A nonlinear optimal control approach for two-class freeway traffic regulation to reduce congestion and emissions

C. Pasquale, I. Papamichail, C. Roncoli, S. Sacone, S. Siri, M. Papageorgiou

Abstract— **This paper proposes a freeway traffic controller with the objective of minimizing, at the same time, congestion phenomena and traffic emissions. A multi-class framework is considered in the paper, i.e two classes of vehicles (cars and trucks) are explicitly modelled and specific control actions for each vehicle class are computed. The controller is based on the formulation and solution of a constrained discrete-time nonlinear optimal control problem for which a specific solution algorithm, the feasible direction algorithm, is used. The effectiveness of the proposed approach is shown and discussed in the paper by means of some simulation results.**

I. INTRODUCTION

In order to reduce congestion in freeway systems, different control measures can be adopted, such as ramp metering which enables to regulate the inflow of vehicles from on-ramps. One of the first effective ramp metering strategies is the local feedback traffic controller ALINEA [1], developed in the Nineties, that has been successfully applied in practice in many cases. During the years, ALINEA has been further extended, resulting in the proportional-integral version PI-ALINEA [2], or through a coordination of the local ramp-metering actions which yields the linked control of the inflow from consecutive on-ramps [3].

Among the existing traffic control methodologies, some approaches are based on optimization or optimal control algorithms. In some cases, the problem of controlling a freeway is formulated as a discrete-time constrained nonlinear optimal control problem (see [4] and the references therein), whose numerical solution is often hard to find by directly using the available Nonlinear Programming codes, because of the problem dimensions and complexity. A very efficient numerical solution has been adopted in the optimal freeway traffic control tool AMOC [5], [6], in which the so-called feasible direction algorithm is used. More recent works propose sophisticated control architectures, again based on AMOC, such as the three-layer hierarchical control approach described in [7] and the mainstream traffic flow control scheme proposed in [8]. Optimal control algorithms may be also embedded in Model Predictive Control (MPC) schemes, using real-time measurements as initial states. For instance, in [9], [10] nonlinear MPC frameworks adopting the macroscopic model METANET [11] for prediction are presented. Also in these works, nonlinear optimization problems have to be solved, since the prediction model is nonlinear, thus efficient numerical solution algorithms are needed to enable on-line applications for large freeway networks.

Most of the research works on traffic control are devised to minimize congestion, often measured in terms of total time spent by the drivers, whereas in few recent works other cost terms are considered, such as the minimization of traffic emissions (see [12], [13], [14]). In order to explicitly consider traffic emissions as an objective to be minimized, models for evaluating hot exhaust emissions are needed. Among others, simple but very widespread models are the so-called average-speed emission models which assume that the average emission factor for a certain pollutant and a given type of vehicle only depends on the average speed during a trip [15]. These models have been studied and periodically updated on the basis of real measured data obtained from different sources [16].

In this paper, we propose a freeway traffic control approach which considers, as control objectives, both the minimization of traffic emissions and the reduction of traffic congestion. A similar idea was proposed in [14], [17], where the control scheme was based on a local regulator inspired by ALINEA. In this work, instead, an optimal control problem is formulated and solved by applying the feasible direction algorithm and, in particular, a specific version of this algorithm which considers the derivative backpropagation method RPROP. A further characteristic of this work is that a two-class macroscopic traffic model (based on METANET model) and a two-class controller are considered. First of all, considering a two-class traffic model allows to represent the system behaviour more accurately than with a one-class macroscopic model. Secondly, it is possible to devise separate control actions for the two vehicle classes, properly actuated with separate on-ramp lanes and signals for cars and trucks. The two-class model considered in this paper has already been adopted in [14], [17]. Other multi-class models have been studied ([18], [19]) and in some cases adopted in MPC approaches [20], [21].

The paper is organized as follows. Section II introduces the modeling framework. In Section III the optimal control problem is stated while the adopted numerical solution algorithm is described in Section IV. In Section V simulation results are discussed, and the conclusions are drawn in Section VI.

