Π Ο Λ Υ Τ Ε Χ Ν Ε Ι Ο Κ Ρ Η Τ Η Σ

ΣΧΟΛΗ ΜΗΧΑΝΙΚΩΝ ΠΑΡΑΓΩΓΗΣ ΚΑΙ ΔΙΟΙΚΗΣΗΣ

ΠΡΟΓΡΑΜΜΑ ΔΙΔΑΚΤΟΡΙΚΩΝ ΣΠΟΥΔΩΝ

ΔΙΔΑΚΤΟΡΙΚΗ ΔΙΑΤΡΙΒΗ

Mathematical Modelling of Cyber-Attacks and Proactive Defenses

Διατριβή που υπεβλήθη για τη μερική ικανοποίηση των απαιτήσεων για την απόκτηση διδακτορικού διπλώματος

Υπό

ΥΔ: Αργύριος Αλεξόπουλος

ΧΑΝΙΑ, ΙΟΥΝΙΟΣ 2022

Page intentionally left blank

Page intentionally left blank

ΔΙΔΑΚΤΟΡΙΚΗ ΔΙΑΤΡΙΒΗ

MATHEMATICAL MODELLING OF CYBER- ATTACKS AND PROACTIVE DEFENSES

Όνομα ΥΔ: ΑΡΓΥΡΙΟΣ ΑΛΕΞΟΠΟΥΛΟΣ

ΕΠΤΑΜΕΛΗΣ ΕΞΕΤΑΣΤΙΚΗ ΕΠΙΤΡΟΠΗ

Καθηγητής Ματσατσίνης Νικόλαος (Επιβλέπων, ΜΠΔ)	«Υπογραφή»
Καθηγητής Δάρρας Νικόλαος (Μέλος Τριμελούς, ΣΣΕ)	«Υπογραφή»
Αναπληρωτής Καθηγητής Νικόλαος Παπαδάκης (Μέλος Τριμελούς, ΣΣΕ)	«Υπογραφή»
Αναπληρωτής Καθηγητής Στέλιος Τσαφαράκης (ΜΠΔ)	«Υπογραφή»
Αναπληρωτής Καθηγητής Κωνσταντίνος Καραματσούκης (ΣΣΕ)	«Υπογραφή»
Καθηγητής Απόστολος Μπουρνέτας (Τμήμα Μαθηματικών Ε.Κ.Π.Α.)	«Υπογραφή»
Επίκουρος Καθηγήτρια Ευαγγελία Σ. Αθανασιάδου - Κόττα (Τμήμα Μαθηματικών Ε.Κ.Π.Α.)	«Υπογραφή»

Ημερομηνία εξέτασης ΗΗ/ΜΜ/ΕΕΕΕ

ABSTRACT

The main purpose of this dissertation is to document a holistic modelling background and set up a corresponding mathematical theory in order to provide a rigorous description of cyber-attacks and cyber-security. Proactiveness of cyber-security is the foremost and paramount concern of the current research approach. The starting point is to determine the critical assets of cyberspace, define them consistently and elaborate the attack vectors that may affect them. Concepts as node constituent, valuations and vulnerabilities of parts of a node constituent are cornerstones throughout the dissertation. Based on fundamental concepts, one may be led to consider the concept of node supervision and subsequently to give the definition of cyber-effects and from this the definition of cyber-interaction.

We describe the germ of cyber-attack that can be viewed as a family of cyber-interactions with coherence properties and depending strongly on subjective purposes, information and/or estimates on the valuations and the vulnerabilities of parts of the involved nodes. In general, the germs of cyber-attacks can be distinguished in three types: the germs of correlated cyber-attacks, the germs of absolute cyber-attacks and the germs of partial cyber-attacks. This approach provides immediate possibility of rigorous determination of the concepts of proactive cyber defence and proactive cyber protection.

Enumerating and describing a non-exhaustive list of attack vectors using the approach of the dissertation, we propose adequate proactive mitigation measures. We then try to elaborate a holistic mathematical approach to a rigorous description of Advanced Persistent Threat (APT) actors' modus operandi through various scenarios and Cyber Kill Chain stages. APT focused approach is tried due to competency, high intention and capabilities of these actors, likely using attack vectors at the threshold of defensive ecosystems. Relevant elements of Cyber-Attacks conducted by APT actors presented and proposals of some techniques (via 5 scenarios) of tracking the modus operandi of these sophisticated and non-linear cyber actors. Threat hunting techniques for these competent and highly sophisticated actors are also analysed using Domain Name Systems (DNS) approach.

Key Words: Cyber security, attack vectors, cyber defence, APT actors

Page intentionally left blank

I owe my greatest gratitude to my family and to God. To my family for supporting me throughout the whole process that took much longer that it should and to the God Who helped me to be scared of my dreams[1] *and see always positively the advantages and dis-advantages of this long journey.*

[1] If dreams do not scare you are not big enough…

Page intentionally left blank

ΛΙΣΤΑ ΔΗΜΟΣΙΕΥΣΕΩΝ

- Daras, N.J and Alexopoulos, A. (2017). Mathematical description of cyber-attacks and proactive defenses, Journal of Applied Mathematics & Bioinformatics, Vol.7, no.1, pp. 71-142. Scienpress Ltd, 2017.
- Daras, N.J and Alexopoulos, A. (2017). Modelling Cyber-Security, Journal of Applied Mathematics & Bioinformatics, Vol. 7, no.1, 2017, pp. 71-142. Scienpress Ltd, 2017.
- Alexopoulos, A. and Daras, N. (2018). Mathematical Study of Various Types of Cyber-Attacks and Protection, Journal of Computations & Modelling, Vol.8, no.2, 2018 pp. 1-61. Scienpress Ltd, 2018.
- Alexopoulos, A. and Daras, N. (2020). Mathematical Study of Advanced Persistent Threat (APT) Hunting Techniques, Journal of Computations & Modelling, Vol. 10, no. 2, pp. 1-24. Scientific Press Ltd, 2020.
- Alexopoulos A. (2021). Entropy-Based Evaluation of DNS Activity for Threat Hunting, Journal of Applied Mathematics and Bioinformatics, Vol. 11, No. 1, 2021, 25-31

Page intentionally left blank

ΠΕΡΙΕΧΟΜΕΝΑ

Page intentionally left blank

LIST OF FIGURES

Page intentionally left blank

LIST OF TABLES

Page intentionally left blank

1. Introduction

It is generally weird that despite the fact that increasing efforts and resources are dedicated on cyber security without at the end to have proportional results on defenses. It seems that the lack of consistent mathematical description and orientation of cyberspace is one of the main reasons for this situation.

Therefore, the main innovation of this research dissertation is the questions that answer and mainly triggered the foundation of the research. The initiative for this holistic approach derives from the following questions: Why despite the fact that nations, organizations and entities keep on spending more and more money and resources on Cyber Security, exploitations and critical compromises are proportionally increase? Why lessons identified and lessons learnt from recent decade severe Cyber-attacks did not ameliorate the situation and on the contrary raised the consequences. Why some entities seem to “get” security differently than others and for what reason is this ambiguity? Why is it that, several decades into the digital revolution, some entities still deliver digital products with serious vulnerabilities and inconsistencies in them that leak sensitive data or act as a conduit to unauthorized system access. Why do the weakest part of security chain (humans) keeps on engaging in risky behavior despite the strict Policies, Frameworks, Directions and Guidelines?

The unique answer that this dissertation dealt with in a very innovative way is that all these facts are due to the lack of comprehensive mathematical structure of Cyberspace. Cyberspace has been and is being built at an ad-hoc manner using means and concepts not strictly defined. This definition that is missing from the Cyber researchers of last decade as well as the complex description of interrelations and interdependencies of all Cyberspace components in a strict mathematical foundation is the main innovative contribution of this dissertation.

This dissertation proposes holistically and comprehensively a mathematical approach that gives a consistent description of all key stages in Cyberspace answering all above questions. Building consistently the paramount basis of Cyberspace that are nodes and parts of nodes, valuations and vulnerabilities of parts of a node constituent the whole mathematical structure. This consistent basis gives the yeast to describe almost exhaustively the Cyber-attack vectors and the mitigation measures that should be applied. This unique approach provides immediate possibility of rigorous determination of the concepts of proactiveness in cyber defense and protection. This mathematical approach minimizes the ambiguity that existed and has not been solved by the recent researches. Nonetheless, the theoretical framework that has been built through the dissertation has also been applied in real assets with great success. A stern threat hunting solutions through some scenarios that were built give the applicability of the dissertation.

In many modern scientific studies, quantifying assumptions, data and variables can contribute to the accurate description of the phenomena through appropriate mathematical models. So, in many disciplines, the analysts resort to a mathematical foundation of the concepts, in order to create a solid base for the theoretical formulation and solving all relevant problems. As classic examples of such an integrated mathematization, we can mention Mechanics, Physics, Biology, Earth Science, Meteorology, Medicine, Statistics and Operations Research. In recent years, an effort has initiated to mathematical modeling of the social sciences, such as Economics ([3-5, 14, 15, 22 and 24], Psychology (see, for instance, [*6,* 18 and 19]), Sociology (see, indicatively, [7]), Political Science (see, for instance, [17 and 32]) and Geopolitics ([12-13]).

In this direction, there have been numerous significant contributions on the mathematical modeling of several branches of Theoretical Engineering disciplines, such as Theoretical Computer Science, Network Security, Electronics, and Artificial Intelligence etc. Especially, in the case of cyber-security, we may mention several descriptive papers ([21]) or papers containing several partial research results. All these scientific approaches emphasize mainly on some of stochastic modeling applications, leaving open the question of introducing a full mathematical theory of cyber-security. See, for instance, the papers [23, 27, 29-31]. One can also consult the books [1 and 20] and the references therein. These two books provide in-depth coverage of the mathematical prerequisites and assemble a complete presentation of how computer networks function. The interested reader may also consult the chapter [28] and the references therein and/or the report of President's Information Technology Advisory Committee ([25]) which explicitly states that "*we urgently need to expand our focus on short-term patching to also include longer-term development of new methods for designing and engineering secure systems. Addressing cyber security for the longer term requires a vigorous ongoing program of fundamental research to explore the science and develop the technologies necessary to design security into computing and networking systems and software* *from the ground up. Fundamental research is characterized by its potential for broad, rather than specific, application and includes farsighted, high-payoff research that provides the basis for technological progress*". Indeed, starting from this consideration, Daniel M. Dunlavy, Bruce Hendrickson, and Tamara G. Kolda gave three challenge areas that are, in their opinion, the major mathematical challenges in cyber security ([16]).

