Highway Traffic State Estimation With Mixed Connected and Conventional Vehicles Using Speed Measurements

Nikolaos Bekiaris-Liberis, Claudio Roncoli, and Markos Papageorgiou

Abstract— **We present a macroscopic model-based approach for estimation of the total density and flow of vehicles, for the case of "mixed" traffic, i.e., traffic comprising both ordinary and connected vehicles, utilizing only average speed measurements reported by connected vehicles and a minimum number (sufficient to guarantee observability) of spot sensor-based total flow measurements. The approach is based on the realistic assumption that the average speed of conventional vehicles is roughly equal to the average speed of connected vehicles, and consequently, it can be obtained at the (local or central) traffic monitoring and control unit from connected vehicles' reports. Thus, complete traffic state estimation (for arbitrarily selected segments in the network) may be achieved by estimating the total density of vehicles. Recasting the dynamics of the total density of vehicles, which are described by the well-known conservation law equation, as a linear time-varying system, we employ a Kalman filter for the estimation of the total density. We demonstrate the fact that the developed approach allows a variety of different measurement configurations, by also considering the case in which additional mainstream total flow measurements are employed to replace a corresponding number of total flow measurements at on-ramps or off-ramps. We validate the performance of the developed estimation scheme through simulations using a well-known second-order traffic flow model as ground truth for the traffic state.**

I. INTRODUCTION

The introduction of various Vehicle Automation and Communication Systems (VACS), which, besides their potentials in improving driving safety and convenience have great potentials in mitigating traffic congestion, creates the need for the development of advanced traffic management methodologies that efficiently exploit VACS capabilities [7]. For this reason, numerous papers are dealing with the modeling and control of traffic flow in presence of VACS, employing both macroscopic and microscopic approaches, such as [5], [10], [18], [19], [20], [22], [24], [25], [26], among others.

Due to the high purchase, installation, and maintenance costs of the large amount of spot sensors needed for traffic monitoring, traffic estimation utilizing a limited amount of sensors is preferable for achieving a cost-efficient solution to the highway traffic surveillance and control problems. As a result, several papers are devoted to the development of traffic state estimation algorithms employing a limited number of conventional detector measurements, such as, for example, [1], [9], [11], [13], [14], [28]. The communication capabilities of VACS-equipped vehicles can be exploited for further reduction of the implementation and maintenance costs that the use of conventional road-side detectors entails. Since in the presence of VACS vehicles may become "connected", i.e., enabled to send (and receive) real-time information to a local or central monitoring and control unit (MCU), connected vehicles may communicate their position, speed and other relevant information, i.e., they can act as mobile sensors. Consequently, exploiting different, less costly data sources such as mobile phone, or GPS (Global Positioning System), or even vehicle speed data, for travel time or highway traffic state estimation is the subject of various works; see, e.g., [2], [6], [8], [15], [17], [21], [23], [29], [30]; employing various kinds of traffic or statistic models.

In this paper, we address the problem of estimation of the total density and flow of vehicles in highway segments of arbitrary length (typically around 500 m) for a mixed traffic flow that includes both conventional and connected vehicles, exploiting the information provided by connected vehicles, thus reducing substantially the need for spot sensor measurements. The developments rely on the realistic assumption that the average speed of conventional vehicles is roughly equal to the average speed of connected vehicles, and consequently, the average speed of all vehicles on an arbitrary segment of the highway can be readily obtained at the local or central MCU from connected vehicles reports. This assumption relies on the fact that, even at very low densities, there is no reason for connected vehicles to feature a systematically different mean speed than conventional vehicles; while at higher densities, the assumption is further reinforced due to increasing difficulty of overtaking. As a consequence of this assumption, complete traffic state estimation (of the total density and flow in arbitrary segments in the highway) may be achieved via estimating the traffic density (see [4] for an alternative approach to estimating the traffic state via estimation of the percentage of connected vehicles with respect to the total number of vehicles) and by utilizing only average speed measurements from connected vehicles together with a minimum (necessary to guarantee observability) amount of conventional measurements of traffic volumes, e.g., at all entries and exits of the considered highway stretches.