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II. THE ADOPTED MODELS

The considered two-class macroscopic traffic flow model, derived from METANET [11], is based on the division of the freeway stretch in N sections ($i = 1, \dots, N$ indicates the section) and the discretization of the time horizon in K time steps ($k = 0, \dots, K$ denotes the temporal stage). Moreover, $c = 1, 2$ represents the vehicle class (cars and trucks, respectively), T is the sample time interval and L_i is the length of section i.

The main aggregate variables of the considered model are the following:

- $\rho_{i,c}(k)$ is the traffic density of class c in section i at time kT (expressed in vehicles of class c per kilometre);
- $\rho_i(k)$ is the total traffic density in section i at time kT (expressed in cars per kilometre);
- $v_{i,c}(k)$ is the mean traffic speed of class c in section i at time kT (expressed in kilometre per hour);
- $q_{i,c}(k)$ is the traffic volume of class c leaving section i during time interval $[kT, (k+1)T)$ (expressed in vehicles of class c per hour);
- $l_{i,c}(k)$ is the queue length of vehicles of class c waiting on the on-ramp of section i at time kT (expressed in vehicles of class c);
- $d_{i,c}(k)$ is the traffic volume of class c requiring to access section i from the on-ramp during time interval $[kT, (k+1)T)$ (expressed in vehicles of class c per hour);
- $r_{i,c}(k)$ is the on-ramp traffic volume of class c entering section i during time interval $[kT, (k+1)T)$ (expressed in vehicles of class c per hour);
- $r_i(k)$ is the total on-ramp traffic volume entering section i during time interval $[kT, (k+1)T)$ (expressed in cars per hour);
- $s_{i,c}(k)$ is the off-ramp traffic volume of class c exiting section i during time interval $[kT, (k+1)T)$ (expressed in vehicles of class c per hour).

The considered model includes some traffic parameters. Specifically, $v^{\mathrm{f}}_{i,c}$ and $r^{\max}_{i,c}$ are, respectively, the free-flow speed (expressed in kilometres per hour) and the on-ramp capacity (expressed in vehicles of class c per hour) referred to class $c = 1, 2$ and section $i = 1, \dots, N$. Moreover, ρ^{cr}_i and $\rho^{\max}_i$ represent, respectively, the critical density and the jam density (expressed in cars per kilometre) of section $i = 1, \dots, N$. Finally the parameter ς is a conversion factor between cars and trucks. Its meaning is analogous to the definition of passenger car equivalents (PCE), as better detailed in [22]. In this work a constant factor ς is considered, assuming that it has been suitably estimated on the basis of real data. The two-class dynamic model is given by the following equations

$$\rho_{i,c}(k+1) = \rho_{i,c}(k) + \frac{T}{L_i}\Big[q_{i-1,c}(k) - q_{i,c}(k) + r_{i,c}(k) - s_{i,c}(k)\Big] \quad (1)$$

$$\begin{aligned} v_{i,c}(k+1) = {} & v_{i,c}(k) + \frac{T}{\tau_c}\Big[V_{i,c}(k) - v_{i,c}(k)\Big] \\ & + \frac{T}{L_i} v_{i,c}(k)\big(v_{i-1,c}(k) - v_{i,c}(k)\big) \\ & - \frac{\nu_c T\big(\rho_{i+1}(k) - \rho_i(k)\big)}{\tau_c L_i\big(\rho_i(k) + \chi_c\big)} - \delta^{\mathrm{on}}_c T \frac{v_{i,c}(k) r_i(k)}{L_i(\rho_i(k) + \chi_c)} \end{aligned} \quad (2)$$

$$l_{i,c}(k+1) = l_{i,c}(k) + T\big[d_{i,c}(k) - r_{i,c}(k)\big] \quad (3)$$

$c = 1, 2$, $i = 1, \dots, N$, $k = 0, \dots, K-1$, where τ_c, ν_c, χ_c, $\delta_{\mathrm{on},c}$, $c = 1, 2$, are suitable parameters.

The traffic flow is obtained as

$$q_{i,c}(k) = \rho_{i,c}(k) \cdot v_{i,c}(k) \quad (4)$$

$c = 1, 2$, $i = 1, \dots, N$, $k = 0, \dots, K-1$, and the steady-state speed density relation $V_{i,c}(k)$ can be expressed as

$$V_{i,c}(k) = v^{\mathrm{f}}_{i,c} \cdot \left[1 - \left(\frac{\rho_i(k)}{\rho^{\max}_i}\right)^{l_c}\right]^{m_c} \quad (5)$$

$c = 1, 2$, $i = 1, \dots, N$, $k = 0, \dots, K-1$, where l_c, m_c, $c = 1, 2$, are other model parameters specific for each class.