Indicative of the great interest shown for the mathematization of cyber-security is the regular organization of international conferences of major interest. Examples include the two Workshops "*Mathematics of Data Analysis in Cyber-Security"* (https://icerm.brown.edu/topical_workshops/tw14-8-mdac/) and "*Mathematics of Lattices and Cyber Security*" (https://icerm.brown.edu/topical_workshops/tw15-7-mlc/ ; also in https://sinews.siam.org/DetailsPage/tabid/607/ArticleID/397/ICERM-Workshop-Mathematics-of-Lattices-and-Cybersecurity.aspx) held in Brown University, at *October 22-24, 2014 and April 21-24, 2015, respectively*. The purpose of first workshop was to bring together mathematical scientists and cyber- security practitioners with expertise in several main areas, including especially high dimensional data analysis and cryptography, to establish a road map for bringing more mathematicians into the field of cyber-security. The goal of the second workshop was on the one hand to stimulate activity between different groups interested in lattice problems, such as mathematicians, computer scientists, and experts in cyber-security, and, on the other hand, to give recent results on densest lattice packings, the geometry of lattice moduli space and its connections with automorphic forms and algebraic number theory, cryptographic applications of lattices, and the state of the art of lattice reduction in high dimensions.

However, many authors do not fail to highlight the importance of creating a ***whole*** mathematical theory of cyber-security. For instance, one can mention the abstract [26] in a workshop sponsored by the Department of Energy (DOE) Office of Advanced Scientific Computing, Applied Mathematics Research Program, where Dwayne Ramsey of Lawrence Berkeley National Laboratory found that "*significant fundamental mathematical research is needed to characterize the network in new meaningful ways and subsequently assess risk for the DOE cyber infrastructure in order to make informed decisions with regard to cyber security policy*". In the same spirit, Wendelberger, Griffin, Wilder, Yu Jiao and Kolda made a remarkable comment on the Current Landscape and Need for Fundamental Research. In this comment, it was pointed out that "*cyber-security, as currently practiced, is a mixed bag of electronic patches and reactionary physical and administrative controls aimed at fixing the crisis of the day. …. As the cyber threat continues to grow, it becomes increasingly clear that the Department of Energy (DOE) must embark on a scientific process of inquiry, investigation, and sound decision-making. Rather than waiting to discover a cyber-attack (perhaps days, weeks, or months after it has happened), we need to implement a science-based approach to cyber-security with a rigorous technical foundation. Here, we propose a mathematical research that will pave the way for the interdisciplinary advances needed to thwart the growing cyber threat and transform the DOE approach for protecting electronic resources"* ([33]). Finally, Juan Meza, Scott Campbell and David Bailey noted that "*the role of mathematics in a complex system such as the Internet has yet to be deeply explored. In this paper, we summarize some of the important and pressing problems in cyber security from the viewpoint of open science environments. We start by posing the question \What fundamental problems exist within cyber security research that can be helped by advanced mathematics and statistics?" Our first and most important assumption is that access to real-world data is necessary to understand large and complex systems like the Internet. Our second assumption is that many proposed cyber security solutions could critically damage both the openness and the productivity of scientific research. After examining a range of cyber security problems, we come to the conclusion that the field of cyber security poses a rich set of new and exciting research opportunities for the mathematical and statistical sciences*" ([23]).

Although these presentations are innovative and promising, it seems that they lack a holistic view of the cyberspace ecosystem. Moreover, there is no predictability of cyber-attacks, nor any opportunity to have given a strict definition of defensive protection so that we can look for an optimal design and organization of cyber defense. As a consequence, thereof, one cannot build a solid foundation for a complete theory containing assumptions, definitions, theorems and conclusions. But this prevents the researchers and planners to understand deeper behaviours, and requires limiting ourselves solely to practical techniques.

The aim of the present dissertation is to document a holistic modeling background and set up a corresponding mathematical theory in order to provide a rigorous description of cyber-attacks and cyber-security. The text that follows comes as a follow-up of the article [9] in which it has been given a mathematical definition of cyberspace.

2. General Assumptions and Basic Notations

Having already mentioned in [9] an adequate supportive theoretical background for cyberspace modeling, we can proceed to the consideration of the concepts of cyber-attack and cyber-defense. In order to rigorously define these two concepts, we will adopt the following approach. At any moment t, a node $V = V_{(x_1,x_2,x_3,t,)}$ in location (x_1, x_2, x_3) of the cyber-domain $\left(|ob(W_e)|, d_{W_e}\right)$ is composed of cyber constituents (or cyber characteristics) consisting in devices $dev_j^{(V)}$ (:sensors, regulators of information flow, etc) and resource elements $res_k^{(V)}$ (:services, data, messages etc), the number of which depend potentially from the three geographical coordinates x_1, x_2, x_3 and the time t. Here, the order of any used quote of devices $dev_1^{(V)}$, $dev_2^{(V)}$,… and the order of any used quote of resource elements $res_1^{(V)}$, $res_2^{(V)}$,… are assumed to be given, pre-assigned and well defined. For instance, one can order the devices $dev_1^{(V)}$, $dev_2^{(V)}$,… as well as the resource elements $res_1^{(V)}$, $res_2^{(V)}$,… alphabetically.

Assumption 1 We will assume uninterruptedly that:

- the potential number of all possible devices of V is equal to $\mathcal{M}_V \gg 0$, while
- the number of V's available devices is only $m_V = m_V(t)$, with $m_V < \mathcal{M}_V$.

Similarly, we will assume that

- the potential quantity (or number) of all possible resource elements of V is equal to $\mathcal{L}_V \gg 0$, while
- the quantity (or number) of V's available resource elements is only $\ell_V = \ell_V(t)$, in the sense that $\ell_V < \mathcal{L}_V$.

3. Mathematical definition of cyberspace

As detailed described in [9], a multilayered weighted (finite or infinite) graph $\boldsymbol{\mathcal{X}}$ with $\boldsymbol{N}$ interconnected layers is said to be an $\boldsymbol{N}-$ **cyber-archetype germ**. An $\boldsymbol{e}-$**manifestation** gives a geographical qualifier at each node of $\boldsymbol{\mathcal{X}}$. It is an embedding of $\boldsymbol{\mathcal{X}}$ into a Cartesian product of $\boldsymbol{N}$ complex projective spaces $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}} \equiv \mathbf{P}(\mathbb{C}^{\boldsymbol{n_k}+\mathbf{1}})$, such that all nodes of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}-$layer, called $\boldsymbol{e}-$**node manifestations**, are illustrated at weighted points of the set $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$ and all directed edges(flows) of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}-$layer, called $\boldsymbol{e}-$**edge manifestations**, are given by simple weighted edges, i.e. by weighted homeomorphic images of the closed interval$[\mathbf{0},\mathbf{1}]$ on $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$, so that, for any $\boldsymbol{k}=\mathbf{1},\mathbf{2},\dots,\boldsymbol{N}$,

- the end points of each $e-$edge manifestation on $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$ must be images of end points of a corresponding original directed edge of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}-$layer
- there should not be any $e-$edge manifestation on $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$ derived from directed $e-$edge of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}-$layer into which belong points of $e-$edge manifestations that are defined by other nodes of $\boldsymbol{\mathcal{X}}$ in the same layer.

The set $\boldsymbol{\mathcal{S}_e}=\boldsymbol{\mathcal{S}_e}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ of $e-$manifestations of $\boldsymbol{N}-$cyber archetype germs is the $\boldsymbol{e}-$ **superclass** in $\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}}$.An$\boldsymbol{e}$—**graph category** $\boldsymbol{\mathcal{E}_{\mathcal{C}}}=\boldsymbol{\mathcal{E}_{\mathcal{C}}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ is a category consisting of the class $\boldsymbol{ob}(\boldsymbol{\mathcal{E}_{\mathcal{C}}})$, whose elements, called $\boldsymbol{e}$—**objects**,are the pairs $\boldsymbol{\mathcal{X}}=(\boldsymbol{V},\boldsymbol{E})\in\boldsymbol{\mathcal{S}_e}$, endowed with a class $\boldsymbol{hom}(\boldsymbol{\mathcal{E}_{\mathcal{C}}})$ of $\boldsymbol{e}$—**morphisms** on $\boldsymbol{ob}(\boldsymbol{\mathcal{E}_{\mathcal{C}}})$ and an associative binary operation $\circ$ with identity.

Generalizing, one may consider additionally the following other four basic $\boldsymbol{e}$—**categories**: The $\boldsymbol{e}-$**set category** $\boldsymbol{e_{Set}}=\boldsymbol{e_{Set}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ where the objects are subsets of $\boldsymbol{\mathcal{E}_{\mathcal{C}}}$, the $\boldsymbol{e}-$**homomorphism category** $\boldsymbol{e_{Hom}}=\boldsymbol{e_{Hom}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ where the objects are sets of homomorphisms between subsets of $\boldsymbol{e_{Set}}$, the $\boldsymbol{e}-$**group category** $\boldsymbol{e_{Grp}}=\boldsymbol{e_{Gpr}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ where the objects are the groups of $\boldsymbol{\mathcal{E}_{\mathcal{C}}}$ and the $\boldsymbol{e}-$**topological category** $\boldsymbol{e_{Top}}=\boldsymbol{e_{Top}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ where the objects are topological subcategories of $\boldsymbol{\mathcal{E}_{\mathcal{C}}}$. For reasons of homogenization of symbolism, we will adopt the following common notation $\boldsymbol{\mathcal{W}_e}=\{\boldsymbol{\mathcal{E}_{\mathcal{C}}},\boldsymbol{e_{Set}},\boldsymbol{e_{Hom}},\boldsymbol{e_{Grp}},\boldsymbol{e_{Top}}\}$. The objects of each $e-$category $\boldsymbol{W_e}=\boldsymbol{W_e}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\dots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})\in\boldsymbol{\mathcal{W}_e}$ will be called $\boldsymbol{e}-$**manifestations**.

An easy **algebraic** structure in the (infinite) set of all these e—manifestations

$(\boldsymbol{V},\boldsymbol{E})$ and simultaneously, a compatible **topological** structure to allow for a detailed analytic study of $\boldsymbol{S_e}$ is given in [9]. Further, [9] investigates the possibility of allocating suitable vector weights to all the objects and morphisms of any e –category $\boldsymbol{W_e} \in \boldsymbol{\mathcal{W}_e} = \{\boldsymbol{\mathcal{E}_{\mathcal{C}}}, \boldsymbol{e_{Set}}, \boldsymbol{e_{Grp}}, \boldsymbol{e_{Top}}\}$.