The developed estimation methodology allows a variety of different measurement configurations. We demonstrate this fact by also considering the case in which traffic state estimation is achieved when additional mainstream total flow measurements are employed to replace a corresponding number of total flow measurements at on-ramps or off-ramps. The performance of the developed estimation scheme is validated

N. Bekiaris-Liberis, C. Roncoli, and M. Papageorgiou are with the Department of Production Engineering and Management, Technical University of Crete, Chania, 73100, Greece. Email addresses: nikos.bekiaris@gmail.com, croncoli@dssl.tuc.gr, and markos@dssl.tuc.gr.

through simulations using a well-known METANET traffic flow model as ground truth for the traffic state.

More specifically, the dynamics of the total traffic density, as described by the well-known (discrete-time) conservation law equation, are recast as a linear time-varying system with known parameters that depend on the real-time average speed measurements (Section II-A), thus removing the requirement of (empirical, hence uncertain) traffic speed modeling, such as the fundamental diagram. The observability properties of this system are studied (Section II-B) and a Kalman filter is employed for the estimation of the total density of vehicles (Section II-C). The effectiveness of the proposed estimation design is illustrated in simulation with a second-order macroscopic model as ground truth (Section II-D). The estimation approach is then extended to the case of unmeasured total flows at on-ramps or off-ramps (by incorporating additional mainstream total flow measurements that replace a corresponding number of total flow measurements at on-ramps or off-ramps) and its performance in this case is also illustrated in simulation (Section III).

II. Traffic Estimation Using Average Speed Measurements from Connected Vehicles

A. The Dynamics of Traffic Density as a Linear Time-Varying System

We consider the following discrete-time equations that describe the dynamics of the total density ρ of vehicles on a highway (see, e.g., [16]; see also the upper part of Fig. 1)

$$\begin{aligned}\rho_i(k+1) \quad &= \quad \rho_i(k)+\frac{T}{\Delta_i}\left(q_{i-1}(k)-q_i(k)\right.\\ &\quad \left.+r_i(k)-s_i(k)\right), \end{aligned} \tag{1}$$

where $i=1,\ldots,N$ is the index of the specific segment at the highway, N being the number of segments on the highway; for all traffic variables, we denote by index sub-i its value at the segment i of the highway; q_i is the total flow at segment i; T is the time-discretization step, Δ_i is the length of the discrete segments of the highway, and $k=0,1,\ldots$ is the discrete time index. The variables r_i and s_i denote the inflow and outflow of vehicles at on-ramps and off-ramps, respectively, at segment i. Using the known relation

$$q_i \quad = \quad \rho_i v_i, \tag{2}$$

where v_i is the average speed in segment i, we write (1) as

$$\begin{aligned}\rho_i(k+1) \; &= \; \frac{T}{\Delta_i}v_{i-1}(k)\rho_{i-1}(k)+\left(1-\frac{T}{\Delta_i}v_i(k)\right)\rho_i(k)\\ &\quad +\frac{T}{\Delta_i}\left(r_i(k)-s_i(k)\right). \end{aligned} \tag{3}$$

Assuming that the average speed of conventional vehicles is roughly equal to the average speed of connected vehicles, and hence, it can be reported to the traffic authority from the connected vehicles, one can conclude that v_i, $i=1,\ldots,N$, are measured. Therefore, defining the state

$$x \quad = \quad \left(\rho_1,\ldots,\rho_N\right)^T, \tag{4}$$

system (3) can be written in the form of a known linear time-varying system of the form

$$x(k+1) \quad = \quad A(k)x(k)+Bu(k) \tag{5}$$

$$y(k) \quad = \quad Cx(k), \tag{6}$$

where

$$A(k) \; = \; \left\{\begin{array}{ll} a_{ij}=\frac{T}{\Delta_i}v_{i-1}(k), & \text{if } i-j=1 \\ & \text{and } i\geq 2 \\ a_{ij}=1-\frac{T}{\Delta_i}v_i(k), & \text{if } i=j \\ a_{ij}=0, & \text{otherwise} \end{array}\right\} \tag{7}$$

$$B \; = \; \left\{\begin{array}{ll} b_{ij}=\frac{T}{\Delta_i}, & \text{if } i=1 \text{ and } j=1,2 \\ & \text{or } j-i=1 \text{ and } i\geq 2 \\ b_{ij}=0, & \text{otherwise} \end{array}\right\}, \tag{8}$$

$$u(k) \; = \; \left[\begin{array}{llll} q_0(k) & r_1(k)-s_1(k) & \ldots & r_N(k)-s_N(k) \end{array}\right]^T \tag{9}$$

$$C \; = \; \left[\begin{array}{llll} 0 & \ldots & 0 & 1 \end{array}\right], \tag{10}$$

with $A\in\mathbb{R}^{N\times N}$, $B\in\mathbb{R}^{N\times(N+1)}$, where q_0 denotes the total flow of vehicles at the entry of the considered highway stretch and acts as an input to system (5), along with r_i and s_i; while v_i, $i=1,\ldots,N$, are viewed as time-varying parameters of system (5). The variable ρ_N at the exit of the considered highway stretch is viewed as the output of the system and may be obtained via $\rho_N=\frac{q_N}{v_N}$, using total flow measurements q_N at the exit of the considered stretch.

