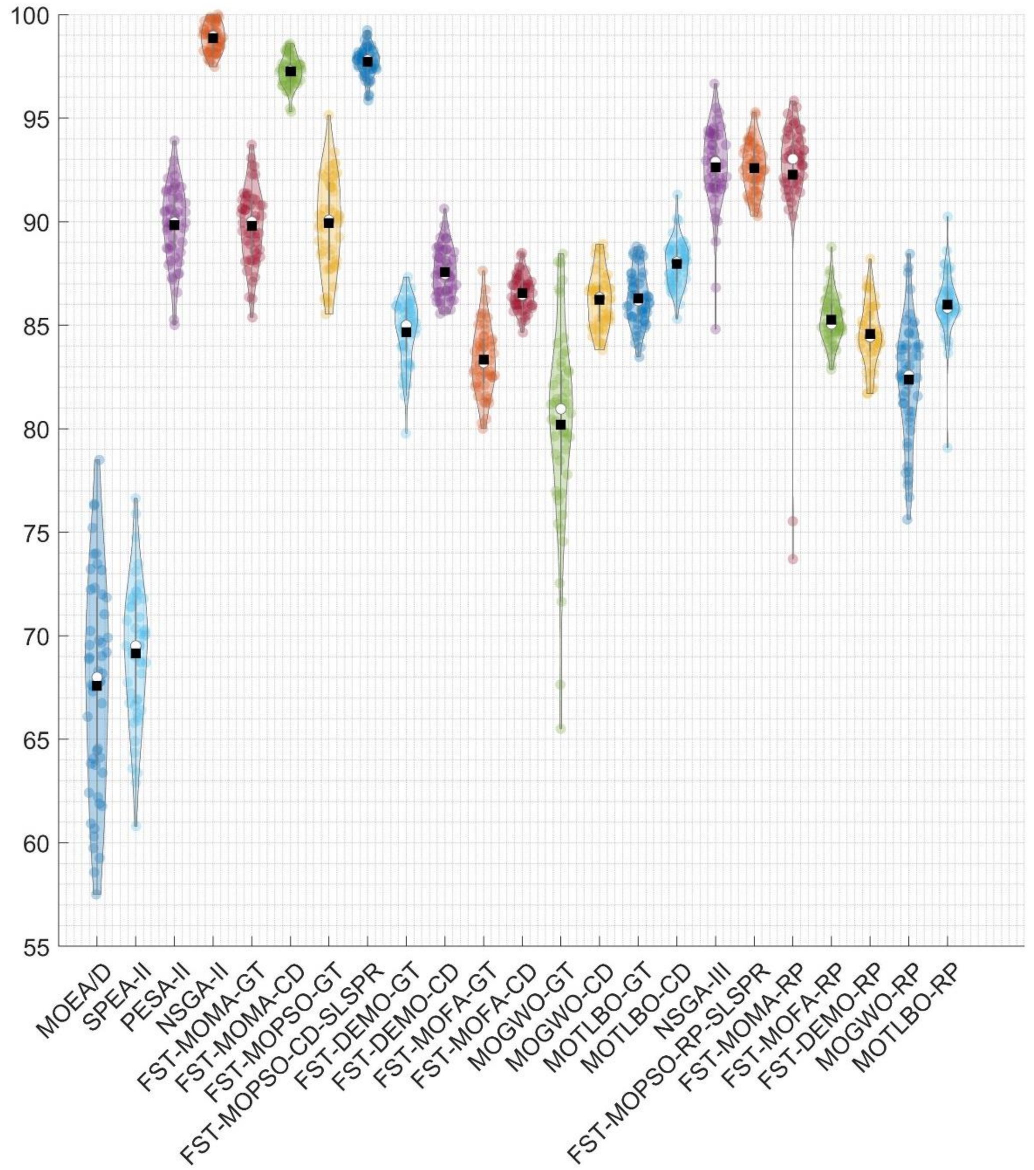


Figure 5.17: Violin plots of M_3^* metric values when using three objectives on the Timbuk2 data

Table 5.30: Statistics of M_3^* metric values when using three objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	78.49	**67.59**	**68.00**	**57.51**	20.98	**5.17**
SPEA-II	**76.64**	69.16	69.51	60.81	15.83	3.33
PESA-II	93.91	89.84	89.96	85.01	8.89	1.89
NSGA-II	**100.00**	**98.87**	**98.96**	**97.48**	**2.52**	**0.67**
FST-MOMA-GT	93.71	89.80	90.00	85.38	8.33	1.82
FST-MOMA-CD	98.59	97.25	97.30	95.31	3.28	0.73
FST-MOPSO-GT	95.14	89.93	90.07	85.53	9.60	2.22
FST-MOPSO-CD-SLSPR	99.24	97.71	97.80	95.86	3.39	0.73
FST-DEMO-GT	87.32	84.66	84.99	79.77	7.56	1.57
FST-DEMO-CD	90.62	87.56	87.46	85.56	5.06	1.18
FST-MOFA-GT	87.62	83.33	83.19	80.01	7.61	1.66
FST-MOFA-CD	88.48	86.54	86.46	84.65	3.83	0.82
MOGWO-GT	88.45	80.19	80.96	65.51	**22.94**	4.74
MOGWO-CD	88.92	86.22	86.33	83.80	5.11	1.35
MOTLBO-GT	88.80	86.29	86.19	83.46	5.34	1.33
MOTLBO-CD	91.30	87.95	88.06	85.30	5.99	1.11
NSGA-III	96.66	92.62	92.89	84.80	11.85	2.08

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	95.29	92.60	92.60	90.27	5.01	1.19
FST-MOMA-RP	95.84	92.26	93.03	73.70	22.13	3.87
FST-MOFA-RP	88.77	85.27	85.06	82.86	5.91	1.16
FST-DEMO-RP	88.20	84.57	84.44	81.70	6.50	1.50
MOGWO-RP	88.44	82.37	82.57	75.62	12.82	2.89
MOTLBO-RP	90.24	85.99	85.83	79.08	11.15	1.55

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.31, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.31: Pairwise comparisons of M_3^* metric when using three objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	0	0	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	0	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	0	0	1	1	1	1	1	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	0	1	-	0	1	1	1	1	0	1	1	0
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	0	1	0	-	1	1	1	1	1	1	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	0	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	0	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	-	0	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	0	1	1	-

Figure 5.17 and Table 5.30 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while MOEA/D provides the worst. Table 5.31 reveals that MOEA/D does not have statistically significant differences with SPEA-II.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.18.

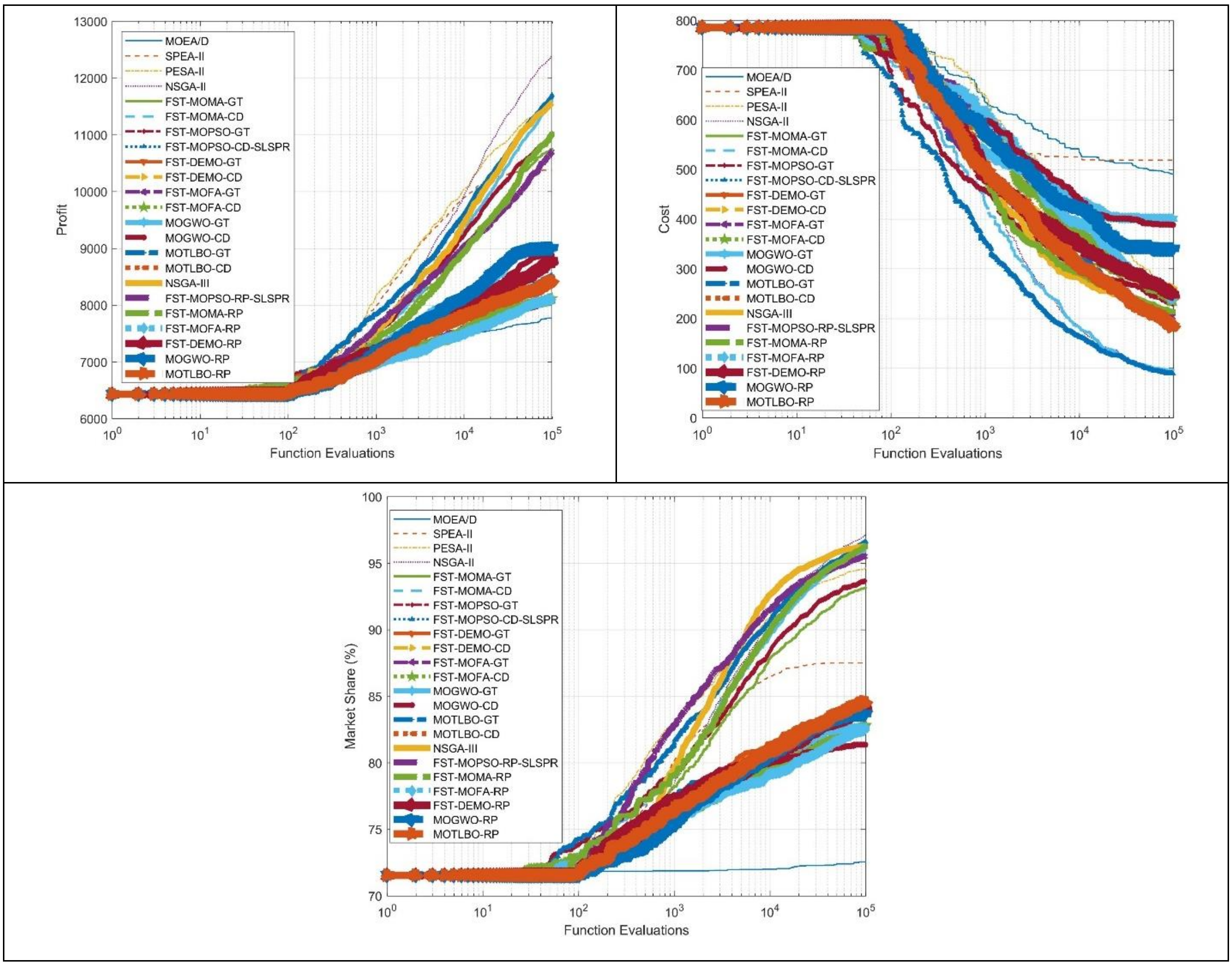


Figure 5.18: Average convergence characteristic curves when using three objectives on the Timbuk2 data

5.2.2 *Results on the Olive oil data set when using three objectives*

Regarding the performance of the comparing algorithms while performing on the olive oil data set when using three objectives, in Figure 5.19, the sets of non-dominated solutions, provided from each algorithm in a random run, are presented.

Tables 5.32-5.33 show the results of the average *C* metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U Test.

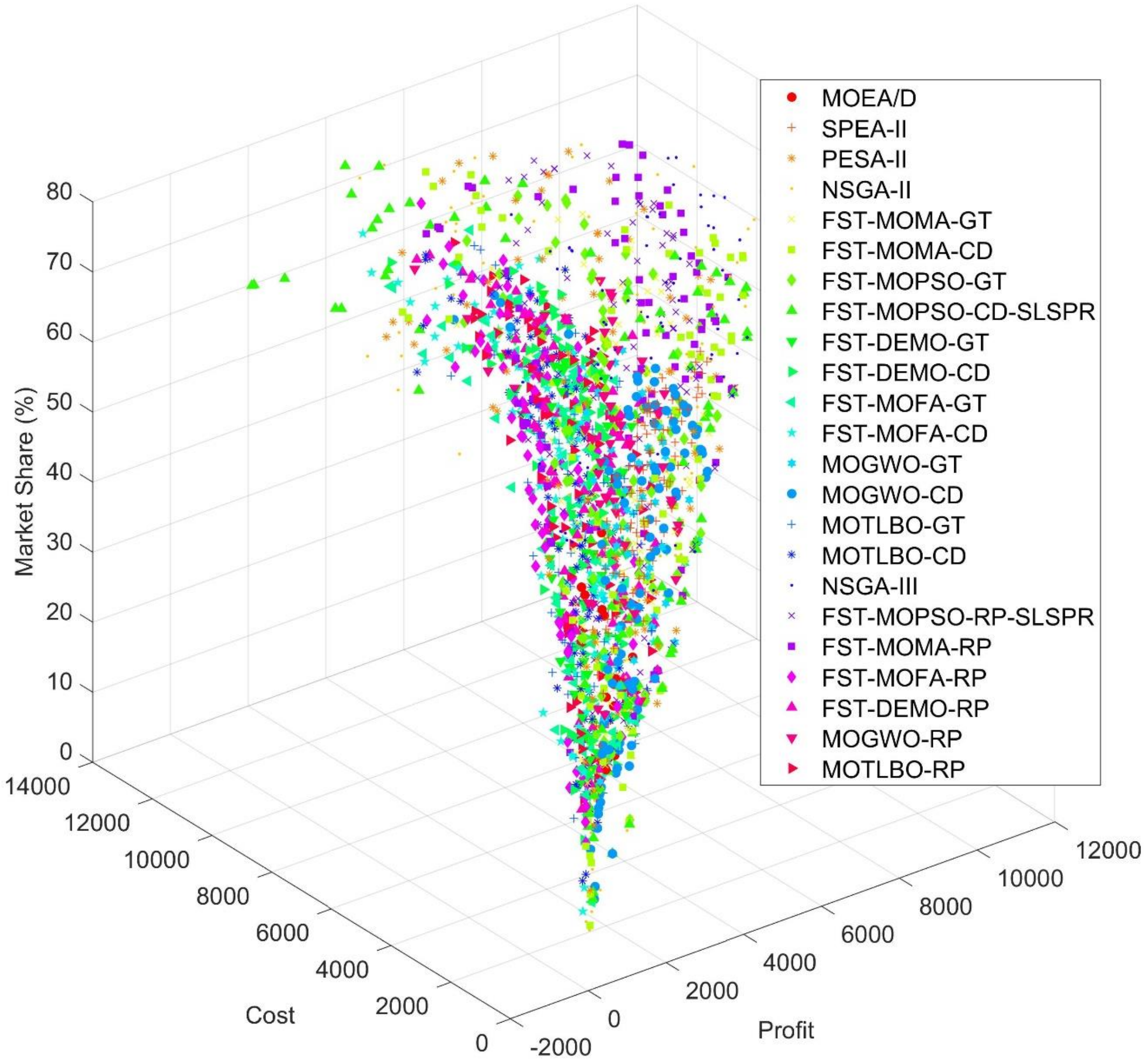


Figure 5.19: Detected non-dominated sets when using three objectives on the Olive oil data

Table 5.32: Comparison of methods according their average C metric values when using three objectives on the Olive oil data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.00	0.02	0.03	0.07	0.02	0.06	0.04	0.19	0.14	0.26	0.26
SPEA-II	0.56	1.00	0.54	0.42	0.72	0.59	0.71	0.65	0.90	0.89	0.95	0.95
PESA-II	0.10	0.00	1.00	0.15	0.22	0.18	0.25	0.24	0.31	0.34	0.48	0.49
NSGA-II	0.12	0.01	0.18	1.00	0.25	0.21	0.26	0.25	0.27	0.30	0.38	0.40
FST-MOMA-GT	0.12	0.00	0.10	0.13	1.00	0.13	0.20	0.19	0.33	0.33	0.50	0.51
FST-MOMA-CD	0.09	0.00	0.13	0.10	0.22	1.00	0.24	0.21	0.26	0.30	0.41	0.43
FST-MOPSO-GT	0.11	0.00	0.08	0.11	0.14	0.11	1.00	0.16	0.29	0.30	0.46	0.47
FST-MOPSO-CD-SLSPR	0.09	0.00	0.11	0.09	0.18	0.14	0.19	1.00	0.23	0.26	0.36	0.38
FST-DEMO-GT	0.09	0.00	0.07	0.17	0.10	0.11	0.14	0.19	1.00	0.18	0.40	0.39
FST-DEMO-CD	0.08	0.00	0.06	0.13	0.10	0.10	0.13	0.17	0.23	1.00	0.42	0.42
FST-MOFA-GT	0.04	0.00	0.03	0.09	0.04	0.05	0.07	0.11	0.10	0.07	1.00	0.19
FST-MOFA-CD	0.04	0.00	0.02	0.08	0.04	0.05	0.07	0.10	0.10	0.09	0.21	1.00
MOGWO-GT	0.30	0.01	0.34	0.36	0.49	0.40	0.51	0.50	0.66	0.63	0.82	0.78
MOGWO-CD	0.29	0.02	0.30	0.27	0.43	0.36	0.47	0.42	0.65	0.67	0.77	0.77

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOTLBO-GT	0.07	0.00	0.03	0.11	0.05	0.06	0.09	0.13	0.15	0.12	0.29	0.28
MOTLBO-CD	0.06	0.00	0.04	0.10	0.05	0.06	0.09	0.12	0.15	0.12	0.30	0.30
NSGA-III	0.09	0.00	0.27	0.21	0.34	0.29	0.37	0.34	0.30	0.32	0.47	0.49
FST-MOPSO-RP-SLSPR	0.06	0.00	0.17	0.13	0.24	0.16	0.26	0.21	0.21	0.25	0.39	0.41
FST-MOMA-RP	0.04	0.00	0.21	0.14	0.28	0.19	0.29	0.25	0.20	0.24	0.36	0.40
FST-MOFA-RP	0.03	0.00	0.04	0.10	0.04	0.06	0.08	0.12	0.09	0.08	0.23	0.18
FST-DEMO-RP	0.07	0.00	0.07	0.14	0.09	0.10	0.14	0.17	0.23	0.19	0.40	0.38
MOGWO-RP	0.20	0.01	0.26	0.23	0.34	0.29	0.39	0.34	0.62	0.58	0.75	0.74
MOTLBO-RP	0.05	0.00	0.04	0.10	0.05	0.06	0.08	0.12	0.14	0.11	0.28	0.26

Table 5.33: Comparison of methods according their average C metric values when using three objectives on the Olive oil data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.04	0.00	0.21	0.15	0.01	0.10	0.06	0.24	0.17	0.03	0.21
SPEA-II	0.38	0.29	0.94	0.92	0.35	0.67	0.52	0.91	0.87	0.33	0.94
PESA-II	0.06	0.07	0.43	0.43	0.07	0.20	0.15	0.47	0.31	0.06	0.43
NSGA-II	0.06	0.05	0.34	0.37	0.09	0.21	0.18	0.38	0.27	0.07	0.36
FST-MOMA-GT	0.07	0.04	0.45	0.43	0.06	0.18	0.14	0.49	0.32	0.05	0.45
FST-MOMA-CD	0.05	0.04	0.36	0.38	0.05	0.18	0.14	0.42	0.28	0.06	0.38
FST-MOPSO-GT	0.06	0.04	0.40	0.40	0.05	0.16	0.11	0.47	0.29	0.05	0.41
FST-MOPSO-CD-SLSPR	0.05	0.04	0.32	0.35	0.05	0.16	0.12	0.38	0.24	0.06	0.34
FST-DEMO-GT	0.04	0.03	0.33	0.29	0.04	0.14	0.10	0.38	0.21	0.03	0.31
FST-DEMO-CD	0.04	0.03	0.35	0.30	0.03	0.13	0.10	0.39	0.20	0.02	0.33
FST-MOFA-GT	0.02	0.02	0.16	0.12	0.02	0.06	0.04	0.21	0.08	0.01	0.14
FST-MOFA-CD	0.01	0.02	0.15	0.14	0.01	0.05	0.04	0.21	0.09	0.02	0.15
MOGWO-GT	1.00	0.16	0.76	0.75	0.18	0.44	0.30	0.75	0.62	0.15	0.73
MOGWO-CD	0.18	1.00	0.74	0.74	0.18	0.42	0.32	0.71	0.63	0.17	0.72
MOTLBO-GT	0.02	0.01	1.00	0.18	0.02	0.09	0.06	0.28	0.12	0.02	0.20
MOTLBO-CD	0.03	0.02	0.24	1.00	0.02	0.09	0.06	0.30	0.13	0.01	0.22
NSGA-III	0.06	0.06	0.42	0.47	1.00	0.30	0.29	0.50	0.35	0.08	0.46
FST-MOPSO-RP-SLSPR	0.04	0.04	0.34	0.37	0.09	1.00	0.15	0.43	0.24	0.06	0.38
FST-MOMA-RP	0.04	0.04	0.33	0.37	0.10	0.20	1.00	0.42	0.24	0.06	0.38
FST-MOFA-RP	0.01	0.02	0.17	0.14	0.02	0.05	0.05	1.00	0.10	0.02	0.15
FST-DEMO-RP	0.03	0.03	0.33	0.29	0.03	0.13	0.11	0.40	1.00	0.03	0.33
MOGWO-RP	0.16	0.13	0.69	0.68	0.20	0.37	0.29	0.75	0.58	1.00	0.72
MOTLBO-RP	0.03	0.02	0.23	0.19	0.02	0.07	0.06	0.30	0.13	0.02	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.20 demonstrates the violin plots which show the distribution of time each method needed.

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,133.10, p<0.001, ε^2=0.99. Their Median values along with the rest of statistics are demonstrated in Table 5.34.

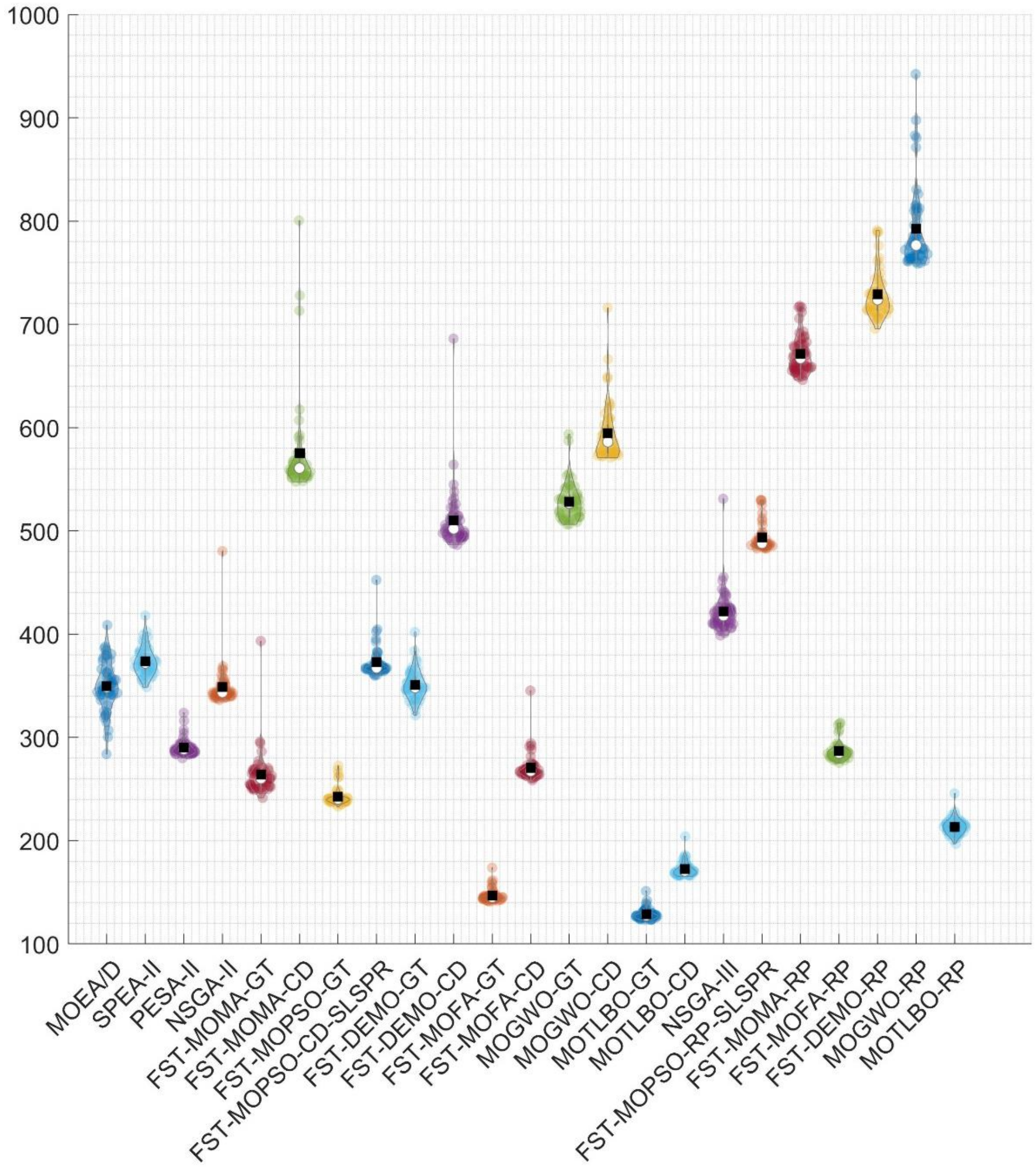


Figure 5.20: Violin plots of *Time* values when using three objectives on the Olive oil data

Table 5.34: Statistics of *Time* when using three objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	408.78	349.47	348.58	283.81	124.96	23.90
SPEA-II	417.99	373.59	372.01	348.70	69.29	13.79
PESA-II	323.79	290.28	288.45	279.93	43.86	8.09
NSGA-II	480.47	348.64	343.64	336.33	144.14	20.51
FST-MOMA-GT	393.37	264.09	260.22	241.43	151.93	21.64
FST-MOMA-CD	800.60	575.54	560.76	547.58	**253.02**	**47.07**
FST-MOPSO-GT	272.57	242.77	240.38	233.05	39.52	8.56
FST-MOPSO-CD-SLSPR	452.42	372.82	367.41	359.84	92.59	15.11
FST-DEMO-GT	402.04	350.71	348.20	321.44	80.60	14.64
FST-DEMO-CD	686.11	510.10	502.06	486.45	199.66	29.78
FST-MOFA-GT	173.82	146.74	145.20	140.96	32.86	5.98
FST-MOFA-CD	345.26	270.94	267.45	258.51	86.75	13.27
MOGWO-GT	593.53	528.45	526.80	506.36	87.17	18.04
MOGWO-CD	715.97	594.80	586.27	570.93	145.04	28.01
MOTLBO-GT	**151.22**	**128.89**	**127.47**	**123.22**	**28.00**	**5.23**
MOTLBO-CD	204.19	172.63	170.66	165.61	38.58	6.99
NSGA-III	531.16	421.89	417.88	398.71	132.45	20.25

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	530.04	493.48	487.92	482.83	47.22	12.78
FST-MOMA-RP	717.78	671.31	667.09	646.13	71.64	18.73
FST-MOFA-RP	314.19	286.93	284.84	275.47	38.72	9.05
FST-DEMO-RP	790.80	729.01	723.88	695.74	95.06	22.65
MOGWO-RP	**942.32**	**792.79**	**776.62**	**758.75**	183.57	40.37
MOTLBO-RP	245.79	213.39	213.18	197.01	48.78	7.94

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.35, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.35: Pairwise comparisons of *Time* when using three objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
NSGA-II	0	1	1	-	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	0	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	0	1	1	0	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.20 and Table 5.34 illustrate that MOTLBO-GT needs the least time, while MOGWO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.21 demonstrates the violin plots of the M_1^* values of each method.

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=845.42$, $p<0.001$, $\varepsilon^2=0.74$. Their Median values along with the rest of statistics are demonstrated in Table 5.36.

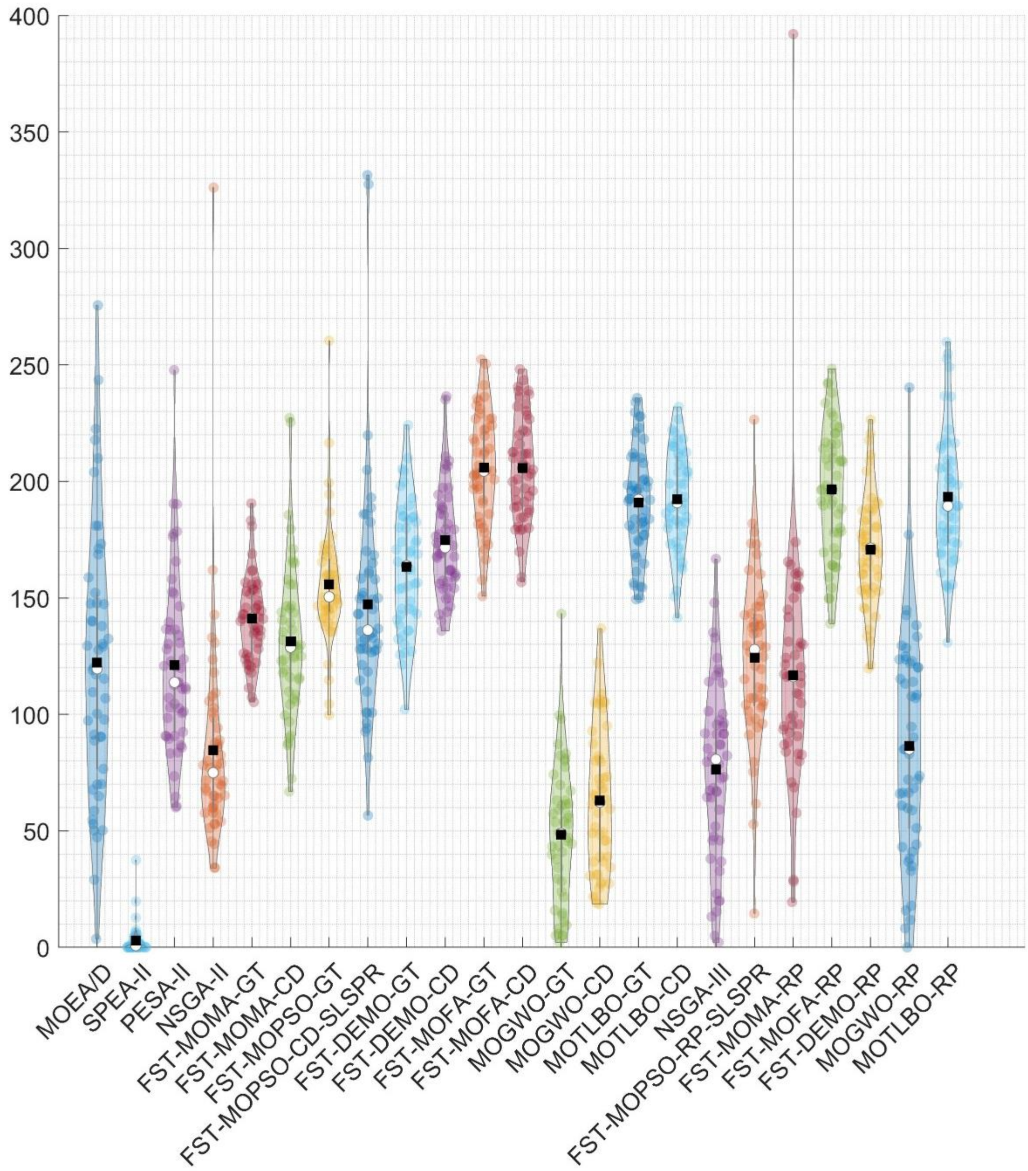


Figure 5.21: Violin plots of M_1^* metric values when using three objectives on the Olive oil data

Table 5.36: Statistics of M_1^* values when using three objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	275.57	122.17	119.59	3.66	271.91	**57.75**
SPEA-II	**37.46**	**2.91**	**0.77**	**0.00**	**37.46**	**6.16**
PESA-II	247.78	121.16	113.77	60.11	187.67	37.30
NSGA-II	326.14	84.62	74.98	34.06	292.08	44.44
FST-MOMA-GT	190.55	141.22	141.34	105.28	85.27	18.42
FST-MOMA-CD	227.26	131.38	128.80	66.90	160.36	32.85
FST-MOPSO-GT	260.37	155.79	150.46	99.71	160.66	26.01
FST-MOPSO-CD-SLSPR	331.44	147.32	136.18	56.62	274.82	49.57
FST-DEMO-GT	224.14	163.27	163.87	102.14	122.00	26.84
FST-DEMO-CD	236.52	174.74	171.60	135.88	100.64	23.05
FST-MOFA-GT	252.37	**205.82**	204.42	150.77	101.60	23.57
FST-MOFA-CD	248.14	205.77	**205.68**	**156.78**	91.36	22.56
MOGWO-GT	143.16	48.34	48.90	2.14	141.02	29.87
MOGWO-CD	136.87	62.99	62.34	18.54	118.33	30.64
MOTLBO-GT	235.78	190.97	192.26	149.38	86.41	22.99
MOTLBO-CD	232.00	192.24	190.82	141.54	90.46	21.06
NSGA-III	166.73	76.46	80.69	2.15	164.58	38.07

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	226.50	124.43	127.84	14.51	211.99	36.10
FST-MOMA-RP	**392.01**	116.94	116.21	19.50	**372.51**	53.55
FST-MOFA-RP	248.34	196.56	196.44	138.96	109.38	27.22
FST-DEMO-RP	226.51	170.60	171.34	119.70	106.80	24.41
MOGWO-RP	240.30	86.41	85.03	**0.00**	240.30	47.43
MOTLBO-RP	259.81	193.43	189.44	130.82	128.99	28.58

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.37, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.37: Pairwise comparisons of M_1^* when using three objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	0	1	0	0	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	0	1	-	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	0	1
FST-MOMA-GT	0	1	1	1	-	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOMA-CD	0	1	0	1	0	-	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-MOPSO-GT	1	1	1	1	0	1	-	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-MOPSO-CD-SLSPR	0	1	0	1	0	0	0	-	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	0	0	-	0	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	0	-	1	1	1	1	0	1	1	1	1	1	0	1	0
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	0	1	1	0	0	1	1	1	0	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	0	-	1	1	0	0	1	1	1	0	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	0	1	1	1	1	1	1	1	1	0	-	1	1	0	1	1	1	1	0	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	0	0	0	1	1	-	0	1	1	1	0	1	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	-	1	1	1	0	1	1	0
NSGA-III	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	-	1	1	1	1	0	1
FST-MOPSO-RP-SLSPR	0	1	0	1	0	0	1	0	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	0	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	0	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0	1	1	1	0	1	1	-

Figure 5.21 and Table 5.36 illustrate that SPEA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-CD provides the farthest ones. Table 5.37 reveals that FST-MOFA-CD does not have statistically significant differences with FST-MOFA-GT, MOTLBO-GT, MOTLBO-CD, FST-MOFA-RP and MOTLBO-RP.

As regards the *Δ* values of each algorithm, Figure 5.22 demonstrates the violin plots of the *Δ* metric values, of each method.

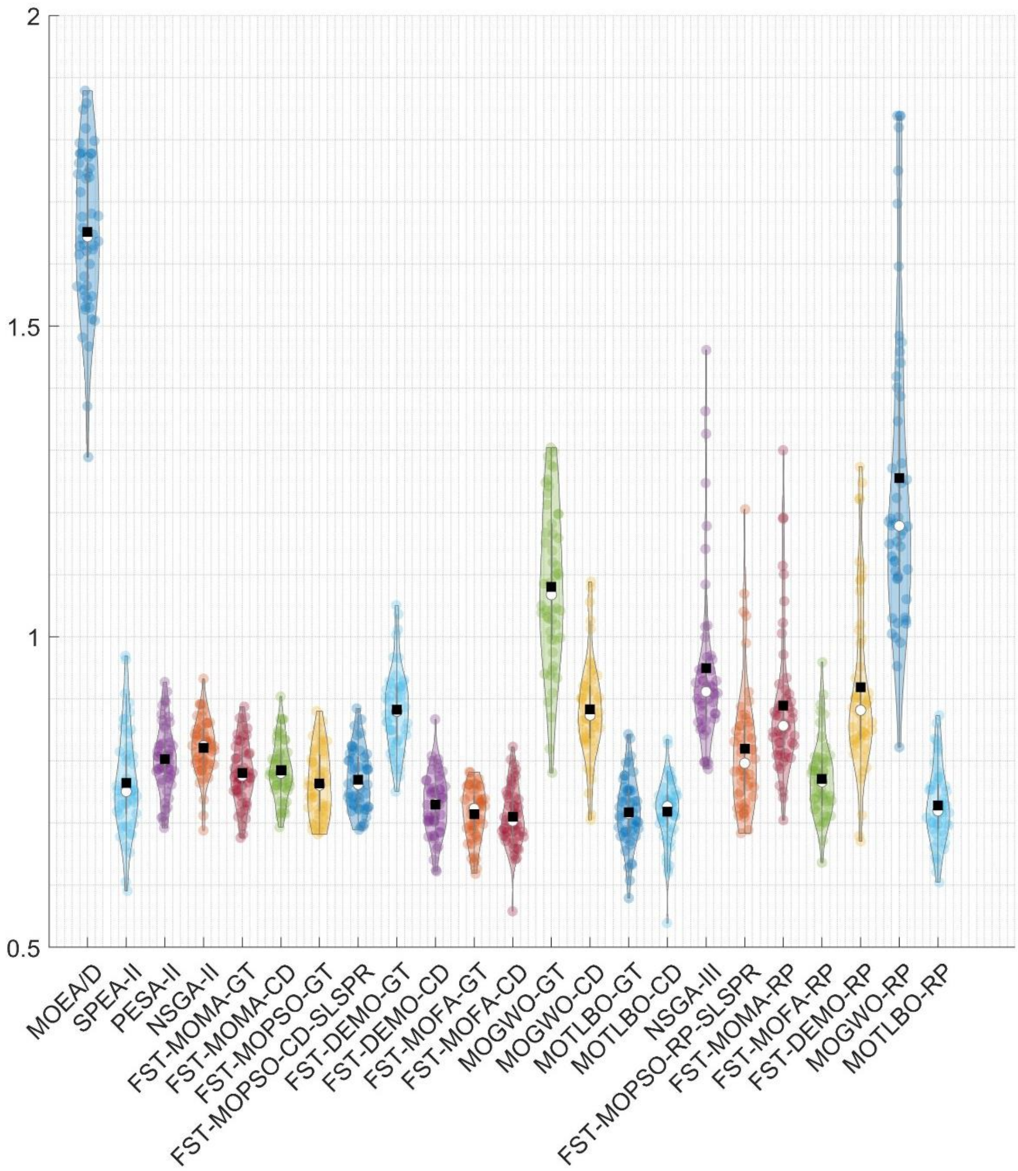


Figure 5.22: Violin plots of Δ metric values when using three objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)$=773.08, p<0.001, ε^2=0.67. Their Median values along with the rest of statistics are demonstrated in Table 5.38.

Table 5.38: Statistics of Δ metric values when using three objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	**1.88**	**1.65**	**1.64**	**1.29**	0.59	0.13
SPEA-II	0.97	0.76	0.75	0.59	0.38	0.08
PESA-II	0.93	0.80	0.80	0.69	0.23	0.06
NSGA-II	0.93	0.82	0.82	0.69	0.24	0.05
FST-MOMA-GT	0.89	0.78	0.78	0.68	0.21	0.05
FST-MOMA-CD	0.90	0.79	0.78	0.69	0.21	0.05
FST-MOPSO-GT	0.88	0.76	0.76	0.68	0.20	0.05
FST-MOPSO-CD-SLSPR	0.88	0.77	0.76	0.69	0.20	0.05
FST-DEMO-GT	1.05	0.88	0.88	0.75	0.30	0.07
FST-DEMO-CD	0.87	0.73	0.73	0.62	0.24	0.05
FST-MOFA-GT	**0.78**	**0.71**	0.72	0.62	**0.16**	**0.04**

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-CD	0.82	**0.71**	**0.70**	0.56	0.26	0.05
MOGWO-GT	1.30	1.08	1.07	0.78	0.52	0.13
MOGWO-CD	1.09	0.88	0.87	0.71	0.38	0.08
MOTLBO-GT	0.84	0.72	0.72	0.58	0.26	0.05
MOTLBO-CD	0.83	0.72	0.73	**0.54**	0.30	0.05
NSGA-III	1.46	0.95	0.91	0.79	0.67	0.14
FST-MOPSO-RP-SLSPR	1.21	0.82	0.80	0.68	0.52	0.10
FST-MOMA-RP	1.30	0.89	0.86	0.70	0.60	0.12
FST-MOFA-RP	0.96	0.77	0.77	0.64	0.32	0.06
FST-DEMO-RP	1.27	0.92	0.88	0.67	0.60	0.14
MOGWO-RP	1.84	1.26	1.18	0.82	**1.02**	**0.25**
MOTLBO-RP	0.87	0.73	0.72	0.60	0.27	0.06

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.39, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.39: Pairwise comparisons of *Δ* metric when using three objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	0	1	0	0	0	0	1	0	0	1	1	1	0	0	1	0	1	0	1	1	0
PESA-II	1	0	-	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1
NSGA-II	1	1	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-MOMA-GT	1	0	0	1	-	0	0	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1
FST-MOMA-CD	1	0	0	1	0	-	0	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1
FST-MOPSO-GT	1	0	0	1	0	0	-	0	1	0	1	1	1	1	1	0	1	0	1	0	1	1	0
FST-MOPSO-CD-SLSPR	1	0	0	1	0	0	0	-	1	1	1	1	1	1	1	1	1	0	1	0	1	1	0
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	0	1	1	0	1	0	1	0	1	1
FST-DEMO-CD	1	0	1	1	1	1	0	1	1	-	0	0	1	1	0	0	1	1	1	0	1	1	0
FST-MOFA-GT	1	0	1	1	1	1	1	1	1	0	-	0	1	1	0	0	1	1	1	1	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	0	0	-	1	1	0	0	1	1	1	1	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	0	1	1	1	1	-	1	1	0	1	0	1	0	1	1
MOTLBO-GT	1	0	1	1	1	1	1	1	1	0	0	0	1	1	-	0	1	1	1	1	1	1	0
MOTLBO-CD	1	0	1	1	1	1	0	1	1	0	0	0	1	1	0	-	1	1	1	1	1	1	0
NSGA-III	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	-	1	0	1	0	1	1
FST-MOPSO-RP-SLSPR	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	-	0	0	1	1	1
FST-MOMA-RP	1	1	1	0	1	1	1	1	0	1	1	1	1	0	1	1	0	0	-	1	0	1	1
FST-MOFA-RP	1	0	0	1	0	0	0	0	1	0	1	1	1	1	1	1	1	0	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	0	1	0	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	1	1	0	1	1	-

Figure 5.22 and Table 5.38 illustrate that FST-MOFA-CD provides the best distribution among the non-dominated solutions, while MOEA/D provides the worst. Table 5.39 reveals that FST-MOFA-CD does not have statistically significant differences with FST-DEMO-CD, FST-MOFA-GT, MOTLBO-GT, MOTLBO-CD and MOTLBO-RP.

Finally, as regards the M_3^* values of each algorithm, Figure 5.23 demonstrates the violin plots of the M_3^* values of each method.

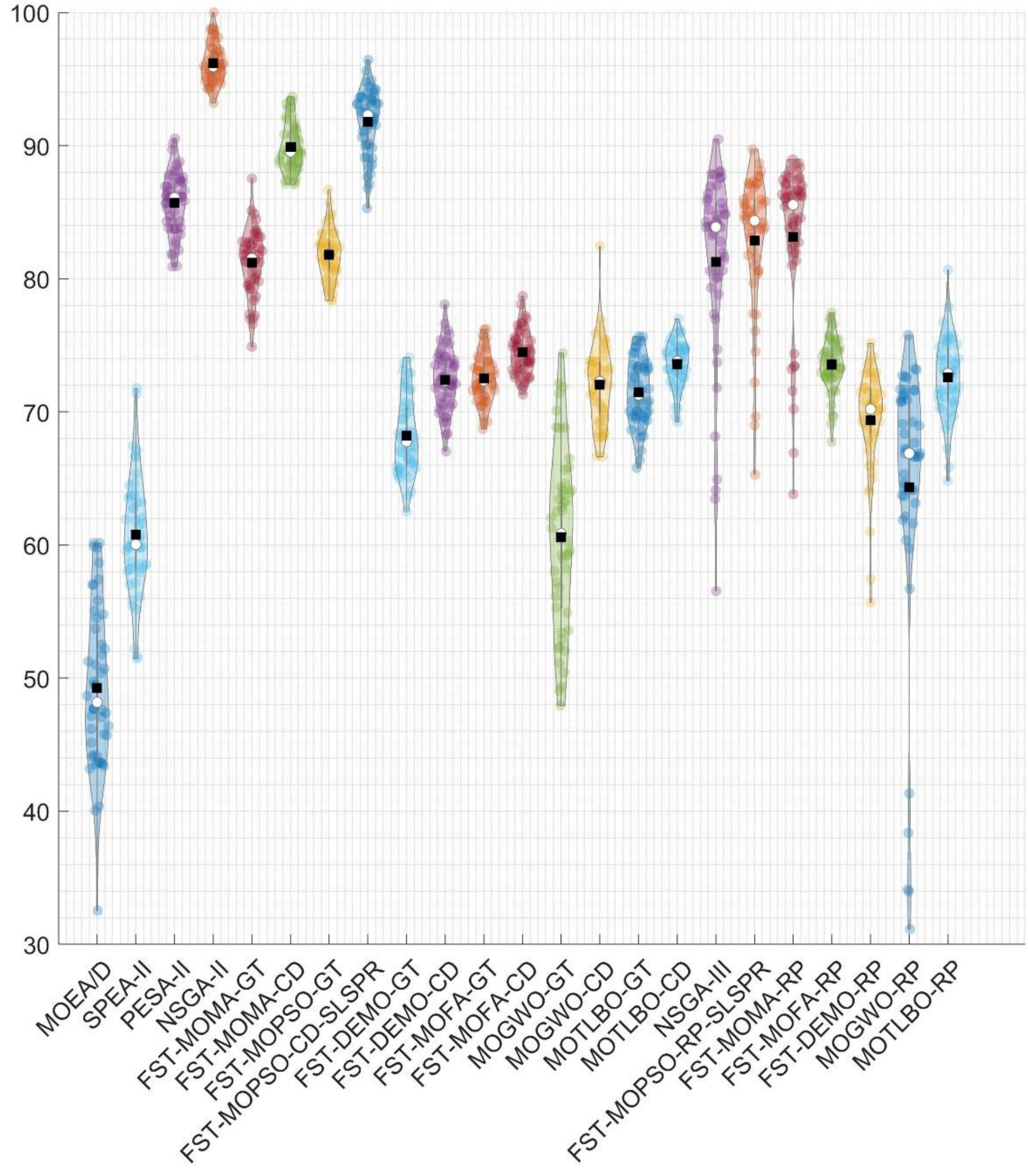


Figure 5.23: Violin plots of M_3^* metric values when using three objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=982.31, p<0.001, ε^2=0.85. Their Median values along with the rest of statistics are demonstrated in Table 5.40.