Towards this end, we consider two types of vector weights that can be attached to any object and/or morphism of such an e –category: the maximum weight and the square weight. Any such weight will be a point in the positive quadrant of the plane. Taking this into account, any e –category $\boldsymbol{W_e} \in \boldsymbol{\mathcal{W}_e} = \{\boldsymbol{\mathcal{E}_{\mathcal{C}}}, \boldsymbol{e_{Set}}, \boldsymbol{e_{Hom}}, \boldsymbol{e_{Gpr}}, \boldsymbol{e_{Top}}\}$ can be viewed as an **infinite** e *–graph*$(\mathbb{V},\mathbb{E})$ with *vector weights*, in such a way that the e –nodes in $\mathbb{V}$ are the e –objects $\boldsymbol{X} \in \boldsymbol{ob}(\boldsymbol{W_e})$, while the e –edges in $\mathbb{E}$ are the e –morphisms $\boldsymbol{h} \in \boldsymbol{hom}(\boldsymbol{W_e})$. For such an e –graph $\mathfrak{G}_{\boldsymbol{W_e}}$ corresponding to an e – category $\boldsymbol{W_e} \in \boldsymbol{\mathcal{W}_e}$, the vector weight of the e –node associated to the e –manifestation $\boldsymbol{\mathcal{X}} = (\boldsymbol{V},\boldsymbol{E}) \in \mathbb{V} \equiv \boldsymbol{ob}(\boldsymbol{W_e})$ is equal to a weight of $\boldsymbol{\mathcal{X}}$. Bearing all this in mind, in [9], we introduced a suitable intrinsic metric $\boldsymbol{d}_{\boldsymbol{W_e}}$ in the set $\boldsymbol{ob}(\boldsymbol{W_e})$ of objects of an e –category $\boldsymbol{W_e}$. The most significant benefits coming from such a consideration can be derived from the definitions of *cyber-evolution* and *cyber-domain*. To do this, we first defined the concept of e*—dynamics*, as a mapping of the form $\mathcal{cy}\colon [0,1] \to \left(\boldsymbol{ob}(\boldsymbol{W_e}), \boldsymbol{d}_{\boldsymbol{W_e}}\right)$; its image is an e*—arrangement*. Each point $\mathcal{cy}(\boldsymbol{t}) \in \mathcal{cy}([0,1])$ is an (instantaneous) local e –node manifestation with an interrelated e –edge manifestation. An e –arrangement together with all of its (instantaneous) e—morphisms is an e *–regularization*. The elements of the completion $\overline{\boldsymbol{ob}(\boldsymbol{W_e})}$ of $\boldsymbol{ob}(\boldsymbol{W_e})$ in $\overline{\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}} \times \ldots \times \mathbb{C}\mathbf{P}^{\boldsymbol{n_N}}}$are the *cyber-elements*, while the topological space $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$ is a *cyber-domain*. With this notation, a continuous e –dynamics $\mathcal{cy}\colon [\boldsymbol{0},\boldsymbol{1}] \to \left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$ is said to be a *cyber-evolutionary path* or simply *cyber-evolution* in the cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$. Its image is said to be a *cyber-arrangement*. A cyber-arrangement together with all of its (instantaneous) cyber-morphisms is called a *cyberspace*.

In view of the above concepts, [9] investigates conditions under which an e –regularization may be susceptible of a projective e –limit. It is important to know if a e *–sub*-regularization is projective e –system. Subsequently, we defined and discussed the concept of the *length* in a cyber-domain. For the intrinsic cyber-metric $\boldsymbol{d}_{\boldsymbol{W_e}}$, the distance between two cyber-elements is the length of the "shortest cyber-track" between these cyber-elements. The term shortest cyber-track is defined and is crucial for understanding the concept of *cyber-geodesic*. Although every shortest cyber track on a cyber-length space is a cyber-geodesic, the reverse argument is not valid. In fact, *some cyber-geodesics may fail to be shortest cyber-tracks on large scales*. However, since each cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$ is a compact, complete metric space, and since for any pair of cyber-elements in $\overline{\boldsymbol{ob}(\boldsymbol{W_e})}$ there is a cyber-evolutionary path of finite length joining them, one can easily ascertain the following converse result: any *pair of two cyber-elements in each cyber-domain* $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$ *has a shortest cyber track joining them*. Finally, [3] gives a discussion about the *speed* (: *cyber-speed*) of a cyber-evolution and the *convergence* of a sequence of cyber-evolutions.

4. Valuations of Parts of a Node Constituent

Let us now turn to the definition of valuation measures, as well as the definition of the vulnerability measures, of an available constituent $\mathcal{A}^{(V)}$ in a cyber node V:

$$\mathcal{A} = \begin{cases} dev, if\ the\ constituent\ is\ a\ device\ , \\ res, if\ the\ constituent\ is\ a\ resource\ element. \end{cases}$$

Obviously, $\mathcal{A}^{(V)}$ may be viewed as a nonempty collection of a number of elements.

Lemma 1 *One can make as much finite σ –algebras as partitions on $\mathcal{A}^{(V)}$. Recall that a partition of a set Σ is defined as a set of nonempty, pairwise disjoint subsets of Σ whose union is Σ.*

Proof. Let $\mathcal{G}$ be the collection of all the algebras over $\mathcal{A}^{(V)}$. Let also Π be the set of all the partitions of $\mathcal{A}^{(V)}$. There is a bijective correspondence between $\mathcal{G}$ and Π. Indeed, for a partition $\mathcal{P} \in \Pi$, consider the algebra $\mathfrak{U}_{\mathcal{P}}$ generated by $\{A_1, \dots, A_k\}$, the elements of $\mathcal{P}$. Then $\mathfrak{U}_{\mathcal{P}}$ consists of the set $\cup_{j\in J} A_j$, where $J \subset \{1, \dots, k\}$. To see that this correspondence is bijective, given an algebra $\mathfrak{U}$, one can define, for all $x \in \mathcal{A}^{(V)}$, the set $A_x := \cap_{A\in\mathfrak{U}, x\in A} A$ (it is a finite intersection), and that will give a unique partition. Indeed, define the equivalence relation $x \sim y$ if and only if $A_x = A_y$. It gives a partition, and it is the unique one. If $\mathcal{P} = \{S_1, \dots, S_m\}$ works, then $A_x = S_{i(x)}$ for some $i(x)$, and you can check that this partition consists of the equivalence classes of $\sim$. So the problem is to enumerate the number of partitions of the set $\mathcal{A}^{(V)}$.

Definition 1 *Let W, $V \in ob\big(cy(t)\big)$ be two cyber nodes and let $\mathcal{A}^{(V)}$ be an* *available constituent in V. For every partition $\mathcal{P}$ of $\mathcal{A}^{(V)}$, let us consider the corresponding σ −algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of $\mathcal{A}^{(V)}$as well as a monotonic measure μ defined on $\mathfrak{U}_{\mathcal{P}}$. Let also $Cr_1, Cr_2, \dots, Cr_{\mathfrak{N}}$ be $\mathfrak{N} = \mathfrak{N}(\mathcal{A}^{(V)}, \mathcal{P})$ objective quantifiable Criteria for the assessment of the points of $\mathcal{A}^{(V)}$. Denoting by $Cr_j(p) = Cr_j[x_1, x_2, x_3, t\,](p) \in \mathbb{R}$ the value of Cr_j on $p \in \mathcal{A}^{(V)}$ at a point $(x_1, x_2, x_3, t\,) \in \mathbb{R}^3 \times [0,1]$, representing location of V at time t, suppose*

1) *the functions $Cr_j(p)$ are measurable and*
2) *an importance of valuation weight $w_j(p)$ is attributed by the (user(s) of) node W to the Criterion Cr_j on $p \in \mathcal{A}^{(V)}$ at $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ (; of course, if the users of W are indifferent or not at all informed on the situation of part p in V relative to the Criterion Cr_j, then the relevant valuation weight $w_j(p)$ will be 0).*

If $E \in \mathfrak{U}_{\mathcal{P}}$ is a part of $\mathcal{A}^{(V)}$and $\mathfrak{n} \leq \mathfrak{N}$, then a relative valuation of E from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ is any vector

$$S_W(E) = S_W[x_1, x_2, x_3, t\,](E) := \left(s_{W,1}(E), s_{W,2}(E), \dots, s_{W,\mathfrak{n}}(E)\right) \in \mathbb{R}^{\mathfrak{n}}$$

where

$$s_{W,j}(E) = s_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t\,](E) := \int_E \; Cr_j(p) w_j(p)\, d\mu(p).$$

Each one indefinite integral

$$s_{W,j} = s_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t\,] = \int Cr_j(p) w_j(p)\, d\mu(p)$$

is called a producing valuation component of part E from the viewpoint of the (user(s) of) node W into the constituent $\mathcal{A}^{(V)}$ at $(x_1, x_2, x_3, t\,)$ with respect to the quantifiable Criterion that represents, while the component values $s_{W,j}(E)$ are called component valuations of E from the viewpoint of the (user(s) of) node W into the constituent $\mathcal{A}^{(V)}$ at the spatiotemporal point $(x_1, x_2, x_3, t\,)$. The number $\mathfrak{n}$ is the dimension of the valuation.

For simplicity and without loss of generality, in what follows, we will always assume that the dimension of the valuation is fixed over the set of all cyber nodes and equal to $\mathfrak{n} = \mathfrak{N}$.

Remark 1 It is possible that all of the components $s_{W,k}(E)$ belong to a fixed discrete or finite set in $\mathbb{R}$. In such a case, the valuation is said to be discrete or finite, respectively. It is also possible to consider the extending of component valuations $s_{W,k}(E)$ onto the Alexandroff one-point compactification $\mathbb{RP}^1$ of $\mathbb{R}$, so that

$s_{W,k}(E) > 0$ means "positive valuation in activated part E"
$s_{W,k}(E) = 0$ means "valuation in disabled /non-existent/non-available part E"
$s_{W,k}(E) < 0$ means "negative valuation in activated part E"
$s_{W,k}(E) = \infty$ means "part E takes its extreme (maximal or minimal) valuation".

If no reference is made to node W and there is no risk of confusion, we can omit the notation of the node W into the indices used.

Let us give an example of the particular case where the component valuations belong to a finite set.

Example 1 Given an available constituent $\mathcal{A}^{(V)}$ (:device $dev^{(V)}$ and/or resource element $res^{(V)}$) in a node V, let us consider a partition $\mathcal{P}$ of $\mathcal{A}^{(V)}$. Let us consider the corresponding σ −algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of $\mathcal{A}^{(V)}$. A valuation of a part $E \in \mathfrak{U}_{\mathcal{P}}$ can be parameterized and measured using segmentation in subparts and issues concerning stochastic as well as administrative processes. Specifically, a valuation of E can be broken down to $\mathfrak{n} = \mathfrak{N} = 22$ component (continuous or discrete) valuations on $\mathfrak{U}_{\mathcal{P}}$: $s_j = s_j^{(\mathcal{A}^{(V)})}$ ($j = 1,2,\ldots,22$ and $\mathcal{A} = dev, res$). In fact, taking equal valuation weights $w_j = 1$ and a normalized measure $\mu(E) = 1$, we may consider the following component valuations, many of which can be the parameters for calculating the reliability of the constituent $\mathcal{A}^{(V)}$.

1) Cr_1: "Aging of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation of part E into the constituent $\mathcal{A}^{(V)}$ is $s_1(E)$, so, if, for instance, $s_1(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for recent, $(1/\kappa)$ stands for not recent and 1 for old.
2) Cr_2: "Level of patching of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_2(E)$, so, if, for instance, $s_2(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε

stands for unpatched, $(1/\kappa)$ for not adequately patched and ν for fully patched.

3) Cr_3 "Number of compromises of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_3(E)$, so, if, for instance, $s_3(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low amount, $(1/\kappa)$ for moderate amount and ν for large amount.