Before studying the observability of (5)–(10), we summarize the assumptions that guarantee that the matrix A is known, as well as that the input u and output y are measured.

- The average speed of all vehicles at a segment of the highway equals the average speed of connected vehicles at the same segment, and hence, it can be obtained from regularly received messages by the connected vehicles.
- The total flow of vehicles at the entry and exit of the considered highway stretch, q_0 and q_N, respectively, are measured via conventional detectors.
- The total flow of vehicles at ramps, i.e., r_i and s_i, $i=1,\ldots,N$, are measured via conventional detectors.

The above formulation may be modified to incorporate different total flow measurement configurations. In Section III we consider the case in which additional mainstream flow measurements (using conventional detectors) are employed to replace a corresponding number of flows at ramps.

B. Observability of the System

System (5) can be viewed as a known linear time-varying system. As it is stated in Section II-A, it is assumed that the quantities q_0, v_i, r_i, and s_i, for all i, are available, which implies that the matrix A and the input u in (5) may be calculated in real time. We show next that system (5)–(10) is observable at $k=k_0+N-1$, for any initial time $k_0\geq 0$. We construct the observability matrix

$$O(k_0,k_0+N)=\left[\begin{array}{c} C \\ CA(k_0) \\ CA(k_0+1)A(k_0) \\ \vdots \\ CA(k_0+N-2)\cdots A(k_0) \end{array}\right]. \tag{11}$$

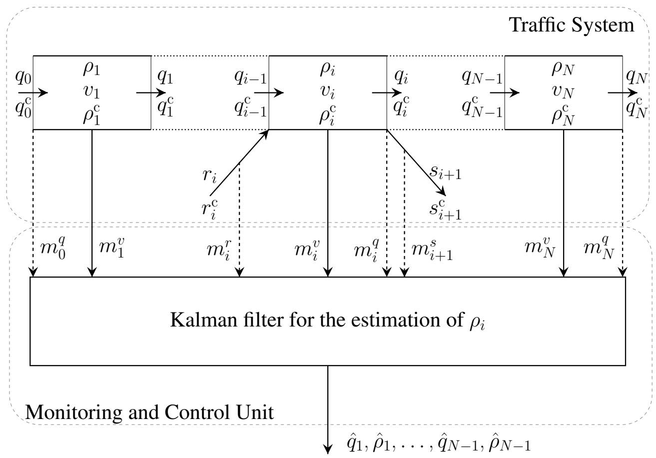

Fig. 1. The traffic system under consideration and the Kalman filter implemented at the MCU. The data used to operate the Kalman filter are either speed measurements coming from connected vehicles (solid lines) or flow measurements coming from fixed sensors (dashed lines). The variable m_i^w denotes the measurement of quantity w at segment i, which might be different than the actual quantity w, due to, for example, the presence of measurement noise. A variable w_i^{c} represents the value of quantity w of connected vehicles at segment i.

From (7), A is lower triangular with non-zero entries only in the main diagonal and the first diagonal below it. Thus, from (10) it follows that O is an anti-lower triangular matrix, namely, a matrix with zero elements above the anti-diagonal. Therefore, relation $\det(O) \neq 0$ holds if the anti-diagonal elements of O are non-zero. The anti-diagonal elements of O are given by $1, \frac{T}{\Delta_N} v_{N-1}(k_0), \frac{T^2}{\Delta_N \Delta_{N-1}} v_{N-1}(k_0 + 1) v_{N-2}(k_0), \ldots, \frac{T^{N-1}}{\Delta_N \Delta_{N-1} \cdots \Delta_2} v_{N-1}(k_0 + N - 2) \cdots v_1(k_0)$. Since v_i, $i = 1, \ldots, N$, are lower and upper-bounded (and positive) for all time, it follows that the matrix O is invertible, and thus, (5)–(10) is observable at $k = k_0 + N - 1$. Note that the measurement of ρ_N (or, equivalently, the measurement of q_N), rather than any other intermediate density, is necessary for system (5)–(10) to be observable. To see this note that if $C = \left\{ \begin{array}{ll} c_{ij} = 1, & \text{if } i = 1 \text{ and } j = J \\ c_{ij} = 0, & \text{otherwise} \end{array} \right\}$ with $J < N$, then the $J+1, \ldots, N$ columns of $O(k_0, k_0 + \bar{N})$ are zero for all $k_0 \geq 0$ and $\bar{N} \geq N$. Thus, the system cannot be observable. In other words, a fixed flow sensor should necessarily be placed at the last segment of the highway in order to guarantee density observability based on model (5)–(10).