Table 5.40: Statistics of M_3^* metric values when using three objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	**60.16**	**49.25**	**48.19**	32.53	27.63	6.00
SPEA-II	71.79	60.77	60.05	51.47	20.31	4.37
PESA-II	90.55	85.73	86.09	80.92	9.63	2.30
NSGA-II	**100.00**	**96.21**	**95.95**	**93.22**	6.78	**1.51**
FST-MOMA-GT	87.53	81.22	81.53	74.90	12.64	2.44
FST-MOMA-CD	93.68	89.90	89.54	87.10	**6.58**	1.81
FST-MOPSO-GT	86.70	81.81	81.83	78.34	8.36	1.88

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-CD-SLSPR	96.43	91.78	92.29	85.31	11.13	2.45
FST-DEMO-GT	74.09	68.22	67.77	62.53	11.57	2.86
FST-DEMO-CD	78.07	72.43	72.34	67.05	11.03	2.30
FST-MOFA-GT	76.22	72.50	72.35	68.71	7.51	1.83
FST-MOFA-CD	78.69	74.49	74.46	71.31	7.38	1.65
MOGWO-GT	74.41	60.58	60.85	47.95	26.46	6.66
MOGWO-CD	82.46	72.05	72.27	66.63	15.83	2.99
MOTLBO-GT	75.67	71.47	71.28	65.81	9.86	2.49
MOTLBO-CD	76.98	73.61	73.81	69.25	7.73	1.72
NSGA-III	90.48	81.27	83.89	56.54	33.94	7.21
FST-MOPSO-RP-SLSPR	89.72	82.86	84.37	65.29	24.43	5.27
FST-MOMA-RP	88.97	83.14	85.56	63.82	25.16	5.92
FST-MOFA-RP	77.46	73.56	73.58	67.76	9.70	2.03
FST-DEMO-RP	75.16	69.38	70.21	55.68	19.48	3.89
MOGWO-RP	75.77	64.34	66.89	**31.15**	**44.62**	**10.48**
MOTLBO-RP	80.67	72.60	72.89	64.82	15.85	2.94

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.41, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.41: Pairwise comparisons of M_3^* metric when using three objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	0	0	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	0	1	1	0	0	0	1	1	1	0	1	1	0
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	0	-	1	1	0	0	0	1	1	1	0	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	0	1	1	1
MOGWO-GT	1	0	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	0	0	1	1	-	0	0	1	1	1	0	0	1	0
MOTLBO-GT	1	1	1	1	1	1	1	1	1	0	0	1	1	0	-	1	1	1	1	1	0	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	0	0	0	1	0	1	-	1	1	1	0	1	1	0
NSGA-III	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	-	0	0	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	0	-	0	1	1	1	1
FST-MOMA-RP	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	0	0	0	1	0	1	0	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1	1	1	-	0	1
MOGWO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	1	1	1	0	1	1	-

Figure 5.23 and Table 5.40 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while MOEA/D provides the worst.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.24.

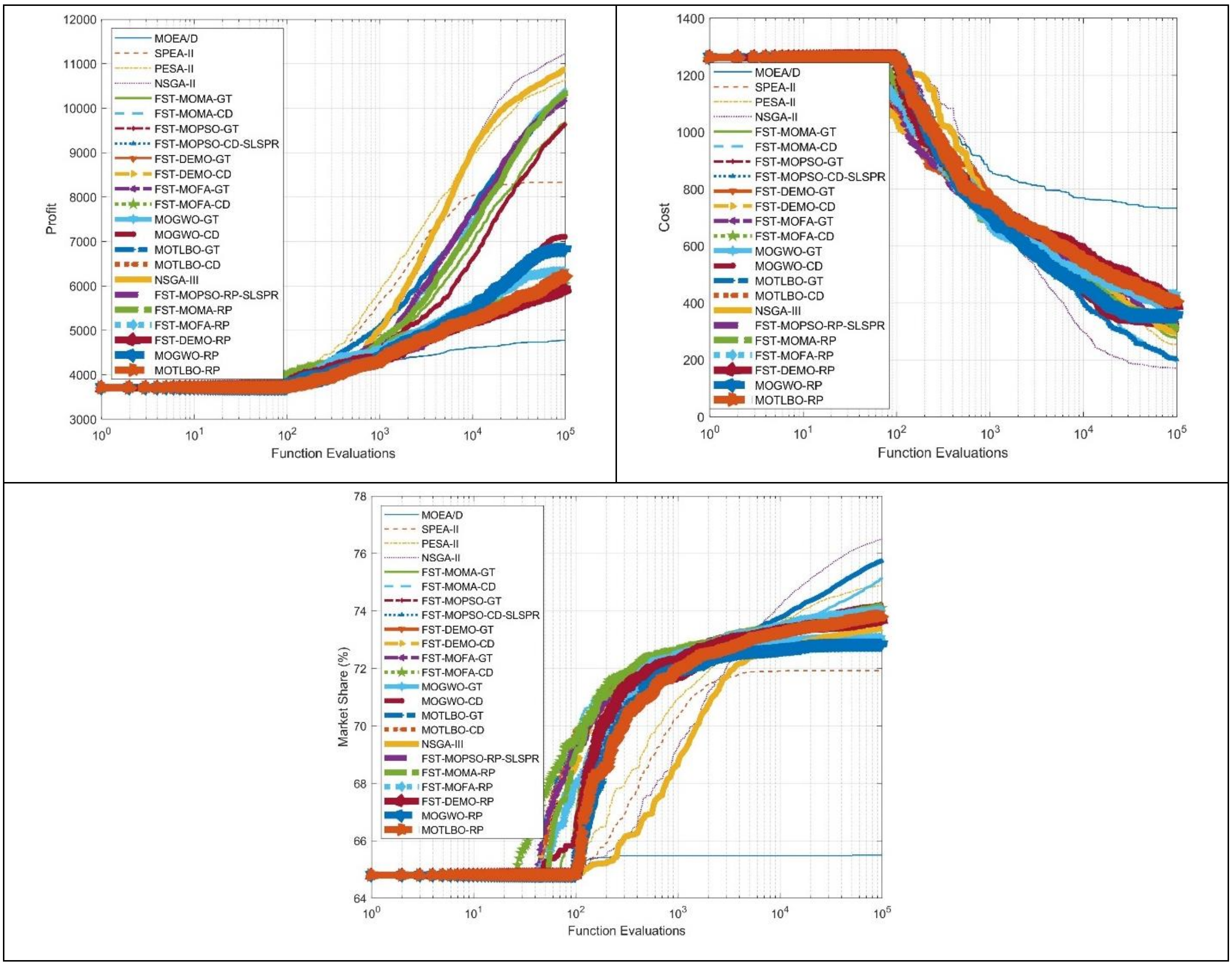


Figure 5.24: Average convergence characteristic curves when using three objectives on the Olive oil data

5.3 Third Scenario: Using four objective functions

In this Subsection the performance of the algorithms when using four objective functions, as presented in Table 5.1, is assessed.

5.3.1 Results on the Timbuk2 data set when using four objectives

Regarding the performance of the comparing algorithms while performing on the Timbuk2 data set when using four objectives, in Tables 5.42 and 5.43 the results of the average *C* metric values of the comparing algorithms, are presented. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U test.

Table 5.42: Comparison of methods according their average C metric values when using four objectives on the Timbuk2 data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.00	0.03	0.00	0.05	0.03	0.10	0.06	0.02	0.02	0.27	0.19
SPEA-II	0.55	1.00	0.47	0.13	0.66	0.51	0.77	0.57	0.52	0.51	0.91	0.88
PESA-II	0.18	0.01	1.00	0.02	0.27	0.12	0.36	0.16	0.14	0.17	0.53	0.53
NSGA-II	0.36	0.05	0.27	1.00	0.41	0.43	0.51	0.50	0.25	0.30	0.60	0.65
FST-MOMA-GT	0.14	0.00	0.05	0.01	1.00	0.06	0.20	0.10	0.06	0.07	0.44	0.42
FST-MOMA-CD	0.18	0.00	0.09	0.04	0.20	1.00	0.29	0.20	0.10	0.13	0.47	0.51
FST-MOPSO-GT	0.11	0.00	0.04	0.00	0.11	0.04	1.00	0.07	0.05	0.06	0.39	0.36
FST-MOPSO-CD-SLSPR	0.18	0.01	0.09	0.04	0.17	0.13	0.25	1.00	0.08	0.11	0.43	0.45
FST-DEMO-GT	0.26	0.01	0.13	0.04	0.24	0.15	0.35	0.21	1.00	0.11	0.64	0.59
FST-DEMO-CD	0.24	0.01	0.14	0.05	0.25	0.17	0.35	0.23	0.16	1.00	0.63	0.64
FST-MOFA-GT	0.05	0.00	0.01	0.00	0.01	0.01	0.06	0.03	0.01	0.00	1.00	0.11
FST-MOFA-CD	0.06	0.00	0.01	0.00	0.02	0.01	0.07	0.03	0.01	0.01	0.21	1.00
MOGWO-GT	0.42	0.04	0.25	0.06	0.42	0.30	0.53	0.34	0.27	0.25	0.79	0.71
MOGWO-CD	0.41	0.03	0.24	0.08	0.39	0.30	0.49	0.37	0.26	0.27	0.72	0.72
MOTLBO-GT	0.09	0.00	0.02	0.01	0.04	0.02	0.09	0.04	0.02	0.01	0.26	0.22
MOTLBO-CD	0.10	0.00	0.03	0.01	0.06	0.03	0.13	0.07	0.04	0.03	0.36	0.34
NSGA-III	0.14	0.02	0.18	0.19	0.29	0.51	0.37	0.51	0.11	0.17	0.34	0.39
FST-MOPSO-RP-SLSPR	0.10	0.00	0.06	0.02	0.14	0.08	0.21	0.11	0.06	0.08	0.28	0.30
FST-MOMA-RP	0.12	0.00	0.09	0.04	0.20	0.15	0.27	0.20	0.08	0.10	0.38	0.40
FST-MOFA-RP	0.06	0.00	0.01	0.00	0.02	0.01	0.05	0.02	0.01	0.00	0.16	0.13
FST-DEMO-RP	0.26	0.00	0.15	0.06	0.28	0.16	0.37	0.25	0.16	0.12	0.66	0.61
MOGWO-RP	0.65	0.09	0.50	0.13	0.64	0.53	0.74	0.59	0.49	0.44	0.91	0.85
MOTLBO-RP	0.11	0.00	0.02	0.00	0.04	0.02	0.10	0.05	0.02	0.01	0.27	0.23

Table 5.43: Comparison of methods according their average C metric values when using four objectives on the Timbuk2 data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.01	0.01	0.16	0.10	0.02	0.18	0.04	0.23	0.02	0.00	0.16
SPEA-II	0.26	0.29	0.87	0.80	0.15	0.59	0.43	0.66	0.43	0.04	0.73
PESA-II	0.06	0.11	0.44	0.40	0.04	0.26	0.13	0.40	0.10	0.01	0.36
NSGA-II	0.10	0.20	0.56	0.56	0.17	0.47	0.31	0.48	0.20	0.02	0.48
FST-MOMA-GT	0.03	0.05	0.33	0.25	0.02	0.22	0.07	0.36	0.05	0.01	0.29
FST-MOMA-CD	0.04	0.09	0.40	0.37	0.04	0.28	0.12	0.39	0.08	0.01	0.33
FST-MOPSO-GT	0.02	0.05	0.28	0.24	0.02	0.17	0.05	0.30	0.04	0.01	0.24
FST-MOPSO-CD-SLSPR	0.03	0.08	0.36	0.32	0.05	0.27	0.10	0.37	0.07	0.01	0.31
FST-DEMO-GT	0.05	0.07	0.52	0.40	0.05	0.38	0.18	0.51	0.12	0.01	0.46
FST-DEMO-CD	0.06	0.10	0.54	0.46	0.06	0.37	0.18	0.52	0.13	0.01	0.47
FST-MOFA-GT	0.00	0.00	0.09	0.05	0.01	0.10	0.01	0.15	0.01	0.00	0.09
FST-MOFA-CD	0.01	0.02	0.14	0.07	0.01	0.11	0.02	0.20	0.01	0.00	0.14
MOGWO-GT	1.00	0.10	0.68	0.57	0.11	0.50	0.28	0.56	0.23	0.02	0.58
MOGWO-CD	0.11	1.00	0.64	0.58	0.11	0.47	0.27	0.57	0.23	0.02	0.56
MOTLBO-GT	0.01	0.01	1.00	0.09	0.01	0.15	0.03	0.24	0.02	0.00	0.16
MOTLBO-CD	0.02	0.04	0.24	1.00	0.02	0.18	0.06	0.31	0.03	0.00	0.22
NSGA-III	0.04	0.11	0.32	0.34	1.00	0.35	0.28	0.26	0.08	0.02	0.25
FST-MOPSO-RP-SLSPR	0.02	0.06	0.22	0.23	0.02	1.00	0.06	0.30	0.05	0.02	0.24

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOMA-RP	0.02	0.07	0.30	0.30	0.03	0.31	1.00	0.40	0.07	0.02	0.33
FST-MOFA-RP	0.00	0.01	0.10	0.05	0.01	0.09	0.01	1.00	0.01	0.00	0.09
FST-DEMO-RP	0.05	0.10	0.54	0.43	0.06	0.37	0.19	0.59	1.00	0.01	0.50
MOGWO-RP	0.21	0.19	0.85	0.80	0.13	0.67	0.35	0.76	0.34	1.00	0.77
MOTLBO-RP	0.01	0.02	0.18	0.10	0.01	0.13	0.03	0.29	0.02	0.00	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.25 demonstrates the violin plots which show the distribution of time each method needed. A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)=1,144.79$, $p<0.001$, $\varepsilon^2=1.00$. Their Median values along with the rest of statistics are demonstrated in Table 5.44.

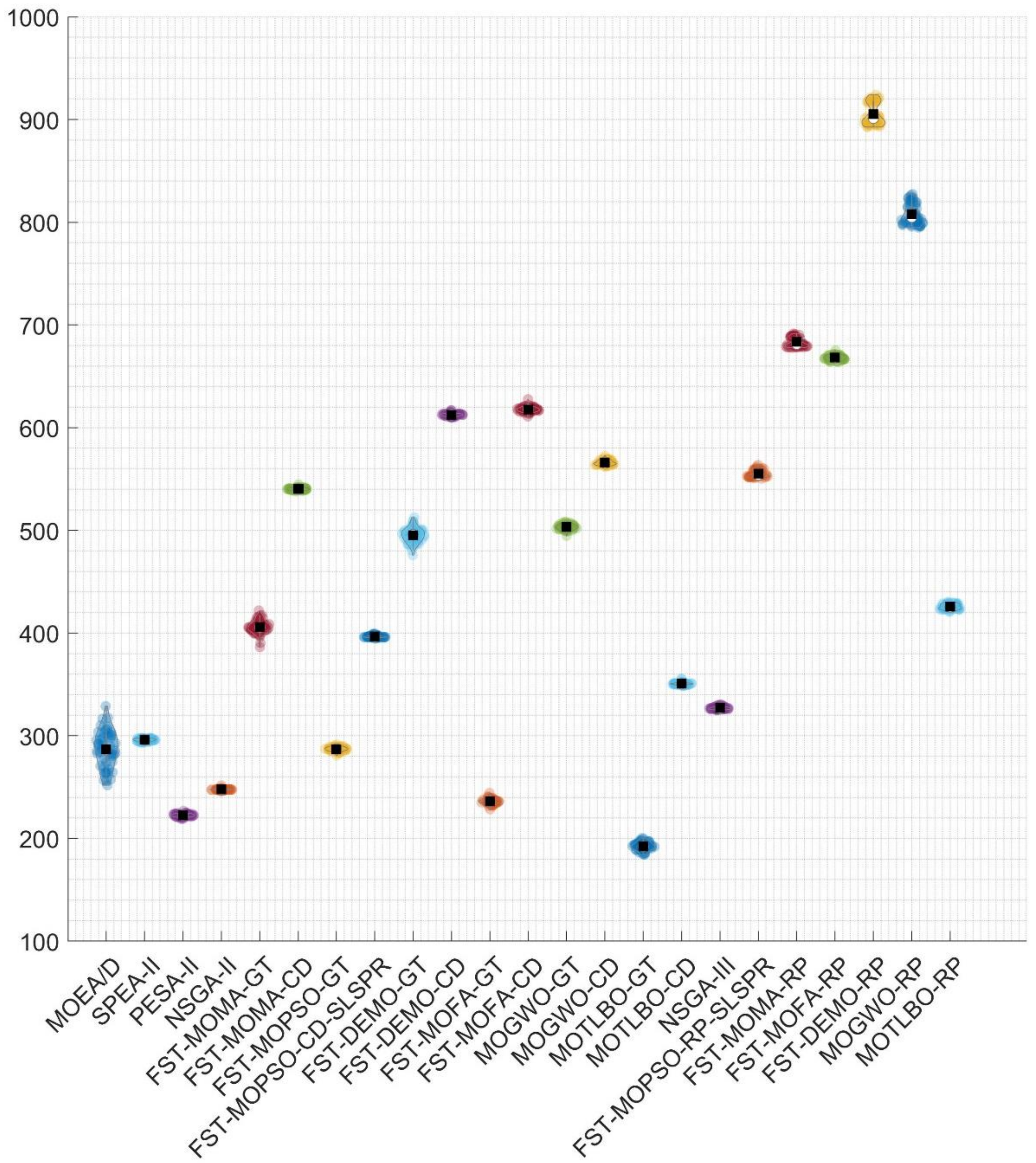


Figure 5.25: Violin plots of *Time* values when using four objectives on the Timbuk2 data

Table 5.44: Statistics of *Time* when using four objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	328.79	286.90	286.52	251.82	**76.97**	**17.79**
SPEA-II	298.02	295.94	295.94	293.17	**4.85**	1.32
PESA-II	226.88	222.61	222.62	219.03	7.85	1.58
NSGA-II	251.67	247.69	247.63	246.16	5.51	**0.83**
FST-MOMA-GT	421.91	406.01	405.58	386.34	35.57	6.66
FST-MOMA-CD	544.83	540.34	540.35	537.88	6.95	1.23
FST-MOPSO-GT	291.49	287.04	287.23	280.78	10.71	2.15
FST-MOPSO-CD-SLSPR	399.72	396.62	396.33	394.64	5.08	1.08
FST-DEMO-GT	512.66	495.32	496.05	475.82	36.84	7.41
FST-DEMO-CD	616.98	612.55	612.56	609.74	7.25	1.57
FST-MOFA-GT	244.35	236.01	236.09	228.41	15.94	2.75
FST-MOFA-CD	627.64	617.77	617.80	610.85	16.80	2.62
MOGWO-GT	508.34	503.14	503.27	494.72	13.62	2.69
MOGWO-CD	572.66	565.97	565.39	561.80	10.86	2.35
MOTLBO-GT	**200.30**	**192.34**	**192.26**	**184.33**	15.97	3.95
MOTLBO-CD	355.68	350.67	350.60	348.56	7.12	1.24
NSGA-III	330.38	327.10	326.80	324.85	5.53	1.37
FST-MOPSO-RP-SLSPR	563.70	555.17	553.95	550.34	13.36	3.51
FST-MOMA-RP	691.55	683.39	681.24	677.83	13.72	4.65
FST-MOFA-RP	675.24	668.16	667.89	663.80	11.44	2.40
FST-DEMO-RP	**924.08**	**905.38**	**901.12**	**892.60**	31.48	10.75
MOGWO-RP	827.38	807.73	804.89	794.81	32.57	10.12
MOTLBO-RP	430.33	425.72	425.16	420.66	9.67	2.55

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.45, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.45: Pairwise comparisons of *Time* when using four objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	0	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.25 and Table 5.44 illustrate that MOTLBO-GT needs the least time, while FST-DEMO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.26 demonstrates the violin plots of the M_1^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)$=885.04, p<0.001, ε^2=0.77. Their Median values along with the rest of statistics are demonstrated in Table 5.46.

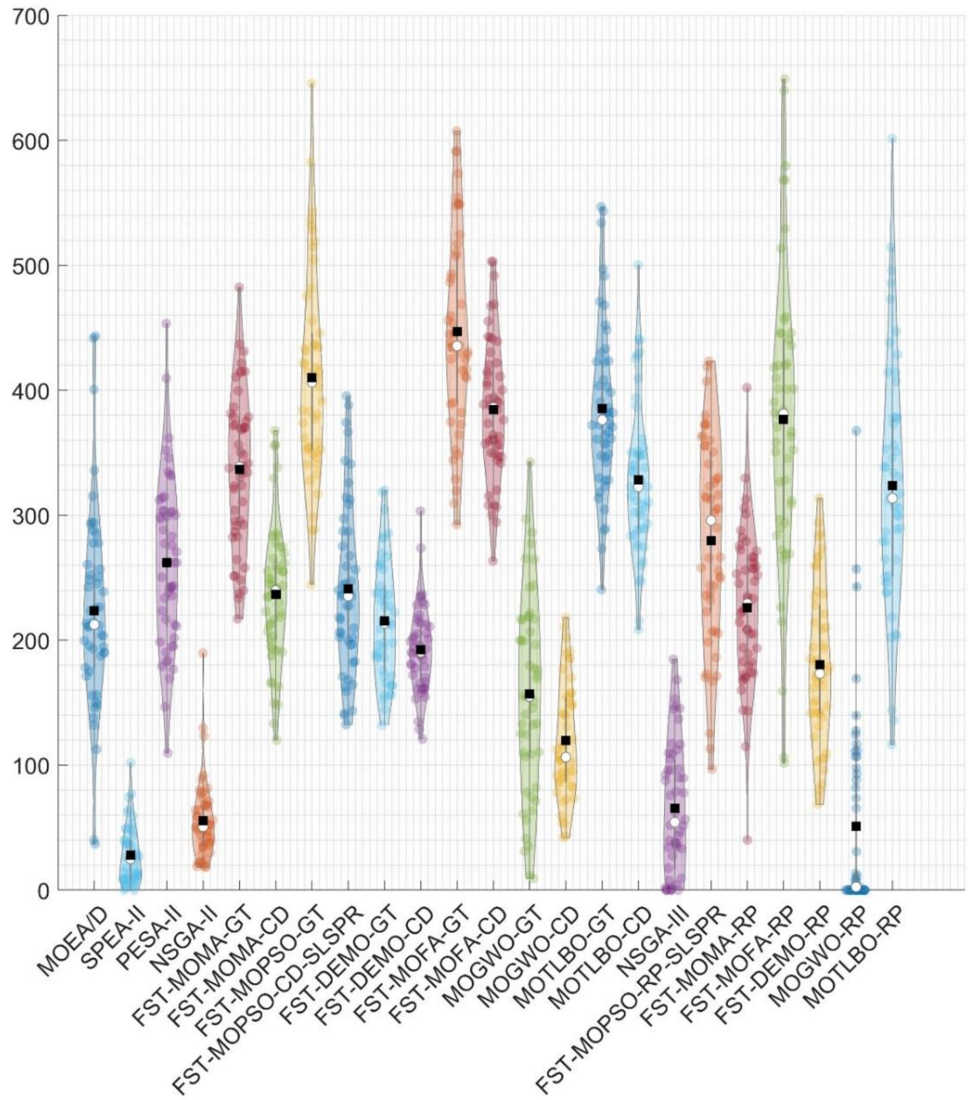


Figure 5.26: Violin plots of M_1^* metric values when using four objectives on the Timbuk2 data

Table 5.46: Statistics of M_1^* values when using four objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	443.29	223.56	212.47	37.00	406.29	80.89
SPEA-II	**101.92**	**27.95**	24.98	**0.00**	**101.92**	**23.05**
PESA-II	453.39	262.21	262.03	109.68	343.71	67.92
NSGA-II	189.57	55.40	50.79	18.25	171.32	31.45
FST-MOMA-GT	482.39	336.66	338.72	217.10	265.30	62.01
FST-MOMA-CD	367.66	236.67	239.69	120.05	247.61	55.74
FST-MOPSO-GT	645.31	409.96	406.44	244.72	400.59	82.08
FST-MOPSO-CD-SLSPR	395.38	241.06	235.36	132.50	262.88	67.42
FST-DEMO-GT	319.91	215.53	213.56	131.93	187.98	47.04
FST-DEMO-CD	303.28	192.43	190.37	120.92	182.36	34.10
FST-MOFA-GT	607.31	**447.10**	**435.54**	**292.67**	314.64	78.09
FST-MOFA-CD	503.06	384.55	386.12	263.38	239.68	55.46
MOGWO-GT	342.55	157.06	154.98	9.45	333.10	79.78
MOGWO-CD	218.08	119.56	106.44	42.06	176.01	45.76
MOTLBO-GT	546.85	385.21	376.27	240.55	306.29	67.27
MOTLBO-CD	500.23	328.53	322.66	208.51	291.72	59.73
NSGA-III	184.72	65.41	54.35	**0.00**	184.72	52.41
FST-MOPSO-RP-SLSPR	423.23	279.73	295.90	96.84	326.39	81.80
FST-MOMA-RP	402.06	226.14	229.21	40.16	361.91	61.02
FST-MOFA-RP	**649.09**	376.71	381.14	101.65	**547.44**	**118.10**
FST-DEMO-RP	313.53	180.46	173.39	68.68	244.86	63.03
MOGWO-RP	367.59	51.31	**2.60**	**0.00**	367.59	79.98
MOTLBO-RP	601.36	323.60	313.65	116.56	484.80	99.84

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.47, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.47: Pairwise comparisons of M_1^* when using four objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	0	1	1	0	1	0	0	0	1	1	1	1	1	1	1	0	0	1	0	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1
PESA-II	0	1	-	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	0
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	0	1	1	0	0	1	0	1	0	1	1	0
FST-MOMA-CD	0	1	0	1	1	-	1	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	0	0	1	1	0	1	1	1	1	0	1	1	1
FST-MOPSO-CD-SLSPR	0	1	0	1	1	0	1	-	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-DEMO-GT	0	1	1	1	1	0	1	0	-	0	1	1	1	1	1	1	1	1	0	1	0	1	1
FST-DEMO-CD	0	1	1	1	1	1	1	1	0	-	1	1	0	1	1	1	1	1	0	1	0	1	1
FST-MOFA-GT	1	1	1	1	1	1	0	1	1	1	-	1	1	1	1	1	1	1	1	0	1	1	1
FST-MOFA-CD	1	1	1	1	0	1	0	1	1	1	1	-	1	1	0	1	1	1	1	0	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	0	1	1	-	0	1	1	1	1	1	1	0	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	0	1	0	1	1	1	1	0	1	1	-	1	1	1	1	0	1	1	1
MOTLBO-CD	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	-	1	0	1	0	1	1	0
NSGA-III	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	0	1
FST-MOPSO-RP-SLSPR	0	1	0	1	0	0	1	0	1	1	1	1	1	1	1	0	1	-	0	1	1	1	0
FST-MOMA-RP	0	1	0	1	1	0	1	0	0	0	1	1	1	1	1	1	1	0	-	1	0	1	1
FST-MOFA-RP	1	1	1	1	0	1	0	1	1	1	0	0	1	1	0	0	1	1	1	-	1	1	0

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-DEMO-RP	0	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	-	1	1
MOGWO-RP	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	-	1
MOTLBO-RP	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	1	1	-

Figure 5.26 and Table 5.46 illustrate that MOGWO-RP provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-GT provides the farthest ones. Table 5.47 reveals that MOGWO-RP does not have statistically significant differences with PESA-II, NSGA-II and NSGA-III, while FST-MOFA-GT does not have statistically significant differences with FST-MOPSO-GT and FST-MOFA-RP.

As regards the *Δ* values of each algorithm, Figure 5.27 demonstrates the violin plots of the *Δ* metric values, of each method.

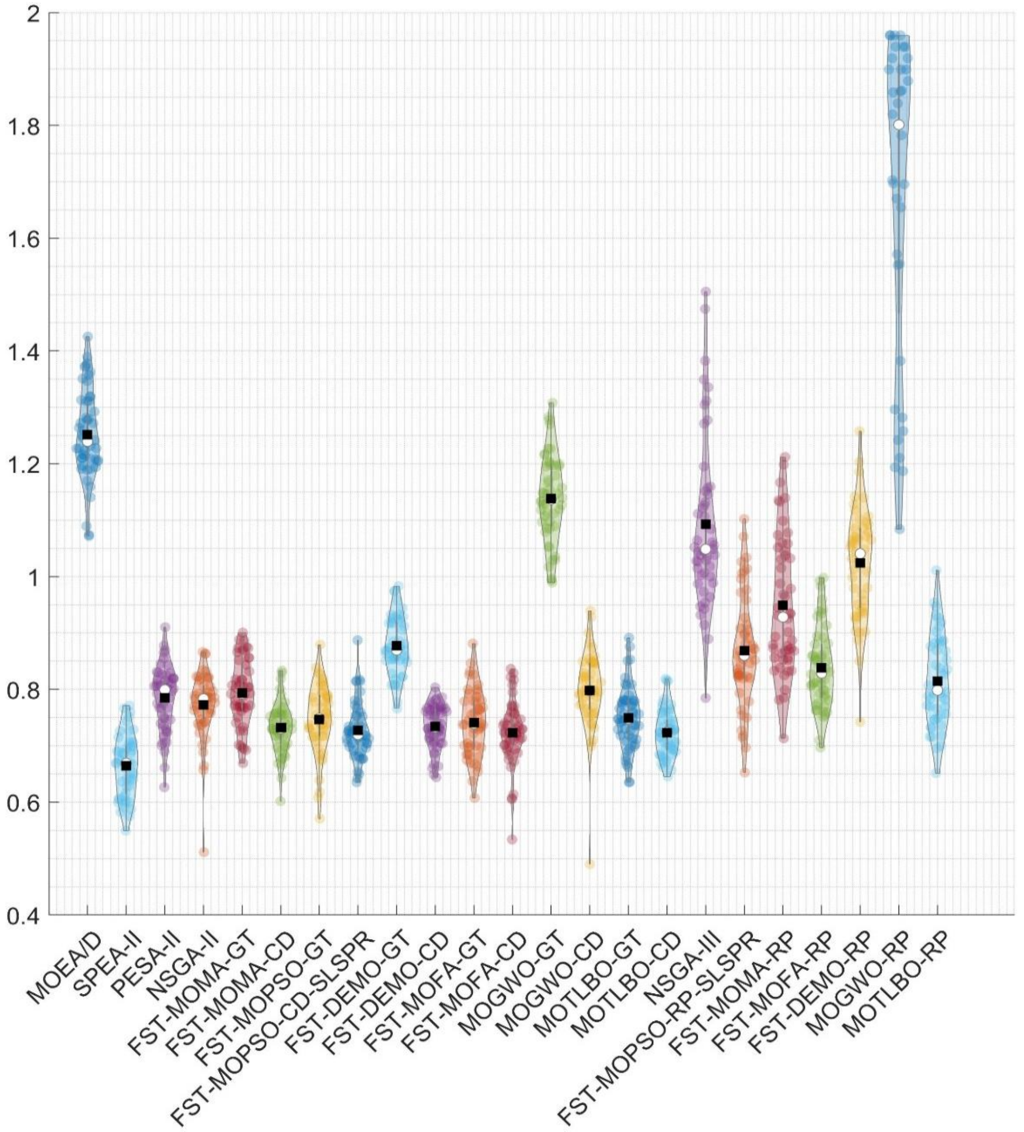


Figure 5.27: Violin plots of *Δ* metric values when using four objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)=837.59$, $p<0.001$, $\varepsilon^2=0.74$. Their Median values along with the rest of statistics are demonstrated in Table 5.48.

Table 5.48: Statistics of *Δ* metric values when using four objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.43	1.25	1.24	1.07	0.35	0.08
SPEA-II	0.77	0.66	0.67	0.55	0.22	0.05
PESA-II	0.91	0.79	0.80	0.63	0.28	0.06
NSGA-II	0.87	0.77	0.78	0.51	0.35	0.06
FST-MOMA-GT	0.90	0.79	0.79	0.67	0.23	0.06
FST-MOMA-CD	0.83	0.73	0.73	0.60	0.23	0.04
FST-MOPSO-GT	0.88	0.75	0.75	0.57	0.31	0.06
FST-MOPSO-CD-SLSPR	0.89	0.73	0.72	0.64	0.25	0.05
FST-DEMO-GT	0.98	0.88	0.87	0.77	0.22	0.05
FST-DEMO-CD	0.80	0.73	0.73	0.64	0.16	0.04
FST-MOFA-GT	0.88	0.74	0.74	0.61	0.27	0.06
FST-MOFA-CD	0.84	0.72	0.72	0.53	0.30	0.06
MOGWO-GT	1.31	1.14	1.14	0.99	0.32	0.08
MOGWO-CD	0.94	0.80	0.80	0.49	0.45	0.07
MOTLBO-GT	0.89	0.75	0.75	0.64	0.26	0.06
MOTLBO-CD	0.82	0.72	0.72	0.65	0.17	0.04
NSGA-III	1.51	1.09	1.05	0.78	0.72	0.15
FST-MOPSO-RP-SLSPR	1.10	0.87	0.86	0.65	0.45	0.10
FST-MOMA-RP	1.21	0.95	0.93	0.71	0.50	0.12
FST-MOFA-RP	1.00	0.84	0.83	0.70	0.30	0.07
FST-DEMO-RP	1.26	1.02	1.04	0.74	0.52	0.10
MOGWO-RP	1.96	1.68	1.80	1.08	0.88	0.28
MOTLBO-RP	1.01	0.81	0.80	0.65	0.36	0.08

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.49, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between groups. Figure 5.27 and Table 5.48 illustrate that SPEA-II provides the best distribution among the non-dominated solutions, while MOEA/D provides the worst.

Table 5.49: Pairwise comparisons of *Δ* metric when using four objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	0	0	1	0	1	1	1	1	1	1	0	0	1	1	1	1	0	1	1	0
NSGA-II	1	1	0	-	0	1	0	1	1	1	0	1	1	0	0	1	1	1	1	1	1	1	0
FST-MOMA-GT	1	1	0	0	-	1	0	1	1	1	1	1	1	0	0	1	1	1	1	0	1	1	0
FST-MOMA-CD	1	1	1	1	1	-	0	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	0	0	0	-	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOPSO-CD-SLSPR	1	1	1	1	1	0	0	-	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	0	0	1	1	1
FST-DEMO-CD	1	1	1	1	1	0	0	0	1	-	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	0	1	0	0	0	1	0	-	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	0	0	0	1	0	0	-	1	1	0	0	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	1	1	1
MOGWO-CD	1	1	0	0	0	1	1	1	1	1	1	1	1	-	1	1	1	1	1	0	1	1	0
MOTLBO-GT	1	1	0	0	0	0	0	0	1	0	0	0	1	1	-	0	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	0	0	0	1	0	0	0	1	1	0	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	1	1	0	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	0	0	1	1	0
FST-MOMA-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	-	1	0	1	1
FST-MOFA-RP	1	1	0	1	0	1	1	1	0	1	1	1	1	0	1	1	1	0	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1	0	1	0	1	1	-

Finally, as regards the M_3^* values of each algorithm, Figure 5.28 demonstrates the violin plots of the M_3^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=887.17, p<0.001, ε^2=0.77. Their Median values along with the rest of statistics are demonstrated in Table 5.50. Figure 5.28 and Table 5.50 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while MOGWO-RP provides the worst. Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.51, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

To check how each algorithm converges towards each objective functions' optima, their average convergence characteristic curves, are demonstrated in Figure 5.29.

Table 5.50: Statistics of M_3^* metric values when using four objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	80.78	72.69	72.05	63.87	16.91	4.28
SPEA-II	**76.42**	68.85	68.89	59.55	16.88	3.89
PESA-II	93.34	87.77	87.62	82.23	11.11	2.05
NSGA-II	**100.00**	**98.69**	**98.78**	**97.01**	**2.99**	0.75
FST-MOMA-GT	90.92	87.36	87.80	83.40	7.52	1.80
FST-MOMA-CD	98.86	96.81	96.77	95.04	3.82	**0.67**
FST-MOPSO-GT	90.20	87.23	87.10	81.90	8.30	1.71
FST-MOPSO-CD-SLSPR	99.73	97.25	97.22	92.52	7.21	1.09
FST-DEMO-GT	85.61	82.26	82.50	78.42	7.19	1.51
FST-DEMO-CD	87.83	85.86	85.80	83.79	4.04	1.10
FST-MOFA-GT	83.81	81.13	81.17	75.24	8.57	1.66
FST-MOFA-CD	87.34	84.87	84.70	82.88	4.46	1.05
MOGWO-GT	82.17	72.24	73.35	59.15	23.02	6.01
MOGWO-CD	90.33	84.71	84.36	82.78	7.55	1.45
MOTLBO-GT	85.99	83.06	82.96	80.15	5.84	1.52

Method	Max	Mean	Median	Min	Range	SD
MOTLBO-CD	88.60	86.27	86.28	83.99	4.61	1.12
NSGA-III	95.15	78.36	86.58	42.77	52.38	17.38
FST-MOPSO-RP-SLSPR	95.51	84.43	85.99	73.25	22.26	7.58
FST-MOMA-RP	96.71	82.86	80.36	64.08	32.63	7.69
FST-MOFA-RP	86.93	77.52	78.28	64.58	22.35	5.07
FST-DEMO-RP	86.77	79.66	79.34	73.55	13.22	3.28
MOGWO-RP	81.55	**41.04**	**53.05**	**0.00**	**81.55**	**29.45**
MOTLBO-RP	85.71	78.77	79.37	69.12	16.59	4.32

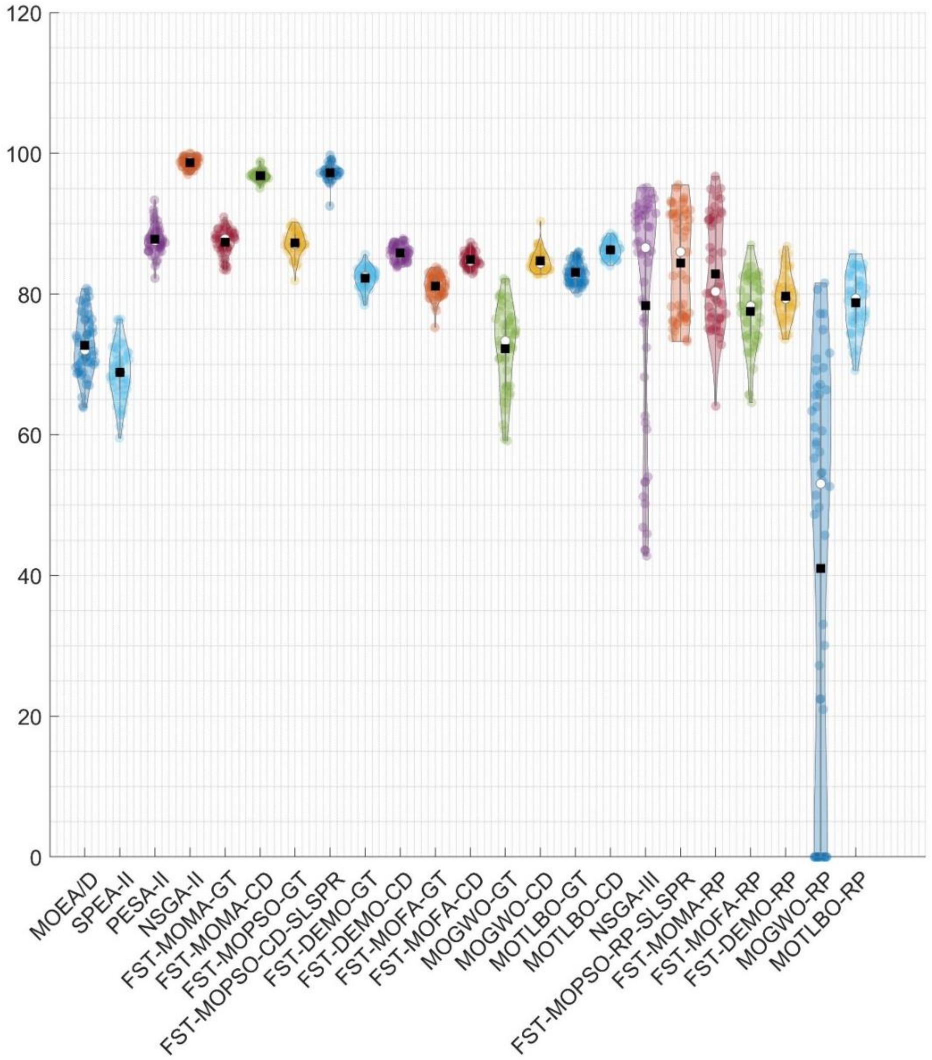


Figure 5.28: Violin plots of M_3^* metric values when using four objectives on the Timbuk2 data

Table 5.51: Pairwise comparisons of M_3^* metric when using four objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	0	1	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	1	0	1	-	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	0	1	1	1	0	1	0	0	0	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	0	0	0	0	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	0	1	-	1	1	1	1	1	0	0	0	1	0	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	0	1	1	0	0	0	1	1	1	1
MOGWO-GT	0	0	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	0	1	-	1	1	0	0	0	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	0	1	1	1	1	1	-	1	0	0	0	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	-	0	0	0	1	1	1	1
NSGA-III	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	-	0	0	0	0	1	0
FST-MOPSO-RP-SLSPR	1	1	0	1	0	1	0	1	0	0	0	0	1	0	0	0	0	-	0	1	0	1	0
FST-MOMA-RP	1	1	0	1	0	1	0	1	0	0	0	0	1	0	0	0	0	0	-	0	0	1	0
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	-	0	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0	-	1	0
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0	0	1	-

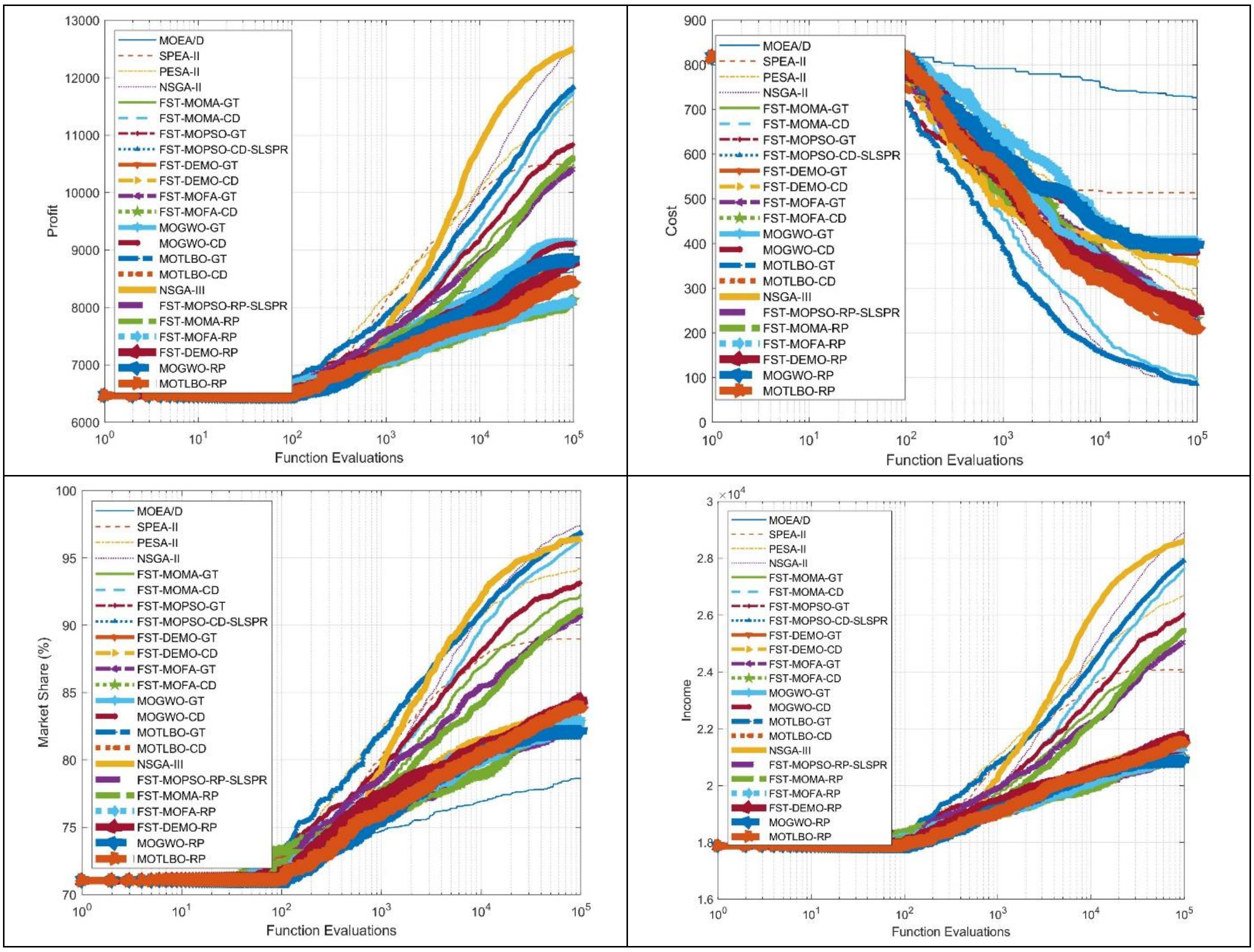


Figure 5.29: Average convergence characteristic curves when using four objectives on the Timbuk2 data

5.3.2 *Results on the Olive oil data set when using four objectives*

Regarding the performance of the comparing algorithms while performing on the olive oil data set when using four objectives, Tables 5.52-5.53 show the results of the average *C* metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U Test.