4) Cr_4: "Criticality of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_4(E)$, so, if, for instance, $s_4(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for trivial, $(1/\kappa)$ for not so critical and ν for very critical.

5) Cr_5: "Indication of over-load of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_5(E)$, so, if, for instance, $s_5(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a limited low, $(1/2)$ for a moderate load and ν for a big load.

6) Cr_6: "Is part E in the constituent $\mathcal{A}^{(V)}$ of node V of known manufacturer/Brand that can support it uninterruptedly?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_6(E)$, so, if, for instance, $s_6(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little-known manufacturer/Brand, $(1/2)$ for a known manufacturer/Brand and ν for a big manufacturer/Brand.

7) Cr_7: "Has part E in the constituent $\mathcal{A}^{(V)}$ of node V been adequately tested?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_7(E)$, so, if, for instance, $s_7(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$ and $\nu, \kappa > 1$, then ε stands for a bit tested, $(1/\kappa)$ for quite tested and ν for too well tested.

8) Cr_8: "Is part E in the constituent $\mathcal{A}^{(V)}$ of node V in the first line of defense? Or is it protected by another defense component?" The corresponding component

valuation for the part E of $\mathcal{A}^{(V)}$ is $s_8(E)$, so, if, for instance, $s_7(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little protected, $(1/\kappa)$ stands for moderately protected and ν for very well protected.

9) Cr_9: "Degree of complexity of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_8(E)$, so, if, for instance, $s_9(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for non-complex, $(1/2)$ for neutral and ν for complex.

10) Cr_{10}: "Is the part E in the constituent $\mathcal{A}^{(V)}$ of node V adequately monitored?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{10}(E)$, so, if, for instance, $s_{10}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little monitorated, $(1/\kappa)$ for moderately monitorated and ν for very well monitorated.

11) Cr_{11}: "What is the price of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{11}(E)$, so, if, for instance, $s_{11}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low cost, $(1/\kappa)$ for moderate cost and ν for high cost.

12) Cr_{12}: "Failure rate of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{12}(E)$, so, if, for instance, $s_{12}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low failure rate, $(1/\kappa)$ for moderate failure rate and ν for high failure rate.

13) Cr_{13}: "Proximity of part E in the constituent $\mathcal{A}^{(V)}$ of node V to its health tolerance". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{13}(E)$, so, if, for instance, $s_{13}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for too close, $(1/\kappa)$ for not so close and ν for far from health tolerance.

14) Cr_{14}: "MTBF (Mean Time Between Failure) of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{14}(E)$, so, if, for instance, $s_{14}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low MTBF, $(1/\kappa)$ for moderate MTBF and ν for high MTBF.

15) Cr_{15}: "Is the average user of part E in the constituent $\mathcal{A}^{(V)}$ of node V trained?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{15}(E)$, so, if, for instance, $s_{15}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for untrained, $(1/\kappa)$ for not so trained and ν for fully trained.

16) Cr_{16}: "Is any Information Awareness training in place into the part E of constituent $\mathcal{A}^{(V)}$ in node V?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{16}(E)$, so, if, for instance, $s_{16}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low Information Awareness training, $(1/\kappa)$ for moderate Information Awareness training and ν for high Information Awareness training.

17) Cr_{17}: "Are all security functions automated or there is human-in-the-loop process?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{17}(E)$, so, if, for instance, $s_{17}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for few automated safety functions, $(1/\kappa)$ for several automated safety functions and ν for many automated safety functions.

18) Cr_{18}: "Is average user of part E in the constituent $\mathcal{A}^{(V)}$ of node V experienced?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{18}(E)$, so, if, for instance, $s_{18}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for little experience of the average user, $(1/\kappa)$ for moderate experience of the average user and ν for great experience of the

average user.

19) Cr_{19}: “Strictness of security Law and regulations in the wide area of part E in the constituent $\mathcal{A}^{(V)}$ of node V”. The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{19}(E)$, so, if, for instance, $s_{19}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for looseness of regulations and security law in the wide area of node, $(1/\kappa)$ for typical regulations and security law in the wide area of node and ν for strictness of regulations and security law in the wide area of node.

20) Cr_{20}: “Is a detailed security policy in place?” The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{20}(E)$, so, if, for instance, $s_{20}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little detailed security police, $(1/\kappa)$ stands for a sufficiently detailed security police and ν for a very detailed security police.

21) Cr_{21}: “Are there any back up processes?” The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{21}(E)$, so, if, for instance, $s_{21}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for the existence of not so successful back up procedures, $(1/\kappa)$ stands for the existence of quite successful back up procedures and ν for the existence of successful back up procedures.

22) Cr_{22}: “How much risk can the organization accept?” The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{22}(E)$, so, if, for instance, $s_{22}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for no risk, $(1/\kappa)$ stands some risk and ν for full risk acceptance.

Both effectiveness states

$$S_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_1^{(V)}\right)\right), \ldots, S_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)$$

and applicability situations

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)$$

are called cyber node valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1,x_2,x_3,t\,)$. They are also denoted separately by $fr\left(\beta_\kappa^{(W\rightsquigarrow V)}\right)=fr\left(\beta_\kappa^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]$, $\kappa=1,2,\dots,\mathcal{M}_V+\mathcal{L}_V$, or by the vector valuation representation

$$fr\left(\beta^{(W\rightsquigarrow V)}\right)\ =fr\left(\beta^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]:=$$
$$\left(fr\left(\beta_1^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,],\dots,fr\left(\beta_{\mathcal{M}_V+\mathcal{L}_V}^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]\right)^T.$$

If there is no risk of confusion, we will prefer write simply $\beta_\kappa^{(W\rightsquigarrow V)}=\beta_\kappa^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]$, $\kappa=1,2,\dots,\mathcal{M}_V+\mathcal{L}_V$, or use by the joint vector valuation representation

$$\beta^{(W\rightsquigarrow V)}=\beta^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]:=$$
$$\left(\beta_1^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\dots,\beta_{\mathcal{M}_V+\mathcal{L}_V}^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]\right)^T.$$

In the total case, the effectiveness states $S_W[x_1,x_2,x_3,t\,]\left(dev_1^{(V)}\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(dev_{\mathcal{M}_V}^{(V)}\right)$ and applicability situations $S_W[x_1,x_2,x_3,t\,]\left(res_1^{(V)}\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(res_{\mathcal{L}_V}^{(V)}\right)$ are called cyber node valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1,x_2,x_3,t\,)$. As above, they are again denoted separately by

$$\beta_\kappa^{(W\rightsquigarrow V)}=\beta_\kappa^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\ \kappa=1,2,\dots,\mathcal{M}_V+\mathcal{L}_V,$$

or jointly by the vector valuation representation

$$\beta^{(W\rightsquigarrow V)}=\beta^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]:=.$$
$$\left(\beta_1^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\dots,\beta_{\mathcal{M}_V+\mathcal{L}_V}^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]\right)^T.$$

By analogy, both available effectiveness states

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right),\dots,\ S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{m_V}^{(V)}\right)\right)$$

and available applicability situations

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,\ S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\ell_V}^{(V)}\right)\right)$$

are called available cyber node fractional valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1,x_2,x_3,t\,)$. They are denoted separately by

$$fr\left(b_{\kappa}^{(W \rightsquigarrow V)}\right) = fr\left(b_{\kappa}^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,],\ \kappa = 1,2, \dots, m_V + \ell_V,$$

or jointly by the available vector valuation representation

$$fr\left(b^{(W \rightsquigarrow V)}\right) = fr\left(b^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,] := .$$
$$\left(fr\left(b_1^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,], \dots, fr\left(b_{m_V+\ell_V}^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]\right)^T.$$

As before, if there is no risk of confusion, we may adopt the simpler notation

$$b_{\kappa}^{(W \rightsquigarrow V)} = b_{\kappa}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,],\ \kappa = 1,2, \dots, m_V + \ell_V,$$

or use the joint vector valuation representation

$$b^{(W \rightsquigarrow V)} = b^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,] :=$$
$$\left(b_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,], \dots, b_{\mathcal{M}_V+\mathcal{L}_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\right)^T.$$

In particular, in total case, the effectiveness states $S_W[x_1, x_2, x_3, t\,]\left(dev_1^{(V)}\right), \dots, S_W[x_1, x_2, x_3, t\,]\left(dev_{m_V}^{(V)}\right)$ and applicability situations $S_W[x_1, x_2, x_3, t\,]\left(res_1^{(V)}\right), \dots, S_W[x_1, x_2, x_3, t\,]\left(res_{\ell_V}^{(V)}\right)$ are called available cyber node valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1, x_2, x_3, t\,)$. They are also denoted separately by

$$b_{\kappa}^{(W \rightsquigarrow V)} = b_{\kappa}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,],\ \kappa = 1,2, \dots, m_V + \ell_V,$$

or jointly by the available vector valuation representation

$$b^{(W \rightsquigarrow V)} = b^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]:$$
$$\left(b_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,], \dots, b_{m_V+\ell_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\right)^T.$$

In order to be more understandable, let us give a schematic example (Figure 1) only for the indicative case of some of the above definitions in the total case.