C. Kalman Filter

We employ a Kalman filter for the estimation of the total density of vehicles on a highway (Fig. 1). Defining $\hat{x} = (\hat{\rho}_1, \ldots, \hat{\rho}_N)^T$, the Kalman filter's equations are (e.g., [3])

$$\begin{aligned} \hat{x}(k+1) &= A(k)\hat{x}(k) + Bu(k) \\ &\quad + A(k)K(k)\left(z(k) - C\hat{x}(k)\right) && (12) \\ K(k) &= P(k)C^T \left(CP(k)C^T + R\right)^{-1} && (13) \\ P(k+1) &= A(k)\left(I - K(k)C\right)P(k)A(k)^T + Q, && (14) \end{aligned}$$

where z is a noisy version of the measurement y, $R = R^T > 0$ and $Q = Q^T > 0$ are tuning parameters. Note that, in the ideal case in which there is additive, zero-mean Gaussian white noise in equations (5) and (6), respectively, R and Q represent the (ideally known) covariance matrices of the measurement and process noise, respectively. Since the system equations here are relatively complex, some tuning of R, Q may be needed for best estimation results. The initial conditions of the filter (12)–(14) are chosen as

$$\hat{x}(k_0) = \mu \quad (15)$$

$$P(k_0) = H, \quad (16)$$

where μ and $H = H^T > 0$, which, in the ideal case in which $x(k_0)$ is a Gaussian random variable, represent the mean and auto covariance matrix of $x(k_0)$, respectively. The Kalman filter (12)–(16) delivers estimates of the total densities $\hat{\rho}_i$ as indicated at the output of the Kalman filter in Fig. 1.

In addition to guaranteeing observability of the system, we impose the conditions that the pair (A, C) is uniformly completely observable (UCO) and that the pair $(A, Q^{\frac{1}{2}})$ is uniformly completely controllable (UCC), which, in combination with the fact that A is uniformly bounded with bounded from below positive determinant (assuming $1 - \frac{T}{\Delta_i} v_i$, $\forall i$, positive and bounded from below), guarantee that the homogenous part of the estimator is exponentially stable and that the covariance of the estimation error is bounded [12]. We show that (A, C) is UCO by showing that $\exists \epsilon_1, \epsilon_2 > 0$ such that $\epsilon_1 I_{N \times N} \leq O^T(k_0, k_0 + N) O(k_0, k_0 + N) \leq \epsilon_2 I_{N \times N}$, $\forall k_0 \geq 0$. Since O is an anti-lower triangular matrix with uniformly bounded from below and above, positive elements on the anti-diagonal, it follows that it has N independent columns, and hence, $O^T(k_0, k_0 + N) O(k_0, k_0 + N) > 0$, $\forall k_0 \geq 0$. Thus, $\epsilon_1 I_{N \times N} \leq O^T(k_0, k_0 + N) O(k_0, k_0 + N) \leq \epsilon_2 I_{N \times N}$, $\forall k_0 \geq 0$, where $\epsilon_1 = \inf_{k_0 \geq 0} \lambda_{\min}\left(O^T(k_0, k_0 + N) O(k_0, k_0 + N)\right)$ and $\epsilon_2 = \sup_{k_0 \geq 0} \lambda_{\max}\left(O^T(k_0, k_0 + N) O(k_0, k_0 + N)\right)$. Note that $\epsilon_1 > 0$ since $\det(O)^2$ is uniformly bounded from below and $\epsilon_2 < \infty$ (since A is bounded). The fact that $(A, Q^{1/2})$ is UCC follows exploiting the choice $Q = \sigma I_{N \times N}$, for some bounded $\sigma > 0$, and the fact that A is lower triangular with bounded from below positive elements on the main diagonal.