Table 5.52: Comparison of methods according their average C metric values when using four objectives on the Olive oil data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.00	0.01	0.01	0.04	0.05	0.05	0.05	0.25	0.22	0.35	0.31
SPEA-II	0.69	1.00	0.43	0.34	0.67	0.62	0.70	0.62	0.92	0.88	0.96	0.93
PESA-II	0.21	0.00	1.00	0.13	0.24	0.19	0.27	0.23	0.44	0.46	0.60	0.59
NSGA-II	0.16	0.01	0.13	1.00	0.23	0.23	0.25	0.25	0.29	0.34	0.39	0.42
FST-MOMA-GT	0.19	0.00	0.07	0.10	1.00	0.14	0.18	0.16	0.42	0.39	0.54	0.54
FST-MOMA-CD	0.14	0.00	0.10	0.10	0.20	1.00	0.22	0.20	0.28	0.33	0.42	0.44
FST-MOPSO-GT	0.18	0.00	0.07	0.09	0.14	0.13	1.00	0.16	0.39	0.37	0.53	0.52
FST-MOPSO-CD-SLSPR	0.13	0.00	0.09	0.09	0.17	0.14	0.19	1.00	0.27	0.30	0.38	0.40
FST-DEMO-GT	0.10	0.00	0.04	0.09	0.06	0.09	0.07	0.11	1.00	0.14	0.32	0.31
FST-DEMO-CD	0.10	0.00	0.03	0.08	0.07	0.09	0.07	0.12	0.24	1.00	0.37	0.34
FST-MOFA-GT	0.05	0.00	0.01	0.06	0.03	0.04	0.03	0.08	0.10	0.07	1.00	0.16
FST-MOFA-CD	0.05	0.00	0.01	0.06	0.03	0.04	0.04	0.08	0.12	0.10	0.22	1.00
MOGWO-GT	0.42	0.01	0.25	0.25	0.39	0.40	0.42	0.46	0.74	0.65	0.84	0.79
MOGWO-CD	0.37	0.03	0.22	0.22	0.40	0.37	0.41	0.39	0.66	0.67	0.76	0.73
MOTLBO-GT	0.08	0.00	0.02	0.07	0.04	0.07	0.05	0.09	0.16	0.11	0.27	0.24
MOTLBO-CD	0.08	0.00	0.02	0.07	0.04	0.06	0.04	0.08	0.18	0.13	0.29	0.25
NSGA-III	0.09	0.00	0.32	0.26	0.43	0.48	0.46	0.54	0.19	0.34	0.45	0.54
FST-MOPSO-RP-SLSPR	0.08	0.00	0.15	0.13	0.24	0.21	0.27	0.28	0.23	0.32	0.37	0.43
FST-MOMA-RP	0.07	0.00	0.19	0.14	0.25	0.25	0.29	0.31	0.22	0.32	0.35	0.43
FST-MOFA-RP	0.05	0.00	0.02	0.07	0.03	0.04	0.04	0.10	0.11	0.08	0.21	0.17
FST-DEMO-RP	0.11	0.00	0.04	0.11	0.05	0.08	0.06	0.14	0.25	0.16	0.38	0.31
MOGWO-RP	0.31	0.00	0.21	0.23	0.34	0.35	0.46	0.31	0.66	0.62	0.75	0.76
MOTLBO-RP	0.09	0.00	0.02	0.07	0.04	0.06	0.05	0.11	0.17	0.11	0.29	0.24

Table 5.53: Comparison of methods according their average C metric values when using four objectives on the Olive oil data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.01	0.01	0.25	0.22	0.01	0.12	0.11	0.23	0.18	0.01	0.21
SPEA-II	0.23	0.24	0.92	0.90	0.26	0.64	0.51	0.75	0.74	0.12	0.86
PESA-II	0.03	0.08	0.50	0.54	0.07	0.26	0.20	0.51	0.35	0.04	0.49
NSGA-II	0.03	0.07	0.33	0.38	0.08	0.23	0.19	0.36	0.25	0.04	0.32
FST-MOMA-GT	0.02	0.04	0.46	0.46	0.05	0.23	0.18	0.45	0.33	0.03	0.44
FST-MOMA-CD	0.02	0.06	0.34	0.39	0.06	0.20	0.17	0.38	0.24	0.03	0.34
FST-MOPSO-GT	0.02	0.05	0.44	0.46	0.05	0.22	0.17	0.44	0.31	0.03	0.43
FST-MOPSO-CD-SLSPR	0.01	0.05	0.32	0.36	0.05	0.18	0.14	0.35	0.24	0.03	0.30

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-DEMO-GT	0.00	0.01	0.25	0.21	0.02	0.15	0.11	0.26	0.19	0.01	0.25
FST-DEMO-CD	0.00	0.01	0.30	0.26	0.02	0.15	0.12	0.31	0.20	0.01	0.29
FST-MOFA-GT	0.00	0.00	0.13	0.10	0.01	0.09	0.08	0.17	0.08	0.00	0.13
FST-MOFA-CD	0.00	0.01	0.16	0.13	0.01	0.08	0.07	0.20	0.10	0.01	0.16
MOGWO-GT	1.00	0.08	0.79	0.74	0.14	0.48	0.36	0.64	0.58	0.05	0.71
MOGWO-CD	0.11	1.00	0.72	0.71	0.13	0.39	0.32	0.62	0.53	0.07	0.65
MOTLBO-GT	0.00	0.00	1.00	0.16	0.02	0.12	0.10	0.24	0.13	0.01	0.19
MOTLBO-CD	0.00	0.01	0.20	1.00	0.02	0.11	0.08	0.26	0.14	0.01	0.21
NSGA-III	0.01	0.07	0.27	0.47	1.00	0.37	0.34	0.45	0.22	0.04	0.35
FST-MOPSO-RP-SLSPR	0.01	0.03	0.28	0.37	0.06	1.00	0.20	0.45	0.24	0.02	0.35
FST-MOMA-RP	0.01	0.03	0.28	0.37	0.07	0.20	1.00	0.44	0.25	0.02	0.34
FST-MOFA-RP	0.00	0.01	0.16	0.12	0.01	0.07	0.06	1.00	0.13	0.01	0.15
FST-DEMO-RP	0.00	0.01	0.31	0.22	0.02	0.15	0.12	0.31	1.00	0.02	0.28
MOGWO-RP	0.06	0.07	0.72	0.73	0.06	0.40	0.31	0.67	0.62	1.00	0.71
MOTLBO-RP	0.00	0.01	0.20	0.17	0.01	0.11	0.09	0.27	0.16	0.01	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.30 demonstrates the violin plots which show the distribution of time each method needed.

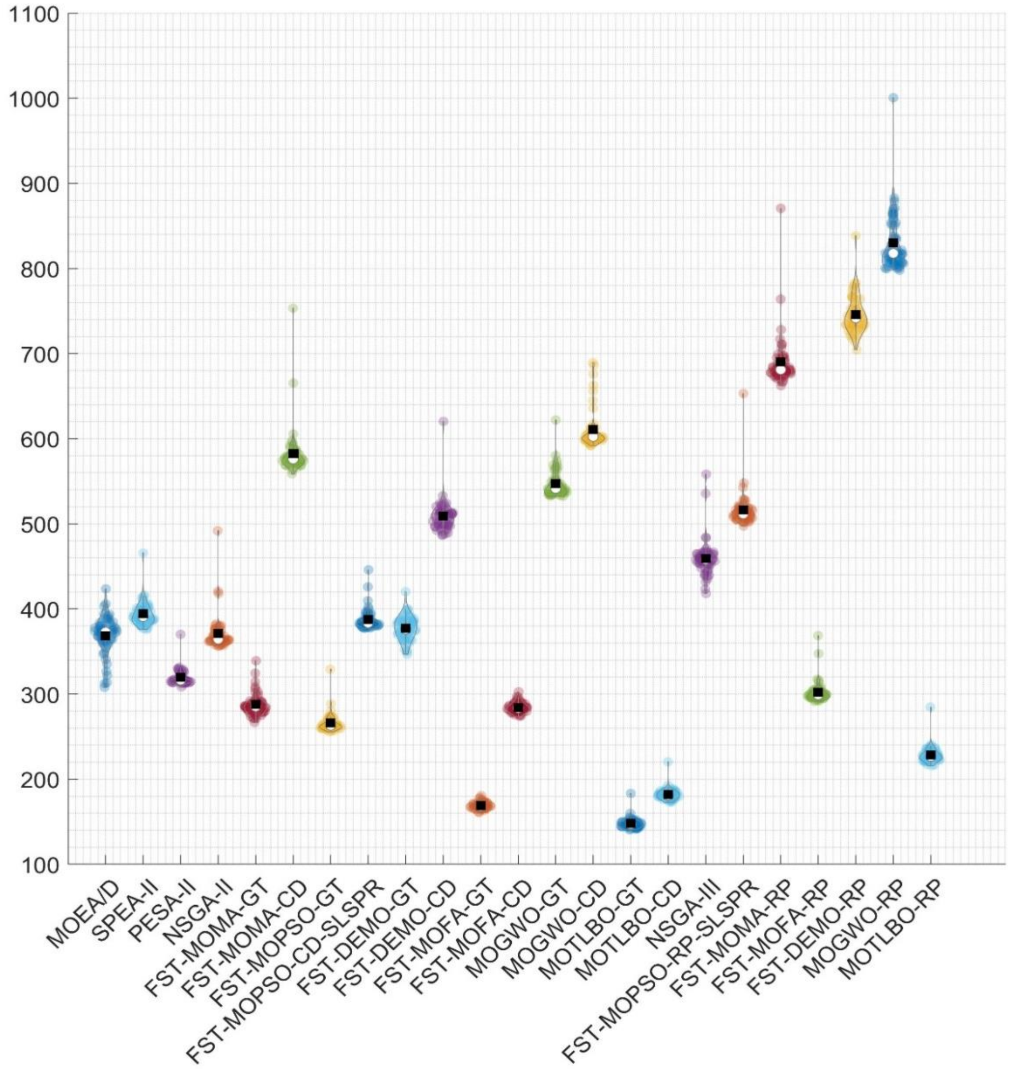


Figure 5.30: Violin plots of *Time* values when using four objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,134.28, p<0.001, ε^2=0.99. Their Median values along with the rest of statistics are demonstrated in Table 5.54.

Table 5.54: Statistics of *Time* when using four objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	423.62	368.40	372.85	307.92	115.70	23.65
SPEA-II	465.66	394.24	390.80	375.91	89.75	14.37
PESA-II	370.16	319.68	316.37	308.48	61.68	9.34
NSGA-II	491.95	371.20	364.56	356.33	135.62	21.49
FST-MOMA-GT	339.06	287.82	285.36	266.66	72.40	13.45
FST-MOMA-CD	753.35	582.75	576.03	558.73	194.62	29.08
FST-MOPSO-GT	329.14	265.88	263.01	255.70	73.44	11.18
FST-MOPSO-CD-SLSPR	445.92	387.57	383.55	377.28	68.64	12.33
FST-DEMO-GT	420.36	377.30	376.17	346.78	73.59	14.19
FST-DEMO-CD	620.35	509.07	508.84	486.39	133.96	19.35
FST-MOFA-GT	180.74	168.99	168.50	160.91	19.83	4.12
FST-MOFA-CD	302.67	284.09	283.43	273.69	28.98	6.02
MOGWO-GT	622.03	547.44	541.47	532.42	89.62	16.55
MOGWO-CD	689.35	610.85	602.54	591.64	97.72	23.64
MOTLBO-GT	183.31	147.99	146.99	140.62	42.69	6.48
MOTLBO-CD	220.42	182.17	181.19	173.03	47.39	7.12
NSGA-III	558.35	459.10	458.06	418.12	140.23	22.34
FST-MOPSO-RP-SLSPR	653.12	516.41	511.61	497.42	155.71	21.96
FST-MOMA-RP	870.73	690.16	681.10	662.26	208.47	31.31
FST-MOFA-RP	368.76	302.29	299.08	291.49	77.27	13.08
FST-DEMO-RP	838.53	745.66	741.82	704.18	134.35	22.61
MOGWO-RP	1000.69	830.34	817.84	798.02	202.68	34.71
MOTLBO-RP	284.59	228.52	226.88	215.80	68.79	10.07

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.55, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.55: Pairwise comparisons of *Time* when using four objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	0	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	0	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	0	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	0	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.30 and Table 5.54 illustrate that MOTLBO-GT needs the least time, while MOGWO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.31 demonstrates the violin plots of the M_1^* values of each method.

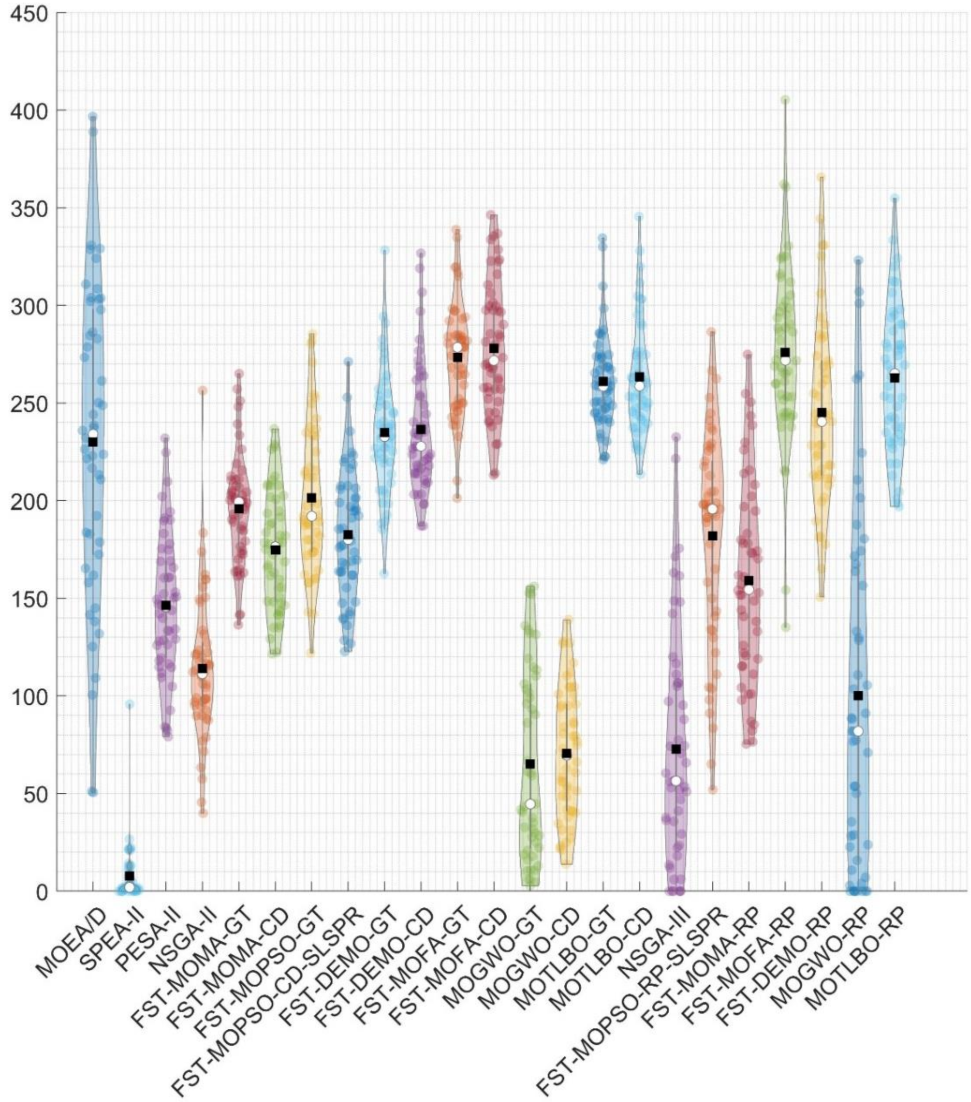


Figure 5.31: Violin plots of M_1^* metric values when using four objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=853.43$, $p<0.001$, $\varepsilon^2=0.74$. Their Median values along with the rest of statistics are demonstrated in Table 5.56.

Table 5.56: Statistics of M_1^* values when using four objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	396.63	230.10	234.02	50.55	**346.09**	79.80
SPEA-II	**95.76**	**7.68**	**2.00**	**0.00**	**95.76**	**14.73**
PESA-II	231.96	146.40	145.67	79.11	152.86	36.48
NSGA-II	256.36	113.99	111.36	39.94	216.42	37.16
FST-MOMA-GT	265.01	195.84	199.29	136.38	128.63	28.66
FST-MOMA-CD	236.84	174.61	176.35	121.58	115.26	30.17
FST-MOPSO-GT	285.36	201.39	192.04	121.93	163.44	38.14
FST-MOPSO-CD-SLSPR	271.13	182.64	180.08	122.74	148.39	32.99
FST-DEMO-GT	328.28	234.80	232.81	162.65	165.63	31.22
FST-DEMO-CD	326.72	236.49	227.81	187.05	139.66	32.40
FST-MOFA-GT	338.72	273.27	**278.49**	201.30	137.41	28.77
FST-MOFA-CD	346.32	**278.18**	271.76	213.06	133.26	33.64
MOGWO-GT	156.06	65.07	44.63	2.79	153.28	48.88
MOGWO-CD	139.07	70.45	69.57	13.74	125.33	33.30
MOTLBO-GT	334.45	261.07	258.54	**220.97**	113.47	24.17
MOTLBO-CD	345.45	263.42	258.68	213.61	131.84	28.68
NSGA-III	232.54	72.80	56.47	**0.00**	232.54	60.03
FST-MOPSO-RP-SLSPR	286.44	181.87	195.70	51.97	234.47	54.90
FST-MOMA-RP	274.85	158.96	154.46	75.36	199.49	50.94
FST-MOFA-RP	**405.25**	275.83	271.84	135.14	270.11	46.25
FST-DEMO-RP	365.64	245.07	240.51	150.60	215.04	46.95
MOGWO-RP	323.08	100.02	81.93	**0.00**	323.08	**92.82**
MOTLBO-RP	354.85	262.78	265.14	196.93	157.92	37.23

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.57, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.57: Pairwise comparisons of M_1^* when using four objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	0	1	0	1	0	0	0	0	1	1	0	0	1	0	1	0	0	1	0
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
FST-MOMA-GT	0	1	1	1	-	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	0	-	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-MOPSO-GT	0	1	1	1	0	0	-	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	0	0	0	-	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-DEMO-GT	0	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	0	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-MOFA-GT	0	1	1	1	1	1	1	1	1	1	-	0	1	1	0	0	1	1	1	0	1	1	0

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOFA-CD	0	1	1	1	1	1	1	1	1	1	0	-	1	1	0	0	1	1	1	0	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	0	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	0	1	1	1	1	0	1
MOTLBO-GT	0	1	1	1	1	1	1	1	1	1	0	0	1	1	-	0	1	1	1	0	0	1	0
MOTLBO-CD	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	-	1	1	1	0	0	1	0
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	-	1	1	1	1	0	1
FST-MOPSO-RP-SLSPR	0	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	0	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1
FST-MOFA-RP	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	-	0	1	0
FST-DEMO-RP	0	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1	0	-	1	0
MOGWO-RP	1	1	0	0	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	-	1
MOTLBO-RP	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	0	0	1	-

Figure 5.31 and Table 5.56 illustrate that SPEA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-GT provides the farthest ones. Table 5.57 reveals that FST-MOFA-GT does not have statistically significant differences with MOEA/D, FST-MOFA-CD, MOTLBO-GT, MOTLBO-CD, FST-MOFA-RP and MOTLBO-RP.

As regards the *Δ* values of each algorithm, Figure 5.32 demonstrates the violin plots of the *Δ* metric values, of each method.

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)$=864.48, p<0.001, ε^2=0.75. Their Median values along with the rest of statistics are demonstrated in Table 5.58.

Table 5.58: Statistics of *Δ* metric values when using four objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.74	1.49	1.48	**1.20**	0.54	0.11
SPEA-II	0.99	0.78	0.78	0.60	0.39	0.09
PESA-II	0.89	0.78	0.78	0.67	0.22	**0.04**
NSGA-II	0.89	0.80	0.79	0.70	**0.19**	**0.04**
FST-MOMA-GT	0.89	0.77	0.77	0.64	0.25	0.05
FST-MOMA-CD	0.88	0.74	0.74	0.62	0.26	0.06
FST-MOPSO-GT	0.87	0.74	0.74	0.62	0.24	0.06
FST-MOPSO-CD-SLSPR	0.84	0.75	0.75	0.62	0.22	0.05
FST-DEMO-GT	0.91	0.83	0.83	0.72	0.20	0.05
FST-DEMO-CD	**0.81**	0.71	0.72	**0.55**	0.25	0.05
FST-MOFA-GT	0.83	0.72	0.71	0.60	0.23	0.05
FST-MOFA-CD	0.83	**0.69**	**0.69**	0.60	0.24	0.05
MOGWO-GT	1.24	1.08	1.08	0.82	0.42	0.09
MOGWO-CD	1.01	0.87	0.89	0.67	0.34	0.08
MOTLBO-GT	0.82	0.70	0.70	0.57	0.25	0.06
MOTLBO-CD	0.82	0.70	**0.69**	0.62	0.21	0.05
NSGA-III	1.70	1.12	1.08	0.86	0.84	0.19
FST-MOPSO-RP-SLSPR	1.11	0.90	0.90	0.66	0.45	0.08
FST-MOMA-RP	1.20	0.94	0.94	0.75	0.45	0.09
FST-MOFA-RP	0.96	0.81	0.80	0.65	0.31	0.07
FST-DEMO-RP	1.29	1.06	1.09	0.70	0.59	0.13

Method	Max	Mean	Median	Min	Range	SD
MOGWO-RP	**1.98**	**1.67**	**1.68**	0.90	**1.08**	**0.20**
MOTLBO-RP	0.98	0.77	0.75	0.63	0.34	0.07

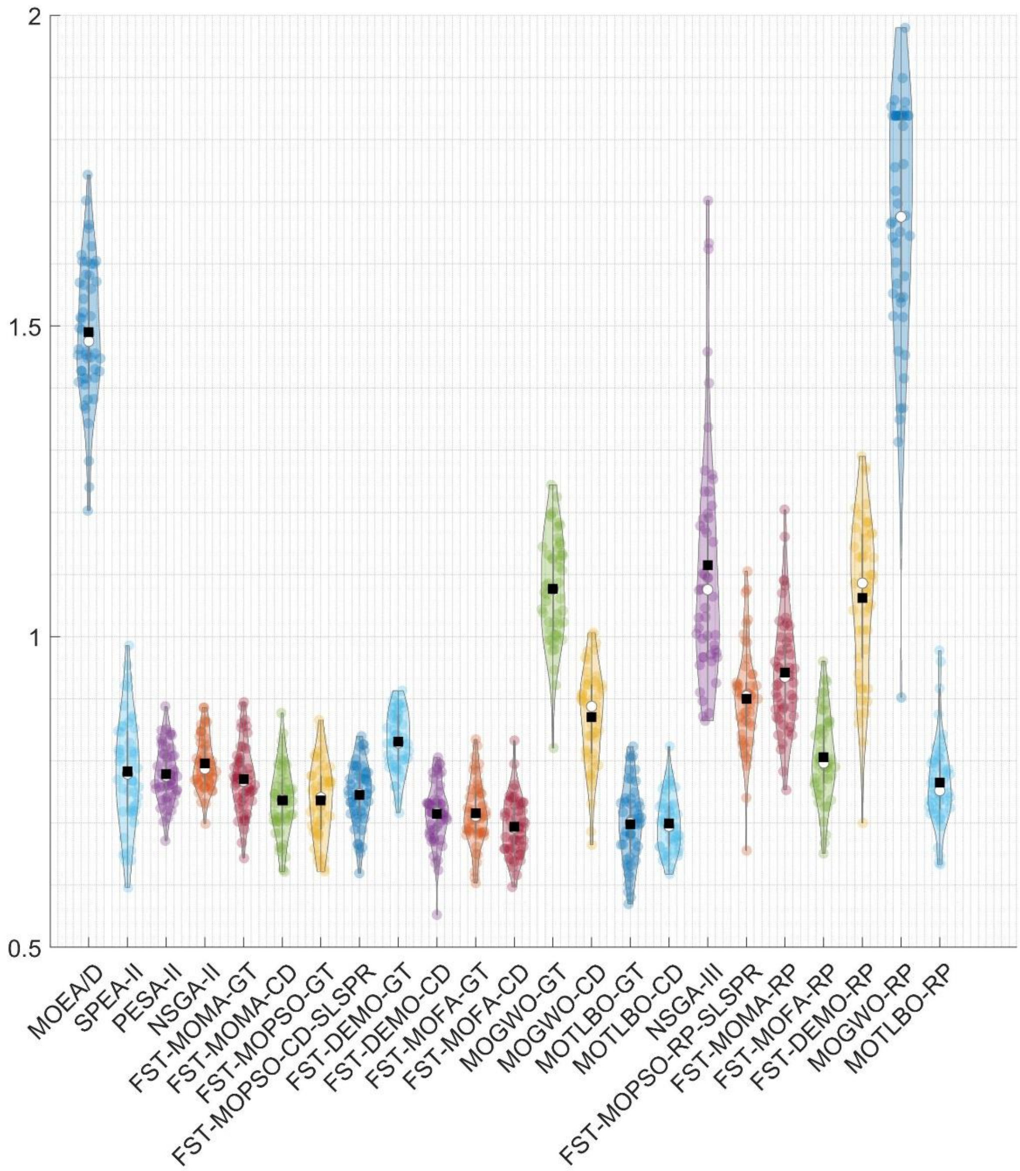


Figure 5.32: Violin plots of *Δ* metric values when using four objectives on the Olive oil data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.59, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.59: Pairwise comparisons of *Δ* metric when using four objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	0
PESA-II	1	0	-	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0
NSGA-II	1	0	0	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0
FST-MOMA-GT	1	0	0	0	-	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0
FST-MOMA-CD	1	0	1	1	0	-	0	0	1	0	0	1	1	1	0	0	1	1	1	1	1	1	0
FST-MOPSO-GT	1	0	0	1	0	0	-	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	0
FST-MOPSO-CD-SLSPR	1	0	0	1	0	0	0	-	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0
FST-DEMO-GT	1	0	1	1	1	1	1	1	-	1	1	1	1	0	1	1	1	1	1	0	1	1	1
FST-DEMO-CD	1	1	1	1	1	0	0	0	1	-	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	0	0	0	1	0	-	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	0	1	1	0	0	-	1	1	0	0	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	0	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	0	1	1	1	1	-	1	1	1	0	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	0	0	1	1	0	0	0	1	1	-	0	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	0	0	1	1	0	0	0	1	1	0	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	1	1	0	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1
FST-MOFA-RP	1	0	0	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	-

Figure 5.32 and Table 5.58 illustrate that FST-MOFA-CD provides the best distribution among the non-dominated solutions, while MOGWO-RP provides the worst. Table 5.59 reveals that FST-MOFA-CD does not have statistically significant differences with FST-MOPSO-GT, FST-DEMO-CD, FST-MOFA-GT, MOTLBO-GT and MOTLBO-CD.

Finally, as regards the M_3^* values of each algorithm, Figure 5.33 demonstrates the violin plots of the M_3^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)=921.64$, $p<0.001$, $\varepsilon^2=0.80$. Their Median values along with the rest of statistics are demonstrated in Table 5.60.

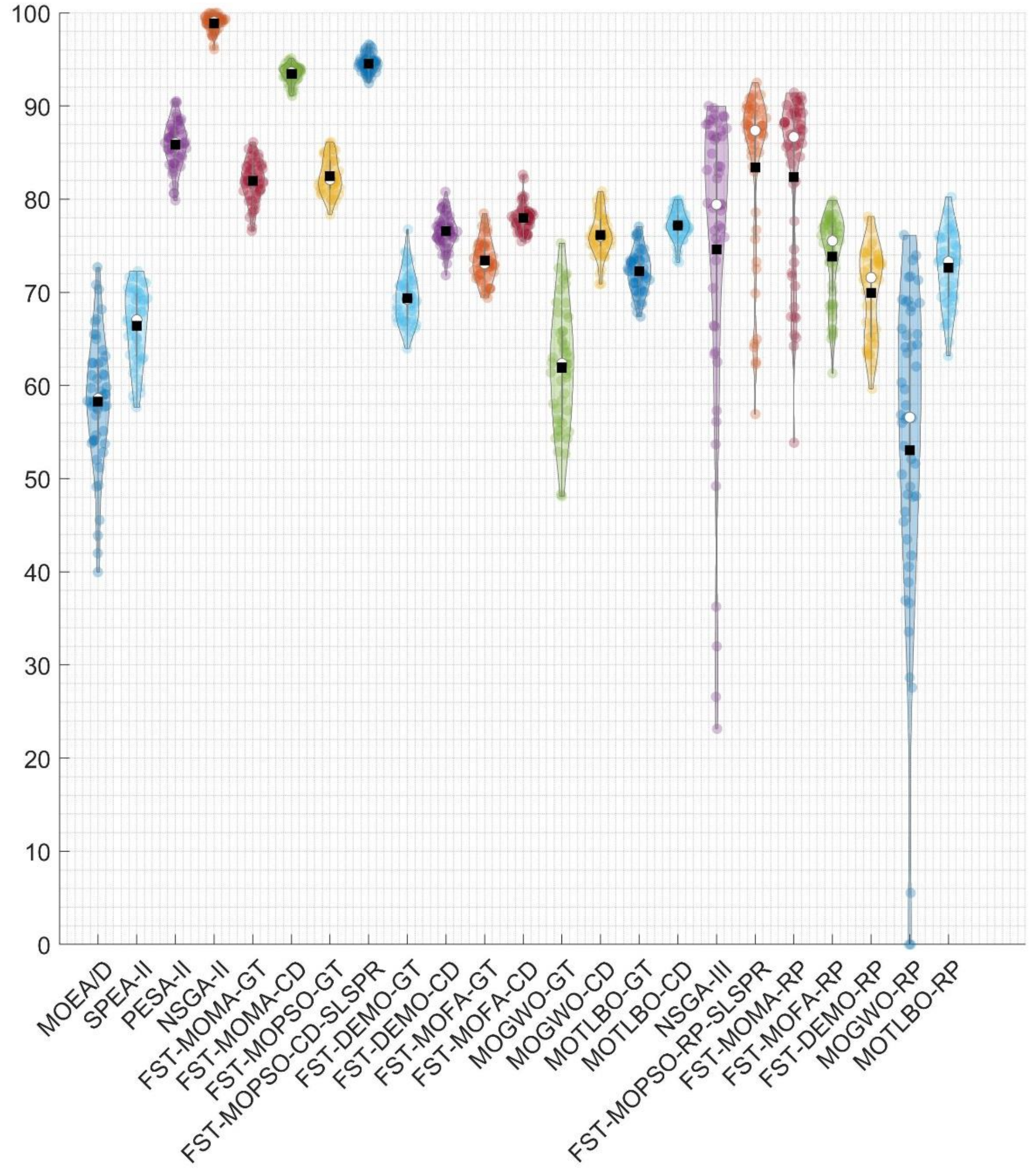


Figure 5.33: Violin plots of $M_3{}^*$ metric values when using four objectives on the Olive oil data

Table 5.60: Statistics of $M_3{}^*$ metric values when using four objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	72.68	58.28	58.59	39.97	32.71	7.06
SPEA-II	**72.27**	66.39	67.04	57.65	14.61	4.01
PESA-II	90.51	85.87	86.01	79.87	10.64	2.47
NSGA-II	**100.00**	**98.86**	**99.03**	**96.09**	**3.91**	**0.87**
FST-MOMA-GT	86.10	81.96	81.95	76.59	9.51	2.17
FST-MOMA-CD	95.12	93.46	93.63	91.08	4.04	0.92
FST-MOPSO-GT	86.11	82.44	82.10	78.33	7.78	1.93
FST-MOPSO-CD-SLSPR	96.60	94.55	94.54	92.44	4.16	0.96
FST-DEMO-GT	76.75	69.37	69.25	63.98	12.77	2.70
FST-DEMO-CD	80.78	76.57	76.60	71.84	8.95	1.78
FST-MOFA-GT	78.46	73.43	73.08	69.40	9.06	2.21
FST-MOFA-CD	82.61	77.99	77.93	75.46	7.15	1.47
MOGWO-GT	75.27	61.95	62.35	48.10	27.17	6.45
MOGWO-CD	80.83	76.17	76.02	70.90	9.93	2.31
MOTLBO-GT	77.05	72.27	72.30	67.39	9.67	2.29
MOTLBO-CD	79.95	77.16	77.19	73.22	6.74	1.60
NSGA-III	90.00	74.60	79.43	23.15	66.85	17.03

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-RP-SLSPR	92.49	83.39	87.38	56.93	35.57	9.25
FST-MOMA-RP	91.43	82.34	86.68	53.86	37.57	9.44
FST-MOFA-RP	79.85	73.82	75.53	61.31	18.53	4.65
FST-DEMO-RP	78.12	69.94	71.57	59.63	18.49	4.84
MOGWO-RP	76.15	**53.06**	**56.58**	**0.00**	**76.15**	**18.03**
MOTLBO-RP	80.24	72.61	73.30	63.17	17.07	3.92

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.61, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.61: Pairwise comparisons of M_3^* metric when using four objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1
SPEA-II	1	-	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	0	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	0	0	1	1	0	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	0	1	1	0	0	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	0	1	1	1	1	1	1
MOGWO-GT	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	0	0	1	1	0	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	1	1	1	0	0	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	0	1	0	1	0	1	-	0	1	1	1	1	1	1
NSGA-III	1	1	1	1	0	1	0	1	1	0	0	0	1	0	1	0	-	0	0	0	1	1	0
FST-MOPSO-RP-SLSPR	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	0	1	1	1	1
FST-MOMA-RP	1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	0	0	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	1	1	1	1	1	-	1	0
MOGWO-RP	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	0	1	1	0	0	1	-

Figure 5.33 and Table 5.60 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while MOGWO-RP provides the worst. Table 5.61 reveals that MOGWO-RP does not have statistically significant differences with MOEA/D and MOGWO-GT.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.34.

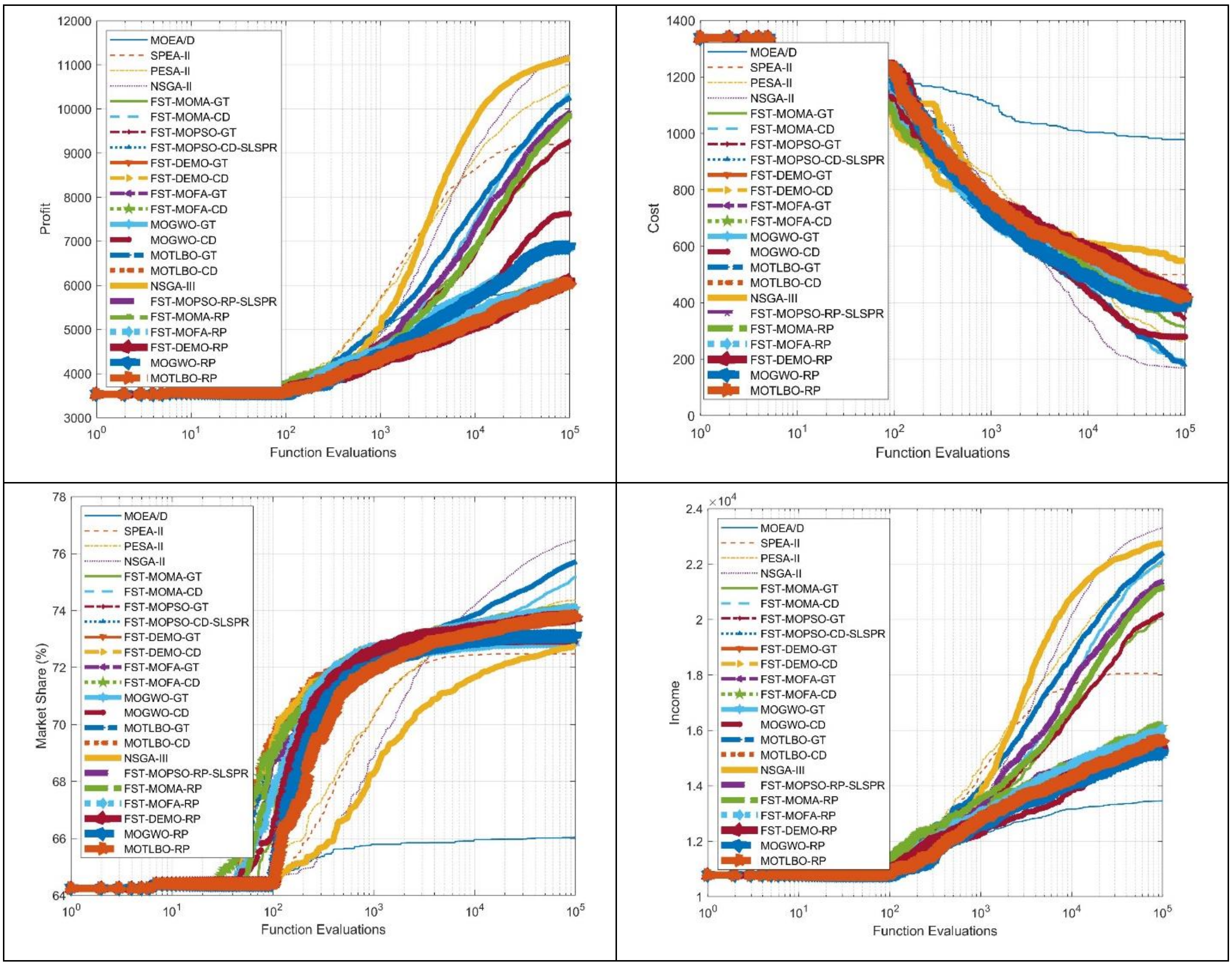


Figure 5.34: Average convergence characteristic curves when using four objectives on the Olive oil data

5.4 Fourth Scenario: Using five objective functions

In this Subsection the performance of the algorithms when using five objective functions, as presented in Table 5.1, is assessed.

5.4.1 Results on the Timbuk2 data set when using five objectives

Regarding the performance of the comparing algorithms while performing on the Timbuk2 data set when using five objectives, in Tables 5.62 and 5.63 the results of the average *C* metric values of the comparing algorithms, are presented. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U test.

Table 5.62: Comparison of methods according their average C metric values when using five objectives on the Timbuk2 data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.05	0.05	0.02	0.05	0.02	0.08	0.04	0.01	0.00	0.07	0.06
SPEA-II	0.01	1.00	0.04	0.02	0.03	0.01	0.04	0.03	0.00	0.00	0.04	0.05
PESA-II	0.01	0.04	1.00	0.02	0.02	0.02	0.05	0.04	0.00	0.00	0.04	0.04
NSGA-II	0.01	0.04	0.04	1.00	0.02	0.03	0.04	0.04	0.00	0.00	0.03	0.05
FST-MOMA-GT	0.02	0.05	0.04	0.03	1.00	0.02	0.07	0.05	0.01	0.01	0.05	0.04
FST-MOMA-CD	0.02	0.05	0.04	0.04	0.03	1.00	0.05	0.05	0.01	0.01	0.03	0.05
FST-MOPSO-GT	0.01	0.03	0.03	0.02	0.02	0.01	1.00	0.03	0.00	0.00	0.04	0.03
FST-MOPSO-CD-SLSPR	0.01	0.04	0.03	0.02	0.02	0.02	0.05	1.00	0.00	0.00	0.03	0.05
FST-DEMO-GT	0.07	0.10	0.11	0.08	0.14	0.09	0.15	0.13	1.00	0.28	0.14	0.11
FST-DEMO-CD	0.06	0.09	0.09	0.08	0.13	0.09	0.14	0.12	0.25	1.00	0.13	0.11
FST-MOFA-GT	0.01	0.03	0.03	0.02	0.02	0.01	0.04	0.03	0.00	0.00	1.00	0.02
FST-MOFA-CD	0.01	0.04	0.04	0.02	0.02	0.02	0.05	0.05	0.00	0.00	0.04	1.00
MOGWO-GT	0.02	0.05	0.04	0.03	0.03	0.02	0.07	0.06	0.01	0.01	0.07	0.05
MOGWO-CD	0.01	0.06	0.05	0.03	0.03	0.02	0.05	0.07	0.00	0.00	0.04	0.07
MOTLBO-GT	0.01	0.03	0.03	0.03	0.02	0.02	0.06	0.04	0.00	0.00	0.05	0.03
MOTLBO-CD	0.02	0.05	0.05	0.03	0.03	0.02	0.06	0.06	0.00	0.01	0.06	0.08
NSGA-III	0.01	0.07	0.07	0.07	0.02	0.06	0.03	0.07	0.00	0.00	0.03	0.06
FST-MOPSO-RP-SLSPR	0.01	0.03	0.03	0.02	0.02	0.01	0.04	0.04	0.00	0.00	0.03	0.05
FST-MOMA-RP	0.02	0.06	0.06	0.05	0.03	0.05	0.04	0.07	0.00	0.01	0.03	0.07
FST-MOFA-RP	0.01	0.03	0.03	0.02	0.03	0.01	0.05	0.04	0.00	0.00	0.05	0.06
FST-DEMO-RP	0.07	0.10	0.11	0.07	0.15	0.11	0.15	0.15	0.25	0.28	0.13	0.11
MOGWO-RP	0.01	0.04	0.05	0.01	0.03	0.01	0.06	0.05	0.00	0.00	0.04	0.06
MOTLBO-RP	0.02	0.04	0.04	0.02	0.04	0.01	0.06	0.05	0.00	0.00	0.05	0.07

Table 5.63: Comparison of methods according their average C metric values when using five objectives on the Timbuk2 data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.02	0.02	0.06	0.04	0.03	0.07	0.02	0.09	0.01	0.01	0.06
SPEA-II	0.02	0.02	0.04	0.03	0.02	0.04	0.01	0.05	0.00	0.01	0.04
PESA-II	0.02	0.02	0.03	0.03	0.01	0.04	0.01	0.04	0.00	0.01	0.04
NSGA-II	0.02	0.02	0.02	0.04	0.01	0.04	0.02	0.04	0.00	0.01	0.03
FST-MOMA-GT	0.03	0.03	0.05	0.03	0.01	0.05	0.01	0.04	0.01	0.01	0.04
FST-MOMA-CD	0.02	0.03	0.03	0.04	0.01	0.04	0.02	0.03	0.01	0.01	0.03
FST-MOPSO-GT	0.02	0.03	0.03	0.02	0.01	0.03	0.01	0.03	0.00	0.01	0.02
FST-MOPSO-CD-SLSPR	0.02	0.03	0.03	0.04	0.01	0.04	0.01	0.04	0.00	0.01	0.03
FST-DEMO-GT	0.07	0.08	0.13	0.11	0.04	0.12	0.06	0.10	0.23	0.03	0.09
FST-DEMO-CD	0.07	0.07	0.11	0.09	0.04	0.10	0.05	0.09	0.24	0.02	0.08
FST-MOFA-GT	0.02	0.02	0.03	0.02	0.01	0.03	0.01	0.02	0.00	0.01	0.01
FST-MOFA-CD	0.02	0.02	0.04	0.04	0.02	0.05	0.01	0.06	0.00	0.01	0.05
MOGWO-GT	1.00	0.02	0.05	0.04	0.01	0.05	0.01	0.05	0.01	0.01	0.04
MOGWO-CD	0.03	1.00	0.04	0.05	0.02	0.06	0.01	0.06	0.00	0.01	0.05
MOTLBO-GT	0.03	0.03	1.00	0.03	0.01	0.03	0.01	0.03	0.00	0.01	0.03
MOTLBO-CD	0.03	0.03	0.04	1.00	0.02	0.06	0.01	0.06	0.00	0.01	0.05
NSGA-III	0.01	0.04	0.02	0.05	1.00	0.09	0.08	0.06	0.00	0.01	0.05
FST-MOPSO-RP-SLSPR	0.02	0.02	0.03	0.03	0.03	1.00	0.01	0.07	0.00	0.01	0.05

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOMA-RP	0.01	0.06	0.03	0.06	0.06	0.10	1.00	0.09	0.01	0.02	0.08
FST-MOFA-RP	0.02	0.02	0.03	0.03	0.02	0.07	0.02	1.00	0.01	0.01	0.08
FST-DEMO-RP	0.07	0.07	0.12	0.10	0.08	0.15	0.07	0.16	1.00	0.04	0.14
MOGWO-RP	0.02	0.02	0.04	0.05	0.03	0.10	0.01	0.12	0.00	1.00	0.09
MOTLBO-RP	0.03	0.02	0.04	0.04	0.04	0.09	0.02	0.11	0.01	0.02	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.35 demonstrates the violin plots which show the distribution of time each method needed.

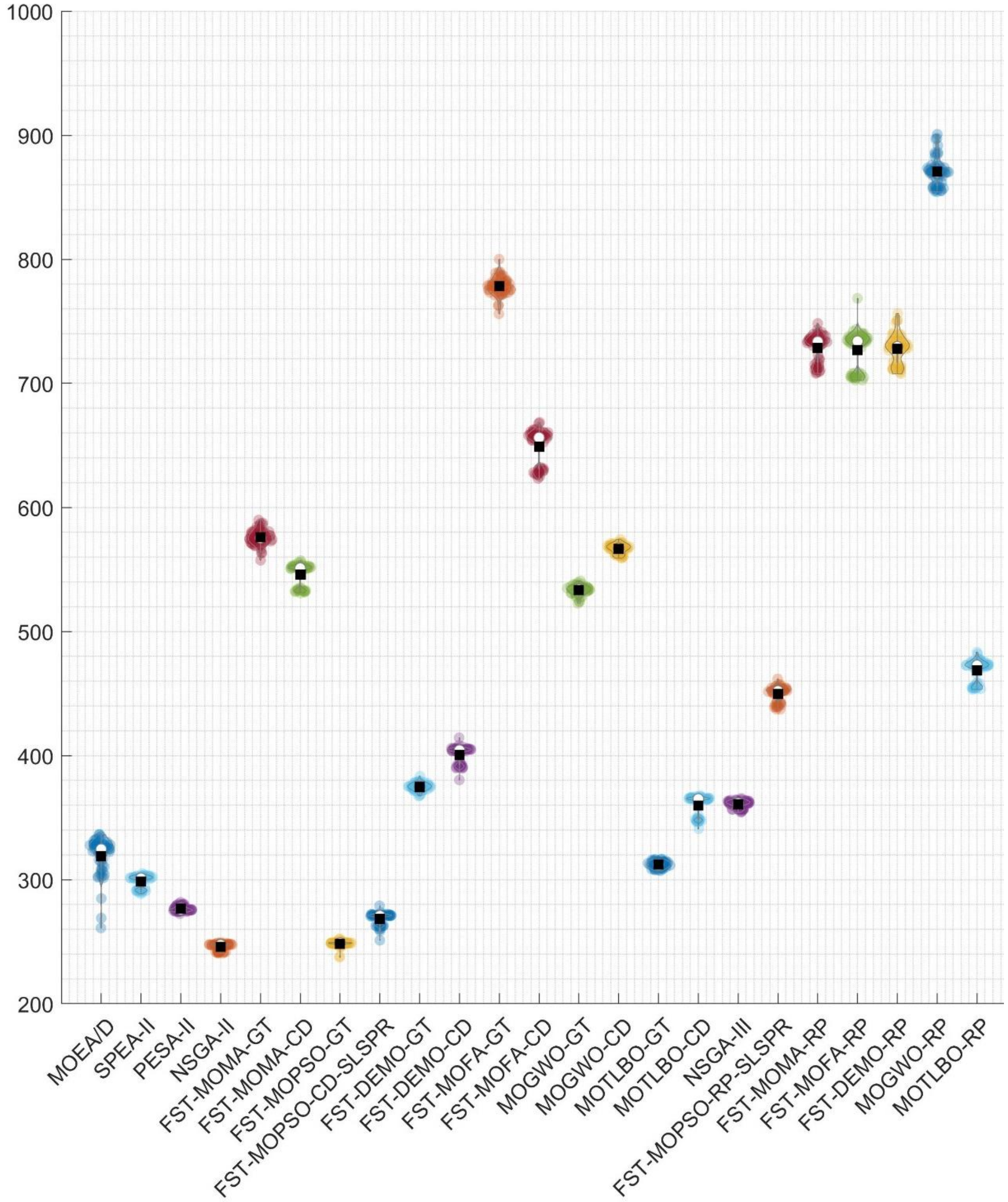


Figure 5.35: Violin plots of *Time* values when using five objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,141.88, p<0.001, ε^2=0.99. Their Median values along with the rest of statistics are demonstrated in Table 5.64.