The total density and the total on-ramp traffic volume, for $i = 1, \dots, N$, $k = 0, \dots, K-1$, can be computed as

$$\rho_i(k) = \rho_{i,1}(k) + \varsigma \rho_{i,2}(k) \quad (6)$$

$$r_i(k) = r_{i,1}(k) + \varsigma r_{i,2}(k) \quad (7)$$

If the freeway system is controlled, the on-ramp entering flow $r_{i,c}(k)$ is a portion $\mu_{i,c}(k)$ of the outflow $\bar{r}_{i,c}(k)$ that should access in the mainstream in the case without ramp metering. Therefore $\mu_{i,c}(k) \in [\mu^{min}_{i,c}, 1]$ is the metering rate for the on-ramp of section i at time step kT for class c, and when $\mu_{i,c}(k)$ is set equal to 1 no ramp metering is applied. Then

$$r_{i,c}(k) = \mu_{i,c}(k) \cdot \bar{r}_{i,c}(k) \quad (8)$$

$$\bar{r}_{i,c}(k) = \min\left\{d_{i,c}(k) + \frac{l_{i,c}(k)}{T}, r^{\max}_{i,c}, r^{\max}_{i,c} \cdot \frac{\rho^{\max}_i - \rho_i(k)}{\rho^{\max}_i - \rho^{\mathrm{cr}}_i}\right\} \quad (9)$$

with $c = 1, 2$, $i = 1, \dots, N$, $k = 0, \dots, K-1$.

In order to consider the traffic emissions in the freeway, the average-speed emission model COPERT proposed in [16] has been adopted. In this paper, we only consider gasoline cars, split in four legislation emission categories (from Euro 1 to Euro 4). For the second class of vehicles, we consider only half loaded trucks in the case of roads with no slope.

Let us start from cars. The hot emissions for a gasoline passenger car of legislation emission category j are calculated as a function of the mean speed v, i.e.

$$\Xi^j_1(v) = \frac{a^j_1 + e^j_1 v + f^j_1 v^2}{1 + b^j_1 v + d^j_1 v^2} \quad (10)$$

where a^j_1, b^j_1, d^j_1, e^j_1 and f^j_1, $j = 1, \dots, 4$, are parameters depending on the considered pollutant. The hot emissions for a truck are given by

$$\Xi_2(v) = a_2 + \frac{b_2}{1 + exp(-c_2 + d_2 \ln(v) + e_2 v)} \quad (11)$$

where a_2, b_2, c_2, d_2 and e_2 are specific parameters.

Finally, let us recall some performance indicators used in the paper. Starting from the indicators of traffic emissions, ME indicate the emissions in the mainstream, which can be computed as

$$ME = \sum_{k=0}^{K-1} \sum_{i=1}^{N} \left[\sum_{j=1}^{4} L_i \cdot \rho_{i,1}(k) \cdot \gamma_1^j \cdot \Xi_1^j\big(v_{i,1}(k)\big) \right] + \left[L_i \cdot \rho_{i,2}(k) \cdot \Xi_2\big(v_{i,2}(k)\big) \right] \quad (12)$$

The first term in (12) considers the average-speed model (10) referred to one single car of category j, multiplied by the number of these cars, where γ_1^j represents the composition rate of cars of legislation emission j. The second term in (12) considers the average-speed model (11) referred to one single truck, multiplied by the number of trucks. The emissions in the on-ramp, denoted as RE, are computed as

$$RE = \sum_{k=0}^{K-1} \sum_{i=1}^{N} \left[\sum_{j=1}^{4} \gamma_1^j \cdot \alpha_1^j \cdot l_{i,1}(k) \right] + \left[\alpha_2 \cdot l_{i,2}(k) \right] \quad (13)$$

where α_1^j, $j = 1, \ldots, 4$, and α_2 are constant emission factors obtained respectively from (10) and (11) in case of minimum average speed. The Total Emissions in the freeway TE can be computed as