D. Evaluation of the Performance of the Estimator Based on a METANET Model as Ground Truth

For preliminary assessment of the developed estimation scheme, we test in this section the performance of the Kalman filter employing the second-order METANET model [16] as ground truth. We employ equation (1) for the total density of vehicles together with relation (2) for the total flow. The average speed at segment i is given by

$$\begin{aligned} v_i(k+1) = & v_i(k) + \frac{T}{\tau}\left(V\left(\rho_i(k)\right) - v_i(k)\right) + \frac{T}{\Delta_i} v_i(k) \\ & \times \left(v_{i-1}(k) - v_i(k)\right) - \frac{\nu T}{\tau \Delta_i} \frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \kappa} \\ & - \frac{\delta T}{\Delta_i} \frac{r_i(k) v_i(k)}{\rho_i(k) + \kappa}, \quad i = 1, \ldots, N, \end{aligned} \quad (17)$$

with $v_0 = v_1$ and $\rho_N = \rho_{N+1}$, where the nominal average speed V is given by $V(\rho) = v_{\mathrm{f}} e^{-\frac{1}{\alpha}\left(\frac{\rho}{\rho_{\mathrm{cr}}}\right)^{\alpha}}$, and τ, ν, κ, δ, v_{f},

TABLE I

PARAMETERS OF THE MODEL (1), (2), (17).

T	$\frac{1}{360}$ (h)	δ	1.4	Δ_i	0.5 (km)	N	20
v_f	$120\left(\frac{\mathrm{km}}{\mathrm{h}}\right)$	τ	$\frac{1}{180}$ (h)	ρ_cr	$33.5\left(\frac{\mathrm{veh}}{\mathrm{km}}\right)$		
ν	$35\left(\frac{\mathrm{km}^2}{\mathrm{h}}\right)$	α	1.4324	κ	$13\left(\frac{\mathrm{veh}}{\mathrm{km}}\right)$		

TABLE II

THE MEASUREMENT NOISE γ_i^w AND THE PROCESS NOISE ξ_i^w, $i = 0, \ldots, N$ AFFECTING THE w VARIABLE AT SEGMENT i.

	γ_i^q	γ_i^r	γ_i^s	γ_i^v	ξ_i^v	ξ_i^q
SD	$25\frac{\mathrm{veh}}{\mathrm{h}}$	$10\frac{\mathrm{veh}}{\mathrm{h}}$	$5\frac{\mathrm{veh}}{\mathrm{h}}$	$3\frac{\mathrm{km}}{\mathrm{h}}$	$5\frac{\mathrm{km}}{\mathrm{h}}$	$25\frac{\mathrm{veh}}{\mathrm{h}}$

ρ_cr, α are positive model parameters. In particular, v_f denotes the free speed, ρ_cr the critical density, and α the exponent of the stationary speed equation. The model parameters, which are taken from [27], are shown in Table I.

The measurements of the total flow of vehicles at the entry and exit of the highway stretch under consideration, the speed measurements stemming from connected vehicles, and the measurements of the total flow at the on-ramps or off-ramps are subject to additive measurement noise. Moreover, there is additive process noise affecting the speed and flow equations, namely, (17) and (2), respectively. All noise used in the simulation is zero-mean Gaussian white with standard deviation (SD) shown in Table II.

The parameters and initial conditions of the Kalman filter (12)–(16), (7)–(10) are shown in Table III. In Fig. 2 we show the employed scenario of total input flow at the entry of the considered highway stretch for our simulation investigation. We assume that there are three on-ramps at segments $2, 6, 10$ with constant inflows satisfying $r_2 = r_6 = r_{10} = 150\frac{\mathrm{veh}}{\mathrm{h}}$. Three off-ramps are supposedly present on the highway under study, specifically at segments $4, 8, 12$. It is assumed

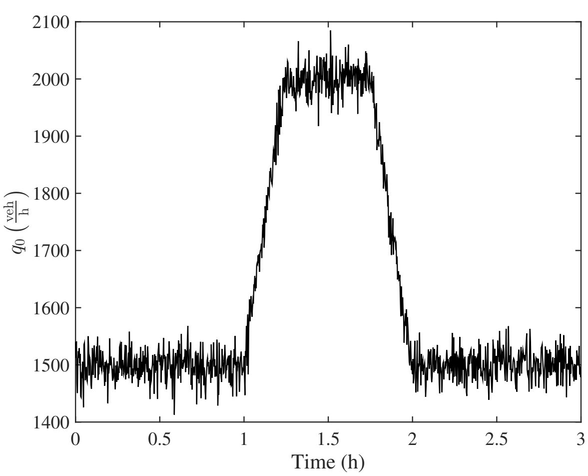

Fig. 2. The total flow of vehicles q_0 at the entry of the highway stretch under consideration.