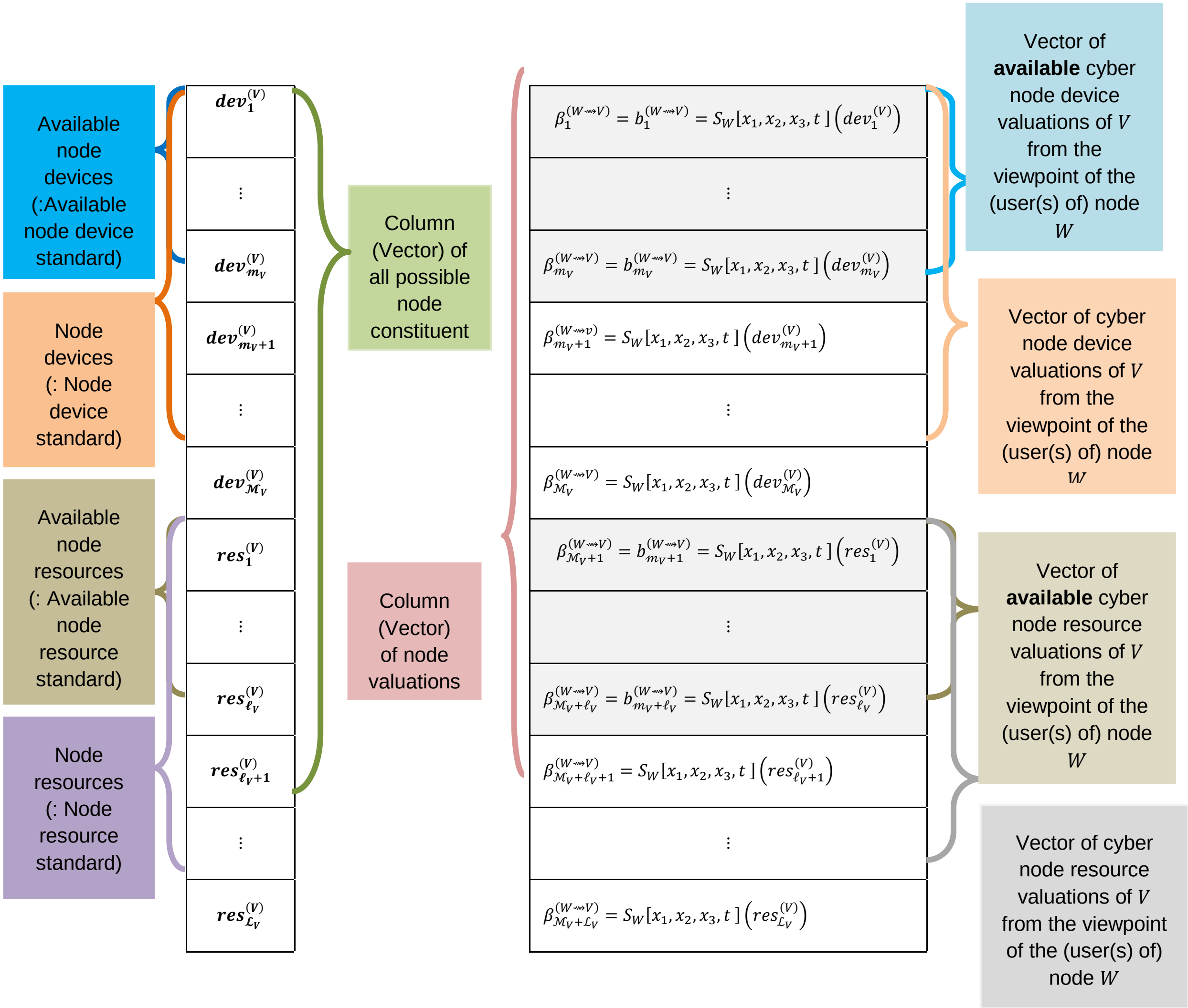

Figure 1 (Schematic Example 1)

5. Vulnerabilities of Parts of a Node Constituent

There is a special category of valuations of particular interest. This category refers to those valuations that are determined in regards to the low degree of "security" of the constituents of the node. The low degree of security is described completely by the concept of vulnerability. Vulnerability, as used in cyber context, is the property of a constituent (device or resource element) in a given state that may be exploited in the relative future. This exploitation at time t may actually lead to a constituent (device or resource element) of any node to be compromised and the valuation of this component to be degraded proportionally.

Definition 2 *Let* $W, V \in ob(cy(t))$ *be two cyber nodes and let* $\mathcal{A}^{(V)}$ *be an available constituent in* V. *For every partition* $\mathcal{P}$ *of* $\mathcal{A}^{(V)}$, *let us consider the* *corresponding* σ *–algebra* $\mathfrak{U}_{\mathcal{P}}$ *of subsets of* $\mathcal{A}^{(V)}$*as well as a monotonic measure* λ *defined on* $\mathfrak{U}_{\mathcal{P}}$*. Let also* $SeCr_1, SeCr_2, \dots, SeCr_{\mathfrak{M}}$ *be* $\mathfrak{M} = \mathfrak{M}(\mathcal{A}^{(V)}, \mathcal{P})$ *objective quantifiable Security Criteria for the security assessment of the points of* $\mathcal{A}^{(V)}$*. Denoting by* $SeCr_j(p) = SeCr_j[x_1, x_2, x_3, t\,](p) \in \mathbb{R}$ *the value of* $SeCr_j$ *on* $p \in \mathcal{A}^{(V)}$ *at a spatiotemporal point* $(x_1, x_2, x_3, t\,) \in \mathbb{R}^3 \times [0,1]$*, representing location of node* V *at time* t*, suppose*

1) *the functions* $SeCr_j(p)$ *are measurable and*
2) *an importance of vulnerability weight* $w_j(p)$ *is attributed by the (user(s) of) node* W *to the Security Criterion* $SeCr_j$ *on* $p \in \mathcal{A}^{(V)}$ *at* $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ *(; of course, if the users of* W *are indifferent or not at all informed on the situation of part* p *in* V *relative to the Criterion* $SeCr_j$*, then* $w_j(p) = 0$*).*

If $E \in \mathfrak{U}_{\mathcal{P}}$ *is a part of* $\mathcal{A}^{(V)}$*and* $\mathfrak{m} \le \mathfrak{M}$*, then a relative vulnerability of* E *from the viewpoint of the (user(s) of) node* W *at* $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ *is any vector*

$$U_W(E) = U_W[x_1, x_2, x_3, t\,](E) := \left(u_{W,1}(E), u_{W,2}(E), \dots, u_{W,\mathfrak{m}}(E)\right) \in \mathbb{R}^m$$

Where:

$$u_{W,j}(E) = u_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t\,](E) := \int_E SeCr_j(p) w_j(p)\, d\lambda(p).$$

Each one indefinite integral

$$u_{W,j} = u_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t\,] = \int SeCr_j(p) w_j(p)\, d\lambda(p)$$

is called a producing vulnerability component of part E *from the viewpoint of the (user(s) of) node* W *into the constituent* $\mathcal{A}^{(V)}$ *at* $(x_1, x_2, x_3, t\,)$ *with respect to the quantifiable Security Criterion that represents, while the component values* $u_{W,j}(E)$ *are called component vulnerabilities of* E *from the viewpoint of the (user(s) of) node* W *into the constituent* $\mathcal{A}^{(V)}$ *at* $(x_1, x_2, x_3, t\,)$*. The number* $\mathfrak{m}$ *is the dimension of the vulnerability.*

For simplicity and without loss of generality, in what follows, we will always assume that the dimension of the vulnerability is fixed over the set of all cyber nodes and equal to $\mathfrak{m} = \mathfrak{M}$.

Remark 2 It is possible that the components $u_{W,j}(E)$ belong to a fixed discrete or finite set in $\mathbb{R}$. In such a case, the vulnerability is said to be discrete or finite, respectively. It is also possible to consider the extending of component vulnerabilities $u_{W,j}(E)$ onto the Alexandroff one-point compactification $\mathbb{RP}^1$ of $\mathbb{R}$, so that

$u_{W,j}(E) > 0$ means "vulnerability in activated part E"
$u_{W,j}(E) = 0$ means "invulnerability in disabled/non-existent/non-available part E"
$u_{W,j}(E) < 0$ means "invulnerability in activated part E"
$u_{W,j}(E) = \infty$ means "extreme vulnerability situation: completely immune part E".

If no reference is made to node W and there is no risk of confusion, we can omit the notation of the node W into the indices used. Let us give an example.

Example 2 Following the notation in the Example 1, and taking equal vulnerability weights $\mathcal{w}_j = 1$ and normalized measure $\lambda(E) = 1$, vulnerability can be broken down to the following 5 parameters.

1) $SeCr_1$: "Level of patching of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_1(E)$ that is the inverse of the valuation $s_2(E)$ in Example 1. In the discrete case, if $s_2(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_1(E) = 1/\varepsilon$ stands great vulnerability for unpatched part E, $u_1(E) = \kappa$ moderate vulnerability for not adequately patched part E and $u_1(E) = 1/\nu$ small vulnerability for fully patched part E.

2) $SeCr_2$: "Number of compromises of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_2(E)$ that is the inverse of the valuation $s_3(E)$ in Example 1. Note that in the discrete case, if $s_2(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_2(E) = 1/\varepsilon$ stands great vulnerability for low amount of compromises of part E, $u_2(E) = \kappa$ moderate vulnerability for moderate amount of compromises of part E and $u_2(E) = 1/\nu$ small vulnerability for large amount of compromises of part E.

3) $SeCr_3$: "Is part E in the constituent $\mathcal{A}^{(V)}$ of node V in the first line of defense? Or is it protected by another defense component? " The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_3(E)$ that is the inverse of the valuation $s_8(E)$ in Example I.1. In the discrete case, if $s_8(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_3(E) = 1/\varepsilon$ stands great vulnerability for a little protected part E, $u_3(E) = \kappa$ moderate vulnerability for a moderately protected part E, while $u_3(E) = 1/\nu$ small vulnerability for a very well protected part E.
4) $SeCr_4$: "Are all security functions automated or there is human-in-the-loop process?" The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_4(E)$ that is the inverse of the valuation $s_{17}(E)$ in Example 1. In the discrete case, if $s_{17}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_4(E) = 1/\varepsilon$ stands great vulnerability for few automated safety functions, $u_4(E) = \kappa$ moderate vulnerability for several automated safety functions and $u_4(E) = 1/\nu$ small vulnerability for many automated safety functions.
5) $SeCr_5$: "Is any security police (cryptographic process) in place?" The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_5(E)$ that is the inverse of the valuation $s_{20}(E)$ in Example 1. In the discrete case, if $s_{20}(E) \in \{\varepsilon, (1/\kappa), \nu\}$ with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_5(E) = 1/\varepsilon$ stands great vulnerability for a little detailed security police, $u_5(E) = \kappa$ moderate vulnerability for a sufficiently detailed security police and $u_5(E) = 1/\nu$ small vulnerability for a very detailed security police.

Remark 3 A basic and reasonable question arises immediately and may be constitute the central subject of discussion in subsequent additional scientific studies. The question relates to the objectivity and/or subjectivity in the choice of the numerical characteristics (:objective quantifiable Criteria) of a device and a resource element: given that it is very doubtful whether the considered set of numerical characteristics could be considered as exhaustive, one wonders if the above approach is ultimately reliable. Equivalently, *if a scientific entity considers a set of numerical characteristics and if another scientific entity considers a different set of numerical characteristics, then how much the two approaches will differ or diverge?* Certainly, the issue of rational choice of specifications, characteristics and criteria is more general. An initial attempt to set up an appropriate theory has begun in [13] for the choice of characteristics and associated numerical values in a systemic geopolitical modeling. However, the question is much general and as such will be considered at a forthcoming article. At present, for the purposes of the present work, we will make the following technical and often realistic assumption.