Table 5.64: Statistics of *Time* when using five objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	336.44	318.71	324.49	261.02	**75.42**	**15.78**
SPEA-II	304.96	298.44	301.04	288.60	16.36	5.10
PESA-II	281.82	276.63	276.01	272.54	9.28	**2.11**
NSGA-II	**249.30**	**245.87**	**247.44**	240.70	**8.60**	2.93
FST-MOMA-GT	589.82	575.91	575.41	557.43	32.39	6.48
FST-MOMA-CD	557.12	546.00	551.01	531.00	26.12	9.10
FST-MOPSO-GT	252.21	248.36	248.80	**237.40**	14.82	2.48
FST-MOPSO-CD-SLSPR	278.89	268.20	270.79	251.10	27.79	5.11
FST-DEMO-GT	383.32	374.88	375.12	367.47	15.85	3.05
FST-DEMO-CD	414.37	400.41	404.22	380.35	34.02	7.23
FST-MOFA-GT	800.09	778.21	778.33	756.05	44.04	7.45
FST-MOFA-CD	668.61	648.94	656.20	623.38	45.23	14.49
MOGWO-GT	540.85	533.23	533.95	522.67	18.18	3.84
MOGWO-CD	574.32	566.64	567.12	559.26	15.06	3.82
MOTLBO-GT	316.46	312.13	312.19	307.21	9.25	2.57
MOTLBO-CD	367.90	359.81	364.93	340.96	26.93	8.46
NSGA-III	365.55	360.65	361.54	354.37	11.18	3.06
FST-MOPSO-RP-SLSPR	461.82	449.52	451.98	437.30	24.52	6.33
FST-MOMA-RP	748.32	728.66	733.53	708.10	40.22	11.10
FST-MOFA-RP	768.57	726.83	733.92	702.55	66.02	15.73
FST-DEMO-RP	756.49	727.95	729.41	707.75	48.74	11.78
MOGWO-RP	**900.79**	**870.67**	**870.32**	**854.41**	46.38	11.52
MOTLBO-RP	483.36	468.69	472.60	453.20	30.16	8.84

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.65, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.65: Pairwise comparisons of *Time* when using five objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	0	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	0	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.35 and Table 5.64 illustrate that NSGA-II needs the least time, while MOGWO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.36 demonstrates the violin plots of the M_1^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=497.49$, $p<0.001$, $\varepsilon^2=0.43$. Their Median values along with the rest of statistics are demonstrated in Table 5.66.

Table 5.66: Statistics of M_1^* values when using five objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	278.86	132.40	126.63	8.94	269.91	72.31
SPEA-II	344.01	173.77	174.51	57.21	286.79	67.43
PESA-II	275.02	178.13	170.89	82.43	192.59	46.90
NSGA-II	173.86	114.42	111.63	49.56	124.30	**26.70**
FST-MOMA-GT	216.96	141.01	140.36	65.53	151.43	34.18
FST-MOMA-CD	190.36	99.51	97.98	35.84	154.52	32.11
FST-MOPSO-GT	336.70	205.82	200.13	75.26	261.45	60.36
FST-MOPSO-CD-SLSPR	256.74	174.92	174.00	111.92	144.81	31.52
FST-DEMO-GT	167.93	101.64	106.13	23.01	144.92	34.10
FST-DEMO-CD	**156.16**	96.74	95.19	39.13	**117.04**	28.60
FST-MOFA-GT	247.59	173.38	169.59	110.05	137.55	35.51
FST-MOFA-CD	299.82	201.17	194.80	111.62	188.20	42.25
MOGWO-GT	430.81	172.72	143.56	18.44	412.37	117.83
MOGWO-CD	250.03	121.28	114.82	16.26	233.76	59.62
MOTLBO-GT	242.49	148.54	144.30	88.65	153.84	38.63
MOTLBO-CD	222.58	154.89	156.99	87.30	135.28	31.52
NSGA-III	203.34	**85.47**	81.20	5.68	197.66	51.33
FST-MOPSO-RP-SLSPR	320.66	222.64	223.57	**143.18**	177.48	44.01
FST-MOMA-RP	196.05	86.38	**79.40**	7.87	188.18	45.20
FST-MOFA-RP	361.77	**233.73**	**238.13**	91.47	270.30	58.53
FST-DEMO-RP	282.60	110.97	105.90	27.13	255.47	48.31
MOGWO-RP	**1,117.83**	176.15	153.54	**0.00**	**1,117.83**	**186.30**
MOTLBO-RP	299.32	193.27	194.24	90.97	208.35	50.79

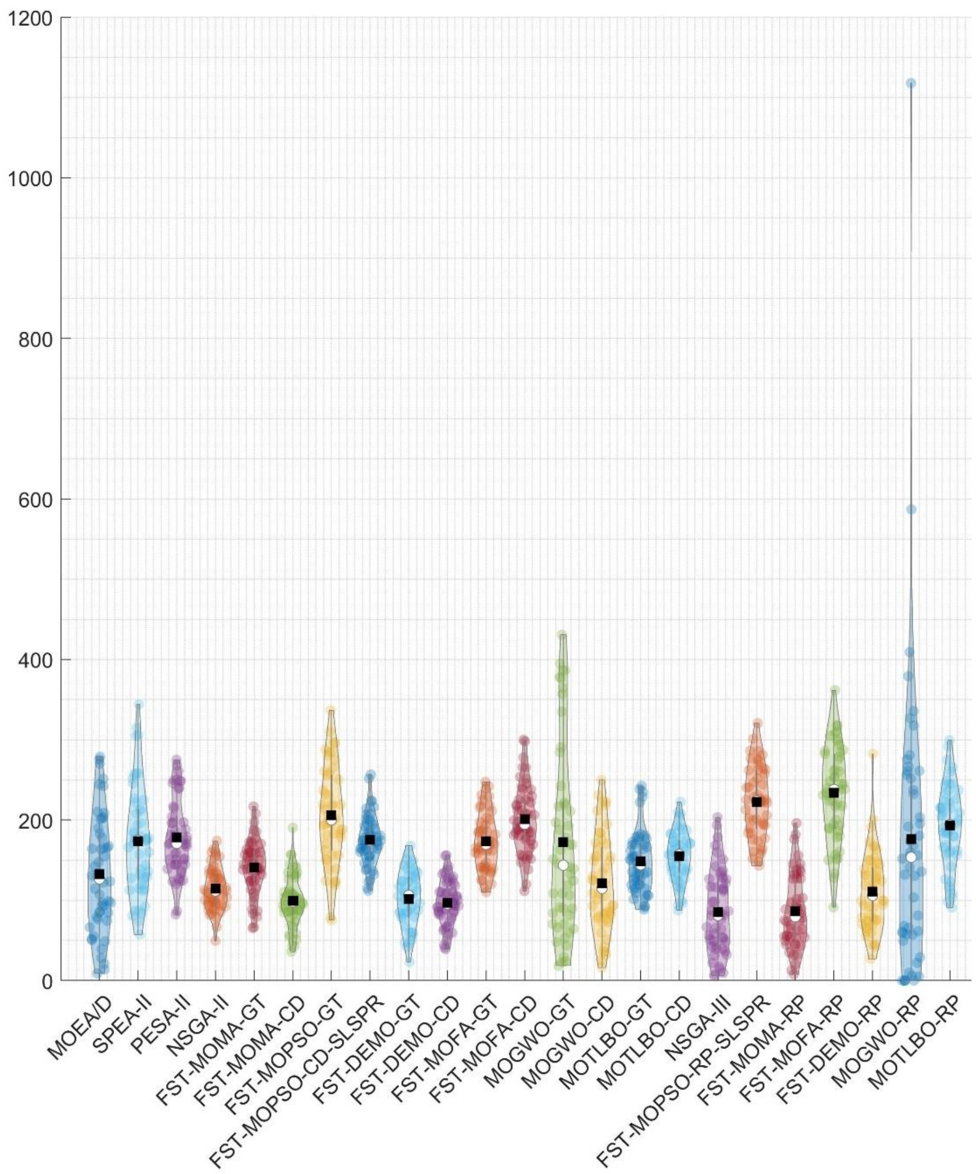


Figure 5.36: Violin plots of M_1^* metric values when using five objectives on the Timbuk2 data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.67, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.67: Pairwise comparisons of M_1^* when using five objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1
SPEA-II	0	-	0	1	0	1	0	0	1	1	0	0	0	1	0	0	1	1	1	1	1	0	0
PESA-II	0	0	-	1	1	1	0	0	1	1	0	0	0	1	0	0	1	1	1	1	1	0	0
NSGA-II	0	1	1	-	1	0	1	1	0	0	1	1	0	0	1	1	0	1	1	1	0	0	1
FST-MOMA-GT	0	0	1	1	-	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	0	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOMA-CD	0	1	1	0	1	-	1	1	0	0	1	1	0	0	1	1	0	1	0	1	0	0	1
FST-MOPSO-GT	1	0	0	1	1	1	-	0	1	1	0	0	0	1	1	1	1	0	1	0	1	0	0
FST-MOPSO-CD-SLSPR	0	0	0	1	1	1	0	-	1	1	0	0	0	1	1	0	1	1	1	1	1	0	0
FST-DEMO-GT	0	1	1	0	1	0	1	1	-	0	1	1	0	0	1	1	0	1	0	1	0	0	1
FST-DEMO-CD	0	1	1	0	1	0	1	1	0	-	1	1	0	0	1	1	0	1	0	1	0	0	1
FST-MOFA-GT	0	0	0	1	1	1	0	0	1	1	-	0	0	1	0	0	1	1	1	1	1	0	0
FST-MOFA-CD	1	0	0	1	1	1	0	0	1	1	0	-	0	1	1	1	1	0	1	0	1	0	0
MOGWO-GT	0	0	0	0	0	0	0	0	0	0	0	0	-	0	0	0	1	0	1	1	0	0	0
MOGWO-CD	0	1	1	0	0	0	1	1	0	0	1	1	0	-	0	0	0	1	0	1	0	0	1
MOTLBO-GT	0	0	0	1	0	1	1	1	1	1	0	1	0	0	-	0	1	1	1	1	1	0	1
MOTLBO-CD	0	0	0	1	0	1	1	0	1	1	0	1	0	0	0	-	1	1	1	1	1	0	1
NSGA-III	0	1	1	0	1	0	1	1	0	0	1	1	1	0	1	1	-	1	0	1	0	0	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1	1	-	1	0	1	0	0
FST-MOMA-RP	0	1	1	1	1	0	1	1	0	0	1	1	1	0	1	1	0	1	-	1	0	0	1
FST-MOFA-RP	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	0	1	-	1	0	0
FST-DEMO-RP	0	1	1	0	1	0	1	1	0	0	1	1	0	0	1	1	0	1	0	1	-	0	1
MOGWO-RP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0
MOTLBO-RP	1	0	0	1	1	1	0	0	1	1	0	0	0	1	1	1	1	0	1	0	1	0	-

Figure 5.36 and Table 5.66 illustrate that FST-MOMA-RP provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-RP provides the farthest ones. Table 5.67 reveals that FST-MOMA-RP does not have statistically significant differences with MOEA/D, FST-MOMA-CD, FST-DEMO-GT, FST-DEMO-CD, MOGWO-CD, NSGA-III, FST-DEMO-RP and MOGWO-RP, while FST-MOFA-RP does not have statistically significant differences with FST-MOPSO-GT, FST-MOFA-CD, FST-MOPSO-RP-SLSPR, MOGWO-RP and MOTLBO-RP.

As regards the Δ values of each algorithm, Figure 5.37 demonstrates the violin plots of the Δ metric values, of each method.

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)=988.39$, $p<0.001$, $\varepsilon^2=0.86$. Their Median values along with the rest of statistics are demonstrated in Table 5.68.

Table 5.68: Statistics of Δ metric values when using five objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.47	1.23	1.23	1.11	0.36	0.07
SPEA-II	**0.66**	**0.56**	**0.57**	**0.44**	0.22	0.05
PESA-II	0.71	0.62	0.63	0.52	0.19	0.04
NSGA-II	0.74	0.67	0.66	0.58	0.16	0.04
FST-MOMA-GT	0.84	0.69	0.68	0.60	0.24	0.05
FST-MOMA-CD	0.76	0.65	0.65	0.56	0.19	0.04
FST-MOPSO-GT	0.79	0.66	0.66	0.58	0.21	0.04
FST-MOPSO-CD-SLSPR	0.76	0.64	0.64	0.54	0.22	0.05
FST-DEMO-GT	1.37	1.29	1.30	1.19	0.18	0.04
FST-DEMO-CD	1.32	1.25	1.24	1.20	**0.12**	**0.03**
FST-MOFA-GT	0.76	0.63	0.63	0.52	0.24	0.06
FST-MOFA-CD	0.83	0.74	0.74	0.63	0.19	0.05

Method	Max	Mean	Median	Min	Range	SD
MOGWO-GT	1.28	1.15	1.16	0.97	0.31	0.06
MOGWO-CD	0.81	0.72	0.72	0.58	0.23	0.05
MOTLBO-GT	0.81	0.63	0.64	0.51	0.30	0.06
MOTLBO-CD	0.78	0.66	0.67	0.57	0.21	0.04
NSGA-III	1.41	0.99	0.97	0.76	0.65	0.13
FST-MOPSO-RP-SLSPR	1.00	0.78	0.77	0.62	0.38	0.08
FST-MOMA-RP	1.13	0.91	0.89	0.75	0.38	0.09
FST-MOFA-RP	0.95	0.78	0.78	0.61	0.34	0.09
FST-DEMO-RP	1.49	1.35	1.34	**1.22**	0.28	0.06
MOGWO-RP	**1.98**	**1.63**	**1.62**	1.18	**0.79**	**0.21**
MOTLBO-RP	1.05	0.80	0.81	0.64	0.41	0.10

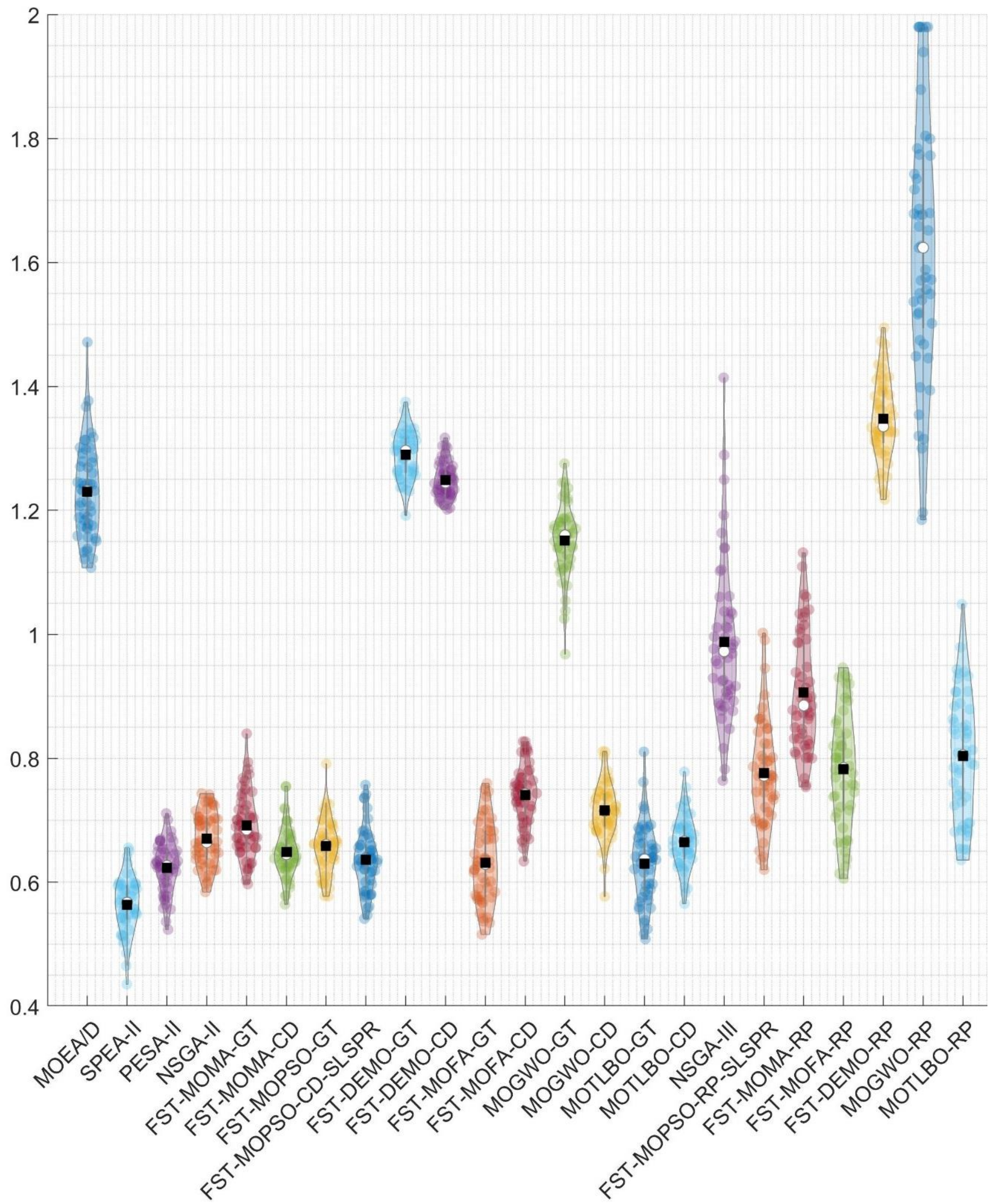


Figure 5.37: Violin plots of Δ metric values when using five objectives on the Timbuk2 data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.69, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.69: Pairwise comparisons of *Δ* metric when using five objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	0	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	0	0	0	0	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	0	-	1	0	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	0	0	1	-	0	0	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	0	0	0	-	0	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	0	0	1	0	0	-	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	0	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	0	0	1	0	0	0	1	1	-	1	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	0	1	1	1	0	1	0	1	1	0
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	0	1	1	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	0	1	1	0	0	0	1	1	0	1	1	1	-	0	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	0	0	0	0	0	1	1	0	1	1	1	0	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	0	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	-	1	0	1	1	0
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	0	1	1	-

Figure 5.37 and Table 5.68 illustrate that SPEA-II provides the best distribution among the non-dominated solutions, while MOGWO-RP provides the worst.

Finally, as regards the M_3^* values of each algorithm, Figure 5.38 demonstrates the violin plots of the M_3^* values of each method.

A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)=881.35$, $p<0.001$, $\varepsilon^2=0.77$. Their Median values along with the rest of statistics are demonstrated in Table 5.70.

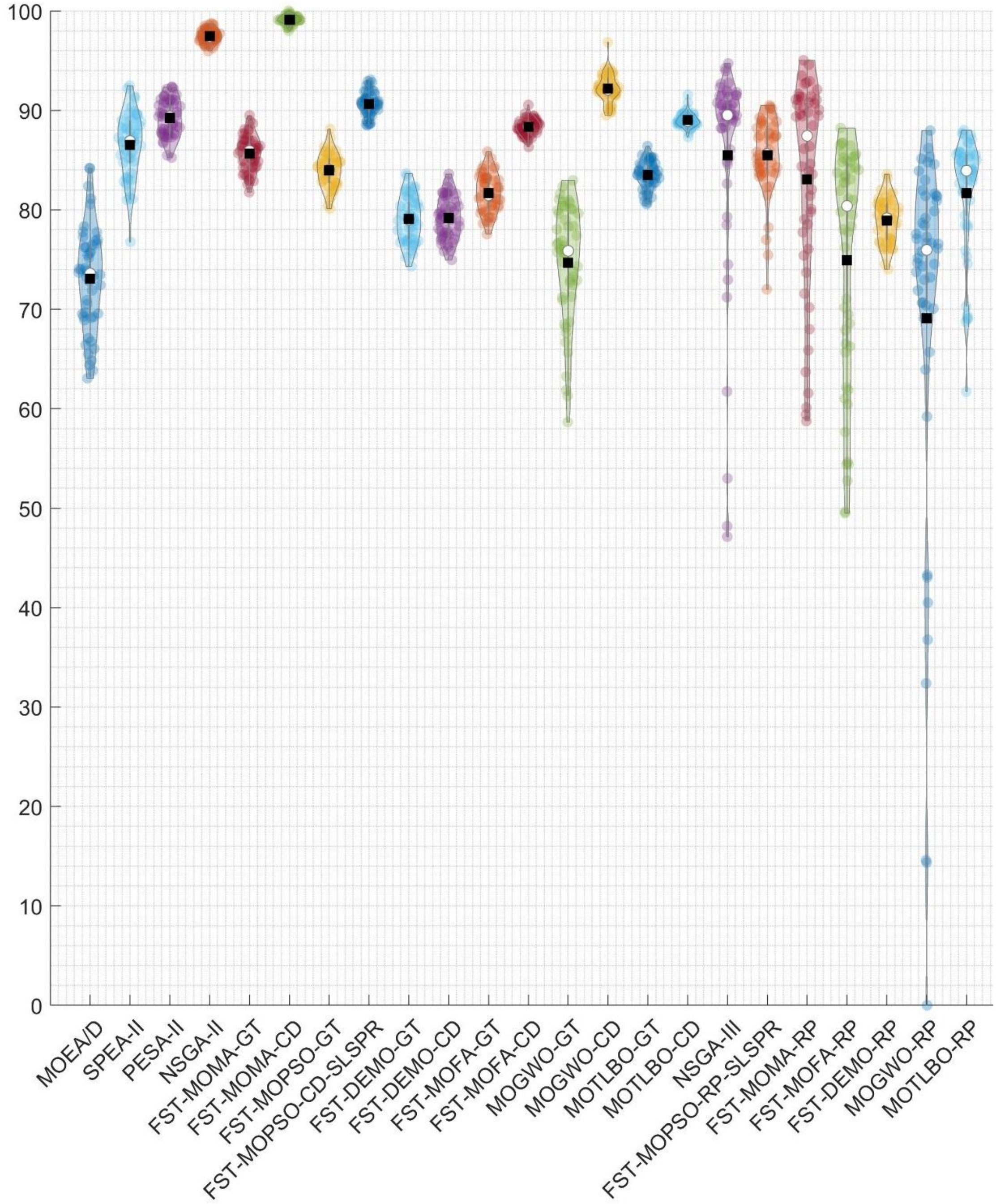


Figure 5.38: Violin plots of M_3^* metric values when using five objectives on the Timbuk2 data

Table 5.70: Statistics of M_3^* metric values when using five objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	84.20	73.08	**73.59**	63.05	21.15	5.18
SPEA-II	92.49	86.54	86.92	76.77	15.72	3.26
PESA-II	92.35	89.25	89.39	85.26	7.10	1.85
NSGA-II	98.73	97.48	97.42	95.97	2.76	0.67
FST-MOMA-GT	89.50	85.67	85.93	81.77	7.74	1.78
FST-MOMA-CD	**100.00**	**99.13**	**99.12**	**98.00**	**2.00**	**0.41**
FST-MOPSO-GT	88.12	83.97	84.03	80.12	7.99	1.73
FST-MOPSO-CD-SLSPR	93.07	90.64	90.64	88.52	4.55	1.18
FST-DEMO-GT	83.65	79.08	79.04	74.31	9.34	2.26
FST-DEMO-CD	83.61	79.18	79.18	74.97	8.64	2.07
FST-MOFA-GT	85.85	81.69	81.48	77.57	8.28	1.89

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-CD	90.55	88.33	88.36	86.33	4.22	0.80
MOGWO-GT	**82.97**	74.68	75.88	58.66	24.31	5.91
MOGWO-CD	96.86	92.19	92.07	89.51	7.35	1.42
MOTLBO-GT	86.41	83.51	83.52	80.57	5.83	1.38
MOTLBO-CD	91.58	89.03	88.90	87.31	4.26	0.71
NSGA-III	94.72	85.50	89.53	47.13	47.60	11.19
FST-MOPSO-RP-SLSPR	90.51	85.47	85.62	72.01	18.50	3.84
FST-MOMA-RP	95.04	83.06	87.44	58.76	36.27	10.48
FST-MOFA-RP	88.23	74.92	80.39	49.49	38.74	11.86
FST-DEMO-RP	83.58	78.90	79.16	74.02	9.55	2.19
MOGWO-RP	87.97	**69.09**	75.97	**0.00**	**87.97**	**19.94**
MOTLBO-RP	88.01	81.67	83.94	61.70	26.31	6.04

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.71, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.71: Pairwise comparisons of M_3^* metric when using five objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	1
SPEA-II	1	-	1	1	0	1	1	1	1	1	1	0	1	1	1	1	0	0	0	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	0	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	0	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	0	1	1	0	0	0	1	1	0
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	0	0	0	0	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	0	0	0	0	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	0	0	1	1	0
FST-MOFA-CD	1	0	0	1	1	1	1	1	1	1	1	-	1	1	1	1	0	1	0	1	1	1	1
MOGWO-GT	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	0	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	0	1	1	1	1	1	1	1	-	1	1	1	0	0	1	1	0
MOTLBO-CD	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	0	1	1	1	1
NSGA-III	1	0	0	1	1	1	1	0	1	1	1	0	1	1	1	0	-	1	0	1	1	1	1
FST-MOPSO-RP-SLSPR	1	0	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	-	1	0	1	0
FST-MOFA-RP	0	1	1	1	1	1	0	1	0	0	0	1	0	1	0	1	1	1	1	-	0	0	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	-	0	1
MOGWO-RP	0	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	1	0	0	-	1
MOTLBO-RP	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	0	1	1	-

Figure 5.38 and Table 5.70 illustrate that FST-MOMA-CD provides the best extend of the non-dominated solutions, while MOEA/D provides the worst. Table 5.71 reveals that MOEA/D does not have statistically significant differences with MOGWO-GT, FST-MOFA-RP and MOGWO-RP.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.39.

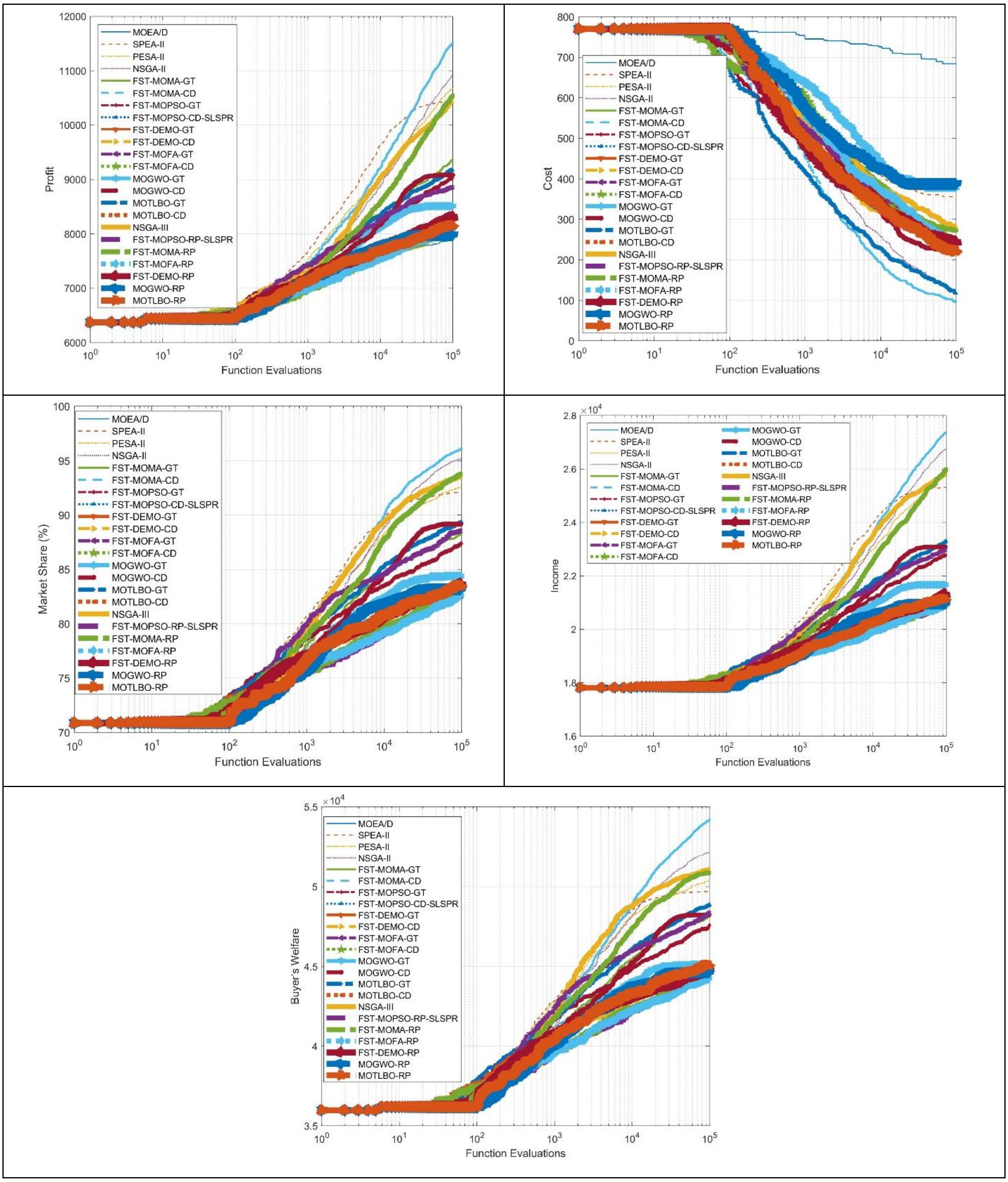


Figure 5.39: Average convergence characteristic curves when using five objectives on the Timbuk2 data

5.4.2 *Results on the Olive oil data set when using five objectives*

Regarding the performance of the comparing algorithms while performing on the olive oil data set when using five objectives, Tables 5.72 and 5.73 show the results of the average *C* metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U Test.

Table 5.72: Comparison of methods according their average C metric values when using five objectives on the Olive oil data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.00	0.06	0.23	0.14	0.23	0.16	0.40	0.24	0.14	0.37	0.31
SPEA-II	0.13	1.00	0.28	0.37	0.53	0.55	0.58	0.62	0.81	0.80	0.89	0.88
PESA-II	0.08	0.01	1.00	0.18	0.25	0.27	0.29	0.34	0.46	0.49	0.60	0.62
NSGA-II	0.01	0.01	0.06	1.00	0.12	0.16	0.14	0.20	0.15	0.22	0.21	0.28
FST-MOMA-GT	0.05	0.01	0.05	0.12	1.00	0.18	0.16	0.26	0.35	0.33	0.46	0.46
FST-MOMA-CD	0.01	0.01	0.05	0.08	0.10	1.00	0.12	0.17	0.14	0.19	0.21	0.27
FST-MOPSO-GT	0.05	0.00	0.04	0.10	0.12	0.17	1.00	0.21	0.33	0.32	0.43	0.44
FST-MOPSO-CD-SLSPR	0.01	0.01	0.05	0.08	0.08	0.10	0.11	1.00	0.12	0.18	0.19	0.24
FST-DEMO-GT	0.04	0.00	0.01	0.08	0.06	0.12	0.06	0.19	1.00	0.15	0.28	0.25
FST-DEMO-CD	0.03	0.00	0.01	0.07	0.06	0.10	0.06	0.16	0.19	1.00	0.27	0.25
FST-MOFA-GT	0.02	0.00	0.01	0.05	0.03	0.06	0.03	0.12	0.10	0.07	1.00	0.14
FST-MOFA-CD	0.01	0.00	0.01	0.04	0.02	0.05	0.03	0.09	0.08	0.08	0.15	1.00
MOGWO-GT	0.21	0.04	0.19	0.33	0.41	0.48	0.44	0.54	0.74	0.68	0.82	0.79
MOGWO-CD	0.07	0.03	0.11	0.19	0.29	0.32	0.31	0.38	0.52	0.53	0.61	0.61
MOTLBO-GT	0.02	0.00	0.01	0.07	0.04	0.08	0.05	0.15	0.14	0.11	0.21	0.19
MOTLBO-CD	0.02	0.00	0.01	0.05	0.04	0.08	0.05	0.13	0.13	0.11	0.20	0.20
NSGA-III	0.03	0.00	0.10	0.28	0.12	0.34	0.17	0.38	0.13	0.22	0.23	0.31
FST-MOPSO-RP-SLSPR	0.02	0.00	0.07	0.14	0.09	0.19	0.13	0.25	0.13	0.19	0.23	0.28
FST-MOMA-RP	0.03	0.00	0.07	0.14	0.08	0.21	0.12	0.26	0.11	0.19	0.21	0.27
FST-MOFA-RP	0.02	0.00	0.01	0.05	0.03	0.07	0.04	0.13	0.08	0.08	0.17	0.15
FST-DEMO-RP	0.06	0.00	0.02	0.08	0.07	0.13	0.08	0.20	0.22	0.16	0.30	0.26
MOGWO-RP	0.20	0.02	0.16	0.20	0.34	0.35	0.35	0.45	0.62	0.59	0.72	0.72
MOTLBO-RP	0.03	0.00	0.01	0.06	0.03	0.09	0.05	0.15	0.11	0.10	0.19	0.20

Table 5.73: Comparison of methods according their average C metric values when using five objectives on the Olive oil data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.00	0.02	0.38	0.25	0.07	0.22	0.14	0.25	0.14	0.00	0.31
SPEA-II	0.11	0.20	0.84	0.85	0.17	0.49	0.34	0.63	0.64	0.10	0.77
PESA-II	0.03	0.09	0.55	0.55	0.08	0.28	0.20	0.45	0.37	0.05	0.50
NSGA-II	0.01	0.04	0.19	0.25	0.04	0.14	0.12	0.21	0.15	0.02	0.20
FST-MOMA-GT	0.02	0.04	0.41	0.39	0.04	0.21	0.14	0.32	0.27	0.03	0.37
FST-MOMA-CD	0.01	0.04	0.18	0.24	0.03	0.13	0.09	0.20	0.14	0.02	0.20
FST-MOPSO-GT	0.01	0.04	0.38	0.38	0.04	0.18	0.11	0.31	0.24	0.03	0.34

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOPSO-CD-SLSPR	0.01	0.03	0.15	0.20	0.03	0.12	0.09	0.18	0.12	0.02	0.17
FST-DEMO-GT	0.00	0.01	0.22	0.19	0.02	0.14	0.08	0.19	0.14	0.01	0.21
FST-DEMO-CD	0.00	0.01	0.23	0.18	0.02	0.11	0.07	0.19	0.13	0.01	0.20
FST-MOFA-GT	0.00	0.00	0.13	0.09	0.01	0.08	0.04	0.11	0.06	0.00	0.11
FST-MOFA-CD	0.00	0.01	0.12	0.10	0.01	0.06	0.04	0.12	0.07	0.00	0.11
MOGWO-GT	1.00	0.14	0.76	0.73	0.13	0.42	0.31	0.57	0.53	0.04	0.63
MOGWO-CD	0.04	1.00	0.57	0.58	0.08	0.27	0.20	0.43	0.39	0.04	0.50
MOTLBO-GT	0.00	0.01	1.00	0.13	0.02	0.10	0.06	0.13	0.09	0.01	0.15
MOTLBO-CD	0.00	0.01	0.18	1.00	0.01	0.08	0.05	0.15	0.10	0.01	0.15
NSGA-III	0.00	0.03	0.19	0.29	1.00	0.26	0.21	0.25	0.17	0.03	0.25
FST-MOPSO-RP-SLSPR	0.00	0.03	0.19	0.25	0.05	1.00	0.12	0.30	0.16	0.03	0.26
FST-MOMA-RP	0.00	0.04	0.17	0.24	0.06	0.19	1.00	0.29	0.16	0.03	0.25
FST-MOFA-RP	0.00	0.01	0.15	0.11	0.02	0.08	0.04	1.00	0.11	0.01	0.14
FST-DEMO-RP	0.00	0.01	0.26	0.19	0.04	0.14	0.09	0.27	1.00	0.02	0.26
MOGWO-RP	0.04	0.09	0.66	0.61	0.13	0.38	0.32	0.63	0.48	1.00	0.67
MOTLBO-RP	0.00	0.01	0.17	0.13	0.02	0.09	0.06	0.18	0.12	0.01	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.40 demonstrates the violin plots which show the distribution of time each method needed.

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,116.39, p<0.001, ε^2=0.97. Their Median values along with the rest of statistics are demonstrated in Table 5.74.

Table 5.74: Statistics of *Time* when using five objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	649.74	471.29	470.91	380.86	268.88	39.81
SPEA-II	730.63	440.90	431.59	410.49	320.14	46.38
PESA-II	600.55	362.76	353.21	346.24	254.32	39.05
NSGA-II	557.40	394.00	387.70	379.36	178.05	28.08
FST-MOMA-GT	602.71	335.83	327.54	306.50	296.21	41.23
FST-MOMA-CD	888.55	609.83	599.98	586.65	301.90	45.64
FST-MOPSO-GT	402.59	303.63	300.04	292.16	110.44	16.99
FST-MOPSO-CD-SLSPR	524.87	420.95	416.72	409.51	115.36	18.25
FST-DEMO-GT	600.25	433.25	428.79	397.66	202.59	29.16
FST-DEMO-CD	798.31	548.52	541.35	515.76	282.56	38.49
FST-MOFA-GT	338.18	205.97	201.06	194.76	143.42	23.91
FST-MOFA-CD	541.75	309.41	300.44	292.04	249.71	39.78
MOGWO-GT	773.48	606.84	587.85	571.86	201.61	48.73
MOGWO-CD	777.76	650.14	637.79	628.51	149.25	33.13
MOTLBO-GT	**203.54**	**172.57**	**171.24**	**163.02**	**40.53**	**8.24**
MOTLBO-CD	234.76	204.53	202.52	193.19	41.57	9.41
NSGA-III	581.22	517.92	512.18	479.07	102.14	24.52
FST-MOPSO-RP-SLSPR	788.50	585.13	572.41	558.36	230.14	40.31
FST-MOMA-RP	999.60	753.27	734.95	715.98	283.62	54.43
FST-MOFA-RP	485.47	341.64	333.81	324.78	160.69	28.49
FST-DEMO-RP	1,173.21	852.22	834.90	803.26	**369.95**	66.66
MOGWO-RP	**1,241.95**	**931.21**	**903.72**	**887.31**	354.64	**69.20**
MOTLBO-RP	361.68	266.59	262.59	251.69	109.99	19.69

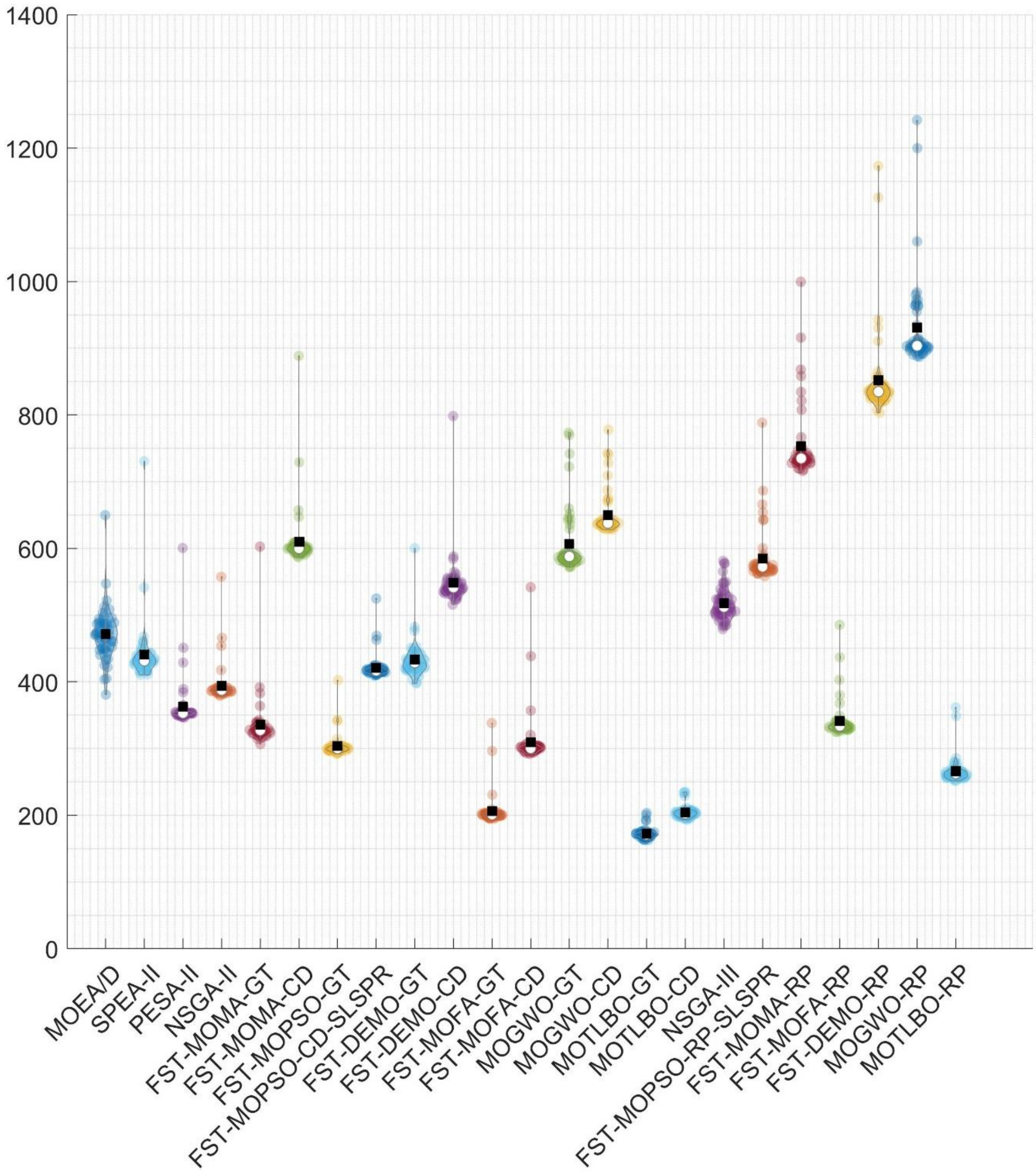


Figure 5.40: Violin plots of *Time* values when using five objectives on the Olive oil data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.75, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.75: Pairwise comparisons of *Time* when using five objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	0	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	0	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.40 and Table 5.74 illustrate that MOTLBO-GT needs the least time, while MOGWO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.41 demonstrates the violin plots of the M_1^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)$=932.28, p<0.001, ε^2=0.81. Their Median values along with the rest of statistics are demonstrated in Table 5.76.

Table 5.76: Statistics of M_1^* values when using five objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1,727.50	597.17	631.73	9.94	1,717.56	**408.69**
SPEA-II	**384.28**	**86.52**	**53.48**	**0.00**	384.28	**88.45**
PESA-II	816.65	474.66	465.79	190.94	625.71	134.45
NSGA-II	745.42	455.04	441.43	321.42	424.00	99.38
FST-MOMA-GT	1,019.33	752.42	734.46	323.58	695.75	142.55
FST-MOMA-CD	1,105.98	700.27	691.54	429.52	676.46	130.67
FST-MOPSO-GT	1,447.89	879.77	868.44	597.61	850.28	177.01
FST-MOPSO-CD-SLSPR	1,159.70	810.79	811.65	535.96	623.73	143.54
FST-DEMO-GT	1,426.75	1,106.07	1,109.44	818.57	608.18	132.16
FST-DEMO-CD	1,361.77	1,061.94	1,057.82	821.26	540.50	112.85
FST-MOFA-GT	1,503.13	**1,305.33**	**1,299.41**	**1,139.44**	**363.69**	89.33
FST-MOFA-CD	1,486.91	1,290.59	1,293.69	1,037.61	449.30	115.37
MOGWO-GT	681.88	202.69	140.44	**0.00**	681.88	204.28

Method	Max	Mean	Median	Min	Range	SD
MOGWO-CD	677.52	343.64	313.21	99.77	577.75	142.53
MOTLBO-GT	1,493.24	1,210.70	1,200.97	949.03	544.21	146.32
MOTLBO-CD	1,402.74	1,184.31	1,188.55	877.69	525.06	116.62
NSGA-III	1,078.09	293.41	286.21	12.36	1,065.73	213.57
FST-MOPSO-RP-SLSPR	1,248.13	782.55	794.06	315.81	932.32	217.88
FST-MOMA-RP	1,126.01	620.61	597.45	159.27	966.74	201.64
FST-MOFA-RP	1,625.93	1,203.02	1,213.19	730.18	895.75	221.17
FST-DEMO-RP	**2,532.42**	1,133.06	1,068.35	785.40	**1,747.02**	302.21
MOGWO-RP	1,074.20	428.07	384.36	**0.00**	1,074.20	294.00
MOTLBO-RP	1,676.69	1,203.41	1,193.80	610.47	1,066.22	203.87

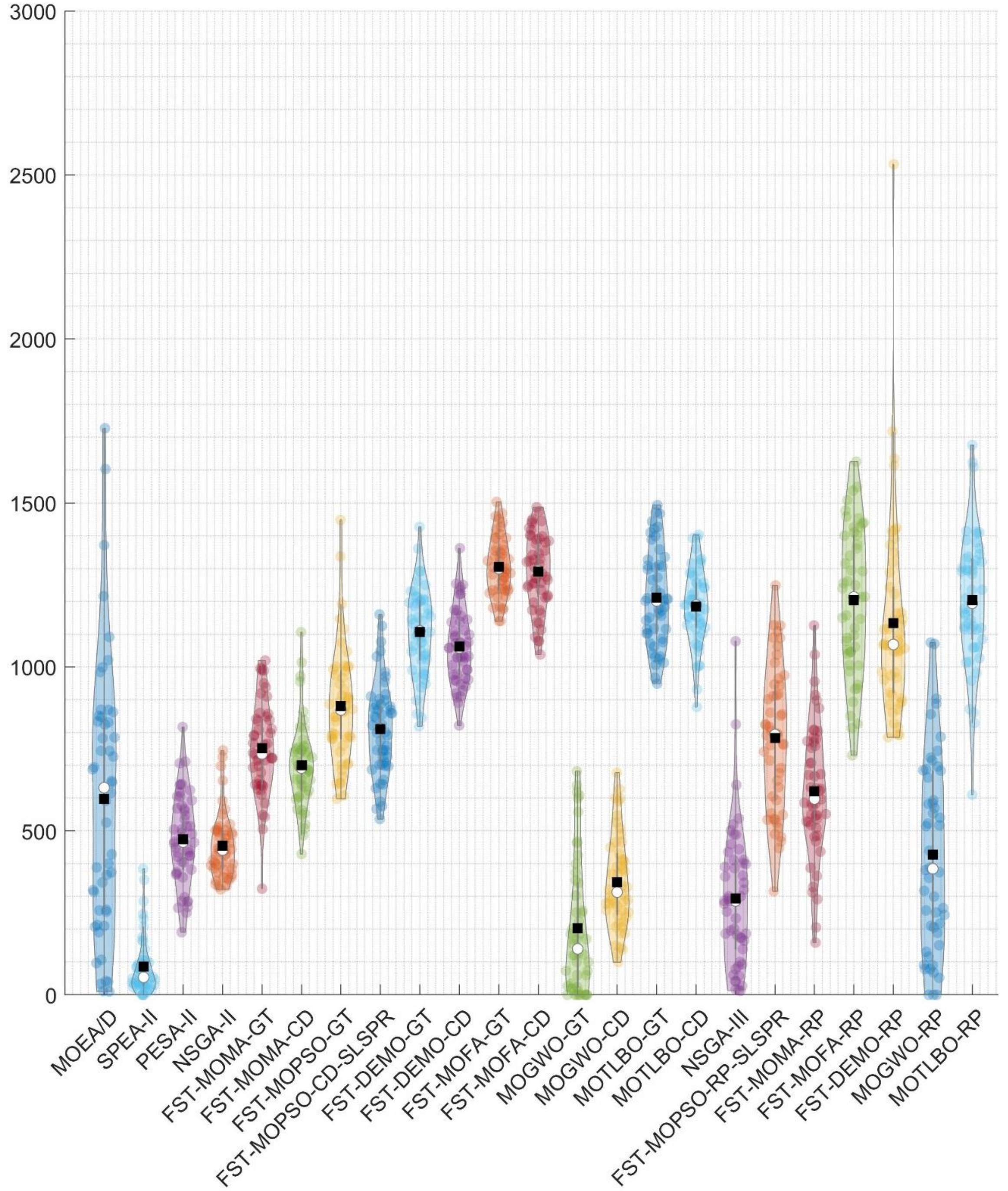


Figure 5.41: Violin plots of M_1^* metric values when using five objectives on the Olive oil data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.77, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.77: Pairwise comparisons of M_1^* when using five objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	0	0	0	0	1	0	1	1	1	1	1	0	1	1	1	0	0	1	1	0	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
PESA-II	0	1	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
NSGA-II	0	1	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
FST-MOMA-GT	0	1	1	1	-	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOMA-CD	0	1	1	1	0	-	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
FST-MOPSO-GT	1	1	1	1	0	1	-	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-MOPSO-CD-SLSPR	0	1	1	1	0	1	0	-	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	0	1	1	1	1	0	0	1	1	1	0	0	1	0
FST-DEMO-CD	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1	0	0	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	0	1	1	0	1	1	1	1	0	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	0	-	1	1	0	1	1	1	1	0	1	1	0
MOGWO-GT	1	0	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	1	1	1
MOGWO-CD	0	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	0	1	1	1	1	0	1
MOTLBO-GT	1	1	1	1	1	1	1	1	0	1	0	0	1	1	-	0	1	1	1	0	0	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	-	1	1	1	0	0	1	0
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	-	1	1	1	1	0	1
FST-MOPSO-RP-SLSPR	0	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	0	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	1	1	1	-	0	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1	0	-	1	0
MOGWO-RP	0	1	0	0	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	0	1	0	0	1	1	0	0	1	1	1	0	0	1	-

Figure 5.41 and Table 5.76 illustrate that SPEA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-GT provides the farthest ones. Table 5.77 reveals that SPEA-II does not have statistically significant differences with MOGWO-GT, while FST-MOFA-GT does not have statistically significant differences with FST-MOFA-CD, MOTLBO-GT, FST-MOFA-RP and MOTLBO-RP.