$$TE = ME + RE \quad (14)$$

The Total Time Spent (TTS), extended to the two-class case, can be seen as the sum of the Total Travel Time (TTT) and the Total Waiting Time (TWT) and is computed as

$$TTS = TTT + TWT = \sum_{k=0}^{K-1} \sum_{i=1}^{N} T L_i \left[\rho_{i,1}(k) + \varsigma \cdot \rho_{i,2}(k) \right] + \sum_{k=0}^{K-1} \sum_{i=1}^{N} T \left[l_{i,1}(k) + \varsigma \cdot l_{i,2}(k) \right] \quad (15)$$

The Total Travelled Distance (TTD) is given by

$$TTD = \sum_{k=0}^{K-1} \sum_{i=1}^{N} \left[L_i \cdot T \Big(q_{i,1}(k) + \varsigma \cdot q_{i,2}(k) \Big) \right] \quad (16)$$

III. Optimal control problem

The objective of the present work is to define a coordinated ramp metering strategy in order to reduce traffic congestion in the freeway and, at the same time, to minimize the total emissions (both in the mainstream and at the on-ramps). The control strategy is sought by defining and solving a finite horizon nonlinear optimal control problem with constrained control variables.

Combining the equations from (1) to (9), a discrete-time dynamic system of the form

$$\underline{x}(k+1) = \underline{f}\big[\underline{x}(k), \underline{u}(k)\big] \quad (17)$$

can be obtained for the considered freeway system, where $\underline{x}(k)$ is the state vector with $\underline{x}(0) = \underline{x}_0$ and $\underline{u}(k)$ is the vector of the control variables. In particular the state vector consists of the densities $\rho_{i,c}(k)$, the mean speeds $v_{i,c}(k)$, and the queues $l_{i,c}(k)$ for every section $i = 1, \ldots, N$ and for each class $c = 1, 2$. The control vector corresponds to the ramp metering rates $\mu_{i,c}(k)$, $i = 1, \ldots, N$, $c = 1, 2$.

The general formulation of the optimization problem over a finite horizon of K time steps is the following.

Problem 1: Given the system initial conditions $\underline{x}(0) = \underline{x}_0$, find the control sequence $\underline{u}(k), k = 0, \ldots, K-1$, that minimizes

$$J = \vartheta[\underline{x}(K)] + \sum_{k=0}^{K-1} \varphi\big[\underline{x}(k), \underline{u}(k)\big] \quad (18)$$

subject to (17) and

$$\underline{u}^{min} \leq \underline{u}(k) \leq \underline{u}^{max} \qquad k = 0, \ldots, K-1 \quad (19)$$

□

In accordance with the purposes of the considered approach, the chosen objective function is defined as

$$J = \beta \cdot \Gamma \cdot TE + (1-\beta) \cdot TTS + \sum_{k=1}^{K-1} \sum_{i=1}^{N} \sum_{c=1}^{2} w_{i,c}^{\mu} \cdot \big[\mu_{i,c}(k) - \mu_{i,c}(k-1)\big]^2 \quad (20)$$

with

$$\mu_{i,c}^{\min} \leq \mu_{i,c}(k) \leq 1 \qquad c = 1, 2, \quad i, \ldots, N, \quad k = 0, \ldots, K-1 \quad (21)$$

The first two terms in cost function (20) are the Total Emissions and the Total Time Spent, given respectively by (14) and (15), reported to the same order of magnitude thanks to coefficient Γ and arbitrarily weighted by $\beta \in [0, 1]$. The third term in (20), with the weights $w_{i,c}^{\mu}$, $i = 1, \ldots, N$, $c = 1, 2$, is introduced in order to prevent oscillations of the control trajectories.

IV. Numerical solution algorithm

The numerical solution of Problem 1 may be obtained by direct use of available Nonlinear Programming codes, but this approach often presents unsurmountable difficulties in the case of large freeway infrastructures, due to the problem dimensions and complexity. A much more efficient numerical solution is obtained by use of the feasible direction algorithm which is adopted within the optimal freeway traffic control tool AMOC [5] and leads to low computation times even for very large-scale freeway traffic control problems, (see e.g. [4], [8]). A version of this algorithm is therefore used for the present problem. The solution determined by the feasible direction algorithm consists of the optimal control actions $\underline{u}(k), k = 0, \ldots, K-1$, and the corresponding optimal state trajectories $\underline{x}(k)$, $k = 1, \ldots, K$, over the whole time horizon.