TABLE III

PARAMETERS OF THE KALMAN FILTER (12)–(16) AND (7)–(10).

Q	R	μ	H
$I_{N\times N}$	100	$(15, \ldots, 15)^T$	$I_{N\times N}$

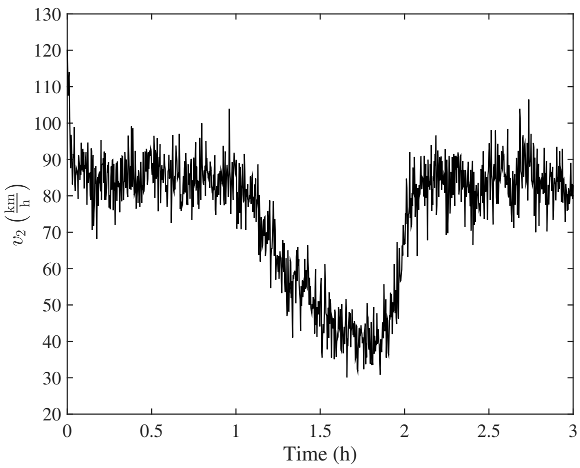

Fig. 3. The average speed v_2 of the second segment of the highway as it is produced by the METANET model (1), (2), (17), with parameters given in Table I and additive process noise given in Table II.

that $s_i = 0.1q_{i-1}$, $i = 4, 8, 12$. The average speed at segment 2 (where the first on-ramp is located) is shown in Fig. 3. It is evident from Fig. 3 that a congestion is created between the first and second hour of our test, whereas free-flow conditions are prevailing at the first and last hour. Congestion starts approximately at the location of the second on-ramp, i.e., at the sixth segment of the highway, and propagates backwards all the way to the input of the highway.

In both traffic conditions, our estimator successfully estimates the total density of vehicles on the highway, as it is evident from Fig. 4, which displays the actual density and its estimate at segment 2 (at which congested conditions prevail for one hour). Note the very fast convergence of the produced density estimates, starting from remote initial values.

III. TRAFFIC ESTIMATION FOR UNMEASURED TOTAL FLOW AT ON-RAMPS AND OFF-RAMPS

In the case that the total flow at some on-ramps or off-ramps is not directly measured, we treat these flows as unmeasured states to be estimated by a Kalman filter. Hence, we augment the state (4) as

$$\bar{x} = (\rho_1, \ldots, \rho_N, \theta_1, \ldots, \theta_{l_r+l_s})^T, \tag{18}$$

where l_r and l_s are the number of unmeasured flows at on-ramps and off-ramps, respectively, and $\theta_i = \left\{ \begin{array}{ll} \frac{T}{\Delta_i} r_{n_i}, & \text{if } n_i \in L_r \\ \frac{T}{\Delta_i} s_{n_i}, & \text{if } n_i \in L_s \end{array} \right\}$, for all $i = 1, \ldots, l_r + l_s$, with $L_r = \{n_1, \ldots, n_{l_r}\}$ and $L_s = \{n_{l_r+1}, \ldots, n_{l_r+l_s}\}$, being the collection of segments, denoted by n_i, which have an on-ramp and an off-ramp, respectively, whose flows are not directly measured. Assuming that at a segment i there can

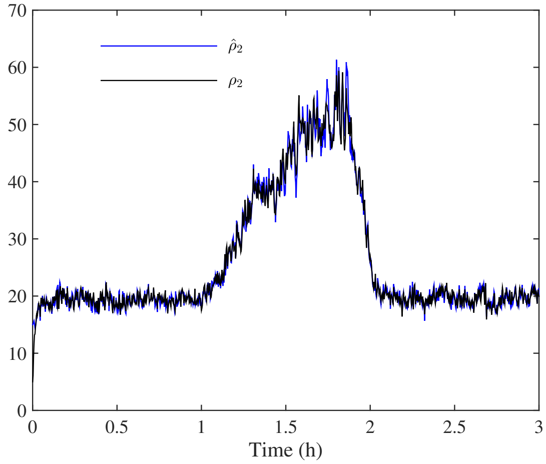

Fig. 4. The total density of vehicles ρ_2 $\left(\text{in } \frac{\text{veh}}{\text{km}}\right)$ at the second segment of the highway (black line) as it is produced by the METANET model (1), (2), (17) with parameters given in Table I and additive process noise given in Table II, and its estimate $\hat{\rho}_2$ (blue line) as it is produced by the Kalman filter (12)–(16) and (7)–(10) with parameters given in Table III.