As regards the *Δ* values of each algorithm, Figure 5.42 demonstrates the violin plots of the *Δ* metric values, of each method.

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)=869.22$, $p<0.001$, $\varepsilon^2=0.76$. Their Median values along with the rest of statistics are demonstrated in Table 5.78.

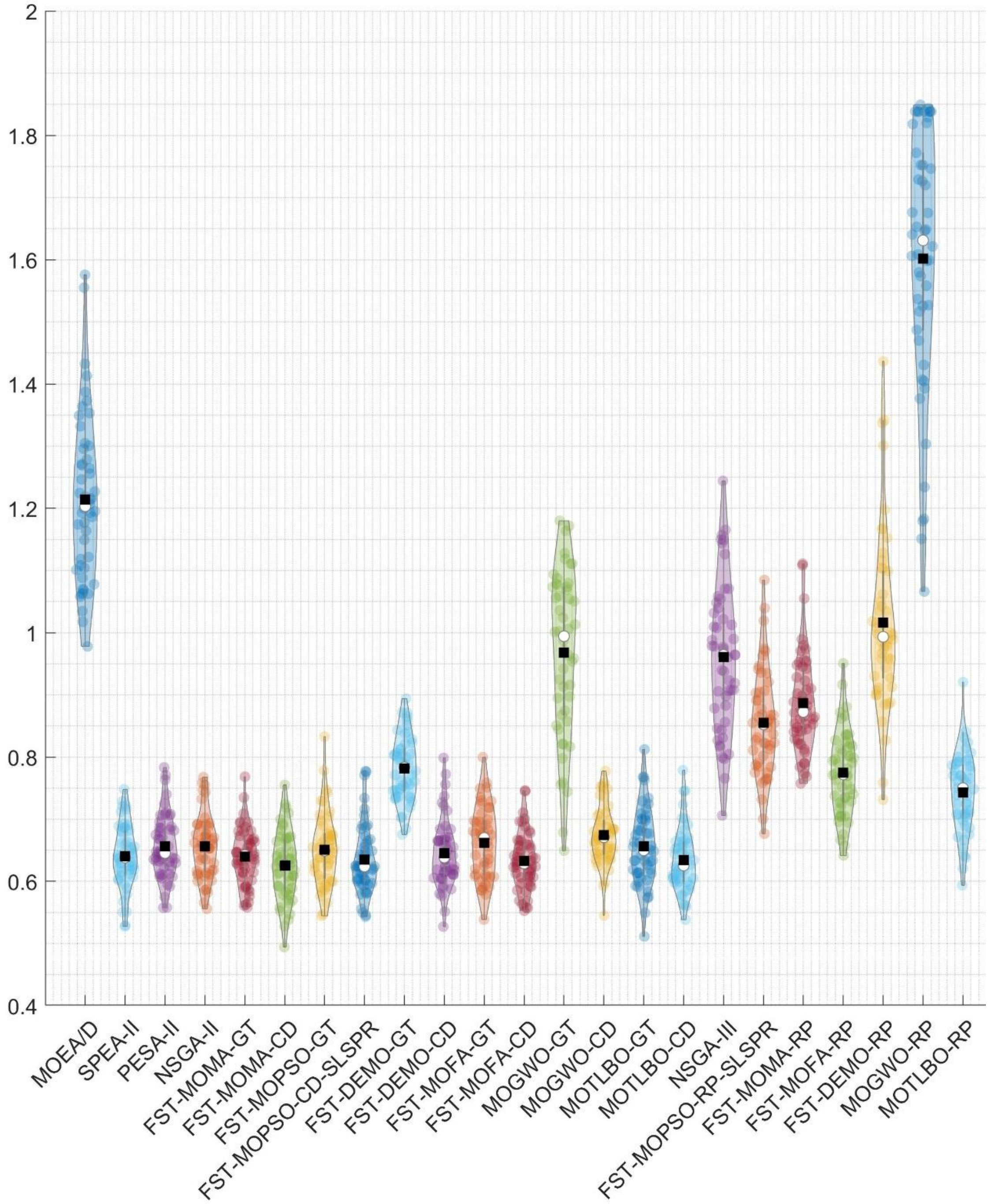


Figure 5.42: Violin plots of Δ metric values when using five objectives on the Olive oil data

Table 5.78: Statistics of Δ metric values when using five objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.58	1.21	1.20	0.98	0.60	0.13
SPEA-II	**0.75**	0.64	0.64	0.53	0.22	0.05
PESA-II	0.78	0.66	0.65	0.56	0.23	0.05
NSGA-II	0.77	0.66	0.66	0.56	0.21	0.05
FST-MOMA-GT	0.77	0.64	0.64	0.56	0.21	**0.04**
FST-MOMA-CD	0.76	**0.63**	0.63	**0.49**	0.26	0.05
FST-MOPSO-GT	0.83	0.65	0.65	0.54	0.29	0.06
FST-MOPSO-CD-SLSPR	0.78	0.64	**0.62**	0.54	0.23	0.06
FST-DEMO-GT	0.89	0.78	0.78	0.68	0.22	0.05
FST-DEMO-CD	0.80	0.65	0.64	0.53	0.27	0.05
FST-MOFA-GT	0.80	0.66	0.67	0.54	0.26	0.06

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-CD	**0.75**	**0.63**	0.63	0.55	**0.19**	0.05
MOGWO-GT	1.18	0.97	0.99	0.65	0.53	0.13
MOGWO-CD	0.78	0.67	0.67	0.55	0.23	0.05
MOTLBO-GT	0.81	0.66	0.65	0.51	0.30	0.06
MOTLBO-CD	0.78	**0.63**	0.63	0.54	0.24	0.05
NSGA-III	1.24	0.96	0.96	0.71	0.54	0.12
FST-MOPSO-RP-SLSPR	1.08	0.86	0.85	0.68	0.41	0.08
FST-MOMA-RP	1.11	0.89	0.87	0.76	0.35	0.08
FST-MOFA-RP	0.95	0.77	0.77	0.64	0.31	0.07
FST-DEMO-RP	1.44	1.02	0.99	0.73	0.70	0.14
MOGWO-RP	**1.85**	**1.60**	**1.63**	**1.07**	**0.78**	**0.21**
MOTLBO-RP	0.92	0.74	0.75	0.59	0.33	0.06

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.79, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.79: Pairwise comparisons of *Δ* metric when using five objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	1	1	1	1
PESA-II	1	0	-	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	1	1	1	1
NSGA-II	1	0	0	-	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOMA-GT	1	0	0	0	-	0	0	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOMA-CD	1	0	0	0	0	-	0	0	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-MOPSO-GT	1	0	0	0	0	0	-	0	1	0	0	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	0	0	0	0	0	0	-	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	0	1	1	0
FST-DEMO-CD	1	0	0	0	0	0	0	0	1	-	0	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	0	0	0	0	0	0	0	1	0	-	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	0	0	0	0	0	0	0	1	0	0	-	1	1	0	0	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	0	1	0	1	1
MOGWO-CD	1	0	0	0	1	1	0	1	1	0	0	1	1	-	0	1	1	1	1	1	1	1	1
MOTLBO-GT	1	0	0	0	0	0	0	0	1	0	0	0	1	0	-	0	1	1	1	1	1	1	1
MOTLBO-CD	1	0	0	0	0	0	0	0	1	0	0	0	1	1	0	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	0	1	0	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	-

Figure 5.42 and Table 5.78 illustrate that FST-MOPSO-CD-SLSPR provides the best distribution among the non-dominated solutions, while MOGWO-RP provides the worst. Table 5.79 reveals that FST-MOPSO-CD-SLSPR does not have statistically significant differences with

SPEA-II, PESA-II, NSGA-II, FST-MOMA-GT, FST-MOMA-CD, FST-MOPSO-GT, FST-DEMO-CD, FST-MOFA-GT, FST-MOFA-CD, MOTLBO-GT and MOTLBO-CD.

Finally, as regards the M_3^* values of each algorithm, Figure 5.43 demonstrates the violin plots of the M_3^* values of each method.

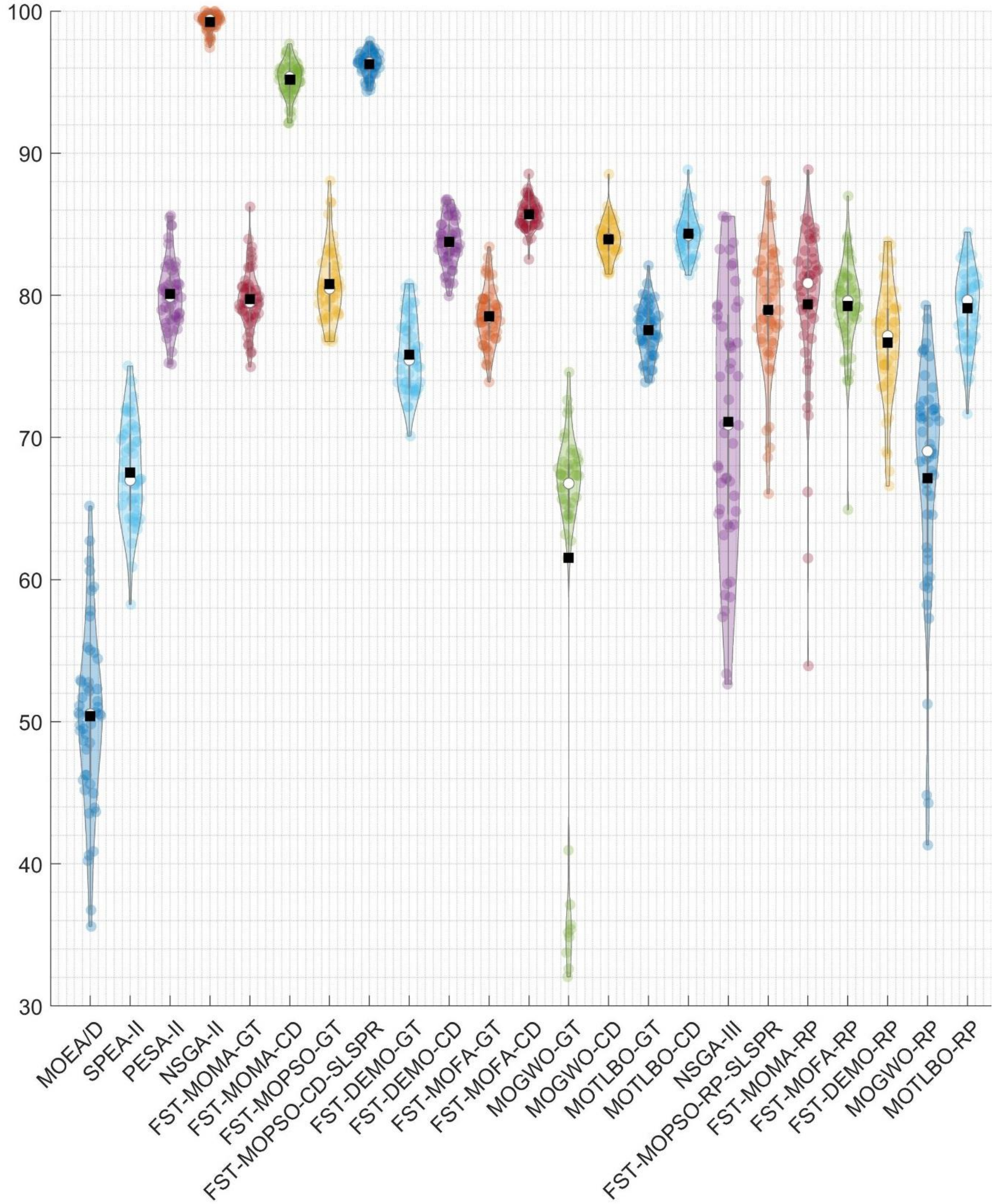


Figure 5.43: Violin plots of M_3^* metric values when using five objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=953.76, p<0.001, ε^2=0.83. Their Median values along with the rest of statistics are demonstrated in Table 5.80.

Table 5.80: Statistics of M_3^* metric values when using five objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	**65.17**	**50.39**	**50.54**	35.60	29.57	6.31
SPEA-II	75.03	67.53	66.99	58.24	16.79	3.51
PESA-II	85.60	80.08	79.90	75.15	10.44	2.45
NSGA-II	**100.00**	**99.21**	**99.38**	**97.43**	**2.57**	**0.60**
FST-MOMA-GT	86.22	79.73	79.52	74.97	11.25	2.14
FST-MOMA-CD	97.69	95.16	95.35	92.12	5.57	1.27
FST-MOPSO-GT	88.05	80.78	80.47	76.75	11.31	2.58
FST-MOPSO-CD-SLSPR	97.89	96.25	96.35	94.36	3.53	0.85
FST-DEMO-GT	80.82	75.81	75.46	70.10	10.72	2.58
FST-DEMO-CD	86.75	83.76	83.71	79.95	6.79	1.70
FST-MOFA-GT	83.41	78.51	78.46	73.90	9.51	2.01
FST-MOFA-CD	88.54	85.69	85.60	82.53	6.01	1.03
MOGWO-GT	74.59	61.53	66.76	**32.05**	**42.54**	**12.69**
MOGWO-CD	88.52	83.95	83.97	81.49	7.03	1.35
MOTLBO-GT	82.09	77.54	77.59	73.88	8.21	1.82
MOTLBO-CD	88.83	84.32	84.22	81.42	7.41	1.52
NSGA-III	85.56	71.09	70.89	52.64	32.93	8.86
FST-MOPSO-RP-SLSPR	88.05	78.97	78.92	66.04	22.00	4.54
FST-MOMA-RP	88.83	79.37	80.85	53.92	34.91	6.06
FST-MOFA-RP	86.98	79.24	79.59	64.92	22.06	3.36
FST-DEMO-RP	83.79	76.66	77.14	66.59	17.20	3.99
MOGWO-RP	79.29	67.13	69.04	41.32	37.97	8.48
MOTLBO-RP	84.44	79.09	79.64	71.65	12.79	2.77

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.81, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

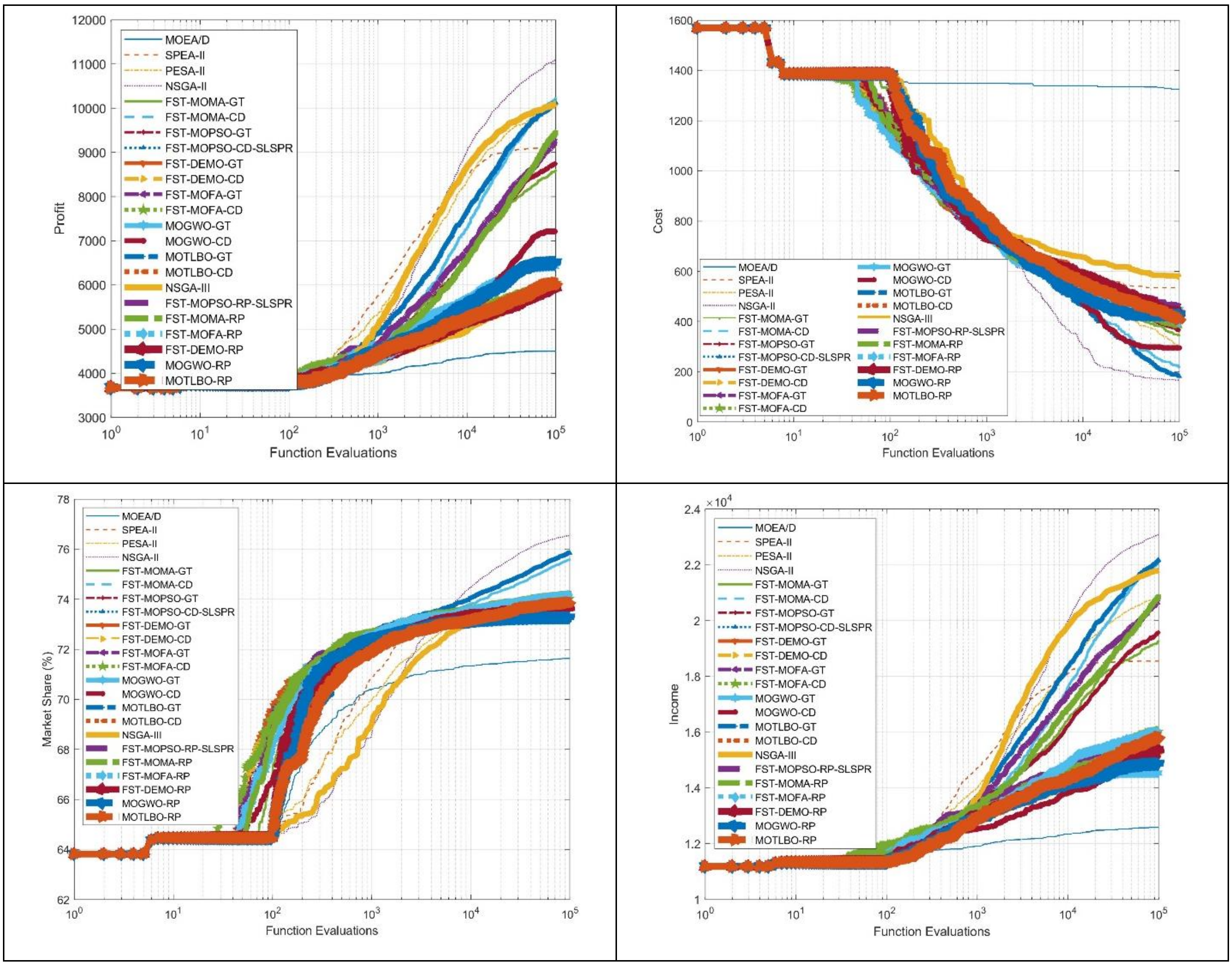
Table 5.81: Pairwise comparisons of M_3^* metric when using five objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0	1
PESA-II	1	1	-	1	0	1	0	1	1	1	0	1	1	1	1	1	1	0	0	0	1	1	0
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	1	1	1	0	1	1	1	1	1	1	0	0	0	1	1	0
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	1	0	1	-	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	0	1	0	1	1	1	0	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	0	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	0	1	0	1	1	1	1	1	-	1	1	1	0	1	1	0	0	0	0	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	0	1	1	1	1	1	1	1	1	1	1	-	1	1	1	0	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	0	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	0	1	0	1	1	1	-	1	0	0	1	1	0	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	-	1	1	1	1	1	1	1
NSGA-III	1	0	1	1	1	1	1	1	0	1	1	1	0	1	0	1	-	1	1	1	0	0	1
FST-MOPSO-RP-SLSPR	1	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	1	-	0	0	0	1	0

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOMA-RP	1	1	0	1	0	1	0	1	1	1	0	1	1	1	1	1	1	0	-	0	1	1	0
FST-MOFA-RP	1	1	0	1	0	1	0	1	1	1	0	1	1	1	1	1	1	0	0	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	1	0	0	1	1	-	1	0
MOGWO-RP	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	-	1
MOTLBO-RP	1	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	1	0	0	0	0	1	-

Figure 5.43 and Table 5.80 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while MOEA/D provides the worst.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.44.



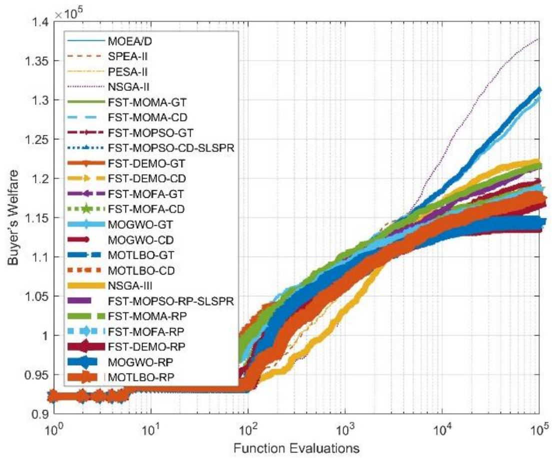


Figure 5.44: Average convergence characteristic curves when using five objectives on the Olive oil data

5.5 Fifth Scenario: Using six objective functions

In this Subsection the performance of the algorithms when using six objective functions, as presented in Table 5.1, is assessed.

5.5.1 *Results on the Timbuk2 data set when using six objectives*

Regarding the performance of the comparing algorithms while performing on the Timbuk2 data set when using six objectives, in Figure 5.45 the non-dominated solutions obtained from each algorithm are graphically demonstrated.

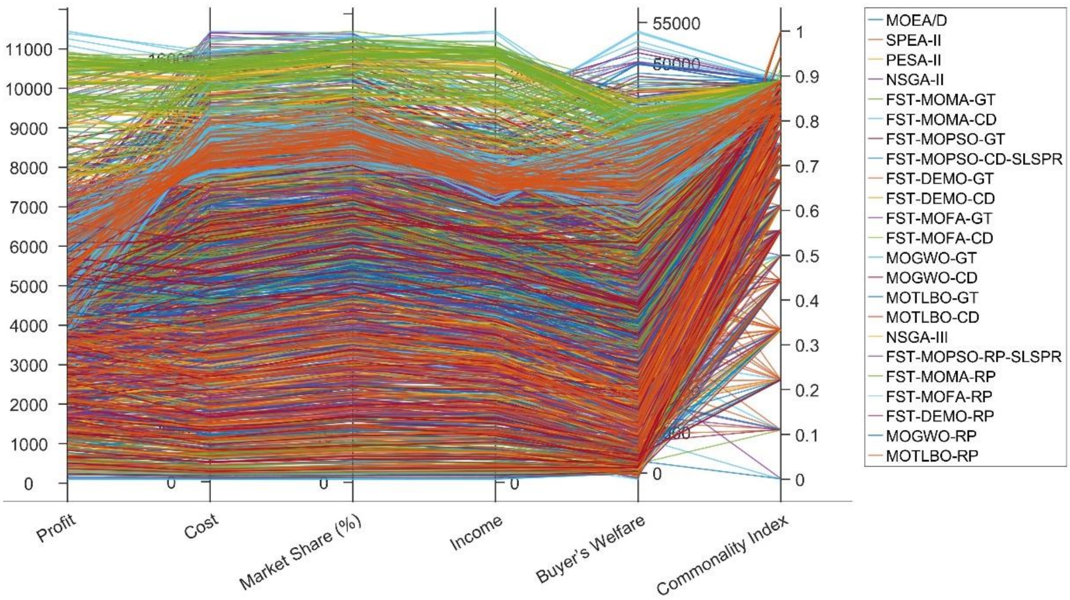


Figure 5.45: Detected non-dominated solutions when using five objectives on the Olive oil data

In Tables 5.82 and 5.83 the results of the average *C* metric values of the comparing algorithms, are presented. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U test.

Table 5.82: Comparison of methods according their average C metric values when using six objectives on the Timbuk2 data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.16	0.07	0.03	0.04	0.02	0.05	0.06	0.01	0.01	0.06	0.07
SPEA-II	0.00	1.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PESA-II	0.00	0.07	1.00	0.02	0.02	0.01	0.03	0.03	0.00	0.00	0.02	0.01
NSGA-II	0.01	0.06	0.02	1.00	0.01	0.03	0.02	0.03	0.00	0.00	0.02	0.04
FST-MOMA-GT	0.01	0.10	0.05	0.02	1.00	0.02	0.05	0.04	0.02	0.02	0.05	0.03
FST-MOMA-CD	0.01	0.07	0.03	0.03	0.02	1.00	0.03	0.03	0.01	0.01	0.02	0.03
FST-MOPSO-GT	0.01	0.07	0.03	0.01	0.02	0.01	1.00	0.03	0.00	0.00	0.02	0.02
FST-MOPSO-CD-SLSPR	0.01	0.08	0.03	0.02	0.01	0.01	0.04	1.00	0.00	0.00	0.02	0.04
FST-DEMO-GT	0.06	0.19	0.11	0.07	0.16	0.10	0.13	0.11	1.00	0.28	0.11	0.09
FST-DEMO-CD	0.06	0.17	0.11	0.06	0.15	0.11	0.11	0.09	0.27	1.00	0.11	0.08
FST-MOFA-GT	0.01	0.07	0.03	0.01	0.01	0.01	0.04	0.03	0.00	0.00	1.00	0.01
FST-MOFA-CD	0.01	0.09	0.03	0.02	0.02	0.01	0.03	0.04	0.00	0.00	0.03	1.00
MOGWO-GT	0.01	0.10	0.04	0.02	0.03	0.02	0.07	0.04	0.00	0.00	0.04	0.02
MOGWO-CD	0.01	0.08	0.03	0.03	0.02	0.02	0.04	0.05	0.00	0.00	0.02	0.05
MOTLBO-GT	0.01	0.08	0.04	0.02	0.02	0.01	0.04	0.03	0.00	0.00	0.03	0.02
MOTLBO-CD	0.01	0.10	0.04	0.03	0.02	0.02	0.04	0.04	0.00	0.01	0.04	0.06
NSGA-III	0.01	0.08	0.02	0.09	0.01	0.08	0.02	0.07	0.00	0.00	0.02	0.08
FST-MOPSO-RP-SLSPR	0.01	0.08	0.03	0.02	0.01	0.01	0.03	0.03	0.00	0.00	0.03	0.04
FST-MOMA-RP	0.01	0.09	0.03	0.04	0.02	0.03	0.03	0.06	0.01	0.01	0.02	0.08
FST-MOFA-RP	0.01	0.10	0.03	0.02	0.01	0.01	0.03	0.03	0.00	0.00	0.03	0.04
FST-DEMO-RP	0.06	0.21	0.12	0.06	0.17	0.12	0.14	0.11	0.25	0.29	0.11	0.12
MOGWO-RP	0.03	0.11	0.08	0.01	0.03	0.02	0.03	0.04	0.00	0.00	0.04	0.07
MOTLBO-RP	0.01	0.11	0.04	0.02	0.02	0.01	0.03	0.04	0.00	0.00	0.03	0.05

Table 5.83: Comparison of methods according their average C metric values when using six objectives on the Timbuk2 data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.02	0.03	0.05	0.03	0.02	0.06	0.01	0.06	0.00	0.00	0.06
SPEA-II	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PESA-II	0.01	0.02	0.02	0.01	0.00	0.02	0.00	0.01	0.00	0.00	0.01
NSGA-II	0.01	0.03	0.02	0.03	0.01	0.03	0.01	0.02	0.00	0.00	0.02
FST-MOMA-GT	0.01	0.04	0.04	0.02	0.01	0.03	0.01	0.02	0.02	0.01	0.02
FST-MOMA-CD	0.01	0.03	0.02	0.02	0.01	0.03	0.01	0.03	0.01	0.00	0.02
FST-MOPSO-GT	0.01	0.02	0.02	0.01	0.00	0.02	0.00	0.01	0.00	0.00	0.01
FST-MOPSO-CD-SLSPR	0.01	0.03	0.02	0.03	0.01	0.03	0.01	0.03	0.00	0.00	0.03
FST-DEMO-GT	0.04	0.09	0.10	0.08	0.02	0.09	0.06	0.07	0.25	0.02	0.08
FST-DEMO-CD	0.03	0.09	0.10	0.08	0.02	0.09	0.07	0.07	0.25	0.01	0.07
FST-MOFA-GT	0.01	0.03	0.02	0.01	0.01	0.02	0.00	0.01	0.00	0.00	0.01
FST-MOFA-CD	0.01	0.03	0.02	0.03	0.01	0.04	0.01	0.04	0.00	0.00	0.03
MOGWO-GT	1.00	0.05	0.03	0.02	0.00	0.03	0.01	0.01	0.00	0.00	0.02

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOGWO-CD	0.01	1.00	0.03	0.04	0.01	0.04	0.01	0.03	0.00	0.01	0.03
MOTLBO-GT	0.01	0.03	1.00	0.02	0.00	0.03	0.01	0.01	0.00	0.00	0.02
MOTLBO-CD	0.01	0.04	0.03	1.00	0.01	0.04	0.01	0.04	0.00	0.01	0.04
NSGA-III	0.00	0.06	0.02	0.06	1.00	0.10	0.08	0.09	0.00	0.01	0.07
FST-MOPSO-RP-SLSPR	0.01	0.02	0.02	0.02	0.01	1.00	0.01	0.06	0.00	0.00	0.04
FST-MOMA-RP	0.01	0.05	0.02	0.06	0.02	0.11	1.00	0.10	0.01	0.01	0.09
FST-MOFA-RP	0.01	0.03	0.03	0.02	0.01	0.06	0.01	1.00	0.00	0.01	0.06
FST-DEMO-RP	0.04	0.09	0.11	0.10	0.03	0.13	0.09	0.13	1.00	0.01	0.11
MOGWO-RP	0.02	0.05	0.03	0.03	0.05	0.09	0.02	0.06	0.00	1.00	0.05
MOTLBO-RP	0.01	0.03	0.02	0.03	0.02	0.07	0.02	0.09	0.00	0.01	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.46 demonstrates the violin plots which show the distribution of time each method needed.

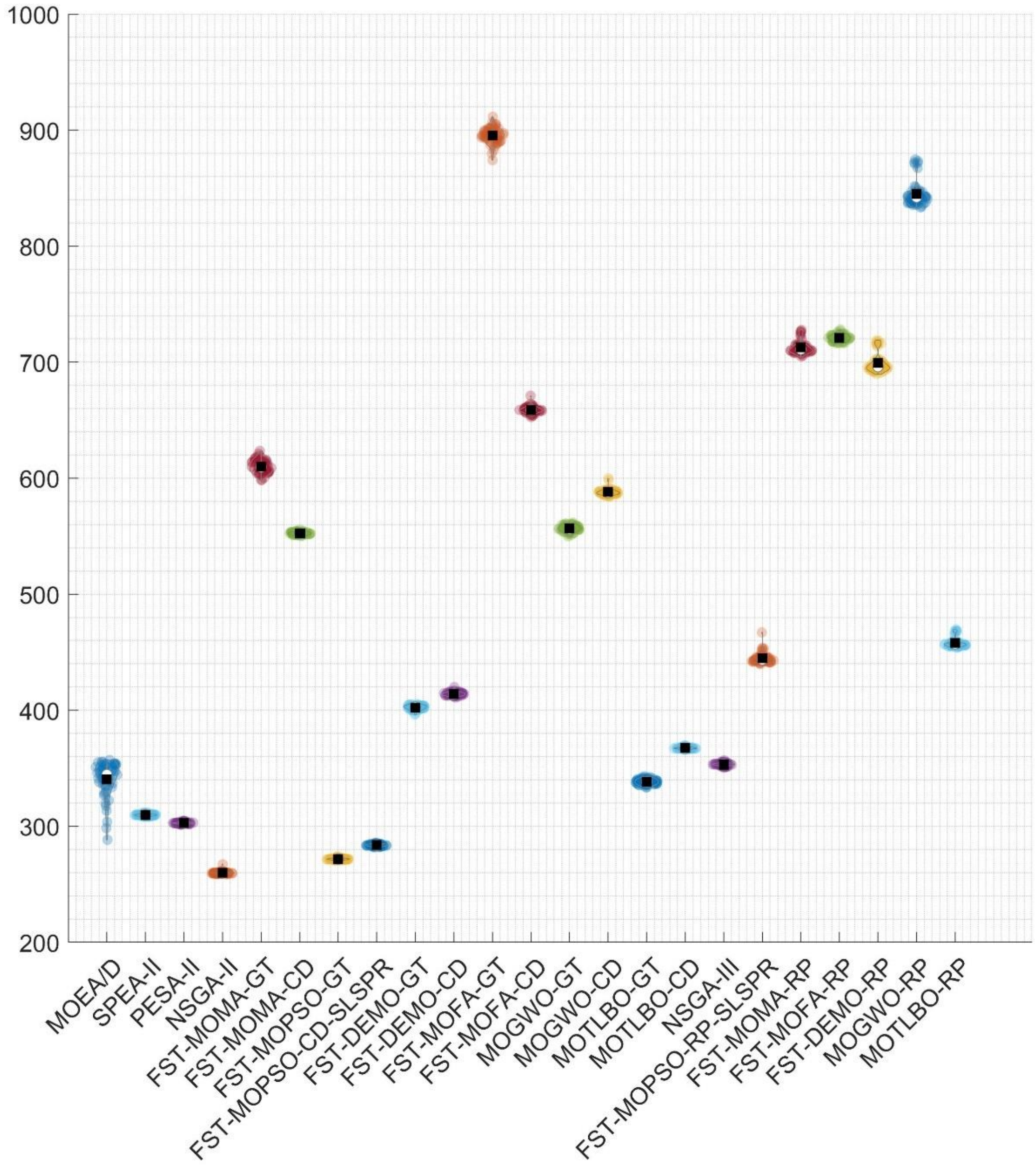


Figure 5.46: Violin plots of *Time* values when using six objectives on the Timbuk2 data

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,144.55, p<0.001, ε^2=1.00. Their Median values along with the rest of statistics are demonstrated in Table 5.84.

Table 5.84: Statistics of *Time* when using six objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	357.13	340.39	344.54	288.39	**15.42**	**68.74**
SPEA-II	311.89	309.75	309.73	308.15	0.93	3.74
PESA-II	304.67	302.90	302.88	301.39	0.84	**3.28**
NSGA-II	**267.22**	**259.90**	**259.71**	**258.55**	1.25	8.67
FST-MOMA-GT	623.51	610.06	609.59	598.36	5.62	25.15
FST-MOMA-CD	555.54	552.59	552.57	550.02	1.21	5.52
FST-MOPSO-GT	274.14	271.82	271.76	270.36	**0.78**	3.79
FST-MOPSO-CD-SLSPR	285.75	283.72	283.72	281.93	0.93	3.81
FST-DEMO-GT	405.35	402.35	402.73	396.15	1.95	9.19
FST-DEMO-CD	420.01	414.12	414.05	411.00	1.61	9.01
FST-MOFA-GT	**911.53**	**895.24**	**895.38**	**874.05**	6.80	37.48
FST-MOFA-CD	671.10	658.78	658.69	652.63	3.05	18.47
MOGWO-GT	561.72	556.70	556.26	549.89	2.80	11.83
MOGWO-CD	599.98	588.04	587.65	584.26	2.87	15.72
MOTLBO-GT	343.07	338.39	338.49	333.56	2.33	9.51
MOTLBO-CD	369.89	367.40	367.25	366.00	0.83	3.89
NSGA-III	356.95	353.33	353.31	350.45	1.35	6.50
FST-MOPSO-RP-SLSPR	467.05	444.77	443.49	439.68	4.60	27.37
FST-MOMA-RP	727.73	712.72	710.98	704.98	5.52	22.75
FST-MOFA-RP	728.21	720.95	720.89	716.44	2.86	11.77
FST-DEMO-RP	718.93	699.33	696.02	689.75	8.21	29.18
MOGWO-RP	874.75	844.99	842.20	833.34	10.80	41.42
MOTLBO-RP	469.87	458.11	457.01	453.94	4.10	15.93

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.85, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.85: Pairwise comparisons of *Time* when using six objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	0	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.46 and Table 5.84 illustrate that NSGA-II needs the least time, while FST-MOFA-GT needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.47 demonstrates the violin plots of the M_1^* values of each method.

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)=489.09$, $p<0.001$, $\varepsilon^2=0.43$. Their Median values along with the rest of statistics are demonstrated in Table 5.86.

Table 5.86: Statistics of M_1^* values when using six objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	415.21	95.89	82.11	**0.00**	415.21	76.79
SPEA-II	393.56	**232.78**	**222.99**	**116.64**	276.92	75.58
PESA-II	211.98	139.32	133.81	80.18	131.80	29.69
NSGA-II	223.19	99.11	91.04	48.49	174.71	36.97
FST-MOMA-GT	159.55	97.77	97.50	42.89	116.66	**24.54**
FST-MOMA-CD	178.80	88.34	82.85	31.71	147.09	32.84
FST-MOPSO-GT	272.75	150.55	147.27	66.19	206.56	38.03
FST-MOPSO-CD-SLSPR	220.67	148.12	149.80	58.87	161.80	37.34
FST-DEMO-GT	182.84	81.43	78.50	37.77	145.07	30.12
FST-DEMO-CD	175.92	84.44	79.45	42.54	133.38	26.38
FST-MOFA-GT	207.00	128.07	123.18	76.66	130.33	30.83
FST-MOFA-CD	242.18	157.95	153.95	68.37	173.81	37.60
MOGWO-GT	**532.48**	100.15	67.31	**0.00**	**532.48**	98.62
MOGWO-CD	428.97	125.80	101.66	4.62	424.35	86.70
MOTLBO-GT	216.00	119.21	118.35	52.69	163.31	32.72
MOTLBO-CD	177.63	124.14	120.12	76.88	**100.75**	25.87
NSGA-III	**129.89**	**51.83**	**48.75**	**0.00**	129.89	35.45
FST-MOPSO-RP-SLSPR	310.70	180.71	167.90	72.90	237.80	58.08
FST-MOMA-RP	171.15	74.18	67.86	5.64	165.50	36.31
FST-MOFA-RP	313.49	176.91	167.68	63.92	249.58	68.60
FST-DEMO-RP	245.00	95.26	89.69	20.16	224.84	46.01
MOGWO-RP	406.75	98.95	66.35	**0.00**	406.75	**105.63**
MOTLBO-RP	275.87	156.60	147.36	62.52	213.35	60.75

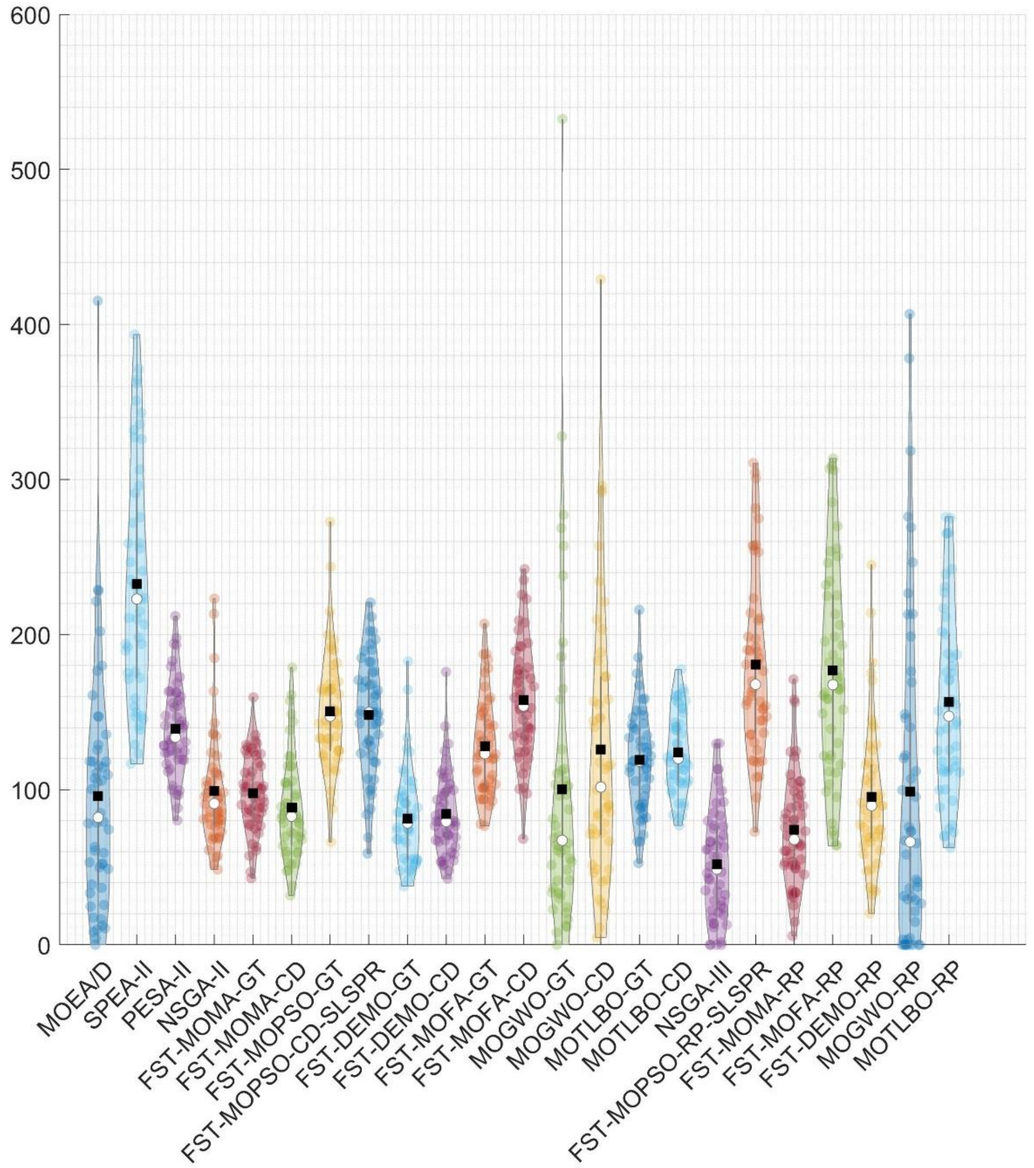


Figure 5.47: Violin plots of M_1^* metric values when using six objectives on the Timbuk2 data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.87, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.87: Pairwise comparisons of M_1^* when using six objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1
PESA-II	1	1	-	1	1	1	0	0	1	1	0	0	1	0	0	0	1	1	1	0	1	1	0
NSGA-II	0	1	1	-	0	0	1	1	0	0	1	1	0	0	0	1	1	1	0	1	0	0	1
FST-MOMA-GT	0	1	1	0	-	0	1	1	0	0	1	1	0	0	0	1	1	1	1	1	0	0	1

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
FST-MOMA-CD	0	1	1	0	0	-	1	1	0	0	1	1	0	0	1	1	1	1	0	1	0	0	1
FST-MOPSO-GT	1	1	0	1	1	1	-	0	1	1	0	0	1	0	1	1	1	0	1	0	1	1	0
FST-MOPSO-CD-SLSPR	1	1	0	1	1	1	0	-	1	1	0	0	1	0	1	0	1	0	1	0	1	1	0
FST-DEMO-GT	0	1	1	0	0	0	1	1	-	0	1	1	0	0	1	1	1	1	0	1	0	0	1
FST-DEMO-CD	0	1	1	0	0	0	1	1	0	-	1	1	0	0	1	1	1	1	0	1	0	0	1
FST-MOFA-GT	0	1	0	1	1	1	0	0	1	1	-	1	1	0	0	0	1	1	1	1	1	0	0
FST-MOFA-CD	1	1	0	1	1	1	0	0	1	1	1	-	1	0	1	1	1	0	1	0	1	1	0
MOGWO-GT	0	1	1	0	0	0	1	1	0	0	1	1	-	0	1	1	0	1	0	1	0	0	1
MOGWO-CD	0	1	0	0	0	0	0	0	0	0	0	0	0	-	0	0	1	1	0	1	0	0	0
MOTLBO-GT	0	1	0	0	0	1	1	1	1	1	0	1	1	0	-	0	1	1	1	1	0	0	0
MOTLBO-CD	0	1	0	1	1	1	1	0	1	1	0	1	1	0	0	-	1	1	1	1	1	0	0
NSGA-III	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	-	1	0	1	1	0	1
FST-MOPSO-RP-SLSPR	1	0	1	1	1	1	0	0	1	1	1	0	1	1	1	1	1	-	1	0	1	1	0
FST-MOMA-RP	0	1	1	0	1	0	1	1	0	0	1	1	0	0	1	1	0	1	-	1	0	0	1
FST-MOFA-RP	1	0	0	1	1	1	0	0	1	1	1	0	1	1	1	1	1	0	1	-	1	1	0
FST-DEMO-RP	0	1	1	0	0	0	1	1	0	0	1	1	0	0	0	1	1	1	0	1	-	0	1
MOGWO-RP	0	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	1	0	1	0	-	1
MOTLBO-RP	1	1	0	1	1	1	0	0	1	1	0	0	1	0	0	0	1	0	1	0	1	1	-

Figure 5.47 and Table 5.86 illustrate that NSGA-III provides the closest to the optimal pareto front non-dominated solutions, while SPEA-II provides the farthest ones. Table 5.87 reveals that NSGA-III does not have statistically significant differences with MOEA/D, MOGWO-GT, FST-MOMA-RP and MOGWO-RP, while SPEA-II does not have statistically significant differences with FST-MOPSO-RP-SLSPR and FST-MOFA-RP.

As regards the *Δ* values of each algorithm, Figure 5.48 demonstrates the violin plots of the *Δ* metric values, of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)$=910.21, p<0.001, ε^2=0.80. Their Median values along with the rest of statistics are demonstrated in Table 5.88.