Both effectiveness states

$$U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)$$

and applicability situations

$$U_W[x_1,x_2,x_3,t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)$$

are called cyber node fractional vulnerabilities of V from the viewpoint of the (user(s) of) node W, at the spatiotemporal point $(x_1,x_2,x_3,t\,)$. They are also denoted separately by $fr\left(\phi_\kappa^{(W\rightsquigarrow V)}\right)=fr\left(\phi_\kappa^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]$, $\kappa=1,2,\dots,\mathcal{M}_V+\mathcal{L}_V$, or by a vector vulnerability representation

$$fr\left(\phi^{(W\rightsquigarrow V)}\right)=fr\left(\phi^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]$$
$$:=\left(fr\left(\phi_1^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,],\dots,fr\left(\phi_{\mathcal{M}_V+\mathcal{L}_V}^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]\right)^T.$$

If there is no risk of confusion, we will prefer write simply $\phi_\kappa^{(W\rightsquigarrow V)}=\phi_\kappa^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]$, $\kappa=1,2,\dots,\mathcal{M}_V+\mathcal{L}_V$, or use the vector vulnerability representation

$$\phi_\kappa^{(W\rightsquigarrow V)}=\phi_\kappa^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]:=$$
$$\left(\phi_1^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\dots,\phi_{\mathcal{M}_V+\mathcal{L}_V}^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]\right)^T.$$

In the total case, effectiveness states $U_W[x_1,x_2,x_3,t\,]\left(dev_1^{(V)}\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(dev_{\mathcal{M}_V}^{(V)}\right)$ and applicability situations

$U_W[x_1, x_2, x_3, t\,]\left(res_1^{(V)}\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(res_{\mathcal{L}_V}^{(V)}\right)$ are called cyber node vulnerabilities of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1, x_2, x_3, t\,)$ and they are again denoted separately by $\phi_\kappa^{(W \rightsquigarrow V)} = \phi_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V$, or by the joint vector vulnerability representation

$$\phi^{(W \rightsquigarrow V)} = \phi^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,] := \left(\phi_{V,1}^{(W)}[x_1, x_2, x_3, t\,], \dots, \phi_{V,\mathcal{M}_V + \mathcal{L}_V}^{(W)}[x_1, x_2, x_3, t\,]\right)^T.$$

By analogy, both available effectiveness states

$$U_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_1^{(V)}\right)\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_{m_V}^{(V)}\right)\right)$$

and available applicability situations

$$U_W[x_1, x_2, x_3, t\,]\left(fr\left(res_1^{(V)}\right)\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(fr\left(res_{\ell_V}^{(V)}\right)\right)$$

are called available cyber node fractional vulnerabilities from the viewpoint of the (user(s) of) node W, at the spatiotemporal point $(x_1, x_2, x_3, t\,)$. They are denoted separately by $fr\left(c_\kappa^{(W \rightsquigarrow V)}\right) = fr\left(c_\kappa^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, m_V + \ell_V$, or jointly by a corresponding available node vector vulnerability representation

$$fr\left(c^{(W \rightsquigarrow V)}\right) = fr\left(c^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,] := \left(fr\left(c_1^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,], \dots, fr\left(c_{m_V+\ell_V}^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]\right)^T.$$

If there is no risk of confusion, we will prefer write simply $c_\kappa^{(W \rightsquigarrow V)} = c_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, m_V + \ell_V$, or adopt the vector vulnerability representation

$$c_\kappa^{(W \rightsquigarrow V)} = c_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,] := \left(c_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,], \dots, c_{m_V+\ell_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\right)^T.$$

In total case, effectiveness states $U_W[x_1, x_2, x_3, t\,]\left(dev_1^{(V)}\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(dev_{m_V}^{(V)}\right)$ and applicability situations $U_W[x_1, x_2, x_3, t\,]\left(res_1^{(V)}\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(res_{\ell_V}^{(V)}\right)$ are called available cyber node vulnerabilities from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1, x_2, x_3, t\,)$ and they are also denoted separately by $c_\kappa^{(W \rightsquigarrow V)} = c_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, m_V + \ell_V$, or jointly by the available cyber node vector vulnerability representation

$$c_V^{(W)} = c_V^{(W)}[x_1, x_2, x_3, t\,] \coloneqq$$
$$\left(c_{V,1}^{(W)}[x_1, x_2, x_3, t\,], \dots, c_{V,m_V+\ell_V}^{(W)}[x_1, x_2, x_3, t\,]\right)^T.$$

In order to be more understandable, let us give a schematic example only for the indicative case of some of the above definitions in the **total** case.

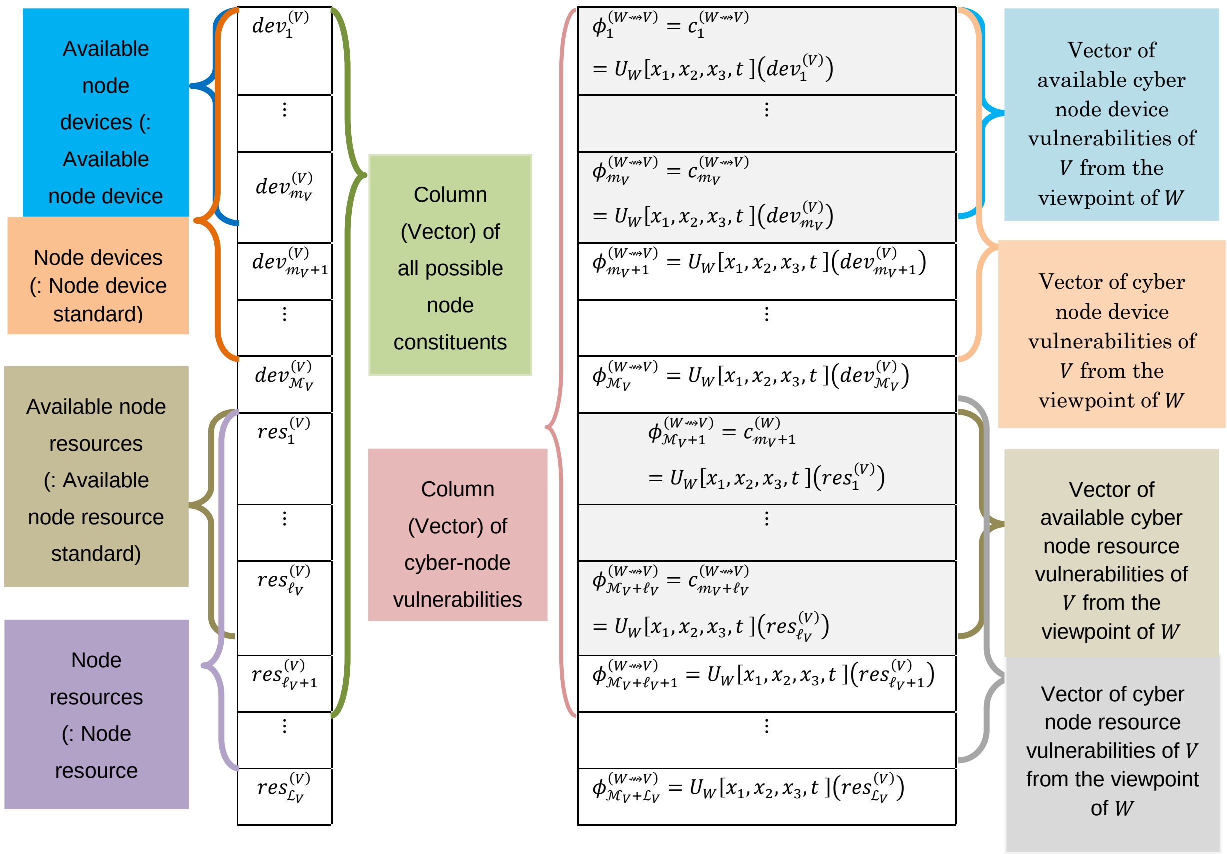

Figure 2 (Schematic Example 2)

6. Node Supervisions

We are now in position to proceed towards a qualitative/quantitative description of homorphisms between cyber nodes. Let W and V be two cyber nodes. We will presume the following notations for the sets of relative valuations of parts (fractions) of possible constituents:

1) $$\mathfrak{C}^{(fraction)}(V) = \left\{\left(fr\left(dev_1^{(V)}\right), \dots, fr\left(dev_{\mathcal{M}_V}^{(V)}\right), fr\left(res_1^{(V)}\right), \dots, fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)^T : \right.$$
$$fr\left(dev_k^{(V)}\right) \mathit{is\ part\ of\ possible\ device}\ dev_k^{(V)}\ \mathit{of}\ V,$$
$$k = 1,2, \dots, \mathcal{M}_V\,, \mathit{with}\ \ \mathcal{M}_V \in \mathbb{N}$$
$$\mathit{and}\ fr\left(res_\xi^{(V)}\right) \mathit{is\ part\ of\ possible\ resource}\ res_\xi^{(V)}\ \mathit{of}\ V,$$
$$\left.\xi = 1,2, \dots, \ \mathcal{L}_V, \mathit{with}\ \ \mathcal{L}_V \in \mathbb{N}\right\}$$: the set of all ordered columns

of possible parts (fractions) of constituents $\left(fr\left(dev_1^{(V)}\right),\dots,fr\left(dev_{\mathcal{M}_V}^{(V)}\right),fr\left(res_1^{(V)}\right),\dots,fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)^T$ of V;

2) $\mathcal{S}_W\mathfrak{C}^{(fraction)}(V)=\Big\{\Big(S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right),$

$$S_W[x_1,x_2,x_3,t,\ id_t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)\Big)^T:$$

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_k^{(V)}\right)\right)\ is\ valuation\ of\ part$$

$$of\ possible\ device\ in\ V\ subject\ to\ W,\ \ k\le\mathcal{M}_V\ with\ \mathcal{M}_V\in\mathbb{N}$$

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_\xi^{(V)}\right)\right)\ is\ valuation$$

$$of\ possible\ resource\ \ in\ V\ subject\ to\ W,\xi\le\mathcal{L}_V\ with\ \ \mathcal{L}_V\in\mathbb{N}\ ,$$

$$at\ the\ spatiotemporal\ point\ (x_1,x_2,x_3,t)\ \in\mathbb{R}^3\times[0{,}1]\Big\}:$$

the set of all ordered columns of relative valuations of parts (fractions) of possible constituents of V, from the viewpoint of the (user(s) of) node W, over the space time $\mathbb{R}^3\times[0{,}1]$;

3) $\mathcal{U}_W\mathfrak{C}^{(fraction)}(V)=$
$\Big\{\Big(U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right),$

$$U_W[x_1,x_2,x_3,t,\ id_t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)\Big)^T:$$

$$U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_k^{(V)}\right)\right)\ is\ vulnerability\ of\ part$$

$$of\ possible\ device\ in\ V\ subject\ to\ W,\ \ k\le\mathcal{M}_V\ with\ \mathcal{M}_V\in\mathbb{N}$$

$$U_W[x_1,x_2,x_3,t\,]\left(fr\left(res_\xi^{(V)}\right)\right)\ is\ vulnerability$$

$$of\ possible\ resource\ \ in\ V\ subject\ to\ W,\xi\le\mathcal{L}_V\ with\ \ \mathcal{L}_V\in\mathbb{N},$$

$$at\ the\ spatiotemporal\ point\ (x_1,x_2,x_3,t)\ \in\mathbb{R}^3\times[0{,}1]\Big\}:$$

the set of all ordered columns of relative vulnerabilities of parts (fractions) of possible constituents in V, from the viewpoint of the (user(s) of) node W, over $\mathbb{R}^3\times[0{,}1]$.