For a given control sequence $\underline{u}(k)$, $k = 0, \ldots, K-1$, and for a given initial condition, the state trajectory $\underline{x}(k+1)$ can be found by applying (17), so that the cost criterion only depends on the control variables $\underline{u}(k)$ that can be considered as the independent optimization variables. Thus, the cost criterion can be expressed as $\bar{J}[\underline{u}(k)]$, and the reduced gradient $\underline{g}(k)$ is given by

$$\underline{g}(k) = \frac{\partial \bar{J}[\underline{u}(k)]}{\partial \underline{u}(k)} + \left(\frac{\partial \underline{f}\left[\underline{x}(k), \underline{u}(k)\right]}{\partial \underline{u}(k)} \right)^T \cdot \underline{\lambda}(k+1) \quad (22)$$

where the vector $\underline{\lambda}(\cdot)$ is calculated via backward integration by using the following

$$\underline{\lambda}(k) = \frac{\partial J\left[\underline{x}(k), \underline{u}(k)\right]}{\partial \underline{x}(k)} + \left(\frac{\partial \underline{f}\left[\underline{x}(k), \underline{u}(k)\right]}{\partial \underline{x}(k)} \right)^T \cdot \underline{\lambda}(k+1)$$
$$k = 0, \ldots, K-1 \quad (23)$$

starting from the final condition

$$\underline{\lambda}(K) = \frac{\partial \vartheta\left[\underline{x}(K)\right]}{\partial \underline{x}(K)} \quad (24)$$

Therefore the optimality conditions that have to be satisfied are (17), (19), (23), (24). Moreover, a saturation vector function $sat(\pi)$ is defined by

$$sat(\pi) = \begin{cases} \pi_{max}, & \text{if } \pi > \pi_{max} \\ \pi_{min}, & \text{if } \pi < \pi_{min} \\ \pi, & \text{else} \end{cases} \quad (25)$$

The adopted numerical algorithm, i.e. the feasible direction algorithm, is widely known in the literature. In this work a specific version of this algorithm is used, that is the derivative backpropagation method RPROP, in the version proposed in [6], since its application to freeway traffic control problems has led to effective results, as described in [6]. The steps of the adopted algorithm follow.

1) Guess a feasible initial control sequence $\underline{u}^{(0)}(k)$, $k = 0, \ldots K-1$, and set the iteration index $\imath = 0$.
2) For each iteration $\imath$, using $\underline{u}^{(\imath)}(k)$ and the initial conditions $\underline{x}(0)$, apply (17) to calculate $\underline{x}^{(\imath)}(k+1)$; then, using $\underline{x}^{(\imath)}(k+1)$ and $\underline{u}^{(\imath)}(k)$, apply (23) via backward integration from the final state (24) to get $\underline{\lambda}^{(\imath)}(k+1)$.
3) Use $\underline{x}^{(\imath)}(k+1)$, $\underline{u}^{(\imath)}(k)$ and $\underline{\lambda}^{(\imath)}(k+1)$ to compute the components of the reduced gradient $\underline{g}^{(\imath)}(k)$.
4) Apply the RPROP method to get a new, improved admissible control sequence $\underline{u}^{(\imath+1)}(k)$ by applying the following relation

$$\underline{u}^{(\imath+1)}(k) = sat\big(\underline{u}^{(\imath)}(k) + \underline{\Delta u}^{(\imath)}(k)\big) \quad (26)$$