be either only one on-ramp or only one off-ramp (which is typically the case on a highway) and that the unmeasured ramp flows are constant (or, effectively, slowly varying), the unmeasured ramp flow dynamics may be reflected by a random walk, i.e., $\theta_i(k+1) = \theta_i(k) + \xi_i^\theta(k)$, where ξ_i^θ is zero-mean white Gaussian noise. Thus, the deterministic part of the total density dynamics (1) and of θ_i are

$$\bar{x}(k+1) = \bar{A}(k)\bar{x}(k) + \bar{B}\bar{u}(k), \tag{19}$$

where

$$\bar{A}(k) = \left\{ \begin{array}{ll} \bar{a}_{ij} = \frac{T}{\Delta_i} v_{i-1}(k), & \text{if } i-j=1 \\ & \text{and } i \geq 2 \\ \bar{a}_{ij} = 1 - \frac{T}{\Delta_i} v_i(k), & \text{if } i=j \\ \bar{a}_{n_i j} = 1, & \text{if } n_i \in L_r \\ & \text{and } j = N+i \\ \bar{a}_{n_i j} = -1, & \text{if } n_i \in L_s \\ & \text{and } j = N+i \\ \bar{a}_{ij} = 1, & \text{if } N < i \leq N_1 \\ & \text{and } j = i \\ \bar{a}_{ij} = 0, & \text{otherwise} \end{array} \right\} \tag{20}$$

$$\bar{B} = \left[\begin{array}{ll} \bar{b}_{ij} = \frac{T}{\Delta_i}, & \text{if } i=1 \text{ and } j=1 \\ \bar{b}_{m_i j} = \frac{T}{\Delta_{m_i}}, & \text{if } m_i \notin \bar{L},\ 1 \leq m_i \leq N, \\ & 1 \leq i \leq N_2, \text{ and } j = i+1 \\ \bar{b}_{ij} = 0, & \text{otherwise} \end{array} \right] \tag{21}$$

$$\bar{u}(k) = \left[\begin{array}{ll} \bar{u}_i = q_0(k), & \text{if } i=1 \\ \bar{u}_{i+1} = r_{m_i} - s_{m_i}, & \text{if } m_i \notin \bar{L} \end{array} \right], \tag{22}$$

with $\bar{L} = L_r \cup L_s$, $N_1 = N + l_r + l_s$, $N_2 = N - l_r - l_s$, $\bar{A} \in \mathbb{R}^{N_1 \times N_1}$, $\bar{B} \in \mathbb{R}^{N_1 \times (N_2+1)}$. The measured outputs associated with system (19)–(22) are the density (or, equivalently, the flow) at the exit of the considered highway stretch and at a highway segment between every two consecutive ramps whose flows are not measured. Therefore,

$$\bar{y}(k) = \bar{C}\bar{x}(k), \tag{23}$$

TABLE IV
PARAMETERS OF THE KALMAN FILTER EMPLOYED IN SECTION III.

$\bar{Q}$	$\bar{R}$	$\bar{\mu}$	$\bar{H}$
$I_{(N+2)\times(N+2)}$	$100 I_{2\times 2}$	$(2,\ldots,2)^T$	$I_{(N+2)\times(N+2)}$

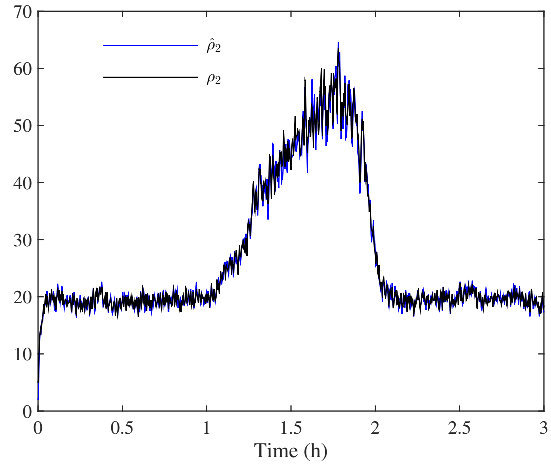

Fig. 5. The total density of vehicles ρ_2 $\left(\text{in } \frac{\text{veh}}{\text{km}}\right)$ at the second segment of the highway (black line) and its estimate $\hat{\rho}_2$ (blue line) as it is produced by the Kalman filter with parameters given in Table IV.