Table 5.88: Statistics of *Δ* metric values when using six objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.40	1.22	1.22	1.06	0.34	0.08
SPEA-II	**0.72**	**0.62**	**0.61**	**0.52**	0.20	0.05
PESA-II	0.76	0.67	0.67	0.57	0.19	0.05
NSGA-II	0.80	0.68	0.68	0.57	0.23	0.05
FST-MOMA-GT	0.83	0.71	0.71	0.53	0.30	0.06
FST-MOMA-CD	0.79	0.68	0.68	0.57	0.23	0.05
FST-MOPSO-GT	0.82	0.70	0.71	0.58	0.24	0.06
FST-MOPSO-CD-SLSPR	0.77	0.64	0.63	0.54	0.23	0.05
FST-DEMO-GT	1.32	1.26	1.26	1.21	**0.11**	**0.03**
FST-DEMO-CD	1.32	1.24	1.24	1.20	0.13	**0.03**
FST-MOFA-GT	0.82	0.66	0.65	0.55	0.28	0.06
FST-MOFA-CD	0.80	0.70	0.70	0.61	0.20	0.05
MOGWO-GT	1.38	1.19	1.18	1.00	0.38	0.08
MOGWO-CD	0.86	0.71	0.70	0.58	0.28	0.05
MOTLBO-GT	0.83	0.67	0.65	0.54	0.28	0.06
MOTLBO-CD	0.76	0.66	0.67	**0.52**	0.24	0.05

Method	Max	Mean	Median	Min	Range	SD
NSGA-III	1.86	1.04	1.00	0.70	**1.16**	**0.20**
FST-MOPSO-RP-SLSPR	0.97	0.78	0.77	0.65	0.32	0.07
FST-MOMA-RP	1.12	0.90	0.90	0.62	0.49	0.10
FST-MOFA-RP	1.05	0.79	0.79	0.55	0.50	0.10
FST-DEMO-RP	1.54	1.32	1.30	1.21	0.32	0.07
MOGWO-RP	**1.98**	**1.68**	**1.66**	**1.33**	0.65	0.16
MOTLBO-RP	1.02	0.77	0.76	0.54	0.48	0.10

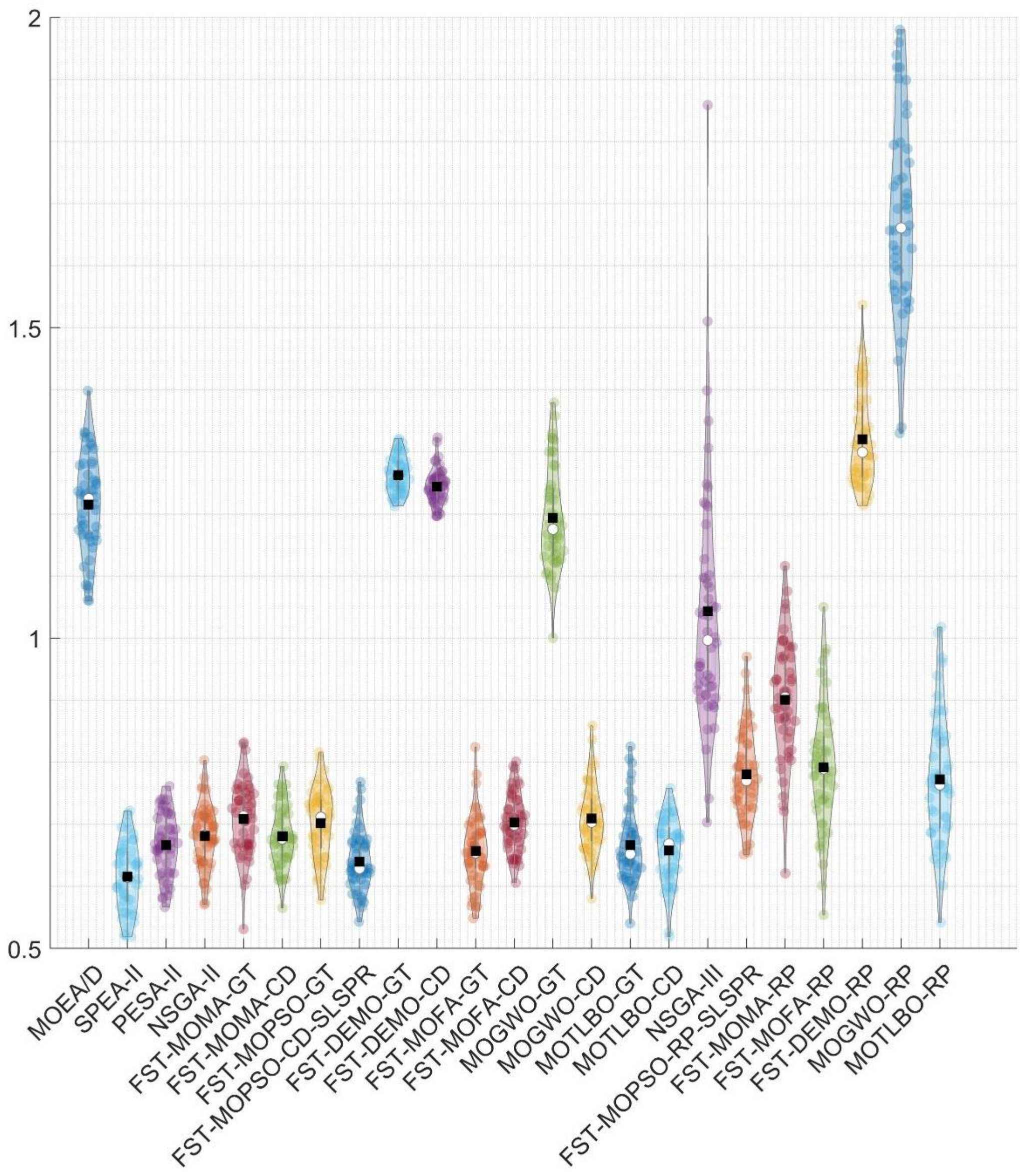


Figure 5.48: Violin plots of *Δ* metric values when using six objectives on the Timbuk2 data

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.89, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between groups.

Table 5.89: Pairwise comparisons of Δ metric when using six objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	0	1	0	0	0	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1
NSGA-II	1	1	0	-	0	0	0	1	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	0	-	0	0	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	0
FST-MOMA-CD	1	1	0	0	0	-	0	1	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	0	0	0	-	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	0	0	1	1	1	1	-	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1
FST-DEMO-GT	0	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	0	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	0	0	0	1	0	1	0	1	1	-	1	1	1	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	0	0	0	0	0	1	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1
MOGWO-GT	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	0	0	0	0	0	1	1	1	1	0	1	-	1	1	1	1	1	1	1	1	0
MOTLBO-GT	1	1	0	0	1	0	0	0	1	1	0	1	1	1	-	0	1	1	1	1	1	1	1
MOTLBO-CD	1	1	0	0	1	0	1	0	1	1	0	1	1	1	0	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	0	1	1	0
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	0	1	0	1	1	-

Figure 5.48 and Table 5.88 illustrate that SPEA-II provides the best distribution among the non-dominated solutions, while MOGWO-RP provides the worst. According to Table 5.89, SPEA-II does not have statistically significant differences with FST-MOPSO-CD-SLSPR and FST-MOFA-GT.

Finally, as regards the M_3^* values of each algorithm, Figure 5.49 demonstrates the violin plots of the M_3^* values of each method. A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)$=831.62, p<0.001, ε^2=0.72. Their Median values along with the rest of statistics are demonstrated in Table 5.90.

Table 5.90: Statistics of M_3^* metric values when using six objectives on the Timbuk2 data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	83.15	72.68	72.57	63.53	19.62	4.31
SPEA-II	87.17	82.12	82.18	75.26	11.91	2.60
PESA-II	85.38	81.25	81.33	76.77	8.61	1.92
NSGA-II	98.48	97.34	97.37	96.31	**2.17**	0.64
FST-MOMA-GT	85.15	82.20	82.19	78.68	6.48	1.66
FST-MOMA-CD	**100.00**	**98.88**	**98.96**	**97.53**	2.47	**0.50**
FST-MOPSO-GT	84.13	79.91	79.99	74.37	9.76	2.33
FST-MOPSO-CD-SLSPR	93.14	90.47	90.54	87.38	5.75	1.44
FST-DEMO-GT	83.14	79.23	79.06	72.81	10.33	2.63
FST-DEMO-CD	83.28	79.15	78.90	73.45	9.83	2.55

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-GT	84.78	80.21	80.22	75.97	8.81	2.02
FST-MOFA-CD	89.78	87.88	87.79	86.83	2.96	0.63
MOGWO-GT	**75.92**	64.14	**64.80**	42.63	33.28	7.34
MOGWO-CD	94.38	91.98	92.06	89.34	5.04	1.11
MOTLBO-GT	85.70	81.47	81.51	76.71	8.99	1.92
MOTLBO-CD	91.10	89.05	89.11	86.73	4.38	0.93
NSGA-III	93.26	78.83	86.48	26.29	66.98	15.91
FST-MOPSO-RP-SLSPR	89.91	84.19	85.20	73.55	16.36	3.97
FST-MOMA-RP	93.30	82.11	87.42	39.75	53.55	11.74
FST-MOFA-RP	86.33	76.04	79.99	47.96	38.36	9.89
FST-DEMO-RP	82.57	78.90	78.79	74.15	8.42	2.55
MOGWO-RP	83.72	**61.01**	72.93	**0.00**	**83.72**	**26.49**
MOTLBO-RP	88.93	78.76	82.18	46.77	42.16	9.10

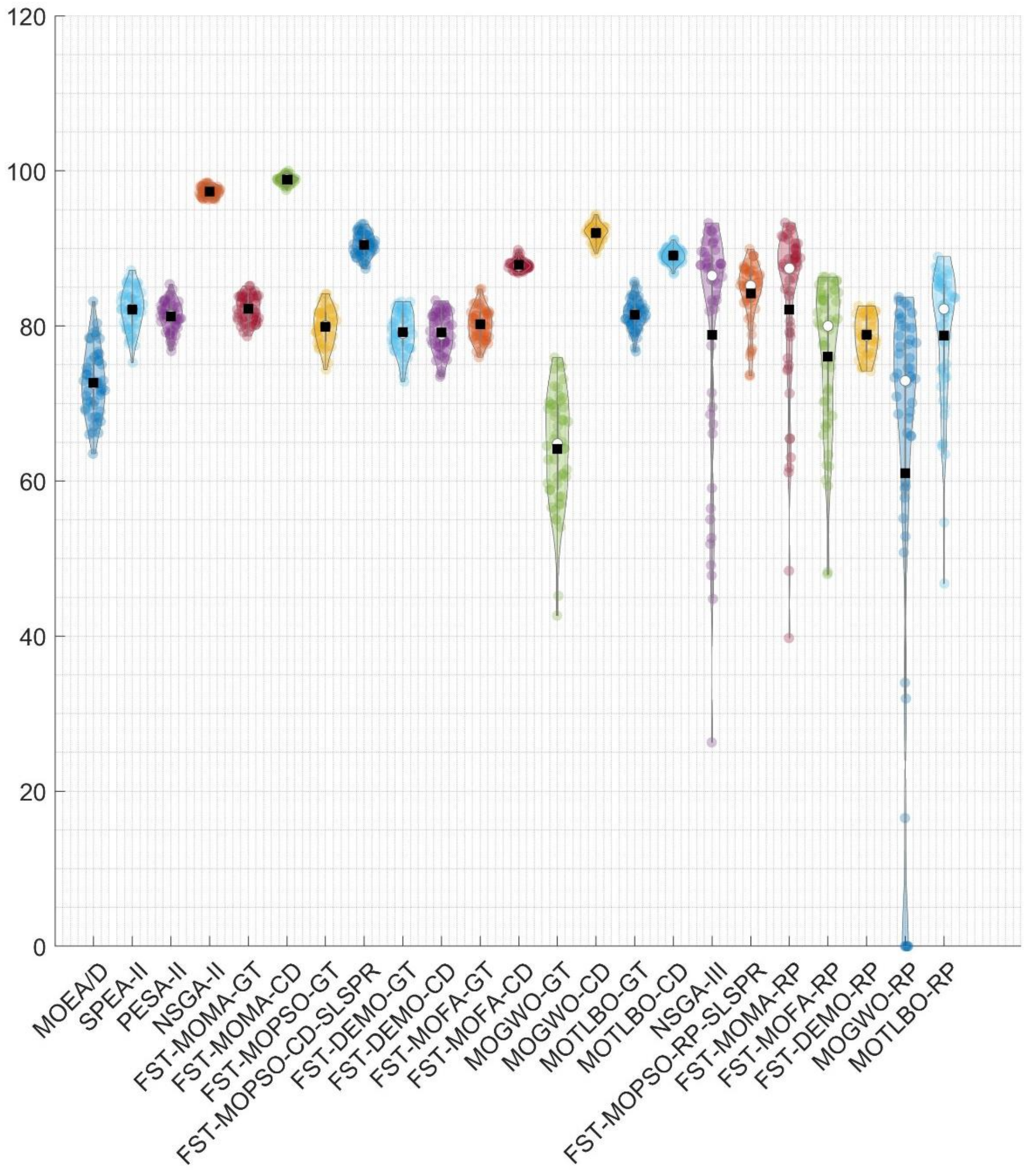


Figure 5.49: Violin plots of M_3^* metric values when using six objectives on the Timbuk2 data

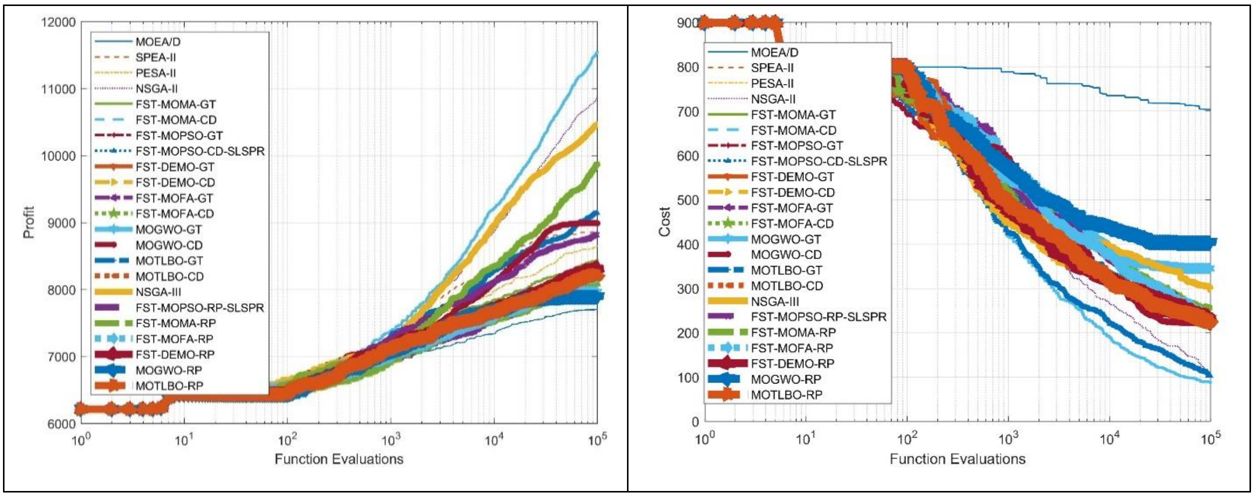
Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.91, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.91: Pairwise comparisons of M_3^* metric when using six objectives on the Timbuk2 data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1
SPEA-II	1	-	0	1	0	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0	1	1	0
PESA-II	1	0	-	1	0	1	0	1	1	1	0	1	1	1	0	1	0	1	0	0	1	1	0
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	0	0	1	-	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0	1	1	0
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	1	1	1	-	1	0	0	0	1	1	1	0	1	0	1	0	0	0	1	0
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	0	1	-	0	0	1	1	1	1	1	0	1	1	0	0	1	0
FST-DEMO-CD	1	1	1	1	1	1	0	1	0	-	0	1	1	1	1	1	0	1	1	0	0	1	0
FST-MOFA-GT	1	1	0	1	1	1	0	1	0	0	-	1	1	1	0	1	0	1	0	0	0	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	0	1	0	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	0	0	1	0	1	0	1	1	1	0	1	1	1	-	1	0	1	0	0	1	1	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	0	1	1	1	1
NSGA-III	1	0	0	1	0	1	0	1	0	0	0	0	1	1	0	1	-	0	0	0	0	1	0
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	0	1	1	1	0
FST-MOMA-RP	1	0	0	1	0	1	0	1	1	1	0	0	1	1	0	0	0	0	-	1	1	1	0
FST-MOFA-RP	0	0	0	1	0	1	0	1	0	0	0	1	1	1	0	1	0	1	1	-	0	1	0
FST-DEMO-RP	1	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1	0	1	1	0	-	1	0
MOGWO-RP	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	0	0	1	0	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0	0	1	-

Figure 5.49 and Table 5.90 illustrate that FST-MOMA-CD provides the best extend of the non-dominated solutions, while MOGWO-GT provides the worst. Table 5.91 reveals that MOGWO-GT does not have statistically significant differences with MOGWO-RP.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.50.



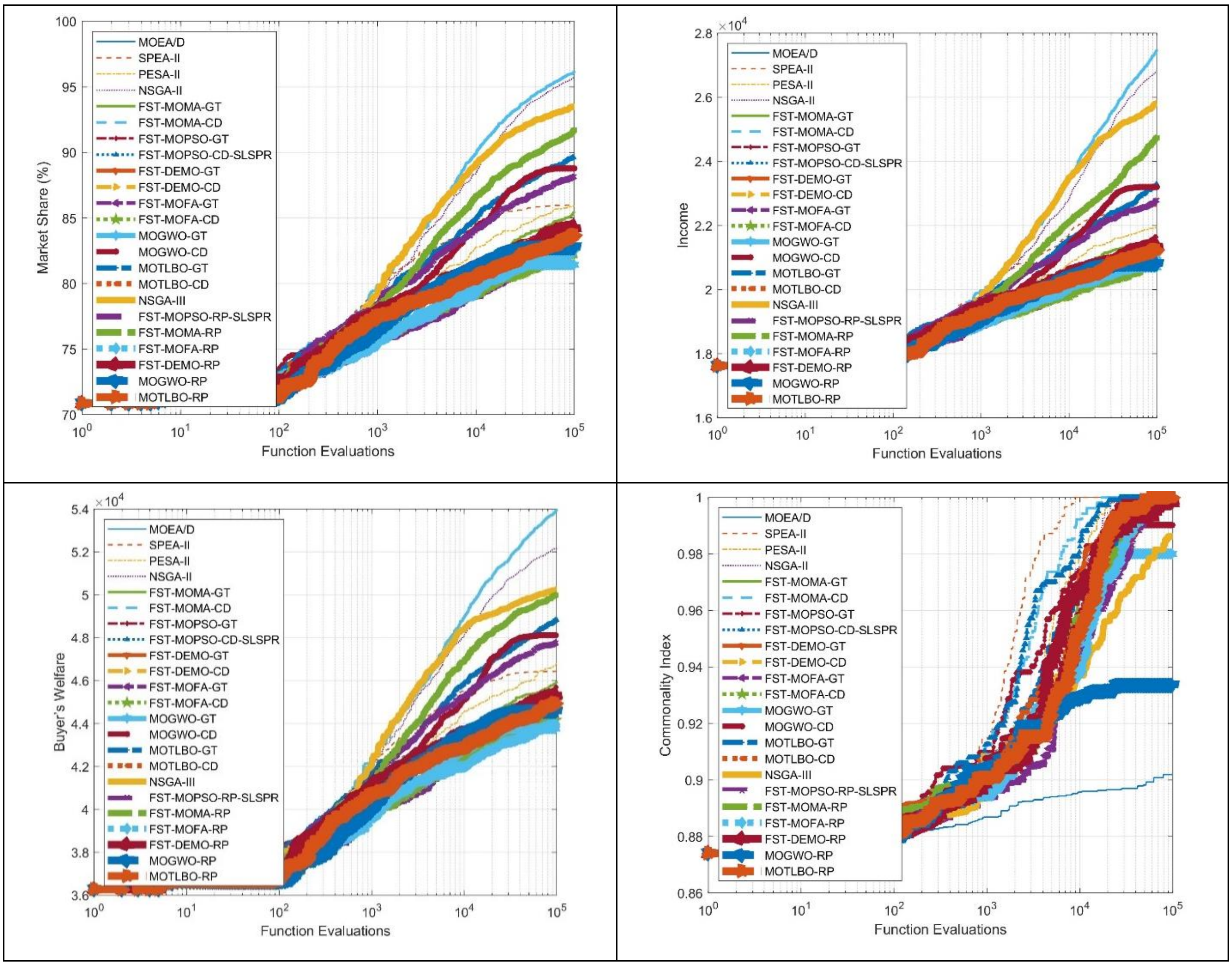


Figure 5.50: Average convergence characteristic curves when using six objectives on the Timbuk2 data

5.5.2 Results on the Olive oil data set when using six objectives

Regarding the performance of the comparing algorithms while performing on the olive oil data set when using six objectives, in Figure 5.51 the non-dominated solutions obtained from each algorithm are graphically demonstrated, while Tables 5.92 and 5.93 show the results of the average *C* metric values of the comparing algorithms. Cells shaded in light blue, indicate whether there is a statistically significant difference between *C*(A,B) and *C*(B,A), according to a Mann Whitney U Test.

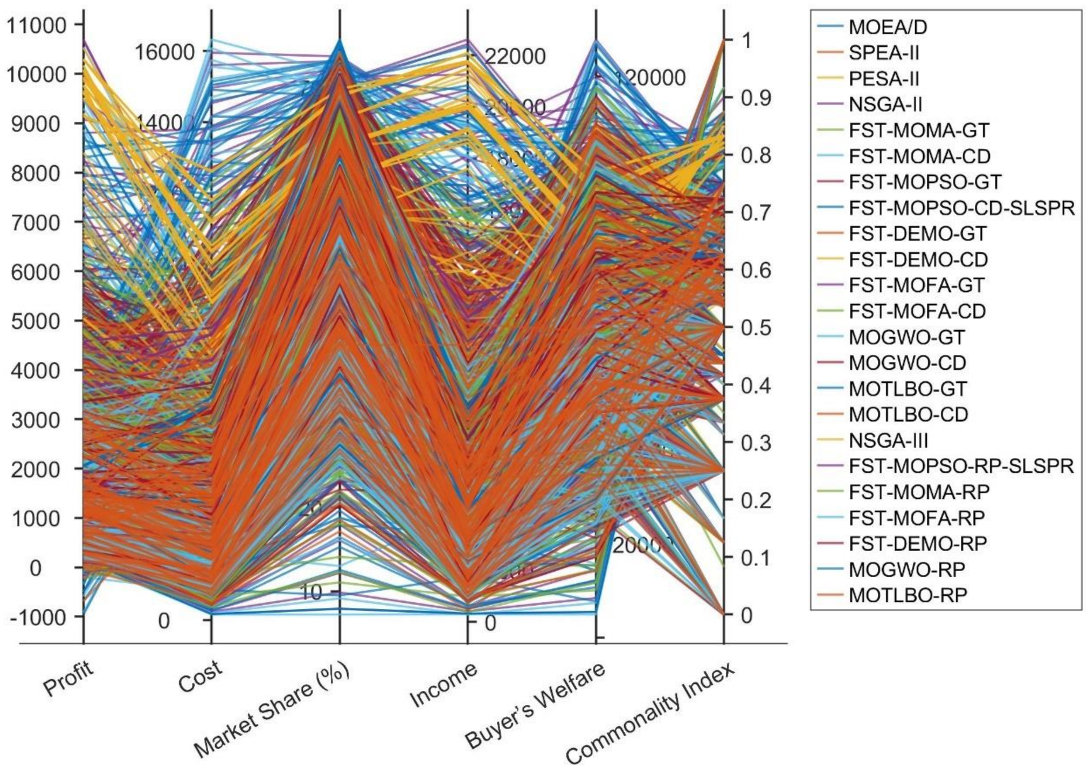


Figure 5.51: Detected non-dominated solutions when using five objectives on the Olive oil data

Table 5.92: Comparison of methods according their average C metric values when using six objectives on the Olive oil data (Part 1)

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOEA/D	1.00	0.01	0.07	0.29	0.09	0.22	0.15	0.30	0.11	0.11	0.36	0.32
SPEA-II	0.05	1.00	0.19	0.44	0.30	0.43	0.34	0.46	0.30	0.31	0.73	0.67
PESA-II	0.04	0.04	1.00	0.32	0.18	0.31	0.22	0.32	0.23	0.23	0.55	0.51
NSGA-II	0.01	0.00	0.04	1.00	0.03	0.12	0.05	0.12	0.03	0.03	0.13	0.21
FST-MOMA-GT	0.06	0.02	0.09	0.29	1.00	0.27	0.17	0.29	0.22	0.21	0.50	0.44
FST-MOMA-CD	0.01	0.01	0.04	0.10	0.04	1.00	0.05	0.12	0.03	0.03	0.15	0.21
FST-MOPSO-GT	0.05	0.02	0.06	0.24	0.09	0.21	1.00	0.24	0.17	0.17	0.44	0.39
FST-MOPSO-CD-SLSPR	0.01	0.00	0.04	0.09	0.04	0.09	0.05	1.00	0.03	0.03	0.15	0.21
FST-DEMO-GT	0.06	0.00	0.04	0.20	0.06	0.16	0.09	0.20	1.00	0.29	0.32	0.29
FST-DEMO-CD	0.05	0.01	0.04	0.19	0.05	0.15	0.09	0.19	0.31	1.00	0.34	0.27
FST-MOFA-GT	0.01	0.00	0.01	0.08	0.01	0.06	0.02	0.07	0.04	0.04	1.00	0.10
FST-MOFA-CD	0.02	0.00	0.02	0.06	0.02	0.05	0.02	0.06	0.03	0.03	0.12	1.00
MOGWO-GT	0.07	0.02	0.07	0.27	0.08	0.25	0.15	0.28	0.19	0.19	0.45	0.40
MOGWO-CD	0.04	0.01	0.06	0.19	0.08	0.14	0.10	0.17	0.13	0.12	0.34	0.33
MOTLBO-GT	0.02	0.00	0.02	0.11	0.02	0.09	0.03	0.10	0.07	0.06	0.20	0.16
MOTLBO-CD	0.02	0.00	0.02	0.09	0.03	0.07	0.03	0.08	0.05	0.05	0.17	0.15
NSGA-III	0.03	0.02	0.09	0.22	0.07	0.27	0.10	0.31	0.05	0.05	0.18	0.32
FST-MOPSO-RP-SLSPR	0.02	0.01	0.04	0.13	0.05	0.12	0.08	0.15	0.05	0.05	0.19	0.26
FST-MOMA-RP	0.02	0.01	0.05	0.16	0.06	0.17	0.08	0.18	0.05	0.05	0.18	0.24
FST-MOFA-RP	0.01	0.00	0.02	0.08	0.02	0.06	0.03	0.07	0.03	0.04	0.14	0.12
FST-DEMO-RP	0.06	0.00	0.04	0.17	0.09	0.16	0.09	0.16	0.28	0.30	0.34	0.26

C(A,B)	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD
MOGWO-RP	0.06	0.01	0.04	0.16	0.03	0.13	0.09	0.16	0.09	0.08	0.29	0.27
MOTLBO-RP	0.02	0.00	0.03	0.11	0.03	0.08	0.05	0.10	0.05	0.05	0.18	0.15

Table 5.93: Comparison of methods according their average C metric values when using six objectives on the Olive oil data (Part 2)

C(A,B)	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	0.05	0.13	0.30	0.22	0.11	0.19	0.11	0.28	0.11	0.08	0.26
SPEA-II	0.19	0.36	0.64	0.64	0.14	0.29	0.19	0.48	0.30	0.16	0.56
PESA-II	0.13	0.25	0.48	0.46	0.12	0.22	0.13	0.36	0.21	0.12	0.42
NSGA-II	0.01	0.07	0.10	0.16	0.04	0.09	0.08	0.16	0.02	0.03	0.14
FST-MOMA-GT	0.12	0.20	0.43	0.38	0.11	0.20	0.11	0.30	0.20	0.09	0.35
FST-MOMA-CD	0.01	0.08	0.11	0.16	0.04	0.10	0.09	0.15	0.03	0.03	0.15
FST-MOPSO-GT	0.08	0.15	0.38	0.34	0.09	0.17	0.10	0.26	0.16	0.08	0.31
FST-MOPSO-CD-SLSPR	0.01	0.08	0.11	0.16	0.04	0.10	0.07	0.15	0.03	0.03	0.15
FST-DEMO-GT	0.05	0.11	0.25	0.21	0.07	0.16	0.07	0.19	0.26	0.04	0.22
FST-DEMO-CD	0.05	0.11	0.27	0.19	0.05	0.15	0.06	0.17	0.27	0.07	0.22
FST-MOFA-GT	0.01	0.03	0.09	0.08	0.02	0.04	0.03	0.08	0.04	0.02	0.09
FST-MOFA-CD	0.01	0.04	0.09	0.09	0.02	0.05	0.02	0.09	0.03	0.03	0.09
MOGWO-GT	1.00	0.11	0.39	0.35	0.11	0.18	0.11	0.23	0.18	0.07	0.30
MOGWO-CD	0.06	1.00	0.29	0.28	0.06	0.13	0.08	0.23	0.11	0.07	0.26
MOTLBO-GT	0.03	0.05	1.00	0.12	0.04	0.07	0.04	0.10	0.05	0.03	0.12
MOTLBO-CD	0.02	0.06	0.14	1.00	0.03	0.07	0.04	0.11	0.04	0.03	0.13
NSGA-III	0.01	0.13	0.14	0.26	1.00	0.27	0.22	0.25	0.05	0.05	0.28
FST-MOPSO-RP-SLSPR	0.02	0.10	0.14	0.21	0.05	1.00	0.09	0.25	0.05	0.04	0.25
FST-MOMA-RP	0.01	0.09	0.15	0.21	0.07	0.20	1.00	0.24	0.05	0.04	0.24
FST-MOFA-RP	0.01	0.05	0.12	0.11	0.03	0.07	0.04	1.00	0.03	0.05	0.12
FST-DEMO-RP	0.03	0.10	0.27	0.20	0.07	0.13	0.07	0.21	1.00	0.08	0.22
MOGWO-RP	0.05	0.12	0.29	0.21	0.05	0.13	0.05	0.22	0.10	1.00	0.23
MOTLBO-RP	0.02	0.05	0.15	0.13	0.04	0.08	0.05	0.15	0.05	0.05	1.00

As regards the time each algorithm needed to complete 100,000 function evaluations, Figure 5.52 demonstrates the violin plots which show the distribution of time each method needed.

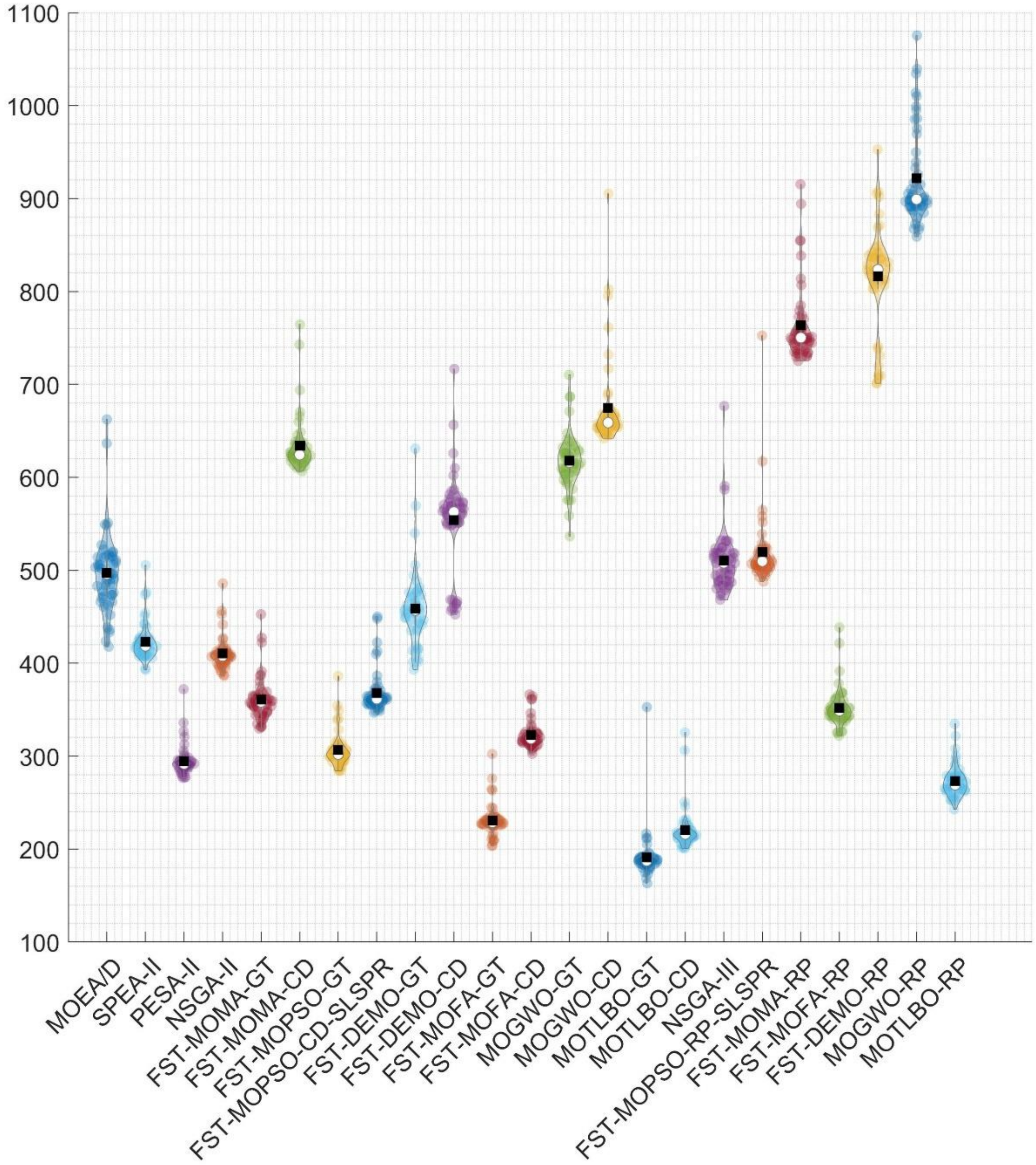


Figure 5.52: Violin plots of *Time* values when using six objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects time needed, $H(22)$=1,121.71, p<0.001, ε^2=0.98. Their Median values along with the rest of statistics are demonstrated in Table 5.94.

Table 5.94: Statistics of *Time* when using six objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	662.36	497.25	495.87	417.71	244.65	43.72
SPEA-II	505.41	422.83	418.59	392.84	112.57	20.31
PESA-II	372.05	294.66	291.68	276.46	95.59	15.92
NSGA-II	485.84	410.44	407.72	386.25	99.59	17.35
FST-MOMA-GT	452.66	360.93	358.31	329.66	123.00	23.49
FST-MOMA-CD	764.51	633.80	624.31	606.51	158.01	29.65
FST-MOPSO-GT	386.09	306.55	302.02	284.15	101.94	18.98
FST-MOPSO-CD-SLSPR	450.17	368.13	361.61	346.76	103.41	22.39

Method	Max	Mean	Median	Min	Range	SD
FST-DEMO-GT	630.95	458.79	457.10	392.93	238.02	40.87
FST-DEMO-CD	716.64	553.80	562.75	452.43	264.21	51.89
FST-MOFA-GT	**302.32**	230.59	228.63	203.31	99.02	16.87
FST-MOFA-CD	366.33	322.75	318.87	302.58	**63.75**	**14.51**
MOGWO-GT	710.26	617.84	616.05	536.56	173.70	30.08
MOGWO-CD	905.34	674.49	658.87	641.95	263.39	47.78
MOTLBO-GT	352.76	**190.77**	**187.33**	**163.17**	189.59	25.22
MOTLBO-CD	325.55	220.33	216.10	200.60	124.95	22.08
NSGA-III	676.75	510.48	507.92	468.25	208.51	33.94
FST-MOPSO-RP-SLSPR	752.52	519.36	509.67	487.89	**264.64**	39.80
FST-MOMA-RP	915.33	763.77	750.26	725.31	190.03	41.41
FST-MOFA-RP	438.61	351.93	348.87	321.99	116.62	20.95
FST-DEMO-RP	952.69	816.40	823.46	701.11	251.58	**55.38**
MOGWO-RP	**1,075.51**	**921.75**	**899.18**	**859.00**	216.51	51.00
MOTLBO-RP	334.66	273.04	269.11	242.71	91.95	17.24

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.95, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between groups.

Table 5.95: Pairwise comparisons of *Time* when using six objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	1	1	-	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
FST-MOMA-CD	1	1	1	1	1	-	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	0	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	0	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1
FST-MOPSO-RP-SLSPR	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1
FST-DEMO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-

Figure 5.52 and Table 5.94 illustrate that MOTLBO-GT needs the least time, while MOGWO-RP needs the most.

As regards the M_1^* metric values of each algorithm, Figure 5.53 demonstrates the violin plots of the M_1^* values of each method.

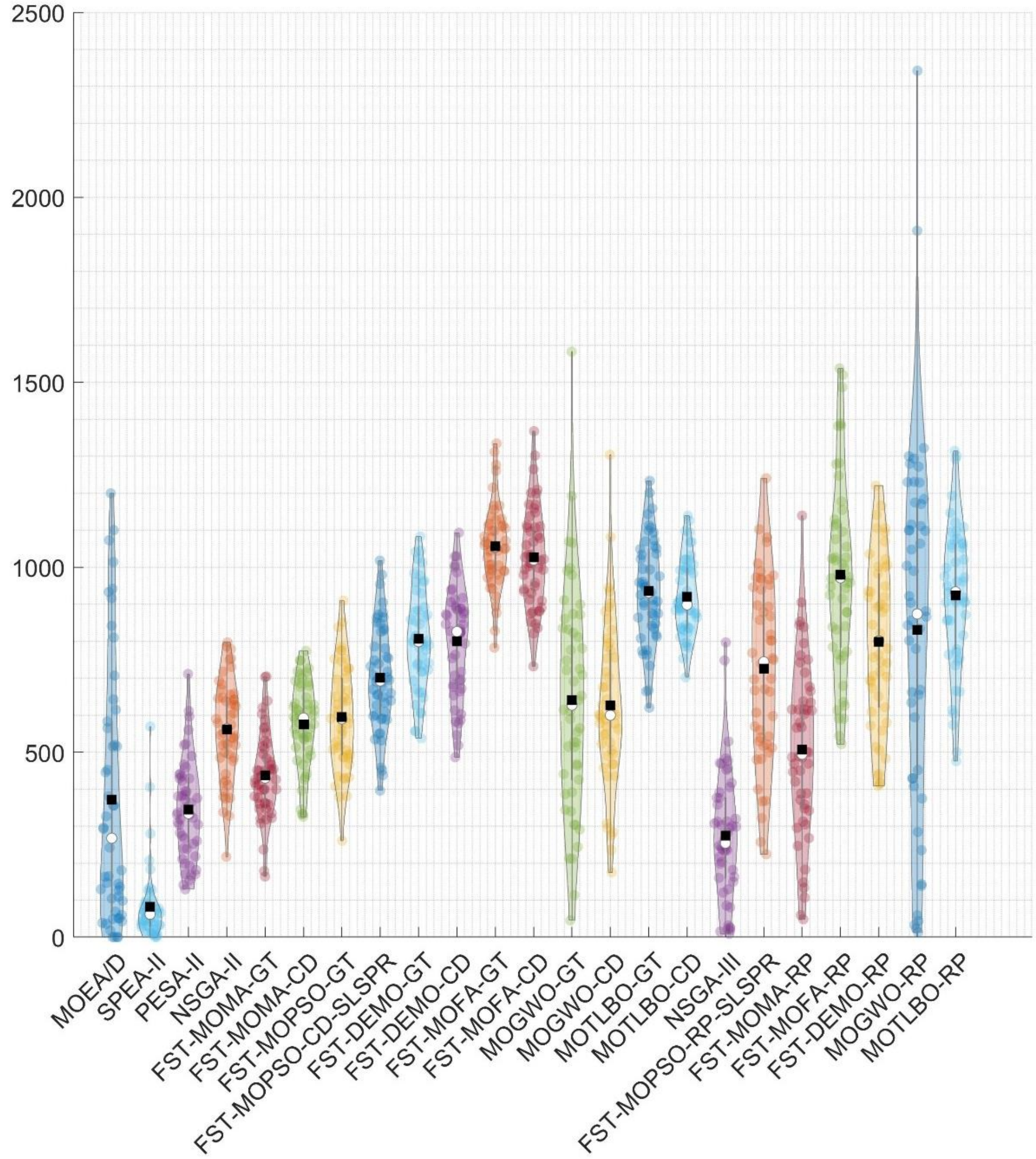


Figure 5.53: Violin plots of M_1^* metric values when using six objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(22)$=700.92, p<0.001, ε^2=0.61. Their Median values along with the rest of statistics are demonstrated in Table 5.96.

Table 5.96: Statistics of M_1^* values when using six objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1,199.87	371.16	267.29	**0.00**	1,199.87	350.78
SPEA-II	**569.56**	**82.08**	**61.94**	**0.00**	569.56	**100.97**
PESA-II	711.51	344.72	334.35	129.21	582.30	132.97
NSGA-II	796.88	561.25	563.35	217.16	579.71	132.37
FST-MOMA-GT	704.69	437.70	430.13	163.75	540.94	115.58
FST-MOMA-CD	773.76	575.40	592.22	325.47	448.29	116.02
FST-MOPSO-GT	909.45	594.85	590.13	260.67	648.79	139.04
FST-MOPSO-CD-SLSPR	1,017.98	701.37	692.86	396.24	621.74	142.33
FST-DEMO-GT	1,082.76	806.40	798.96	537.71	545.04	137.80
FST-DEMO-CD	1,093.47	799.96	826.03	486.14	607.32	146.03
FST-MOFA-GT	1,334.67	**1,056.35**	**1,054.64**	**782.63**	552.04	112.65

Method	Max	Mean	Median	Min	Range	SD
FST-MOFA-CD	1,367.97	1,026.86	1,020.03	732.00	635.97	133.48
MOGWO-GT	1,582.88	640.93	627.60	45.47	1,537.40	301.66
MOGWO-CD	1,304.12	626.28	599.87	175.12	1,129.00	221.03
MOTLBO-GT	1,233.51	935.85	931.74	620.54	612.96	140.77
MOTLBO-CD	1,139.04	919.90	898.52	702.42	**436.62**	101.46
NSGA-III	796.78	274.28	254.37	8.97	787.80	175.96
FST-MOPSO-RP-SLSPR	1,240.58	726.07	744.27	224.14	1,016.44	242.88
FST-MOMA-RP	1,139.54	507.33	494.58	48.14	1,091.41	235.04
FST-MOFA-RP	1,537.58	980.03	971.65	522.01	1,015.58	258.02
FST-DEMO-RP	1,220.43	798.22	800.82	408.78	811.65	224.44
MOGWO-RP	**2,342.27**	830.82	873.85	12.83	**2,329.44**	**489.17**
MOTLBO-RP	1,314.43	923.83	933.37	475.76	838.68	196.15

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.97, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.97: Pairwise comparisons of M_1^* when using six objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PESA-II	0	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
NSGA-II	1	1	1	-	1	0	0	1	1	1	1	1	0	0	1	1	1	0	0	1	1	0	1
FST-MOMA-GT	0	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
FST-MOMA-CD	1	1	1	0	1	-	0	1	1	1	1	1	0	0	1	1	1	0	0	1	1	0	1
FST-MOPSO-GT	1	1	1	0	1	0	-	0	1	1	1	1	0	0	1	1	1	0	0	1	1	0	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	0	-	0	0	1	1	0	0	1	1	1	0	1	1	0	0	1
FST-DEMO-GT	1	1	1	1	1	1	1	0	-	0	1	1	0	1	1	1	1	0	1	1	0	0	0
FST-DEMO-CD	1	1	1	1	1	1	1	0	0	-	1	1	0	1	1	1	1	0	1	1	0	0	0
FST-MOFA-GT	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	0	1	0	1
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	0	-	1	1	0	1	1	1	1	0	1	0	0
MOGWO-GT	1	1	1	0	1	0	0	0	0	0	1	1	-	0	1	1	1	0	0	1	0	0	1
MOGWO-CD	1	1	1	0	1	0	0	0	1	1	1	1	0	-	1	1	1	0	0	1	0	0	1
MOTLBO-GT	1	1	1	1	1	1	1	1	1	1	1	0	1	1	-	0	1	1	1	0	0	0	0
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	0	0	0	0
NSGA-III	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	0	1	0	0	0	0	0	1	1	0	0	1	1	1	-	1	1	0	0	1
FST-MOMA-RP	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	1	1	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	-	0	0	0
FST-DEMO-RP	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	1	0	1	0	-	0	0
MOGWO-RP	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	-	0
MOTLBO-RP	1	1	1	1	1	1	1	1	0	0	1	0	1	1	0	0	1	1	1	0	0	0	-

Figure 5.53 and Table 5.96 illustrate that SPEA-II provides the closest to the optimal pareto front non-dominated solutions, while FST-MOFA-GT provides the farthest ones. Table 5.97 reveals that FST-MOFA-GT does not have statistically significant differences with FST-MOFA-CD, FST-MOFA-RP and MOGWO-RP.

As regards the *Δ* values of each algorithm, Figure 5.54 demonstrates the violin plots of the *Δ* metric values, of each method.

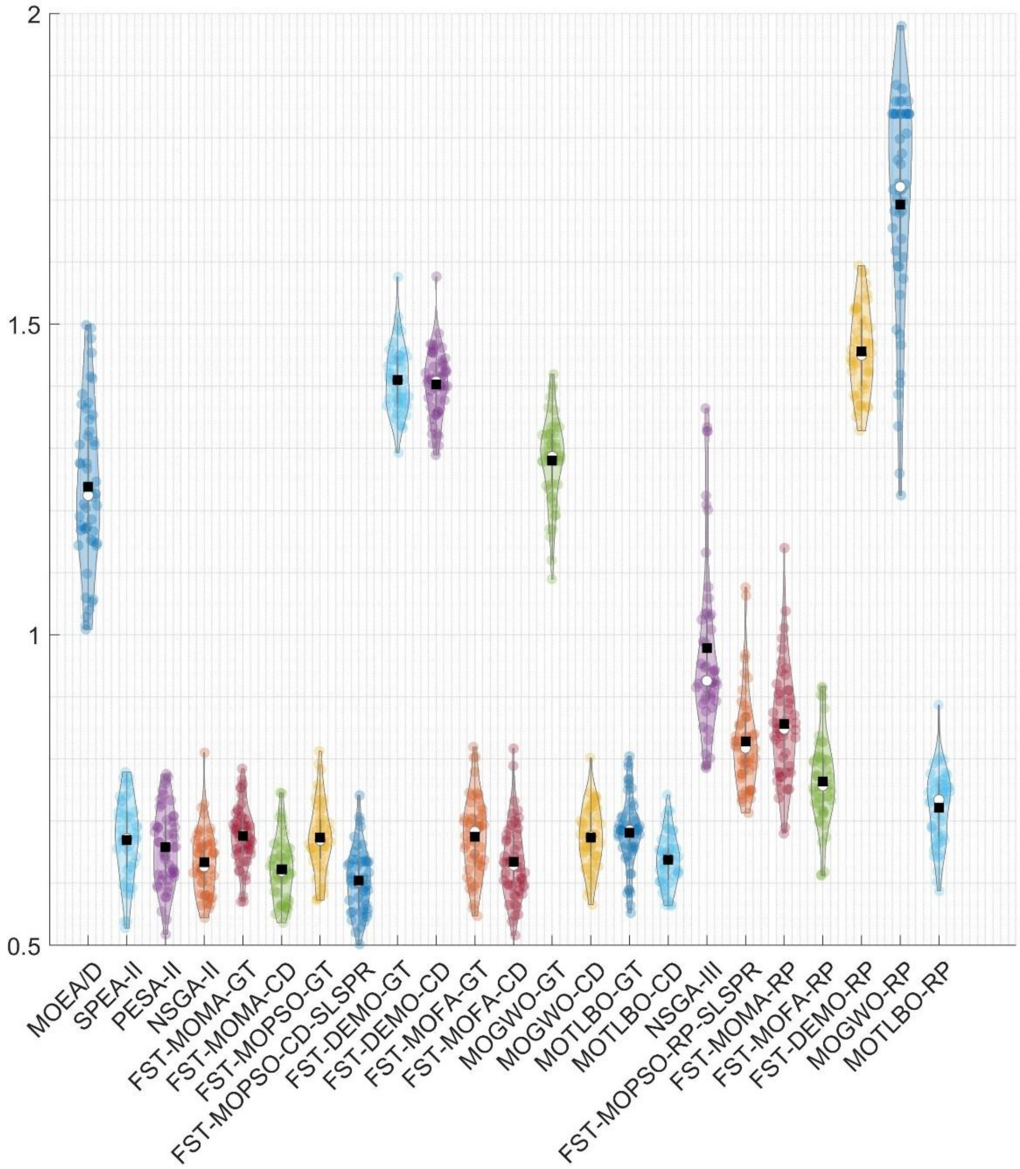


Figure 5.54: Violin plots of *Δ* metric values when using six objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the distribution of the solutions, $H(22)=943.01$, $p<0.001$, $\varepsilon^2=0.82$. Their Median values along with the rest of statistics are demonstrated in Table 5.98.