Each component $\Delta u_i^{(\imath)}(k)$ of the control variable increment $\underline{\Delta u}^{(\imath)}(k)$ is calculated according to the sign of the gradient component $g_i^{(\imath)}(k)$ and the increment component at the previous iteration $\Delta u_i^{(\imath-1)}(k)$, as follows

$$\Delta u_i^{(\imath)}(k) = \begin{cases} -\text{sign}\big(g_i^{(\imath)}(k)\big)\eta^+ \Delta u_i^{(\imath-1)}(k) \\ \qquad\qquad \text{if } g_i^{(\imath-1)}(k) g_i^{(\imath)}(k) > 0 \\ -\text{sign}\big(g_i^{(\imath)}(k)\big)\eta^- \Delta u_i^{(\imath-1)}(k) \\ \qquad\qquad \text{otherwise} \end{cases} \quad (27)$$

where $0 < \eta^- < 1 < \eta^+$. The algorithm starts with $\underline{\Delta u}^{(0)}(k) = \underline{\Delta}$ verifying (26), while in the following iterations (27) is applied. Nevertheless, at each iteration the calculated $\underline{\Delta u}^{(\imath)}(k)$ may be restricted in a specific interval $[\underline{\Delta}_{min}, \underline{\Delta}_{max}]$.
5) If for a given scalar σ the convergence test $|J^{(\imath+1)} - J^{(\imath)}|/J^{(\imath)} < \sigma$ is satisfied, stop; otherwise start a new iteration $\imath = \imath + 1$, and go back to 2).

V. Simulation results

This section presents some simulation results in which the proposed control approach is compared with the two-class PI-ALINEA regulator proposed in [14]. The considered three-lane freeway stretch is composed of $N = 20$ sections, each one with a length $L_i = 500$ [m], $i = 1, \ldots, 20$. Moreover three on-ramps (in sections $i = 12, 14, 16$) and two off-ramps (in sections $i = 13, 15$) are present. The sample time is $T = 10$ [s] and a total time horizon of 2 and half hours ($K = 900$) has been considered for the simulation tests. The case study is characterized by trapezoidal demand profiles for both vehicle classes, as shown in the left side plots in Fig. 1. The mainstream flow is composed by 3900 [cars/h] and 86 [trucks/h] (344 [PCE/h]) and the exit flows from the off-ramps are 5% of the relative mainstream flow. Moreover, the ratio ς has been chosen equal to 4. In order to apply the two-class PI-ALINEA, the set-point value for the density is set equal to 160 [cars/km], while the adopted parameters for the RPROP algorithm are $\eta^+ = 1.2$, $\eta^- = 0.5$, $\Delta_{max} = 0.2$, $\Delta_{min} = 10^{-6}$, $\sigma = 10^{-6}$.

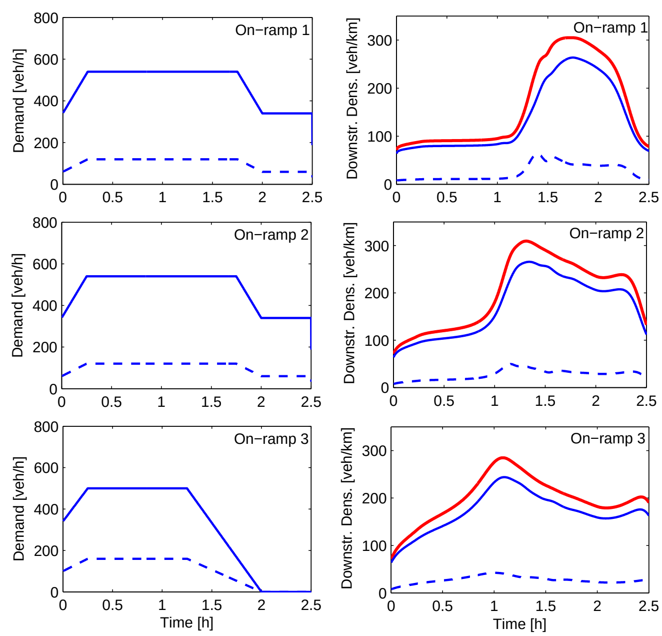

Fig. 1: No-control case (blue solid line for cars, blue dashed line for trucks [PCE], red line for cars plus trucks [PCE]).