Definition 3 *Let W and V be two cyber nodes. The combinatorial triplet*

$$\mathcal{P}=\mathcal{P}(V)=\left(\mathfrak{C}^{(fraction)}(V),\mathcal{S}_W\mathfrak{C}^{(fraction)}(V),\mathcal{U}_W\mathfrak{C}^{(fraction)}(V)\ \right)$$

will be called the cyber-field of V from the viewpoint of the users of W.Its elements are threefold cyber situations which will be represented by $\wp$. Especially, if $W=V$, the cyber-field $\mathcal{P}=\mathcal{P}(V)$ will be called the cyber-purview of V and will be denoted $\mathcal{P}^{(self)}=\mathcal{P}^{(self)}(V)$. Its elements are special threefold cyber situations called self-perceived sites and they are represented by the general form $\hat{\wp}$.

Let now W be a given cyber node and $fr\left(C^{(V)}\right)$ be a given cyber-vector in a *fixed constituent*

$$C^{(V)} = \left(dev_1^{(V)}, \dots, dev_{m_V}^{(V)}, \dots, dev_{\mathcal{M}_V}^{(V)}, res_1^{(V)}, \dots, res_{\ell_V}^{(V)}, \dots, res_{\mathcal{L}_V}^{(V)}\right)^T$$

of V. Its cyber states are

$$\left(dev_1^{(V)}\right), \dots, fr\left(dev_{m_V}^{(V)}\right), \dots fr\left(dev_{\mathcal{M}_V}^{(V)}\right), fr\left(res_1^{(V)}\right), \dots, fr\left(res_{\ell_V}^{(V)}\right), \dots, fr\left(res_{\mathcal{L}_V}^{(V)}\right).$$

Then any two threefold cyber situations $\wp$ and $\hat{\wp}$ on the node $V \in ob\left(cy(t)\right)$ from the viewpoint of the users of node W, situated in the cyber fields

$$\mathcal{P} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{M}_V+\mathcal{L}_V} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{m}} \text{ and}$$

$$\mathcal{P}^{(self)} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{M}_V+\mathcal{L}_V} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{m}}$$

respectively, can simply be viewed as two ordered pairs

$$\wp = (\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left(\left(s_{i,j}\right), \left(u_{i,j}\right)\right) \in \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{m}}$$

and

$$\hat{\wp} = \left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left(\left(\hat{s}_{i,j}\right), \left(\hat{u}_{i,j}\right)\right) \in \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times\mathfrak{m}}$$

respectively, with

$$\mathbb{S}_{W\to V} = \mathbb{S}_{W\to V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
S_W\left(fr\left(dev_1^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\beta_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\beta_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(dev_{m_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\beta_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\beta_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
S_W\left(fr\left(res_1^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_1^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\mathcal{L}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix},$$

Table 1

$$\mathbb{U}_{W\rightarrow V} = \mathbb{U}_{W\rightarrow V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
U_W\left(fr\left(dev_{1}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(dev_{1}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(dev_{1}^{(V)}\right)\right)}_{=:\phi_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(dev_{1}^{(V)}\right)\right)}_{=:\phi_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(dev_{1}^{(V)}\right)\right)}_{=:\phi_{1,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_W\left(fr\left(dev_{m_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\phi_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\phi_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\phi_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_W\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
U_W\left(fr\left(res_{1}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(res_{1}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(res_{1}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(res_{1}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(res_{1}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_W\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_W\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\mathcal{L}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix},$$

Table 2

$$\widehat{\mathbb{S}}_{V\rightarrow V} = \widehat{\mathbb{S}}_{V\rightarrow V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
S_V\left(fr\left(dev_{1}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(dev_{1}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(dev_{1}^{(V)}\right)\right)}_{=:\widehat{\beta}_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(dev_{1}^{(V)}\right)\right)}_{=:\widehat{\beta}_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(dev_{1}^{(V)}\right)\right)}_{=:\widehat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(dev_{m_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
S_V\left(fr\left(res_{1}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(res_{1}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(res_{1}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(res_{1}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(res_{1}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\widehat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix},$$

Table 3

$$\widehat{\mathbb{U}}_{V\rightarrow V} = \widehat{\mathbb{U}}_{V\rightarrow V}\left(fr\left(C^{(V)}\right)\right) =$$

$$
\begin{pmatrix}
U_V\left(fr\left(dev_1^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(dev_1^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{1,1}}, \underbrace{u_{V,2}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{1,2}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{1,\mathfrak{m}}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(dev_{m_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{m_V,1}}, \underbrace{u_{V,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{m_V,2}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{m_V,\mathfrak{m}}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1}}, \underbrace{u_{V,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V,2}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}}}\right) \\
U_V\left(fr\left(res_1^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(res_1^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1}}, \underbrace{u_{V,2}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,2}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}}, \underbrace{u_{V,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,2}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1}}, \underbrace{u_{V,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,2}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\widehat{\phi}^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}}\right)
\end{pmatrix}.
$$

Table 4

Without any loss of generality, we may suppose the numbers $\mathcal{M}_V + \mathcal{L}_V$ and $\mathcal{M}_W + \mathcal{L}_W$ are enough large, so that $\mathcal{M}_V + \mathcal{L}_V = \mathcal{M}_W + \mathcal{L}_W$, for any two cyber nodes W and V. To simplify the notation, we set

$$\mathcal{N} \coloneqq \mathcal{M}_V + \mathcal{L}_V = \mathcal{M}_W + \mathcal{L}_W.$$

Definition 4 *Let W and V be two cyber nodes. The supervision of V in the system of the two nodes V and W at a given time moment $t \in [0,1]$ is defined to be the pair*

$$(z_1, \zeta_1) = (z_1, \zeta_1)(t) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

with

$$z_1 = \mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V},\ \zeta_1 = \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V},$$

and such that

- $i := \sqrt{-1} = (0,1) \in \mathbb{C}$,
- $(\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left(\left(s_{i,j}\right), \left(u_{i,j}\right)\right) \in \mathbb{R}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{R}^{\mathcal{N}\times\mathfrak{m}}$ *and*
- $\left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left(\left(\hat{s}_{i,j}\right), \left(\hat{u}_{i,j}\right)\right) \in \mathbb{R}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{R}^{\mathcal{N}\times\mathfrak{m}}$*.]*

The complex matrices z_1 and ζ_1 are called supervisory perceptions of V in the system of nodes V and W at the moment t. The piecewise continuous mapping $\delta_V \equiv \delta_{[(V,W)\rightsquigarrow V]}$ defined by

$$\delta_V : [0,1] \to \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} : t \mapsto \delta_V(t) = (z_1, \zeta_1)(t)$$
$$\equiv \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right)(t)$$

is the supervisory perception curve of V in the node system (V, W). Its image $\delta_V^* = \delta_V\,([0,1])$ is called universal supervision of V in the node system (V, W), while any subset $\delta_V(I) = \{\delta_V(t) : t \in I \subset [0,1]\}$ of $\delta_V([0,1])$ is said to be a partial supervisory perception of V in the system of the two nodes V and W.

If, according to Remarks 3.3 and 4.2, the component valuations $s_{W,k}\left(fr\left(C^{(V)}\right)\right)$ or vulnerabilities $u_{W,j}\left(fr\left(C^{(V)}\right)\right)$ of a given part $fr\left(C^{(V)}\right)$ in the cyber-node V extent onto the real projective line $\mathbb{RP}^1$ of $\mathbb{R}$, then any two threefold cyber situations $\wp$ and $\hat{\wp}$ in the corresponding cyber fields $\mathcal{P} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{N}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$ and $\mathcal{P}^{(self)} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{N}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$ can be viewed as two ordered pairs

$$\wp = (\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left((s_{i,j}), (u_{i,j})\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}} \text{ and}$$
$$\hat{\wp} = \left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left((\hat{s}_{i,j}), (\hat{u}_{i,j})\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$$

respectively. In such a case, the set δ_V^* of extended universal supervisions of V in the system of the two nodes V and W consists of all ordered pairs $\left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) \in (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}$, which are defined in such a way that a column in the matrices $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}}$ and $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}$ is considered to be infinite if and only if the real or the imaginary part of an element of the column becomes infinite. Here $\mathbb{CP}^1$ denotes, as usually, the complex projective line (: the Riemann sphere S^3). We need the following.

Theorem 1 *The $\mathcal{N}-$fold symmetric product of $\mathbb{CP}^1$ is homeomorphic to $\mathbb{CP}^{\mathcal{N}}$.*

Sketch of Proof. One can be trying to understand the space obtained by taking the Cartesian product $\mathbb{CP}^1 \times \mathbb{CP}^1$ and identifying some of its points by the rule $(x, y) \sim (y, x)$. Viewing $\mathbb{CP}^1$ as a CW complex with one 0-cell and one 2-cell, we can compute the homology of $\mathbb{CP}^1 \times \mathbb{CP}^1/\sim$ which matches that of $\mathbb{CP}^2$ but we can't seem to visualize an "obvious" homeomorphism between the two spaces. The question is the following:

- is $\mathbb{CP}^1 \times \mathbb{CP}^1/\sim$ homeomorphic to $\mathbb{CP}^2$ and,
- if so, how?

We believe we are on the right track, and a homeomorphism from $\mathbb{CP}^1 \times \mathbb{CP}^1/\sim$

to $\mathbb{CP}^2$ is given by

$$[((z_1: z_2), (w_1: w_2))] \mapsto (z_1 w_1: z_2 w_2: z_1 w_2 + z_2 w_1).$$

Note that elements of the form $[(1: z), (1: w)]$ map to $(1: zw: z + w)$, i.e., the coordinates are given by the elementary symmetric functions of z and w, so the map is a homeomorphism restricted to this subspace onto the subspace of $\mathbb{CP}^2$ given by points with non-zero first coordinate. We have not worked out all the details, but we are pretty sure that this argument can be promoted to show that the map is actually a homeomorphism between your spaces. To see this in the 2−fold case: consider homogeneous polynomials of degree two $\mathbb{C}[x, y]^{(2)}$ whose elements are of the form $ax^2 + bxy + cy^2$ and notice that for $\lambda \in \mathbb{C}^*$, it holds

$$\lambda[ax_0^2 + bx_0 y_0 + cy_0^2] = 0 \Leftrightarrow ax_0^2 + bx_0 y_0 + cy_0^2 = 0.$$

This allows us to identify points of $\mathbb{CP}^2$ with elements of $\mathbb{C}[x, y]^{(2)}/\sim$, where $\sim$ identifies polynomials having the same roots. The map from $\mathbb{CP}^2$ to the symmetric product of two copies of $\mathbb{CP}^1$ is then given by

$$(a: b: c) \mapsto ax^2 + bxy + cy^2 = (\alpha x + \beta y)(\alpha' x + \beta' y) \mapsto [(\alpha: \beta), (\alpha': \beta')]$$

where the equality comes from the fundamental theorem of algebra.

In view of this result, we are led to the following definition.

Definition 5 *Let W and V be two cyber nodes. The extended supervision of V in the system of the two nodes V and W at a given time moment $t \in [0,1]$ is defined to be the pair*

$$(z_1, \zeta_1) = (z_1, \zeta_1)(t) \in (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} \times (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}} \equiv (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}$$

with

$$z_1 = \mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V},\ \zeta_1 = \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V},$$

and such that

- $i := \sqrt{-1} = (0,1) \in \mathbb{C}$,
- $(\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left((s_{i,j}), (u_{i,j})\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$ *and*
- $\left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left((\hat{s}_{i,j}), (\hat{u}_{i,j})\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$.