Table 5.98: Statistics of *Δ* metric values when using six objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	1.50	1.24	1.22	1.01	0.49	0.13
SPEA-II	0.78	0.67	0.67	0.53	0.25	0.06
PESA-II	0.78	0.66	0.66	0.52	0.26	0.06
NSGA-II	0.81	0.63	0.63	0.54	0.27	0.05
FST-MOMA-GT	0.78	0.68	0.68	0.57	0.21	0.05
FST-MOMA-CD	0.75	0.62	0.62	0.54	0.21	0.05
FST-MOPSO-GT	0.81	0.67	0.67	0.57	0.24	0.06

Method	Max	Mean	Median	Min	Range	SD
FST-MOPSO-CD-SLSPR	**0.74**	**0.60**	**0.60**	**0.50**	0.24	0.05
FST-DEMO-GT	1.58	1.41	1.41	1.29	0.28	0.05
FST-DEMO-CD	1.58	1.40	1.41	1.29	0.29	0.05
FST-MOFA-GT	0.82	0.67	0.68	0.55	0.27	0.06
FST-MOFA-CD	0.82	0.63	0.63	0.52	0.30	0.06
MOGWO-GT	1.42	1.28	1.29	1.09	0.33	0.07
MOGWO-CD	0.80	0.67	0.68	0.57	0.24	0.05
MOTLBO-GT	0.80	0.68	0.69	0.55	0.25	0.06
MOTLBO-CD	**0.74**	0.64	0.64	0.56	**0.18**	**0.04**
NSGA-III	1.36	0.98	0.93	0.79	0.58	0.15
FST-MOPSO-RP-SLSPR	1.08	0.83	0.82	0.71	0.36	0.08
FST-MOMA-RP	1.14	0.86	0.85	0.68	0.46	0.09
FST-MOFA-RP	0.92	0.76	0.76	0.61	0.30	0.07
FST-DEMO-RP	1.59	1.46	1.45	**1.33**	0.27	0.07
MOGWO-RP	**1.98**	**1.69**	**1.72**	1.22	**0.76**	**0.18**
MOTLBO-RP	0.89	0.72	0.73	0.59	0.30	0.06

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.99, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.99: Pairwise comparisons of *Δ* metric when using six objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	0	0	0	1	0	1	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1
PESA-II	1	0	-	0	0	0	0	1	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1
NSGA-II	1	0	0	-	1	0	0	0	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1
FST-MOMA-GT	1	0	0	1	-	1	0	1	1	1	0	1	1	0	0	1	1	1	1	1	1	1	1
FST-MOMA-CD	1	1	0	0	1	-	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1
FST-MOPSO-GT	1	0	0	0	0	1	-	1	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	0	1	0	1	-	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1
FST-DEMO-GT	1	1	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	1	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOFA-GT	1	0	0	0	0	1	0	1	1	1	-	0	1	0	0	0	1	1	1	1	1	1	1
FST-MOFA-CD	1	0	0	0	1	0	0	0	1	1	0	-	1	0	1	0	1	1	1	1	1	1	1
MOGWO-GT	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1
MOGWO-CD	1	0	0	1	0	1	0	1	1	1	0	0	1	-	0	1	1	1	1	1	1	1	1
MOTLBO-GT	1	0	0	1	0	1	0	1	1	1	0	1	1	0	-	1	1	1	1	1	1	1	0
MOTLBO-CD	1	0	0	0	1	0	0	0	1	1	0	0	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1
FST-MOPSO-RP-SLSPR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	0	1	1	1	1
FST-MOMA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-	1	1	1	1
FST-MOFA-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	0
FST-DEMO-RP	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	-

Figure 5.54 and Table 5.98 illustrate that FST-MOPSO-CD-SLSPR provides the best distribution among the non-dominated solutions, while MOGWO-RP provides the worst. Table

5.99 reveals that FST-MOPSO-CD-SLSPR does not have statistically significant differences with NSGA-II, FST-MOMA-CD, FST-MOFA-CD and MOTLBO-CD.

Finally, as regards the M_3^* values of each algorithm, Figure 5.55 demonstrates the violin plots of the M_3^* values of each method.

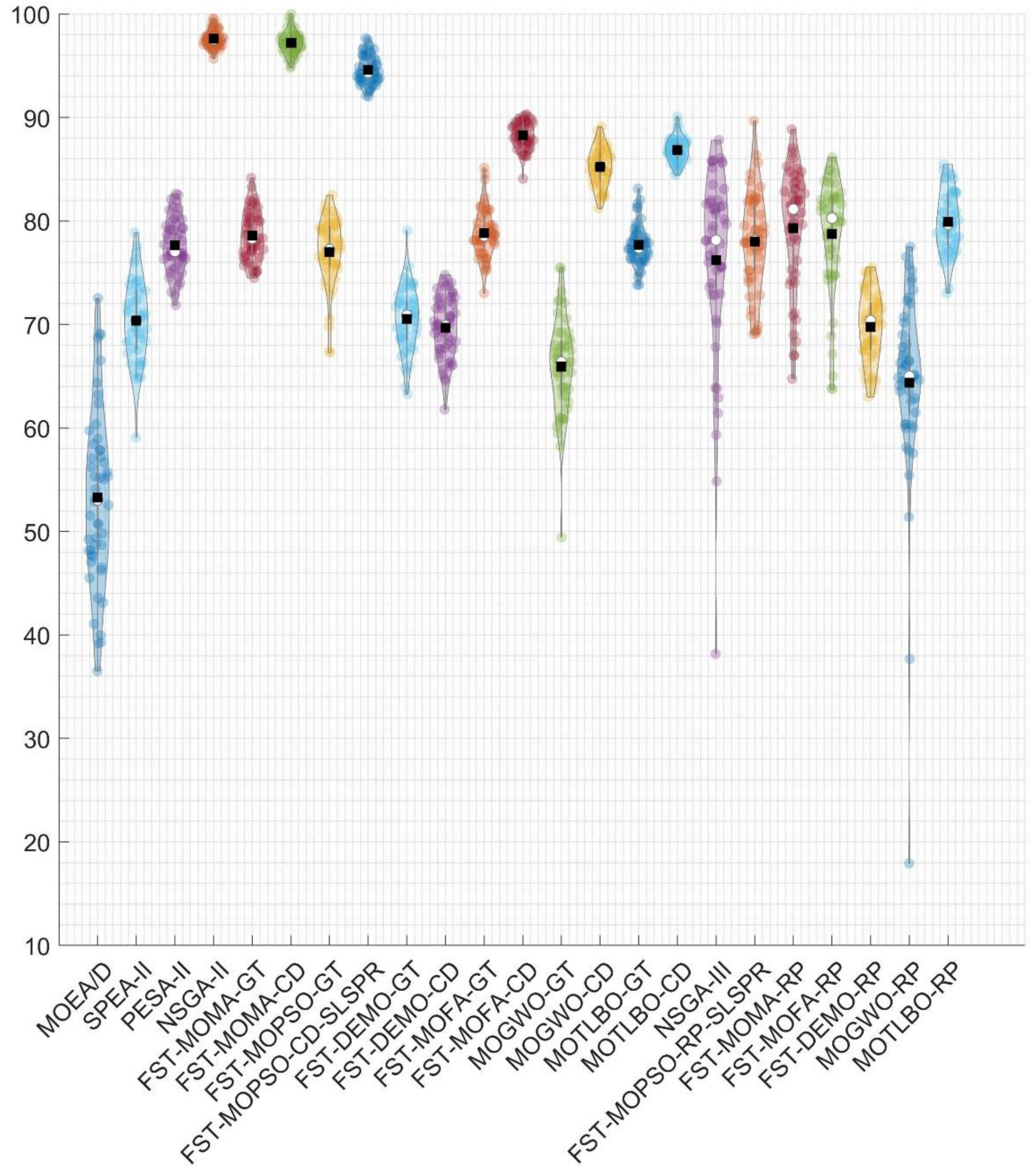


Figure 5.55: Violin plots of M_3^* metric values when using six objectives on the Olive oil data

A Kruskal-Wallis test showed that the optimizer selection significantly affects the extent of the obtained Pareto front, $H(22)=969.75$, $p<0.001$, $\varepsilon^2=0.84$. Their Median values along with the rest of statistics are demonstrated in Table 5.100.

Table 5.100: Statistics of M_3^* metric values when using six objectives on the Olive oil data

Method	Max	Mean	Median	Min	Range	SD
MOEA/D	**72.54**	**53.28**	**53.05**	36.50	36.04	8.41
SPEA-II	78.87	70.37	70.39	59.08	19.79	3.88
PESA-II	82.63	77.66	77.05	71.87	10.75	2.64
NSGA-II	99.56	**97.62**	**97.52**	**95.67**	**3.89**	**0.86**

Method	Max	Mean	Median	Min	Range	SD
FST-MOMA-GT	84.15	78.59	78.31	74.49	9.66	2.37
FST-MOMA-CD	**100.00**	97.21	97.18	94.81	5.19	1.12
FST-MOPSO-GT	82.51	77.00	77.25	67.30	15.21	3.33
FST-MOPSO-CD-SLSPR	97.65	94.60	94.35	92.04	5.61	1.42
FST-DEMO-GT	79.06	70.52	70.88	63.27	15.79	3.06
FST-DEMO-CD	74.78	69.69	69.90	61.79	12.99	2.99
FST-MOFA-GT	85.14	78.83	78.56	73.03	12.12	2.41
FST-MOFA-CD	90.29	88.27	88.28	84.08	6.22	1.30
MOGWO-GT	75.49	65.95	66.37	49.43	26.07	4.68
MOGWO-CD	89.10	85.22	85.24	81.21	7.90	1.86
MOTLBO-GT	83.12	77.69	77.41	73.80	9.32	1.98
MOTLBO-CD	90.10	86.86	86.80	84.44	5.66	1.23
NSGA-III	87.82	76.20	78.15	38.17	49.65	9.35
FST-MOPSO-RP-SLSPR	89.65	78.00	78.12	69.08	20.57	4.77
FST-MOMA-RP	88.85	79.31	81.14	64.74	24.11	5.87
FST-MOFA-RP	86.15	78.73	80.27	63.75	22.40	5.58
FST-DEMO-RP	75.53	69.75	70.37	62.97	12.56	3.34
MOGWO-RP	77.50	64.39	65.00	**17.93**	**59.57**	**9.63**
MOTLBO-RP	85.46	79.91	79.67	73.01	12.45	2.96

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.101, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.101: Pairwise comparisons of M_3^* metric when using six objectives on the Olive oil data

Method	MOEA/D	SPEA-II	PESA-II	NSGA-II	FST-MOMA-GT	FST-MOMA-CD	FST-MOPSO-GT	FST-MOPSO-CD-SLSPR	FST-DEMO-GT	FST-DEMO-CD	FST-MOFA-GT	FST-MOFA-CD	MOGWO-GT	MOGWO-CD	MOTLBO-GT	MOTLBO-CD	NSGA-III	FST-MOPSO-RP-SLSPR	FST-MOMA-RP	FST-MOFA-RP	FST-DEMO-RP	MOGWO-RP	MOTLBO-RP
MOEA/D	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPEA-II	1	-	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1
PESA-II	1	1	-	1	0	1	0	1	1	1	0	1	1	1	0	1	0	0	0	0	1	1	1
NSGA-II	1	1	1	-	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOMA-GT	1	1	0	1	-	1	0	1	1	1	0	1	1	1	0	1	0	0	0	0	1	1	0
FST-MOMA-CD	1	1	1	0	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-MOPSO-GT	1	1	0	1	0	1	-	1	1	1	0	1	1	1	0	1	0	0	0	0	1	1	1
FST-MOPSO-CD-SLSPR	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FST-DEMO-GT	1	0	1	1	1	1	1	1	-	0	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-DEMO-CD	1	0	1	1	1	1	1	1	0	-	1	1	1	1	1	1	1	1	1	1	0	1	1
FST-MOFA-GT	1	1	0	1	0	1	0	1	1	1	-	1	1	1	0	1	0	0	0	0	1	1	0
FST-MOFA-CD	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1	1	1
MOGWO-GT	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	0	1
MOGWO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
MOTLBO-GT	1	1	0	1	0	1	0	1	1	1	0	1	1	1	-	1	0	0	0	0	1	1	1
MOTLBO-CD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1
NSGA-III	1	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	-	0	0	0	1	1	0
FST-MOPSO-RP-SLSPR	1	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	0	-	0	0	1	1	0
FST-MOMA-RP	1	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	0	0	-	0	1	1	0
FST-MOFA-RP	1	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	0	0	0	-	1	1	0
FST-DEMO-RP	1	0	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	-	1	1
MOGWO-RP	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	-	1
MOTLBO-RP	1	1	1	1	0	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0	1	1	-

Figure 5.55 and Table 5.100 illustrate that NSGA-II provides the best extend of the non-dominated solutions, while MOEA/D provides the worst. Table 5.101 reveals that NSGA-II does not have statistically significant differences with FST-MOMA-CD.

To further check the way each algorithm converges towards the optimum values of each objective function, their average convergence characteristic curves through function evaluations, are demonstrated in Figure 5.56.

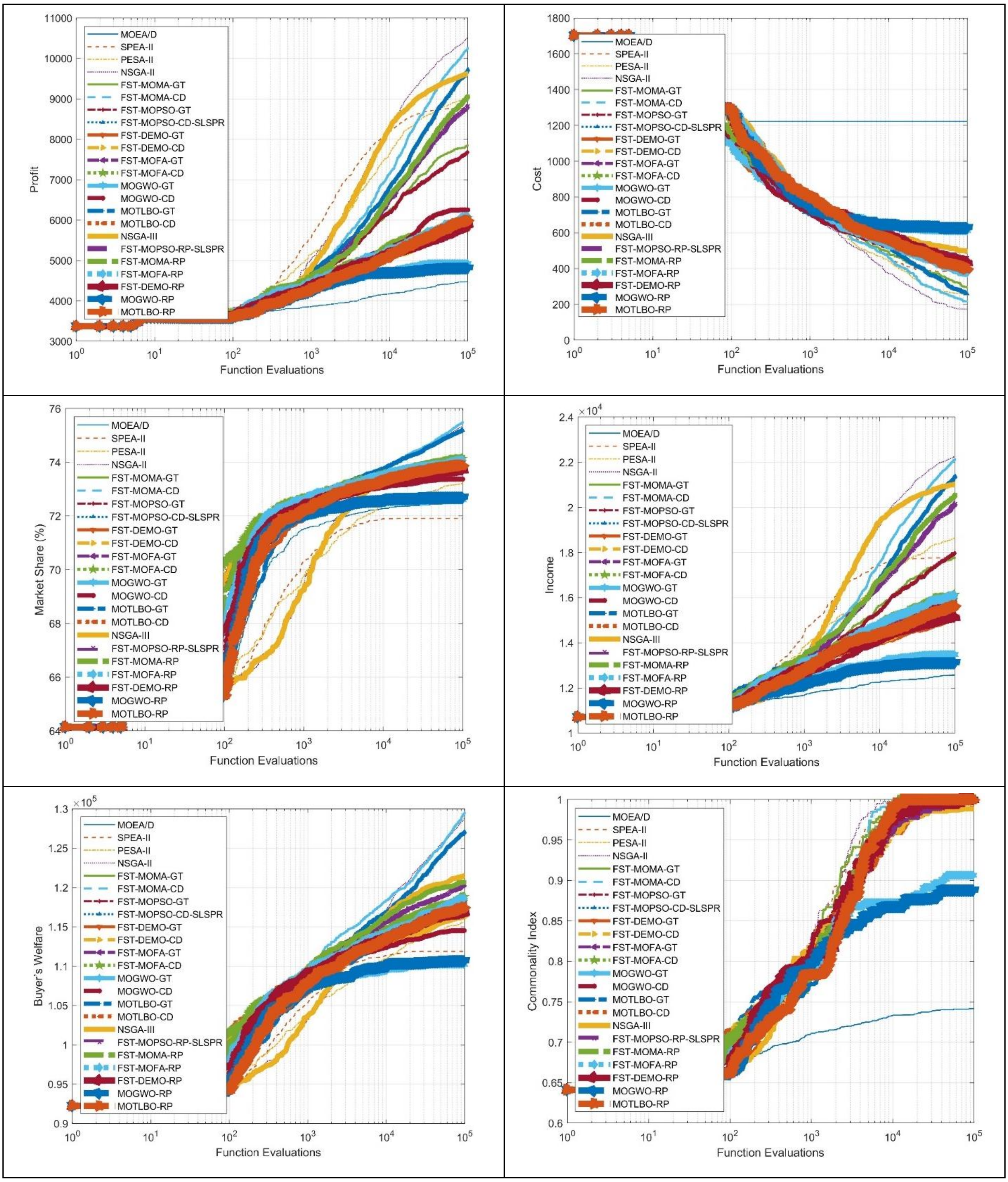


Figure 5.56: Average convergence characteristic curves when using six objectives on the Olive oil data

5.6 Investigating the factors that affect the results of algorithms

In this subsection, possible factors that have a statistically significant effect on the results of the algorithms, are investigated, using statistical analyses.

Initially, the relationships between the number of objectives to be optimized and the time each algorithm needed, the M_1^* values, the Δ values, as well as the M_3^* values, were assessed. The Pearson correlation tests revealed that there was a small possitive correlation between the number of objectives to be optimized and the time needed (r=0.18, p<0.001), a medium possitive correlation between the number of objectives to be optimized and the M_1^* values (r=0.48, p<0.001) and a small negative correlation between the number of objectives to be optimized and the Δ metric values (r=-0.07, p<0.001). Finally, there was no significant correlation between the number of objectives to be optimized and the M_3^* values (r<-0.01, p=0.746).

Furthermore, the impact of the algorithmic structure on performance metrics was assessed. Particularly, the impact of using GA, MA, PSO, DE, FA, GWO and TLBO was assessed. A Kruskal-Wallis test showed that the algorithmic structure selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(6)$=1260.92, p<0.001, ε^2=0.11. Their Median values along with the rest of statistics are demonstrated in Table 5.102.

Table 5.102: Statistics of M_1^* values according to the algorithmic structure

Method	Max	Mean	Median	Min	Range	SD
GA	1,727.50	**142.11**	87.68	**0.00**	1,727.50	**178.14**
MA	**1,139.54**	223.79	149.45	5.64	**1,133.90**	223.81
PSO	1,447.89	280.40	190.13	6.96	1,440.93	269.98
DE	**2,532.42**	292.97	165.36	8.47	**2,523.95**	354.40
FA	1,625.93	**394.92**	**248.06**	10.25	1,615.68	**406.39**
GWO	2,342.27	173.75	**86.88**	**0.00**	2,342.27	248.27
TLBO	1,676.69	360.10	226.27	**11.57**	1,665.12	380.30

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.103, *1* indicates that there were statistically significant differences between groups, while *0* indicates that there were not.

Table 5.103: Pairwise comparisons of M_1^* values according to the algorithmic structure

Method	GA	MA	PSO	DE	FA	GWO	TLBO
GA	-	1	1	1	1	0	1
MA	1	-	1	0	1	1	1
PSO	1	1	-	1	1	1	1
DE	1	0	1	-	1	1	1
FA	1	1	1	1	-	1	1

Method	GA	MA	PSO	DE	FA	GWO	TLBO
GWO	0	1	1	1	1	-	1
TLBO	1	1	1	1	1	1	-

Table 5.102 illustrates that GWO-based structures provide the closest to the optimal pareto front non-dominated solutions, while FA-based structures provide the farthest ones. Table 5.103 reveals that there were not statistically significant differences between GWO-based and GA-based structures.

As regards the Δ values, a Kruskal-Wallis test showed that the algorithmic structure selection significantly affects the distribution of the solutions, $H(6)$=2,347.74, p<0.001, ε^2=0.20. Their Median values along with the rest of statistics are demonstrated in Table 5.104.

Table 5.104: Statistics of Δ metric values according to the algorithmic structure

Method	Max	Mean	Median	Min	Range	SD
GA	**1.98**	0.91	0.81	0.26	**1.72**	0.33
MA	1.39	0.79	0.77	0.27	1.12	0.18
PSO	1.34	0.75	0.73	**0.24**	1.10	0.17
DE	1.59	1.02	0.99	0.27	1.32	0.28
FA	1.33	0.75	0.73	**0.37**	**0.95**	**0.15**
GWO	**1.98**	**1.15**	**1.14**	0.28	1.70	**0.37**
TLBO	**1.27**	**0.74**	**0.71**	0.30	0.97	0.16

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.105, *1* indicates that there were statistically significant differences between groups, while *0* indicates that there were not.

Table 5.105: Pairwise comparisons of Δ metric values according to the algorithmic structure

Method	GA	MA	PSO	DE	FA	GWO	TLBO
GA	-	1	1	1	1	1	1
MA	1	-	1	1	1	1	1
PSO	1	1	-	1	0	1	1
DE	1	1	1	-	1	1	1
FA	1	1	0	1	-	1	1
GWO	1	1	1	1	1	-	1
TLBO	1	1	1	1	1	1	-

Table 5.104 illustrates that TLBO-based algorithms provide the best distribution among the non-dominated solutions, while GWO-based ones provide the worst.

Finally, as regards the M_3^* values, a Kruskal-Wallis test showed that the algorithmic structure selection significantly affects the extent of the obtained Pareto front, $H(6)$=1,537.00, p<0.001, ε^2=0.13. Their Median values along with the rest of statistics are demonstrated in Table 5.106.

Table 5.106: Statistics of M_3^* metric values according to the algorithmic structure

Method	Max	Mean	Median	Min	Range	SD
GA	**100.00**	76.87	80.64	**0.00**	**100.00**	**18.43**
MA	**100.00**	**87.14**	**88.06**	39.75	60.25	9.31
PSO	99.78	86.81	87.64	**56.93**	**42.85**	8.33
DE	**94.93**	77.53	77.85	48.52	46.40	8.20
FA	**100.00**	81.26	81.74	47.96	52.04	**7.22**
GWO	98.58	**73.10**	**75.08**	**0.00**	98.58	16.07
TLBO	98.64	81.67	82.59	46.77	51.86	7.36

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.107, *1* indicates that there were statistically significant differences between groups, while *0* indicates that there were not.

Table 5.107: Pairwise comparisons of M_3^* metric values according to the algorithmic structure

Method	GA	MA	PSO	DE	FA	GWO	TLBO
GA	-	1	1	1	1	1	1
MA	1	-	0	1	1	1	1
PSO	1	0	-	1	1	1	1
DE	1	1	1	-	1	1	1
FA	1	1	1	1	-	1	0
GWO	1	1	1	1	1	-	1
TLBO	1	1	1	1	0	1	-

Table 5.106 illustrates that MA-based algorithms provide the best extend of the non-dominated solutions, while GWO-based ones provide the worst. Table 5.107 reveals that there were not statistically significant differences between MA-based and PSO-based structures.

The third investigated possible factor on affecting the results of the algorithms concerns the data set. As regards the M_1^* metric values obtained in each data set, algorithms' non-dominated solutions retrieved from the Timbuk2 data set (*Mdn*=130.43) were closer to the optimal Pareto front compared to the non-dominated solutions retrieved from the Olive oil data set (*Mdn*=194.74). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Timbuk2}}=5{,}750,\ N_{\text{Olive oil}}=5{,}750)=1.30\text{e}+7$, $p<0.05$, $r=0.22$.

Moreover, regarding the Δ metric values obtained in each data set, algorithms' non-dominated solutions retrieved from the Timbuk2 data set (*Mdn*=0.76) had a better distribution compared to the non-dominated solutions retrieved from the Olive oil data set (*Mdn*=0.82). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Timbuk2}}=5{,}729,\ N_{\text{Olive oil}}=5{,}745)=1.37\text{e}+7$, $p<0.05$, $r=0.17$.

Finally, as regards the M_3^* metric values obtained in each data set, algorithms' non-dominated solutions retrieved from the Timbuk2 data set (*Mdn*=86.56) achieved a better extent compared to the non-dominated solutions retrieved from the Olive oil data set (*Mdn*=76.78). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Timbuk2}}=5{,}750,\ N_{\text{Olive oil}}=5{,}750)=8.54+6$, $p<0.05$, $r=0.48$. The rest of statistics are demonstrated in Table 5.108.

Table 5.108: Statistics of metric values on both data sets

	Dataset	M_1^*	Δ	M_3^*
Mean	Timbuk2	**151.68**	**0.83**	**84.86**
	Olive oil	360.35	0.92	75.74
Median	Timbuk2	**130.43**	**0.76**	**86.56**
	Olive oil	194.74	0.82	76.78
Range	Timbuk2	**1117.83**	1.74	**100**
	Olive oil	2532.42	**1.49**	**100**
Minimum	Timbuk2	**0**	**0.24**	**0**
	Olive oil	**0**	0.49	**0**
Maximum	Timbuk2	**1117.83**	**1.98**	**100**
	Olive oil	2532.42	**1.98**	**100**

The statistically significant differences mentioned above were expected, since the second data set consists of more product characteristics and is therefore characterized by larger objective landscapes.

The fourth investigated possible factor on affecting the results of the algorithms concerns the nature of the algorithm when it comes on operating using Discrete or Continuous values. As regards the M_1^* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from discrete optimization algorithms (*Mdn*=87.68) were closer to the optimal Pareto front compared to the non-dominated solutions retrieved from the continuous ones (*Mdn*=176.25). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Discrete}}=2{,}500,\ N_{\text{Continous}}=9{,}000)=7.60e+6$, $p<0.05$, $r=0.32$. Moreover, regarding the Δ metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from continuous optimization algorithms (*Mdn*=0.78) had a better distribution compared to the non-dominated solutions retrieved from the discrete ones (*Mdn*=0.81). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Discrete}}=2{,}497,\ N_{\text{Continous}}=8{,}977)=1.06e+7$, $p<0.05$, $r=0.06$. Finally, as regards the M_3^* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from continuous optimization algorithms (*Mdn*=82.30) achieved a better extent compared to the non-dominated solutions retrieved from the discrete ones (*Mdn*=80.64). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Discrete}}=2{,}500,\ N_{\text{Continous}}=9{,}000)=1.03+7$, $p<0.05$, $r=0.08$. The rest of statistics are demonstrated in Table 5.109.

Table 5.109: Statistics of metric values on discrete and continous algorithms

	Algorithm Type	M_1^*	Δ	M_3^*
Mean	Discrete	**142.11**	0.91	76.87
	Continuous	287.66	**0.87**	**81.25**
Median	Discrete	**87.68**	0.81	80.64
	Continuous	176.250	**0.78**	**82.30**
Range	Discrete	**1727.50**	**1.72**	**100**
	Continuous	2532.42	1.74	**100**
Minimum	Discrete	**0**	0.26	**0**
	Continuous	**0**	**0.24**	**0**
Maximum	Discrete	**1727.50**	**1.98**	**100**
	Continuous	2532.42	**1.98**	**100**

The fifth investigated possible factor on affecting the results of the algorithms concerns the use of the Crossover technique. As regards the M_1^* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from optimization algorithms using crossover (*Mdn*=111.67) were closer to the optimal Pareto front compared to the ones that did not (*Mdn*=182.24). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Crossover}}=4{,}000, N_{\text{No Crossover}}=7{,}500)=1.14e+7$, $p<0.05$, $r=0.24$. Moreover, regarding the Δ metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from optimization algorithms not using crossover (*Mdn*=0.78) had a better distribution compared to the ones that did not (*Mdn*=0.79). A Mann-Whitney U test indicated that this difference was not statistically significant, $U(N_{\text{Crossover}}=3{,}997, N_{\text{No Crossover}}=7{,}477)=1.48e+7$, $p=0.32$. Finally, as regards the M_3^* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from optimization algorithms using crossover (*Mdn*=83.94) achieved a better extent compared to the ones that did not (*Mdn*=81.42). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Crossover}}=4{,}000, N_{\text{No Crossover}}=7{,}500)=1.31+7$, $p<0.05$, $r=0.13$. The rest of statistics are demonstrated in Table 5.110.

Table 5.110: Statistics of metric values on using the crossover technique

	Crossover	M_1^*	Δ	M_3^*
Mean	No	300.43	0.88	80.07
	Yes	**172.74**	**0.87**	**80.72**
Median	No	182.24	0.78	81.42
	Yes	**111.67**	**0.79**	**83.94**
Range	No	2532.42	1.74	**100**
	Yes	**1727.5**	**1.72**	**100**
Minimum	No	**0**	**0.24**	**0**
	Yes	**0**	0.26	**0**
Maximum	No	2532.42	**1.98**	**100**
	Yes	**1727.5**	**1.98**	**100**

The sixth investigated possible factor on affecting the results of the algorithms concerns the use mutation as a local search technique. As regards the M_1^* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from optimization algorithms not using mutation (*Mdn*=151.24) were closer to the optimal Pareto front compared to the ones that did (*Mdn*=157.97). A Mann-Whitney U test indicated that this difference was not statistically significant, $U(N_{\text{Mutation}}=3{,}000,\ N_{\text{No Mutation}}=8{,}500)=1.27\text{e}+7$, $p=0.60$. Moreover, regarding the *Δ* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from optimization algorithms using mutation (*Mdn*=0.78) had a better distribution compared to the ones that did not (*Mdn*=0.80). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Mutation}}=2{,}977,\ N_{\text{No Mutation}}=8{,}497)=1.15\text{e}+7$, $p<0.05$, $r=0.09$. Finally, as regards the M_3^* metric values obtained from both types of algorithms, algorithms' non-dominated solutions retrieved from optimization algorithms using mutation (*Mdn*=82.66) achieved a better extent compared to the ones that did not (*Mdn*=79.74). A Mann-Whitney U test indicated that this difference was statistically significant, $U(N_{\text{Mutation}}=3{,}000,\ N_{\text{No Mutation}}=8{,}500)=1.03+7$, $p<0.05$, $r=0.19$. The rest of statistics are demonstrated in Table 5.111.

Table 5.111: Statistics of metric values on using mutation as a local search technique

	Mutation	M_1^*	*Δ*	M_3^*
Mean	No	266.93	0.94	77.39
	Yes	**252.16**	**0.85**	**81.33**
Median	No	**151.24**	0.8	79.74
	Yes	157.97	**0.78**	**82.66**
Range	No	**2342.27**	**1.7**	98.64
	Yes	2532.42	1.74	**100**
Minimum	No	**0**	0.28	**0**
	Yes	**0**	**0.24**	**0**
Maximum	No	**2342.27**	**1.98**	98.64
	Yes	2532.42	**1.98**	**100**

Finally, the impact of diversity controlling operator was assessed as the seventh investigated factor affecting the results. Particularly, the impact of using decomposition (DCMP), environmental selection (ENSL), GT, CD and RP, was assessed. A Kruskal-Wallis test showed that the diversity controlling operator selection significantly affects the distance of the obtained non-dominated solutions from the optimal Pareto front, $H(4)=539.65$, $p<0.001$, $\varepsilon^2=0.05$. Their Median values along with the rest of statistics are demonstrated in Table 5.112.

Table 5.112: Statistics of M_1^* values according to the diversity controlling operator

Method	Max	Mean	Median	Min	Range	SD
DCMP	1,727.50	186.77	115.44	**0.00**	1,727.50	246.97
ENSL	**569.56**	**66.74**	**22.13**	**0.00**	**569.56**	**92.32**
GT	1,582.88	**278.70**	**175.16**	**0.00**	1,582.88	311.82
CD	1,486.91	259.93	154.29	**0.02**	1,486.89	309.56
RP	**2,532.42**	266.35	162.97	**0.00**	**2,532.42**	**324.40**

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.113, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between them.

Table 5.113: Pairwise comparisons of M_1^* values according to the diversity controlling operator

Method	DCMP	ENSL	GT	CD	RP
DCMP	-	1	1	1	1
ENSL	1	-	1	1	1
GT	1	1	-	1	1
CD	1	1	1	-	0
RP	1	1	1	0	-

Table 5.112 illustrates that SPEA-II which use ENSL, provides the closest to the optimal pareto front non-dominated solutions, while the algorithms using GT provide the farthest ones.

As regards the *Δ* values, a Kruskal-Wallis test showed that the diversity controlling operator selection significantly affects the distribution of the solutions, $H(4)$=3,396.97, p<0.001, ε^2=0.30. Their Median values along with the rest of statistics are demonstrated in Table 5.114.

Table 5.114: Statistics of *Δ* metric values according to the diversity controlling operator

Method	Max	Mean	Median	Min	Range	SD
DCMP	**1.98**	**1.41**	**1.33**	**0.98**	1.00	0.25
ENSL	**1.28**	**0.68**	**0.65**	0.37	**0.91**	**0.14**
GT	1.73	0.85	0.76	0.51	1.22	0.24
CD	1.58	0.73	0.71	**0.24**	1.34	0.21
RP	**1.98**	1.00	0.91	0.53	**1.45**	**0.29**

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.115, *0* indicates that there were not statistically significant differences between groups, while *1* indicates that there were statistically significant differences between groups.

Table 5.115: Pairwise comparisons of *Δ* metric values according to the diversity controlling operator

Method	DCMP	ENSL	GT	CD	RP
DCMP	-	1	1	1	1
ENSL	1	-	1	1	1
GT	1	1	-	1	1
CD	1	1	1	-	1
RP	1	1	1	1	-

Table 5.114 illustrates that the algorithms using ENSL provide the best distribution among the non-dominated solutions, while algorithms using DCMP provide the worst.

Finally, as regards the M_3^* values, a Kruskal-Wallis test showed that the diversity controlling operator selection significantly affects the extent of the obtained Pareto front, $H(4)$=3073.61, p<0.001, ε^2=0.27. Their Median values along with the rest of statistics are demonstrated in Table 5.116.

Table 5.116: Statistics of M_3^* metric values according to the diversity controlling operator

Method	Max	Mean	Median	Min	Range	SD
DCMP	**84.20**	**57.14**	**59.25**	**0.00**	84.20	**16.09**
ENSL	92.49	66.99	69.07	20.10	72.39	15.18
GT	**100.00**	79.82	80.95	32.05	67.95	**8.75**
CD	**100.00**	**88.01**	**88.77**	**56.32**	**43.68**	8.93
RP	98.96	78.27	80.09	**0.00**	**98.96**	13.26

Post-hoc Dwass-Steel-Critchlow-Fligner pairwise comparisons were used to compare all pairs of groups. In Table 5.117, *1* indicates that there were statistically significant differences between groups, while *0* indicates that there were not.

Table 5.117: Pairwise comparisons of M_3^* metric values according to the diversity controlling operator

Method	DCMP	ENSL	GT	CD	RP
DCMP	-	1	1	1	1
ENSL	1	-	1	1	1
GT	1	1	-	1	0
CD	1	1	1	-	1
RP	1	1	0	1	-

Table 5.116 illustrates that the algorithms using CD provide the best extend of the non-dominated solutions, while algorithms using DCMP provide the worst.

Chapter 6 Decision maker's next moves

In real life, after extracting a Pareto front, the decision maker is called upon to make some important decisions. For instance, product managers may intent to choose one final solution from the non-dominated solution set. Moreover, they may intent to estimate their competitors' possible future moves. The purpose of this chapter it to briefly demonstrate each of those cases.

6.1 Solution ranking using Multi-Criteria Decision Making

Once a Pareto front is extracted, the decision maker is asked to choose which one of the non-dominated solutions is the best choice to be selected. The purpose of this subsection is to demonstrate how an Elimination and Choice Translating Priority III (ELECTRE-III) method (Roy, 1978) can be applied to help product managers in ranking the non-dominated solutions, as demonstrated in the work of Z. Wang and Rangaiah (2017), and therefore, in making their final choice, following Deng et al., (2014) and Shin and Ferguson (2016) who stated that companies should consider introducing novel techniques or adapting new ones to help on selecting a single final solution, when it comes on multi-objective PLD.

By reviewing the literature, various techniques were detected, when it comes on choosing a single solution out of a non-dominated set (Grierson, 2008; Ishibuchi et al., 2014; N. Wang et al., 2017). For instance, detecting a knee point, is a simple efficient way for the decision maker to conclude in a final choice (Branke et al., 2004). It refers to the closest point to the utopian point, in terms of the Euclidean distance, which can be used as an ideal point for the objective values, to pick the best solution from the non-dominated set. A utopian point has coordinates that minimize all objectives at the same time, as depicted in Figure 6.1. In Figure 6.1, blue rhombus represents the utopian point while the blue circle represents the closest non-dominated solution. Even though knee-point-based methods have been widely used in the past, the particular method does not take into account possible importances or preferences of the decision maker, for each criterion.

Another example which is widely used by researchers, is the application of a clustering process to reduce the number of the non-dominated solutions (Faceli et al., 2008). However, this method does not provide the decision maker with a single final solution, nor a proposed ranking order. As a result, the decision maker has to further investigate the non-dominated solutions left, to make his final choice.

To overcome possible disadvantages of the approaches mentioned above, the ELECTRE-III method was adopted in this research, as an outranking method to assist on evaluating non-dominated alternatives and therefore, to provide product managers with a non-dominated solutions ranking.

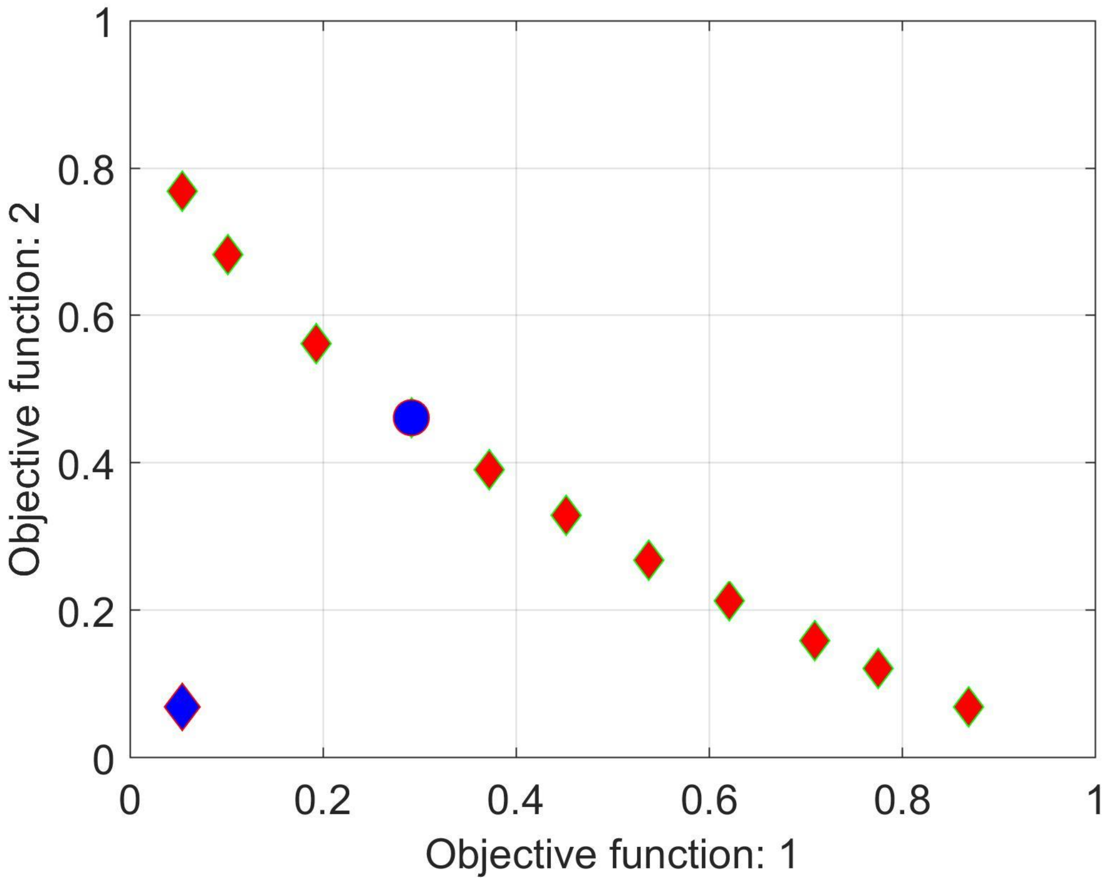


Figure 6.1: Example of utopian and knee points

In Multi-Criteria Decision Making (MCDM), the ELECTRE methods refer to the concept of *outranking relationship*. For instance, the outranking relationship of $A_k \rightarrow A_l$ means that, despite the fact that the two alternatives k and l do not mathematically dominate each other, the decision maker takes the risk of considering A_k as almost surely better than A_l. The introduction of the first method known as ELECTRE I (Gal et al., 1999) was followed by other widely used improved versions, like ELECTRE II (Roy & Bertier, 1971), ELECTRE-III (Roy, 1978), ELECTRE IV (Roy & Hugonnard, 1982) and ELECTRE TRI (W. Yu, 1992).

Even though being based on the same concept, all methods mentioned above differ with each other. Particularly, ELECTRE I was introduced for selection problems, ELECTRE II, III and IV for ranking problems and ELECTRE TRI for assignment problems. When it is possible to quantify the criteria relative importances, ELECTRE-III is in place, while ELECTRE IV is applied when quantification is not feasible.

6.1.1 *The ELECTRE-III method*

Comparing to ELEWCTRE II, in ELECTRE-III (Roy, 1978), a procedure is used to obtain the relationship between alternatives. According to Hashemi et al., (2016) ELECTRE-III's biggest advantage, as an interaction method, is that the decision maker participates directly in the decision process. Furthermore, the indifference and preference thresholds constitute another advantage of ELECTRE-III. In cases of equal and similarly weighted data, the alternatives are also accounted equally. Due to lower accuracy, the indifference and preference thresholds are defined to demonstrate their preference or indifference compared to the rest of alternatives. As a result, when the performance of two alternatives is inferior to the determined quantity in a specific criterion, both are considered to be indifferent in the given criterion. Finally, the particular method supports the decision maker on analyzing both the qualitative and quantitative criteria at different levels of uncertainty.

Considering a MCDM problem with a set of alternatives $A=(a, b, c, ...,n)$ and a set of criteria $(g_1,g_2, ...,g_m)$. $g_j(a_j)$ corresponds to the evaluation (performance) of an alternative $a \in A$ for the g_j criterion. Depending on whether the objective is to maximize or minimize the $g_j(a_j)$ criterion, the higher or lower it is, respectively, the better the alternative meets the specific criterion. As a result, the vector $g(a)=(g_1(a), g_2(a), ...,g_m(a))$ represents the multi-criteria evaluation of the alternative $a \in A$.

According to Tzeng and Huang (2011) ELECTRE-III's evaluation procedures contain the establishment of a threshold function, disclosure of concordance and discordance indices, determination of credibility degree, as well as the alternatives ranking. Let $q(g)$ and $p(g)$ be the indifference and preference thresholds, respectively:

If $g(a) \geq g(b)$, then:

$$aPb \Leftrightarrow g(a) \succ g(b) + p\big(g(b)\big) \tag{6.1}$$

$$aQb \Leftrightarrow g(b) + q\big(g(b)\big) \prec g(a) \prec g(b) + p\big(g(b)\big) \tag{6.2}$$

$$aIb \Leftrightarrow g(b) \prec g(a) \prec g(b) + q\big(g(b)\big) \tag{6.3}$$

where P denotes a strong preference, Q denotes a weak preference, I denotes indifference, and $g(a)$ is the criterion value of the alternative a.

The algorithmic structure of ELECTRE-III as well as all calculations it contains, are demonstrated in Algorithm 6.1, as presented in previous work (Hashemi et al., 2016; Marzouk, 2011; Papadopoulos & Karagiannidis, 2008).

Algorithm 6.1: ELECTRE-III steps

1. For each pair of alternatives, compute the concordance index $C(a,b)$

$$C(a,b) = \frac{\sum_{i=1}^{m} w_i c_i(a,b)}{\sum_{i=1}^{m} w_i} \tag{6.4}$$

where $C_i(a,b) \in [0,1]$ is the outranking degree of the alternatives a and b under the criterion i, defined as:

$$c_i(a,b) = \begin{cases} 0, & \text{if } g_i(b) - g_i(a) \succ p_i\big(g_i(a)\big) \\ 1, & \text{if } g_i(b) - g_i(a) \preccurlyeq q_i\big(g_i(a)\big) \\ \frac{p_i + g_i(a) - g_i(b)}{p_i - q_i}, & \text{otherwise} \end{cases} \tag{6.5}$$

The veto threshold $v_i(g_i(b))$ is defined for each criterion as:

$$v_i\big(g_i(b)\big) = a_v + \beta_v g_i(a) \tag{6.6}$$

It allows for the possibility of aSb to be refused totally if, for any criterion j, $g_i(b) \succ g_i(a) + v_i$. As a result, the case in which a is not preferred over b, is modeled as:

$$a \neg b \Leftrightarrow gi(b) \succ gi(a) + vi \tag{6.7}$$

The statement aSb means that option a outranks option b, when a is at least as good as b in most of the criteria and never significantly worse in the rest of them.

2. For each criterion, the discordance index $d(a,b) \in [0,1]$ is calculated as:

$$d_i(a,b) = \begin{cases} 0, & \text{if } g_i(b) - g_i(a) \preccurlyeq p_i\big(g_i(a)\big) \\ 1, & \text{if } g_i(b) - g_i(a) \succ v_i\big(g_i(a)\big) \\ \frac{g_i(b) - g_i(a) - p_i}{v_i - p_i}, & \text{otherwise} \end{cases} \tag{6.8}$$

3. Finally, the degree of outranking is defined by $S(a,b)$:

$$S(a,b) = \begin{cases} C(a,b), & if\ d_j(a,b) \leq C(a,b),\ \forall j \in J \\ C(a,b) \times \prod_{j \in J(a,b)} \frac{1 - d_j(a,b)}{1 - C(a,b)}, & \text{otherwise} \end{cases} \tag{6.9}$$

where $J(a,b)$ is the set of criteria for which $d_j(a,b) \succ C(a,b)$.

4. To obtain the final alternatives ranking, a structured algorithm via two intermediate ranking procedures is used: The first one contains the classification of the alternatives from the best to the worst (descending distillation), while the second contains the classification of the alternatives from the worst to the best (ascending distillation).