The plots in the right side of Fig. 1 report the density evolution downstream the three on-ramps in case the freeway is not controlled. The no-control case is characterized by a high congestion in these downstream sections, in which for

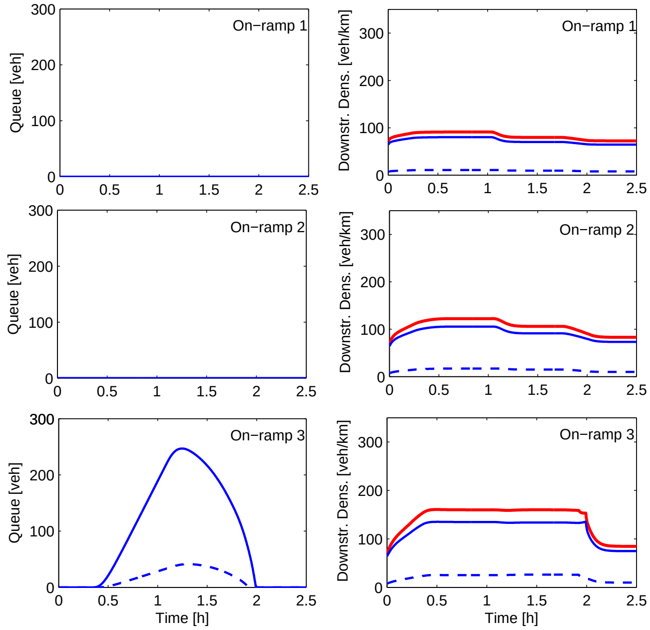

Fig. 2: PI-ALINEA (blue solid line for cars, blue dashed line for trucks [PCE], red line for cars plus trucks [PCE]).

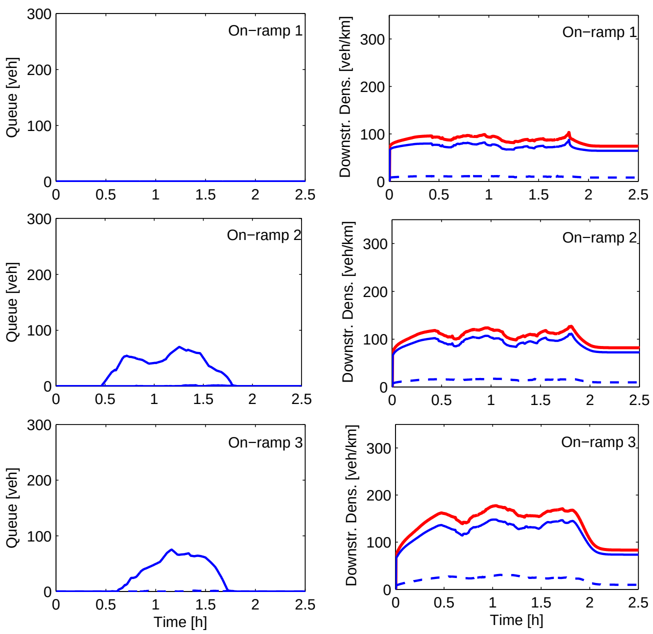

Fig. 3: Optimal solution with $\beta = 0.5$ (blue solid line for cars, blue dashed line for trucks [PCE], red line for cars plus trucks [PCE]).

most of the time the density is much higher than the fixed set-point value. When the system is not controlled the resulting Total Time Spent is $TTS = 3080$ [veh· h], and the Total Emissions are $TE = 907208$ [g].

Fig. 2 shows the behaviour of the queue lengths and the density downstream the three on-ramps in case the two-class PI-ALINEA is applied. The traffic density is highly reduced and a long queue is created only in the third on-ramp (the TWT is equal to 271 [veh·h]). Moreover, the TTS is reduced to 2673 [veh·h], which is a 13.2% reduction compared with the no-control case, whereas the TE are reduced to 732457 [g], i.e. corresponding to a 19.3% reduction in comparison with the no-control case. The variation of the TTD is very slight with respect to the no control case, corresponding to 1.08%.