where $\bar{C} \in \mathbb{R}^{(l_r+l_s)\times(N+l_r+l_s)}$ is defined as

$$\bar{C} = \left[\begin{array}{ll} \bar{c}_{ij} = 1, & \text{for all } i = 1, \ldots, l_r + l_s - 1 \\ & \text{and some } n_i^* \leq j \leq n_{i+1}^* - 1 \\ \bar{c}_{ij} = 1, & \text{if } i = l_r + l_s \text{ and } j = N \\ \bar{c}_{ij} = 0, & \text{otherwise} \end{array} \right], \tag{24}$$

where $\bar{L}^* = \left\{ n_1^*, n_2^*, \ldots, n_{l_r+l_s}^* \right\}$ is the set $\bar{L}$ ordered by $<$.

We employ the Kalman filter (12)–(16) with parameters given in Table IV (in particular, the $\bar{q}_{N+iN+i}$ elements of $\bar{Q}$ represent the filter's anticipation for the covariance of ξ_i^θ), for the estimation of the state $\bar{x}$, defined in (18), of system (19)–(24). We assess the filter's performance, employing the same scenario with the one considered in Section II-D, in the case in which the total flow at on-ramp 6 and off-ramp 8 are not measured. One additional mainstream total flow measurement is available from a fixed detector that is placed at the exit of the seventh segment. We show in Fig. 5 the estimation of the density in segment 2. In Fig 6 we show the estimation of the total flow at on-ramp 6 and off-ramp 8.

IV. Conclusions

A subject of our ongoing research is the performance comparison between the estimation scheme developed in this paper and the alternative estimation algorithm that we recently developed [4], which is based on the estimation of the percentage of connected vehicles, with respect to the total number of vehicles, using a much more detailed microscopic simulation platform, thus considering a more realistic simulation of all involved real-time measurements.

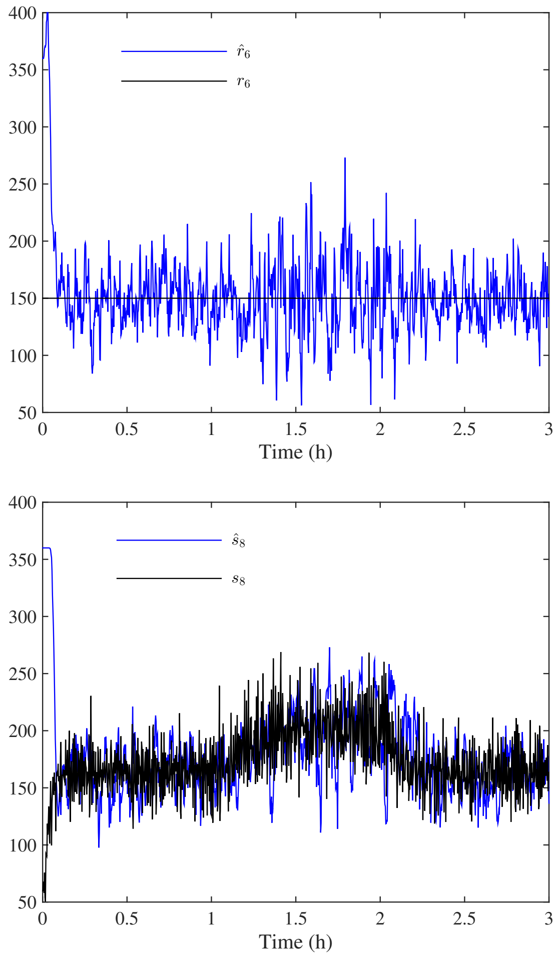

Fig. 6. Top: The total flow of vehicles r_6 $\left(\text{in } \frac{\text{veh}}{\text{h}}\right)$ at on-ramp 6 (black line) and its estimate $\hat{r}_6$ (blue line) as it is produced by the Kalman filter with parameters given in Table IV. Bottom: The total flow of vehicles s_8 $\left(\text{in } \frac{\text{veh}}{\text{h}}\right)$ at off-ramp 8 (black line) and its estimate $\hat{s}_8$ (blue line) as it is produced by the Kalman filter with parameters given in Table IV.

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