6.1.2 Ranking of non-dominated solutions

In this subsection, ELECTRE-III is used to assist product managers in their final choice when it comes on the product line selection.

To perform the experiment, an FST-MOMA-CD was applied on the Timbuk2 data set when using six objectives on optimizing a product line of five products. The retrieved non-dominated solutions are demonstrated in Figure 6.2. To apply the ELECTRE-III method, arbitrary criteria threshold values per objective function, as presented in Table 6.1, were used.

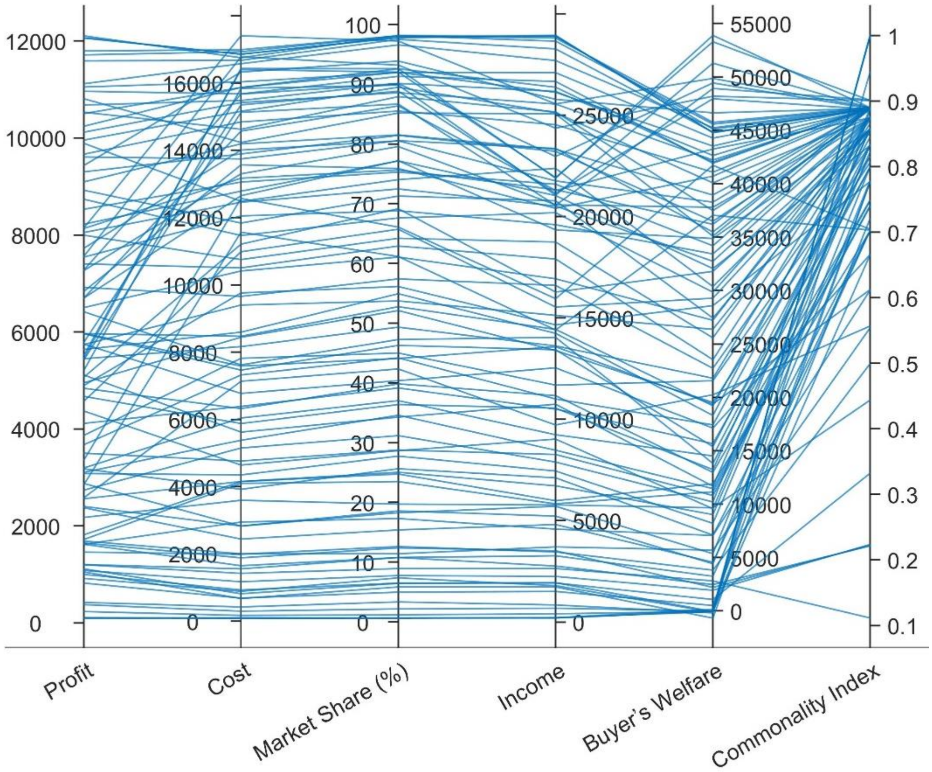


Figure 6.2: Detected non-dominated solutions to be used in the ELECTRE-III method

Table 6.1: ELECTRE-III's criteria threshold values per objective function

Criteria threshold values / Objectives	Profit	Cost	Market Share (%)	Income	Buyer's Welfare	Commonality Index
Weights	0.26	0.09	0.24	0.21	0.18	0.03
Preference (p)	100	100	10	250	5,000	0.30
Indifference (q)	50	30	5	100	1,000	0.50
Veto (v)	120	150	5	1,000	2,000	0.50

Some of the top alternatives resulting from ELECTRE-III's ranking process, are demonstrated in Table 6.2.

Table 6.2: ELECTRE-III's ranking results

Profit	Cost	Market Share (%)	Income	Buyer's Welfare	Commonality Index
11,719.50	16,930.50	98.15	28,650.00	45,596.10	0.89
12,112.00	16,758.00	98.15	28,870.00	45,124.50	0.89
12,062.50	16,857.50	97.84	28,920.00	44,776.40	0.89

Table 6.2 demonstrates that ELECTRE-III is capable of providing product managers a variety of product line alternatives.

6.2 Estimating Competitors’ next moves

After selecting a final solution from the Pareto front, product managers may attempt to model competitive responses. The purpose of this subsection is to estimate competitors’ objective weights in order to predict their future product choices.

By reviewing the literature, previous research on modeling the competitive responses was detected. For example, in the work of Green and Krieger (1997) who was the first study to apply game theory to single-product line design, the Nash equilibrium concept with the use of a divide and conquer metaheuristics was employed. In the same concept, Tsafarakis et al., (2011) also used the Nash equilibrium with the use of a PSO algorithm.

According to Tsafarakis et al., (2011) in game theory, a Nash equilibrium, if it actually exists, is a situation in which none of the competitors that participate in the game, can make any further improvements (when it comes on a specific objective function) by changing their position in the market (changing products or attribute levels). However, as Tsafarakis et al., (2011) mention, a detected Nash equilibrium cannot be proved to be unique. In addition, predicting competitors' movements is an even more complex process when more than one objective functions are taken into account, since an equilibrium in one objective does not necessarily imply the existence of equilibriums in the rest. For this reason, an attempt to calculate the objective weights of competitors' product managers, was made, using their previous movement, which can later be used in a MCDM method like ELECTRE or TOPSIS (Muhsen et al., 2019), to estimate the next product line choice of the competitors.

The followed method is based on previous methods as far as the calculation of weights using information from the objective functions, is concerned (Ridha, Gomes, Hizam, & Ahmadipour, 2020; Ridha, Gomes, Hizam, Ahmadipour, et al., 2020). Initially, using an FST-MOMA-CD multi-objective optimization algorithm, competitors’ previous Pareto front is generated, when using their optimization settings (competitors’ competitive products are used). For instance, considering that the work of Belloni et al., (2008) reflects the optimization problem addressed from the competitors, and that they used a multi-objective optimization algorithm to detect a Pareto front, as depicted in Figure 6.3, by knowing the solution selected by them, their objective weights will be estimated.

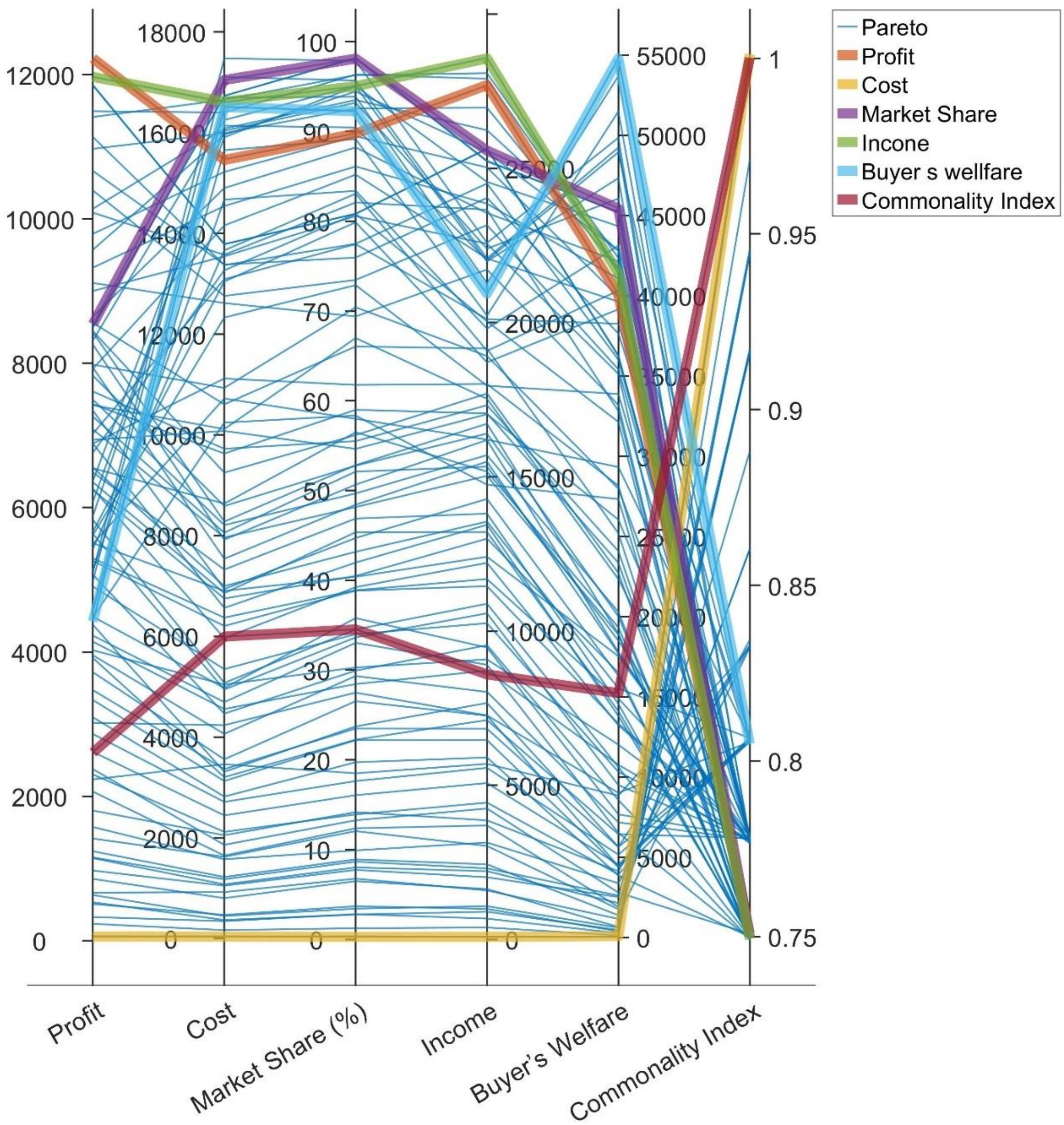


Figure 6.3: Pareto front of competitor and optimal values of each criterion

Even though it is known that Belloni et al., (2008) optimized as regards profit, it is now considered that the objectives used by them are now unknown. As a result, all six objectives used in this research are considered. To estimate the objective weights, the maximum f^{max} and minimum f^{min} values of all objectives are initially detected. For the objectives to be maximized, the objective weights fw_i are then calculated as:

$$fw_i = \frac{f_i - f_i^{min}}{|f_i^{max} - f_i^{min}|} \tag{6.10}$$

while for the objectives to be minimized (Cost), the objective weights fw_i are calculated as:

$$fw_i = \frac{f_i^{max} - f_i}{|f_i^{max} - f_i^{min}|} \tag{6.11}$$

where f_i corresponds to the selected value in objective i.

The final adjusted weights *faw*, so that all have a sum of one, are computed as:

$$faw_i = \frac{fw_i}{\sum_{j=1}^{nObj} fw_j} \tag{6.12}$$

Table 6.3 demonstrates the generated competitor's final weights

Table 6.3: Competitor's weights per objective function

Objective name	Profit	Cost	Market Share (%)	Income	BW	CI
Weight	0.22	0.02	0.20	0.22	0.16	0.17

As a result, by knowing the objective weights of competitors, firms can estimate a choice of their next product line. To do this, firms will have to use their own product line as the competing line in the optimization process to generate a Pareto front. After generating the non-dominated solutions, they can use the calculated objective weights on a MCDM method, to estimate competitors' preference.

To demonstrate this statement, the results presented in Subsection 6.1.1 were used, and an FST-MOMA-CD run for once again, using the first ranking product line as the competitive one. Supposing that the competitor's weights are now known, after generating the non-dominated solutions, an ELECTRE-3 was used, to estimate competitors' preference, as presented in Figure 6.4. *p*, *q* and *v* were chosen arbitrary.

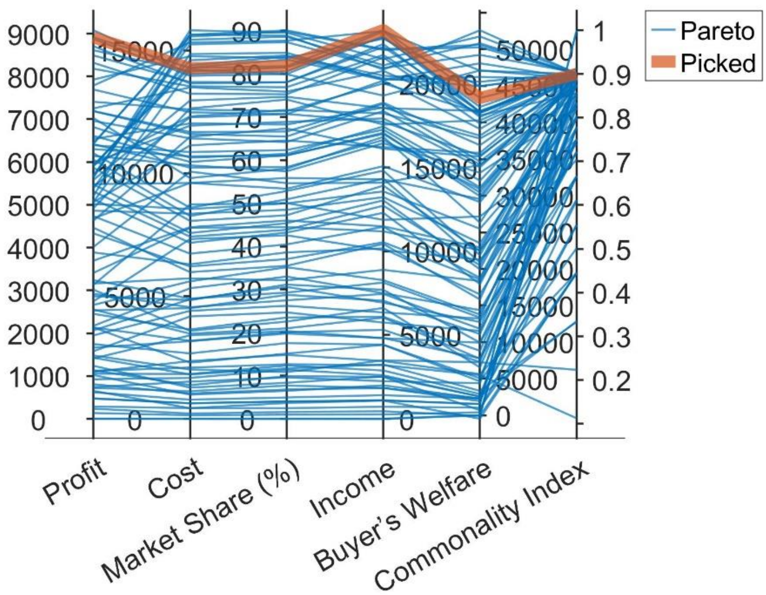


Figure 6.4: Competitor's Preference

Chapter 7 Conclusions and future research suggestions

The main purpose of this research was to address the Product Line Design (PLD) problem, using more than one objectives (Multi-Objective Product Line Design - MOPLD), since PLD is a key decision area that product managers have to deal with in the early stages of product development, to estimate the potential success of a product.

To do that, at a first stage, the need and value of using multi-objective approaches instead of single-objective ones as well as the lack of relevant literature on addressing the MOPLD problem, were demonstrated.

At a second stage, the methodology of the current research was presented. Particularly, the 23 variants of the main seven state-of-the-art metaheuristics that were applied in this research were described, along with the data sets and the used objective functions. The seven main multi-objective metaheuristics used in this research were Genetic Algorithms (GAs), Particle Swarm Optimization (PSO), Firefly Algorithm (FA), Differential Evolution (DE), Grey Wolf Optimizer (GWO), Teaching-Learning Based Optimization (TLBO) and Mayfly Algorithm (MA). Those seven multi-objective algorithms were fully adapted to the MOPLD problem, using popular diversity controlling operators, like Crowding Distance (CD), adaptive Grid Technique (GT) and Reference Points (RP). As a result, three variants of each one, were used in this research. Moreover, other state-of-the-art multi-objective optimizers like MOEA/D (Q. Zhang & Li, 2007) and SPEA-II were used.

Furthermore, the proposed approaches that suffer from initial parameter settings were fully adapted to the problem using an extended to multi-objective optimization fuzzy-self-tuning process, which calculates the settings of parameters independently for each individual, during the optimization process. The main idea of auto tuning process was to help the algorithms overcome specific difficulties when performing on different datasets using the same parameter settings.

At a third stage, the performance of the proposed methods was compared, using statistical analysis, on the values retrieved from popular performance metrics when it comes on multi-objective optimization. Furthermore, possible factors that have a statistically significant effect on the results of the algorithms, were investigated. Finally, a simple novel way of selecting global best particles as well as a new way of mutating design solution vectors for MOPLD, were introduced.

At a fourth and final stage, the adaptation of an ELECTRE-III method was demonstrated, through which the decision maker can rank the retrieved non-dominated solutions, according to his/her preferences. Moreover, an attempt to estimate the weights of competitors' objectives, in order to predict their future moves, was briefly presented.

This research extended previous research in several important ways and provides rich results capable of answering the research questions mentioned in Subsection 3.1. Particularly, considering the first question regarding the proper formulation of the MOPLD problem, Chapter 3 demonstrates how the MOPLD can be modeled successfully, which is verified through the results on Chapter 5. As a result, this research verified the statement of Williams et al., (2011) who showed that the multi-objective optimization approach is very useful to product managers.

Considering the second research question about whether the solutions to the problem (product lines) differ depending on the optimized objective, Table 1.1 and Table 1.2 prove that each criterion has its own overall optimum solution, while considering the third one regarding tradeoffs between the criteria being linear, Figure 5.45, Figure 5.51, Figure 6.2 and Figure 6.3 reveal that tradeoffs not only are not linear, but also are problem-dependent.

When it comes on the fourth research question about addressing the MOPLD without knowing the number of products to be designed, this research revealed that by addressing the MOPLD problem without knowing the specific number of products, product managers, are able to investigate the tradeoffs between different objectives for product lines with different number of products. The impact of this method is demonstrated in Figure 7.1, where the non-dominated solutions obtained when using a fixed number of five products are compared to the non-dominated solutions obtained when using unknown number of products. To obtain Figure 7.1, an FST-MOMA-CD run until 100,000 function evaluations are reached, when performing on the Timbuk2 data set.

According to the C metric, values of *C*(Unknown number of bags, 5 bags)=0.65, *C*(5 bags, Unknown number of bags)=0.04, were found. As a result, the non-dominated solutions obtained when using unknown number of products dominate the ones obtained when using a fixed number of five products, in a wide area of the decision space.

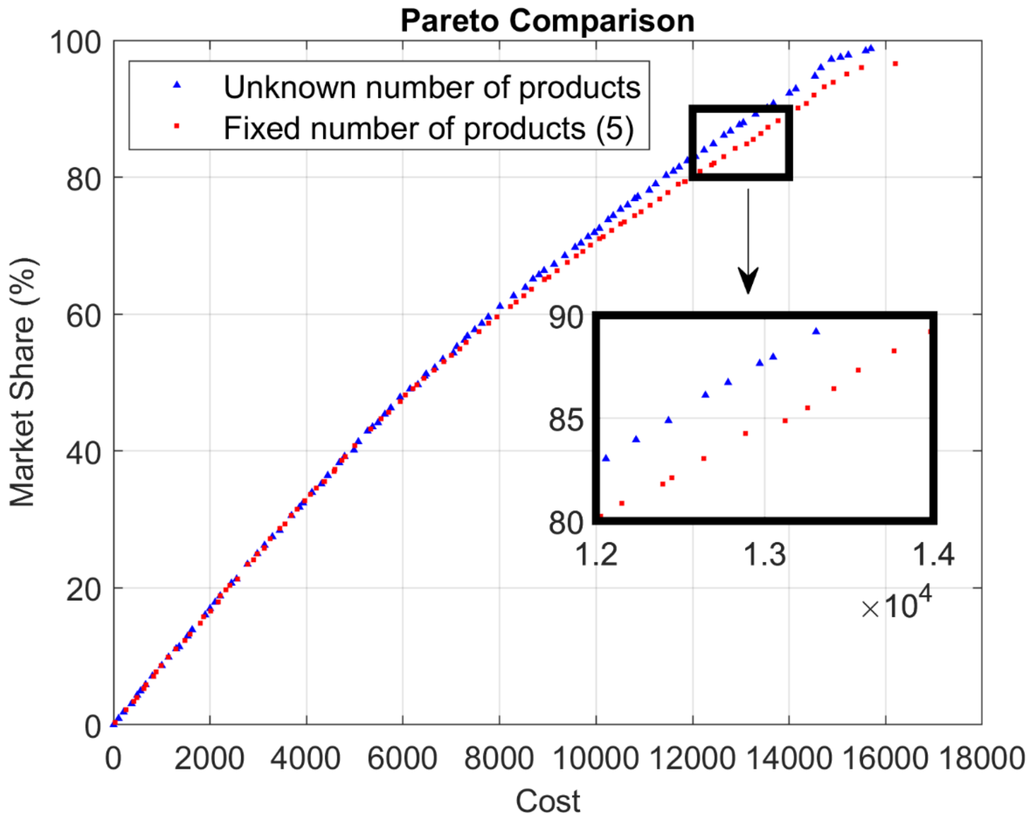


Figure 7.1: Comparison of non-dominated solutions when using unknown and fixed number of products

As far as the fifth research question is concerned, regarding which multi-objective optimization algorithm have the best performance in multi-objective PLD, Subsections 5.1-5.5 reveal that there is no ideal multi-objective optimization algorithm, which perfectly addresses the MOPLD problem, in terms of all metrics. More specifically, it turned out that the performance of the algorithms is directly dependent on the number of objective functions as well as the data set. However, results in Subsection 5.6 revealed that GWO-based and GA-based algorithms detect non-dominated solutions faster, as well as that TLBO-based algorithms provide the best distribution among the non-dominated solutions. Finally, MA-based as well as PSO-based algorithms provide the best extend of the non-dominated solutions.

Regarding the sixth research question about factors affecting the effectiveness of each algorithm, results in Subsection 5.6 revealed that the number of objective functions affects the time each algorithm needs, the M_1^* values and therefore the distance from the optimal Pareto front, as well as the Δ values and therefore the distribution of solutions. Furthermore, it was shown that the dataset to be used affects all three of the M_1^*, Δ and M_3^* metric values as well as that discrete algorithms detect non-dominated solutions faster, even though they appeared to have a worse distribution as well as a worse non-dominated solution extension, compared to the continous ones. Similarly, the crossover technique appeared to assist both the detection of the non-dominated solutions and the non-dominated set extension, while the mutation technique appeared to assist non-dominated solution distribution as well as pareto extension. Finally, diversity controlling operators were proved to affect all M_1^*, Δ and M_3^* metric values.

Regarding the seventh research question about using a MCDM method to evaluate or rank the non-dominated solutions, Subsection 6.1.2 verified the findings of Z. Wang and Rangaiah (2017), when it comes on using an ELECTRE-III for sorting the non-dominated solutions.

Finally, an attempt to estimate competitors' objective weights, using their previous product development selection, in order to predict their future product choices, was made to answer the eighth research question about predicting the future moves of competitors. However, even though the objective weights were successfully retrieved, as Masrah et al., (2011) mention in their research, this process needs to be repeated more than once, taking into account possible occurring errors, which will be reduced more and more, during the repetitions. As a result, an interesting area for future research is to use the method proposed by Masrah et al., (2011) to reduce possible prediction errors, by repeating the prediction process as many times as possible.

Another interesting area for future research is to test the performance of more multi-objective optimization methods or to create hybrid methods of the most successful characteristics, according to the Subsection 5.6. Moreover, researchers can extend the work of T. Wang and Gutierrez (2021) by applying multi-objective optimization approaches to maximize the value of the worst outcome (MaxMin) while minimizing the maximum relative ex-post regret (MinMaxRR). The technique of targeted initial population that Ferguson et al., (2012) and Foster and Ferguson (2013) proposed to advance the convergence behavior of NSGA-II can also be used in future research, to advance the performance of multi-objective optimizers when performing on the MOPLD problem, as well as to reduce the time each algorithm needs.

Finally, researchers are encouraged to test the variants used in this research when performing in other multi-objective engineering problems, like multi-objective vehicle routing problem (Rapanaki et al., 2020; Yan et al., 2019) or multi-objective knapsack problem (Mizobe et al., 2019), as well as they are encouraged to test the performance of algorithms by evaluating the effect of their characteristics on results, as was done in this research, in order to create powerful hybrid algorithmic structures.

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Appendix A Conversion of real values to discrete ones

One of the most critical issues when developing a continuous optimization algorithm to a combinatorial problem like the PLD, is the solution representation. As already mentioned in Chapter 4, various multi-objective optimization algorithms operate in continuous spaces like PSO, FA and MA.

Considering the dataset of Timbuk2, as presented in Subsection 3.2.2.1, a representation of a potential solution vector, rounded at two-decimal points, for a single product could be as:

y = [0.66 1.30 2.33 -0.54 -1.41 0.76 1.62 -0.72 0.10 -1.11 -1.68 -0.04 1.33 -0.95 0.61 0.68 -1.58 -0.83 0.45 -0.15 0.50 -0.07 0.66 -0.44 1.13 0.31 0.11].

A required binary representation is then needed for a solution to be evaluated. Consequently, a rule is needed to convert a real solution vector to a discrete one. In this research, the Smallest Position Value (SPV) rule is in place. As Tsafarakis et al., (2011) report in their research, PSO combined with the SPV rule, has shown the best performance in the optimal PLD problem. However, it was reported, the SPV rule is the most time-consuming method.

According to SPV, the level with the smallest value on each attribute takes a value of 1, with the rest taking a value of 0. By applying the SPV rule in the case of vector y mentioned above, we have:

y = [0.66 1.30 2.33 -0.54 **-1.41** 0.76 1.62 | **-0.72** 0.10 | -1.11 **-1.68** | **-0.04** 1.33 | **-0.95** 0.61 | 0.68 **-1.58** | **-0.83** 0.45 | **-0.15** 0.50 | **-0.07** 0.66 | **-0.44** 1.13 | 0.31 **0.11**],

which is then converted to:

y = [0000100 | 10 | 01 | 10 | 10 | 01 | 10 | 10 | 10 | 10 | 01].

A great advantage of the SPV rule is that it allows an algorithm to search in continuous spaces, without setting any boundaries, or using any rounding off procedures (Tsafarakis et al., 2011, 2020).

Appendix B Crossover and Mutation operator for PLD

As already mentioned, each solution vector is presented as an integer representation same as the solution vectors that Belloni et al., (2008) used in their research for GA. In this Appendix the crossover and mutation operators used in this research are described.

B.1 Crossover operator

As regards the crossover operator, the same crossover method that Belloni et al., (2008) used in their research for PLD, was used.

During crossover, new offspring are generated by combining the genetic information of two parents. A crossover point is picked randomly and values before and after of that point are swapped between the chromosomes of two parent (single-point crossover) (Belloni et al., 2008). As a result, two new offspring, each carrying some genetic information from both parents, are generated. Figure B.1 demonstrates an example of crossover, as it was described in the work of Zervoudakis et al., (2020).

Parent 1

4	0	0	0	1	0	1	0	1	0
1	0	0	0	0	0	0	1	1	0
0	1	1	1	0	0	1	1	1	1
6	0	0	1	1	0	0	1	1	1
4	0	1	1	0	0	0	0	1	1

Parent 2

3	0	0	0	0	0	0	0	0	1
2	0	0	1	1	1	1	0	1	0
4	0	1	1	1	1	0	0	0	1
5	1	0	1	0	0	0	0	0	1
2	1	0	0	1	1	0	0	1	1

Offspring 1

3	0	0	0	0	0	0	0	0	1
2	0	0	1	1	1	1	0	1	0
4	0	1	1	1	1	1	1	1	1
6	0	0	1	1	0	0	1	1	1
4	0	1	1	0	0	0	0	1	1

Offspring 2

4	0	0	0	1	0	1	0	1	0
1	0	0	0	0	0	0	1	1	0
0	1	1	1	0	0	0	0	0	1
5	1	0	1	0	0	0	0	0	1
2	1	0	0	1	1	0	0	1	1

Figure B.1: Example of crossover, using the 3 × 6 cell as a crossover point

Continuous optimization algorithms that use the crossover technique, like the Mayfly Algorithm (MA) (Zervoudakis & Tsafarakis, 2020) use the same crossover technique, as described in Figure B.1, since this technique combined with the SPV rule had the best results on the PLD problem, compared to the continuous crossover (Goldberg, 1989).

B.2 Mutation operator

As regards the mutation process, the same mutation method that Belloni et al., (2008) used in their research for PLD, with a mutation rate of 0.05, was used.

Mutation can introduce traits not in the original population. It alters one or more elements from its initial state, as demonstrated in Figure B.2.

Random solution

0	1	1	0	1	1	1	0	1	1
5	1	0	1	0	0	1	0	0	1
4	1	0	1	0	0	0	1	0	0
2	1	0	1	1	0	0	1	1	0
1	0	0	1	1	1	1	0	0	1

Solution after mutation

0	1	1	0	1	1	1	0	1	0
5	1	0	1	0	1	1	0	0	1
4	1	0	1	1	0	0	1	0	0
3	1	0	1	1	0	0	1	1	1
1	0	0	1	1	1	1	0	0	1

Figure B.2: Example of mutating a solution vector

What is equally important is the fact that the mutation process can change the number of products in a hypothetical product line as demonstrated in Figure B.3.

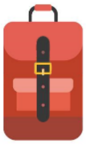
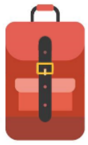
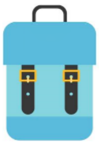
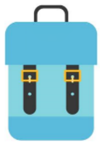
Products	Line before mutation	Mutated Attribute	Line after mutation
Bag 1		Color	
Bag 2		-	
Bag 3		-	
Bag 4	-	Considered	
Bag 5	-	-	-

Figure B.3: Hypothetical bag line, before and after the mutation process

Since numerous of the optimization algorithms used in this research operate in continuous spaces, the mutation process could be accomplished by adding a random term generated in a specific range to an element of an attribute. However, since the algorithms search without setting any boundaries because of the use of the SPV rule, resulting in possible great position values, adding a random term may not change a solution at all.

To be sure that the mutation process keeps changing a solution, the way it works was modified as follows:

First, attributes to be changed are selected based on a probability. Subsequently, all the elements of the selected attributes are circularly shifted by *K* positions, where *K* is a random integer number in the range of [1, M], where *M* is the *number of the specific attribute's level -1*. For instance, suppose mutation is applied to the solution **y** presented in Appendix A and that the third and fifth attributes are mutated. The solution is altered as: y = [0.66 1.30 2.33 -0.54 **-1.41** 0.76 1.62 | **-0.72** 0.10 | **-1.68** -1.11 | **-0.04** 1.33 | 0.61 **-0.95** | 0.68 **-1.58** | **-0.83** 0.45 | **-0.15** 0.50 | **-0.07** 0.66 | **-0.44** 1.13 | 0.31 **0.11**]. As a result, using this method, it is ensured that the levels of the particular features always change.

Appendix C Fuzzy self-tuning method

Since the performance of various algorithms is not only highly dependent on its parameter settings, but also on the dataset and the objectives to be optimized, an automatic parameter tuning method is used to help the algorithms overcome this particular difficulty.

Numerous studies regarding automatic parameter tuning methods for metaheuristics are reported in the literature (Huang et al., 2019; Nobile et al., 2018; Tatsis & Parsopoulos, 2019; Tsafarakis et al., 2020; Zervoudakis & Tsafarakis, 2022). In this research, the fuzzy self-tuning method that Nobile et al., (2018), Tsafarakis et al., (2020) and Zervoudakis and Tsafarakis (2022) proposed for single-objective optimization is extended to multi-objective optimization, where the values of multi-objective algorithms are dynamically controlled by means of Fuzzy Logic (FL). According to literature, FL is already applied to various metaheuristics that suffer from parameter tuning, which is considerably dependent on the problem (Noorbin & Alfi, 2018; Olivas et al., 2018).

To dynamically determine the parameter values in an automatic way, independently for each individual, a Fuzzy Rule-Based System (FRBS) consisting of fuzzy rules is used, as reported in the work of Nobile et al., (2018) and Tsafarakis et al., (2020). These rules are based on the distance of each individual from the chosen leader (δ) solution, and a function measuring the fitness improvement of each solution with respect to the previous iteration (φ).

The main purpose of δ, is to characterize the proximity to the chosen global best (leader) solution, while the purpose of φ is to characterize the improvement of a solution with respect to the fitness values it assumed in the previous iteration.

C.1 Distance between two solutions

As Nobile et al., (2018) mention in their research, the distance between two solutions is calculated by computing the Euclidean distance between them as:

$$\delta(\mathbf{x}_i^t, \mathbf{x}_j^t) = \|\mathbf{x}_i^t - \mathbf{x}_j^t\| = \sqrt{\sum_{k=1}^{M} \left(x_{i,k}^t - x_{j,k}^t\right)^2} \tag{C.1}$$

where $x_{i,k}^t$ and $x_{j,k}^t$ denote the k-th component of the position vectors $\mathbf{x}_i^t$ and $\mathbf{x}_j^t$, respectively.

According to Nobile et al., (2018), the variable of δ corresponds to the interval $[0, g\delta_{max}]$, where $g\delta_{max}$ is the Euclidean distance between the two vectors of best and the worst values of each objective located so far. The term set of this variable is composed by three linguistic values, Same, Near and Far.

The membership function of Same is:

a) 1, if $0 \leq \delta < \delta_1$,

b) $\frac{(\delta_2-\delta)}{(\delta_2-\delta_1)}$, if $\delta_1 \leq \delta < \delta_2$, and

c) 0, if $\delta_2 \leq \delta < g\delta_{max}$.

The membership function of Near is:

a) 0, if $0 \leq \delta < \delta_1$,

b) $\frac{(\delta-\delta_1)}{(\delta_2-\delta_1)}$, if $\delta_1 \leq \delta < \delta_2$,

c) $\frac{(\delta_3-\delta)}{(\delta_3-\delta_2)}$, if $\delta_2 \leq \delta < \delta_3$, and

d) 0, if $\delta_3 \leq \delta < g\delta_{max}$.

The membership function of Far is:

a) 0, if $0 \leq \delta < \delta_2$,

b) $\frac{(\delta-\delta_2)}{(\delta_3-\delta_2)}$, if $\delta_2 \leq \delta < \delta_3$, and

c) 1, if $\delta_3 \leq \delta < g\delta_{max}$.

The values of δ_1, δ_2 and δ_3 are set according to the size of the search space as:

$\delta_1 = 0.2 \cdot g\delta_{max}$, $\delta_2 = 0.4 \cdot g\delta_{max}$ and $\delta_3 = 0.6 \cdot g\delta_{max}$.

As Nobile et al. (Nobile et al. 2018) mention in their research, the general-purpose multipliers are created to avoid any overfitting to the benchmark function and implement a general and fuzzy concept of distance from the best solution found so far.

C.2 Solution Improvement

As Nobile et al., (2018) mention in their research, the normalized fitness incremental factor $\varphi: \mathbb{R}^M \times \mathbb{R}^M \to [-1,1]$, considers the positions of solution i at the current and previous iterations:

$$(\mathbf{x}_i^t, \mathbf{x}_i^{t-1}) = \frac{\sum_{m=1}^{n_{ob}}\left(\frac{fm\left(\mathrm{x}_i^t\right)-fm\left(\mathrm{x}_i^{t-1}\right)}{\delta_{max}^m}\right)}{n_{ob}} \quad \text{(C.2)}$$

where δ_{max} is the length of the diagonal of the hyper-rectangle corresponding to the search space of each objective, and n_{ob} is the number of the objective functions used.

According to Nobile et al., (2018), the variable of φ corresponds to the interval $[-1,1]$. The term set of this variable is composed by three linguistic values, Better, Same and Worse.

The membership function of Better is:

a) 1, if $\varphi = -1$,

b) $-\varphi$, if $-1 < \varphi < 0$, and

c) 0, if $0 \leq \varphi \leq 1$.

The membership function of Same is: $1 - |\varphi|$

The membership function of Worse is:

a) 0, if $-1 \leq \varphi < 0$,

b) φ, if $0 \leq \varphi < 1$, and

c) 1, if $\varphi = 1$.

C.3 Final numerical value

Given a set of a specific number of R rules having the same output variable in their consequent (e.g., rules 1, 2, 3 in Table 4.1 for w, the final numerical value of this output variable is calculated using the Sugeno method (Sugeno, 1985) as:

$$output = \frac{\sum_{r=1}^{R} \rho_r z_r}{\sum_{r=1}^{R} \rho_r} \tag{C.3}$$

where ρ_r denotes the membership degree of the input variable of the r-th rule, and z_r represents the output crisp value for the r-th rule (e.g., as given in Table 4.2).

Appendix D Diversity controlling operators

By reviewing the literature on multi-objective optimization, it was found that researchers have proposed various techniques to maintain diversity within the final set of non-dominated solutions of each algorithm (Coello Coello et al., 2004; Deb et al., 2002; Deb & Jain, 2014; Parsopoulos & Vrahatis, 2002; Q. Zhang & Li, 2007). A briefly description of the most important ones is given below.

D.1 Non-dominated sorting combined with Crowding Distance (CD)

The non-dominated sorting process is a fast and simple approach to compare individuals in a population (Hajiabadi & Zarghami, 2014) according to Definition 1.1. All individuals are ranked into fronts (F_1, F_2,…, etc.) in ascending order according to their fitness function values. Front 1 (F_1) is the front with the best solutions in terms of dominance (Dai et al., 2017).

The Crowding Distance (CD) value provides an estimation of the density of solutions surrounding a specific solution, that is the largest cuboid enclosing a solution without including any other solution (Deb et al., 2002). To perform CD, the population has to be first ranked according to the non-dominated sorting process.

The main core of the CD is to calculate the Euclidian distance of i^{th} individual between its neighbor ones. The boundary solutions which have the lowest and highest objective function values are given an infinite CD value so that they are always selected in the final non-dominated population. This process is repeated for each objective function separately. The final CD value of a solution is computed by summing up the CD values of each objective function as:

$$crowding\ distance(i) = \sum_{k=1}^{K} \frac{\left|f_k^{i+1} - f_k^{i-1}\right|}{f_k^{max} - f_k^{min}} \quad \text{(D.4)}$$

where f_k^{max} and f_k^{min} are the maximum and the minimum values of the k objective function, respectively, and f_k^{i+1} and f_k^{i-1} are i^{th} individual's neighbor solutions, according to the ranking process as presented in the work of Deb et al., (2002). The idea of non-dominated sorting with CD is demonstrated in Figure D.4.

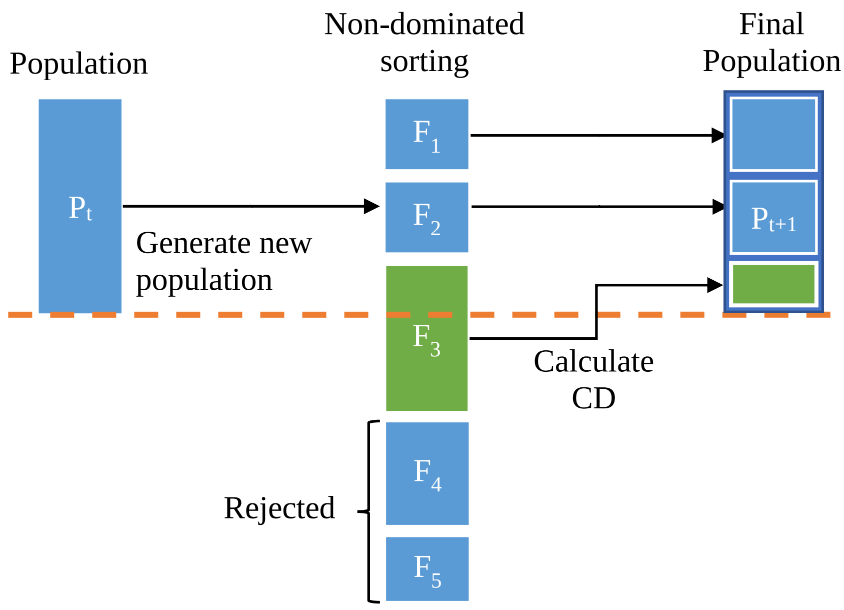


Figure D.4: Non-dominated sorting with CD process

Previous research has already proven high performance results of multi-objective optimization algorithms when combined with non-dominated sorting and CD (Tsai et al., 2014; Yue et al., 2021; Zervoudakis & Tsafarakis, 2020; J. Zhang & Li, 2014).

D.2 Adaptive Grid Technique (GT)

Another widely used diversity controlling operator is the adaptive Grid Technique (GT) (Corne et al., 2001; Knowles & Corne, 2007). According to Coello Coello et al., (2004) its main purpose is to create an external archive for storing all the non-dominated solutions by dividing the objective function space into regions.

The adaptive grid used in this technique is basically a hypercube-formed space. Each hypercube can be considered as a region of the search space that contains a specific number of candidate solutions. The main advantage of GT is that it is distinguished by low computational cost (Knowles & Corne, 2000) in cases where the grid does not need to be updated at each generation. To distribute the largest possible amount of non-dominated solutions in a uniform way, certain information is necessary to be provided which is problem dependent, like the number of grid subdivisions.

Additional details about GT can be found in previous research that has reported high performance results of multi-objective optimization algorithms when combined with GT (Agrawal et al., 2008; Cheng et al., 2016; Coello Coello et al., 2004; Corne et al., 2001; Knowles & Corne, 2007, 2000).

D.3 Non-dominated sorting combined with Reference Points (RP)

According to literature, even though CD has reported high performance, it becomes a computationally expensive operation as the number of objective functions grows, especially when addressing many-objective optimization problems (Deb & Jain, 2014). For that reason, Deb and Jain (2014) replaced the CD operator in NSGA-II with the use of a predefined set of Reference Points (RP) placed on a normalized hyperplane, with a similar implicit principle to the hyper-grid-based techniques, such as GT (Corne et al., 2001; J. Knowles & Corne, 2007), to ensure diversity in obtained solutions (I. Das & Dennis, 1998). Additional details about generating and using the RP technique can be found in the work of Deb and Jain (2014).

Following Deb and Jain (2014), the CD operator was replaced with the use of RP to test its performance on multi-objective PLD, when combined with multi-objective optimization algorithms.

Appendix E Questionnaire

In this appendix, the questionnaire of desirable characteristics for olive oil is demonstrated

Table E.1: Questionnaire of desirable characteristics for olive oil

Question 1: Gender	○ Male	○ Female		
Question 2: Age	○ 15-24 ○ 55-64	○ 25-34 ○ 65+	○ 35-44	○ 45-54
Question 3: Educational Level	○ Junior High School	○ High School	○ Bachelor’s Degree	○ MSc/PhD
Question 4: Occupation	○ Household ○ Rentier	○ State employee ○ Retired	○ Private employee ○ Unemployed	○ Freelance ○ Student
Question 5: Monthly income	○ 0-500 € ○ 3000 €+	○ 501-1000 €	○ 1001-2000 €	○ 2001-3000 €
Question 6: Below are some types of offers. Which do you prefer? Please rate them from the most attractive (value 1) to the least attractive (value 6) ○ Gift packaging (product of the same or another category) ○ Package with free product ○ Gift packaging (no product) ○ Discounted packaging (in product) ○ Package with discount from brochure ○ Participation in a competition				
Question 7: What amount of olive oil do you consume per month	○ Up to 1 liter	○ 2 to 3 liters	○ 4 to 6 liters	○ More than 7 liters
Question 8: To what extent do you want each of the following types of olive oil?				
	Not at all	A little	Absolutely	I have no opinion
Plain	○	○	○	○
Extra Virgin	○	○	○	○
Organic extra virgin	○	○	○	○
POP Kolumpariou	○	○	○	○
Sustainable	○	○	○	○
Question 9: To what extent do you want each of the following package types?				
	Not at all	A little	Absolutely	I have no opinion
Plastic	○	○	○	○
Glass	○	○	○	○
Metal	○	○	○	○
Pouch	○	○	○	○
Question 10: To what extent do you want each of the following package sizes?				
	Not at all	A little	Absolutely	I have no opinion
0.75*lt*	○	○	○	○
1*lt*	○	○	○	○
2*lt*	○	○	○	○
3*lt*	○	○	○	○
5*lt*	○	○	○	○
Question 11: To what extent do you want each of the following brands?				
	Not at all	A little	Absolutely	I have no opinion
Super Market	○	○	○	○
Altis	○	○	○	○
ABEA	○	○	○	○

MINERBA	○	○	○	○

Question 12: If these were your choices, which product would you choose? ○ None

Olive oil type	POP Kolumpariou	Plain	Organic extra virgin	Extra Virgin
Package type	Pouch	Glass	Plastic	Metal
Package size	3*lt*	2*lt*	5*lt*	3*lt*
Brand	Altis	MINERBA	ABEA	ABEA
Price per liter	7.0€	5.3€	4.5€	5.3€
	○	○	○	○

Question 13: If these were your choices, which product would you choose? ○ None

Olive oil type	Extra Virgin	Plain	POP Kolumpariou	Organic extra virgin
Package type	Plastic	Glass	Pouch	Plastic
Package size	1*lt*	0.75*lt*	2*lt*	1*lt*
Brand	ABEA	Super Market	MINERBA	Super Market
Price per liter	7.9€	4.5€	5.3€	7.0€
	○	○	○	○

Question 14: If these were your choices, which product would you choose? ○ None

Olive oil type	Extra Virgin	POP Kolumpariou	Sustainable	POP Kolumpariou
Package type	Plastic	Glass	Pouch	Metal
Package size	2*lt*	0.75*lt*	5*lt*	1*lt*
Brand	Altis	ABEA	Super Market	Altis
Price per liter	4.5€	6.2€	5.3€	7.9€
	○	○	○	○

Question 15: If these were your choices, which product would you choose? ○ None

Olive oil type	Extra Virgin	Extra Virgin	POP Kolumpariou	Plain
Package type	Metal	Glass	Pouch	Plastic
Package size	3*lt*	1*lt*	5*lt*	5*lt*
Brand	Super Market	MINERBA	ABEA	Altis
Price per liter	4.5€	5.3€	7.9€	5.3€
	○	○	○	○

Question 16: If these were your choices, which product would you choose? ○ None

Olive oil type	Sustainable	POP Kolumpariou	Plain	Organic extra virgin
Package type	Glass	Plastic	Metal	Metal
Package size	0.75*lt*	3*lt*	1*lt*	0.75*lt*
Brand	Altis	MINERBA	Super Market	Super Market
Price per liter	7.9€	6.2€	7.0€	5.3€
	○	○	○	○

Question 17: If these were your choices, which product would you choose? ○ None

Olive oil type	POP Kolumpariou	Sustainable	Plain	Extra Virgin
Package type	Pouch	Plastic	Metal	Glass
Package size	2*lt*	1*lt*	5*lt*	0.75*lt*
Brand	Super Market	Altis	MINERBA	ABEA
Price per liter	6.2€	5.3€	4.5€	7.0€
	○	○	○	○

Question 18: If these were your choices, which product would you choose? ○ None

Olive oil type	Extra Virgin	Sustainable	POP Kolumpariou	Organic extra virgin
Package type	Glass	Plastic	Glass	Pouch
Package size	2*lt*	3*lt*	1*lt*	3*lt*
Brand	Altis	Super Market	MINERBA	ABEA
Price per liter	6.2€	6.2€	7.0€	5.3€
	○	○	○	○

Question 19: If these were your choices, which product would you choose? ○ None

Olive oil type	Plain	Plain	Sustainable	Sustainable
Package type	Pouch	Plastic	Metal	Pouch
Package size	2*lt*	0.75*lt*	5*lt*	3*lt*

Brand	ABEA	MINERBA	MINERBA	Altis
Price per liter	7.9€	6.2€	7.0€	7.0€
	○	○	○	○
Question 20: If these were your choices, which product would you choose?				○ None
Olive oil type	Plain	Sustainable	Extra Virgin	Organic extra virgin
Package type	Glass	Metal	Pouch	Metal
Package size	1*lt*	0.75*lt*	5*lt*	2*lt*
Brand	ABEA	MINERBA	Super Market	Altis
Price per liter	4.5€	7.9€	6.2€	7.0€
	○	○	○	○
Question 21: If these were your choices, which product would you choose?				○ None
Olive oil type	Organic extra virgin	Plain	Extra Virgin	Sustainable
Package type	Glass	Pouch	Plastic	Metal
Package size	2*lt*	3*lt*	5*lt*	1*lt*
Brand	Super Market	Altis	MINERBA	ABEA
Price per liter	4.5€	4.5€	7.9€	6.2€
	○	○	○	○
Question 22: If these were your choices, which product would you choose?				○ None
Olive oil type	POP Kolumpariou	Organic extra virgin	Sustainable	Organic extra virgin
Package type	Plastic	Pouch	Glass	Metal
Package size	0.75*lt*	3*lt*	2*lt*	5*lt*
Brand	Super Market	MINERBA	ABEA	Altis
Price per liter	5.3€	7.9€	7.0€	6.2€
	○	○	○	○