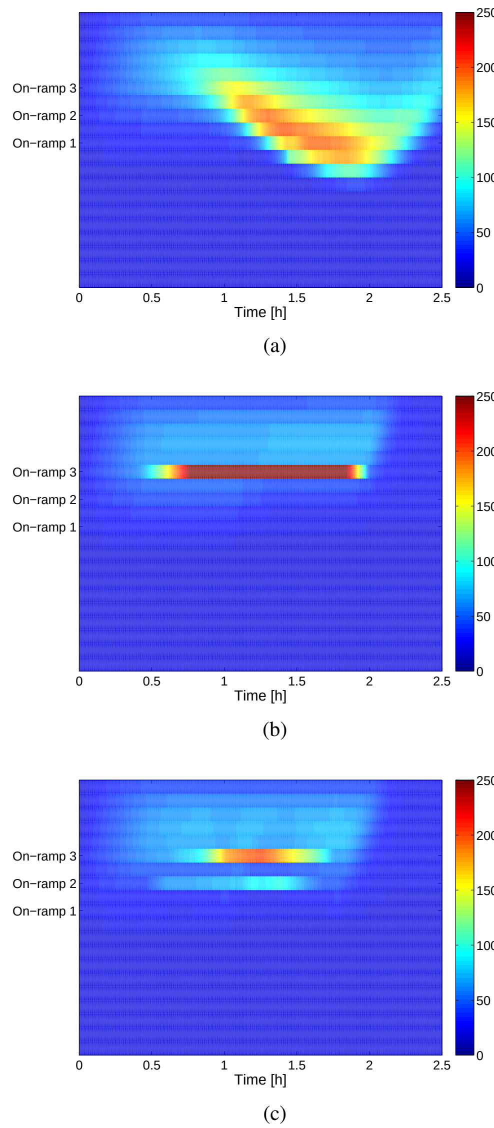

Fig. 4: Total emissions in the no-control case (4a), with PI-ALINEA (4b), and applying the feasible direction algorithm with $\beta = 0.5$ (4c)

In case the feasible direction algorithm is applied with $\beta = 0.5$ (i.e. minimizing both TTS and TE), the queue lengths and the density downstream the three on-ramps are depicted in Fig. 3. In this case, some vehicles are queued in the second and third on-ramps but the queue length is lower than in the case with PI-ALINEA (the TWT is equal to 109 [veh·h]). It is worth noting that in the optimal solution only cars are queued. This is due to the fact that trucks present high emissions in case of low speeds according to

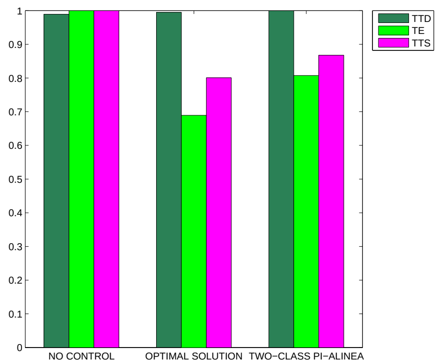

Fig. 5: Performance indicators.

(11). The optimal solution obtained by the application of the feasible direction algorithm reduces the TTS to 2486 [veh·h] and gets a percentage reduction of 19.3%, whereas the TE are reduced to 635582 [g], with a reduction of 29.9%. The variation of the TTD with respect to the no-control case is equal to 0.57%.

To further compare the three considered cases, i.e. no-control, application of PI-ALINEA and optimal solution with $\beta = 0.5$, the total emissions in the freeway along the considered time horizon are reported in Fig. 4. The worst situation is represented by the no-control case in which high emissions are present in a large part of the freeway for more than one hour. The situation with PI-ALINEA is slightly better because the high emissions are more localized only in one freeway section. These high values are mostly due to the emissions at the on-ramps, since with PI-ALINEA vehicles are queued only in the third on-ramp. The best situation is surely guaranteed by the approach proposed in this paper which reduces the total emissions both in space and in time.

Fig. 5 reports the different performance indexes, TTD, TE and TTS, properly normalized, for each of the three considered cases (characterized by very similar values of TTD). Both PI-ALINEA and the feasible direction algorithm guarantee a reduction of TTS and TE but the latter ensures the best performance of the traffic system behaviour.

VI. Conclusion

A two-class traffic regulator has been proposed in the paper with the aim of reducing congestion, on one hand, and reducing traffic emissions, on the other. The control actions to be applied via ramp metering have been computed by formulating a multi-objective nonlinear optimal control problem, which has been solved by adopting the derivative backpropagation method RPROP, a version of the feasible direction algorithm. The effectiveness of the proposed approach has been assessed through simulation results in which it is shown that the two parts of the control function are non conflicting objectives since both the average travel times and the emissions are reduced if the control actions manage to reduce or eliminate traffic congestion.

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