The complex projective points z_1 and ζ_1 are called extended supervisory perceptions of V in the system of nodes V and W at the moment t. The piecewise continuous mapping

$$\delta\mathbb{P}_V \equiv \delta\mathbb{P}_{[(V,W)\rightsquigarrow V]}$$

defined by

$$\delta\mathbb{P}_V: [0,1] \to (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} \times (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}}: t \mapsto \delta\mathbb{P}_V(t) = (z_1, \zeta_1)(t)$$
$$\equiv \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right)(t)$$

is the extended supervisory perception curve of V *in the node system* (V, W)*. Its image* $\delta\mathbb{P}_V([0,1])$ *is called extended universal supervision of* V *in the node system* (V, W)*, while any subset* $\delta\mathbb{P}_V(I) = \{\delta\mathbb{P}_V(t): t \in I \subset [0,1]\}$ *of* $\delta\mathbb{P}_V([0,1])$ *is said to be a partial extended supervisory perception of* V *in the system of nodes* V *and* W.

Provided there is no risk of confusion, we will denote indiscriminately with $\mathbb{CM}$ either $\mathbb{C}$ or $\mathbb{CP}$. Further, in what will follow, we will adopt the common notation

$$\gamma_V \equiv \gamma_{[(V,W)\rightsquigarrow V]}[0,1] \to \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}: t \mapsto \gamma_V(t) = (z_1, \zeta_1)(t)$$
$$\equiv \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right)(t)$$

for the two supervisory perception curves δ_V and $\delta\mathbb{P}_V$. Similarly, we will adopt the common notation $\gamma_V(I) = \{\gamma_V(t): t \in I \subset [0,1]\}$ for the two supervisory perception sets $\delta_V(I)$ and $\delta\mathbb{P}_V(I)$. In particular, we will write γ_V^* for the two universal supervisions $\delta_V([0,1])$ and $\delta\mathbb{P}_V([0,1])$. With this notation, we are now in position to proceed further, as in the following Session.

7. Cyber-Effects

A momentary homomorphism $g: W \to V$ between the two cyber nodes $V, W \in ob\big(cy(t)\big)$ is defined as a collection of mappings from a cyber field of W at time $t \in]\alpha, \beta[\subset\subset [0,1]$ into a cyber field of V at other times $t' \in [\alpha, \beta]$.

Definition 6 *Let us consider the two supervisory perception sets*

$$\Omega_V = \Omega_{[(V,W)\rightsquigarrow V]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}} \text{ and}$$

$$\Omega_W = \Omega_{[(V,W)\rightsquigarrow W]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}.$$

The momentary homomorphism $g: W \to V$ *can be rather understandable as an "adaptive" movement* $\mathscr{g}$ *between time-shifted partial (extended or not) supervisory perceptions of* W *and* V*:*

$$\mathscr{g}: [\alpha, \beta] \mapsto \Omega_W \times \Omega_V: t \mapsto \mathscr{g}(t) := \big(\gamma_W(t), \gamma_V(t + \Delta t)\big).$$

The shifted curve $\mathscr{g}$ *is called cyber-effect of* W *on* V.

It is more appropriate to represent a cyber-effect as a collection of point-wise correspondences

$$\left(\mathscr{g}_t: \gamma_W(t) \longmapsto \gamma_V'(t')\right)_{t\in]\alpha,\beta[} (t' := t + \Delta t),$$

where we denote by $\gamma_W(t)$ and $\gamma_V^{'}(t^{'})$ the curves $\gamma_{[(V,W)\rightsquigarrow W]}(t)$ and $\gamma_{[(V,W)\rightsquigarrow V]}(t+\Delta t)$, respectively. With this notation, at time t, a supervisory perception of W in the system of nodes V, W:

$$\gamma_W(t) = \left(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\right) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{m_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \beta_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{N},1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{N},1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{N},\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{N},\mathfrak{n}}^{(W\rightsquigarrow W)} \end{pmatrix}, \begin{pmatrix} \phi_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{1,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{m_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{m_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \phi_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{N},1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{N},1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{N},\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{N},\mathfrak{m}}^{(W\rightsquigarrow W)} \end{pmatrix}\right)$$

$$\in \Omega_W$$

Table 5

is depicted, by means of the cyber-effect $g = g_t$, at the supervisory perception of V in the system of nodes V and W at a next time $t^{'} := t + \Delta t$:

$$\gamma_V^{'}(t^{'}) = \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{N},1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{N},1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{N},\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{N},\mathfrak{n}}^{(V\rightsquigarrow V)} \end{pmatrix}, \begin{pmatrix} \phi_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{V,1}^{(W\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \phi_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{N},1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{N},1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{N},\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{N},\mathfrak{m}}^{(V\rightsquigarrow V)} \end{pmatrix}\right)$$

Table 6

$$\in \Omega_V.$$

Remark 4 The case $\Delta t = 0$ is not excluded.

Let us give two indicative examples showing the alteration diversity and combinatorial suppleness of this flexible concept.

Example 3 In practice, often, we prefer to reduce only to available constituents and available valuations. Then, the momentary homomorphism g transforms only available quantities of W at a time t into available quantities of V at a next time $t^{'} = t + \Delta t$ and we write $g = g_t\colon \mathcal{Q}_7^{(V)}(W)(t) \to \mathcal{P}_7^{(W)}(V)(t^{'})$, where the combinatorial triplet

$$\mathcal{Q}_7^{(V)}(W) = \mathcal{Q}_7^{(V)}(W)(t) = (\mathfrak{C}_{available}(W), \mathcal{S}_V\mathfrak{C}_{available}(W), \mathcal{U}_V\mathfrak{C}_{available}(W)\ )$$

represents the set of available components of node W at time t, as evaluated in terms of their valuations and their vulnerabilities by the users of node V:

$$\mathfrak{C}_{available}(W) = \Big\{\Big(dev_1^{(W)}, \dots, dev_{m_W}^{(W)}, res_1^{(W)}, \dots, res_{\ell_W}^{(W)}\Big)^T :$$
$$dev_k^{(W)} \text{ is available device of } W, \text{with } m_V \in \mathbb{N} \text{ and}$$
$$res_k^{(W)} \text{ is available resource of } W, \text{with } \ell_V \in \mathbb{N}\Big\}:$$

the set of all ordered columns of available constituents $\Big(dev_1^{(W)}, \dots, dev_{m_W}^{(W)}, res_1^{(W)}, \dots, res_{\ell_V}^{(W)}\Big)^T$ of W,

$$\mathcal{S}_V\mathfrak{C}_{available}(W) = \Big\{\Big(S_V[x_1, x_2, x_3, t\,]\Big(dev_1^{(W)}\Big), \dots, S_V[x_1, x_2, x_3, t\,]\Big(dev_{m_W}^{(W)}\Big),$$
$$S_V[x_1, x_2, x_3, t\,]\Big(res_1^{(W)}\Big), \dots, S_V[x_1, x_2, x_3, t\,]\Big(res_{\ell_W}^{(W)}\Big)\Big)^T :$$
$$S_V[x_1, x_2, x_3, t\,]\Big(dev_k^{(W)}\Big) \text{ is valuation of available device}$$
$$\text{in } W \text{ subject to } V, \qquad k = 1,2, \dots, m_W, \text{with } m_W \in$$
$$\mathbb{N}\; S_V[x_1, x_2, x_3, t\,]\Big(res_\xi^{(W)}\Big) \text{ is valuation of available resource}$$
$$S_V[x_1, x_2, x_3, t\,]\Big(res_\xi^{(W)}\Big) \text{ is valuation of available resource}$$
$$\text{in } W \text{ subject to } V, \xi = 1,2, \dots, \ell_W \text{ with } \ell_W$$
$$\in \mathbb{N}, \text{at the spatiotemporal point } (x_1, x_2, x_3, t)$$
$$\in \mathbb{R}^3 \times [0,1]\}:$$

the set of all ordered columns of relative valuations of available constituents in W, from the viewpoint of the (user(s) of) node V, over the space time $\mathbb{R}^3 \times [0,1]$,

$$\mathcal{U}_V\mathfrak{C}_{available}(W) = \Big\{\Big(U_V[x_1, x_2, x_3, t\,]\Big(dev_1^{(W)}\Big), \dots, U_V[x_1, x_2, x_3, t\,]\big(dev_{m_W}^{W}\big),$$
$$U_V[x_1, x_2, x_3, t\,]\Big(res_1^{(W)}\Big), \dots, U_V[x_1, x_2, x_3, t\,]\Big(res_{\ell_W}^{(W)}\Big)\Big)^T :$$
$$U_V[x_1, x_2, x_3, t\,]\Big(dev_k^{(W)}\Big) \text{ is vulnerability of available device}$$
$$\text{in } W \text{ subject to } V, k = 1,2, \dots, m_W, \text{with } m_W \in$$
$$\mathbb{N}\; S_V[x_1, x_2, x_3, t\,]\Big(res_\xi^{(W)}\Big) \text{ is vulnerability of available resource}$$
$$S_V[x_1, x_2, x_3, t\,]\Big(res_\xi^{(W)}\Big) \text{ is vulnerability of available resource}$$
$$\text{in } W \text{ subject to } V, \qquad \xi = 1,2, \dots, \ell_W \text{ with } \ell_W \in$$
$$\mathbb{N}, \text{at the spatiotemporal point } (x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0,1]\}:$$

the set of all ordered columns of relative vulnerabilities of available constituents in W, from the viewpoint of the (user(s) of) node V, over $\mathbb{R}^3 \times [0,1]$.

Similarly, the combinatorial triplet $\mathcal{P}_7^{(W)}(V) = \mathcal{P}_7^{(W)}(V)(t') =$

$(\mathfrak{C}_{available}(V), \mathcal{S}_W\mathfrak{C}_{available}(V), \mathcal{U}_W\mathfrak{C}_{available}(V))$ represents the set of available components of node V at time t', as evaluated in terms of their valuations and their vulnerabilities by the users of node W. In view of the above Definition 6.1, the correspondence $g = g_t$ can be seen as a mapping between (extended or not) supervisory perceptions $g = g_t\colon \gamma_W(t) \longmapsto \gamma_V^{'}(t^{'})$, in such a way that each (extended or not) supervisory perception of W in the system of nodes V and W at a time moment t, of the form

$$\gamma_W(t) = \left(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\right) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,1}^{(W\rightsquigarrow W)} & \dots & \beta_{1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{m_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \beta_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \dots & \beta_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}, \begin{pmatrix} \gamma_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{1,1}^{(W\rightsquigarrow W)} & \dots & \gamma_{1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \gamma_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \gamma_{m_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{m_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \gamma_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \gamma_{\mathcal{M}_W+1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \gamma_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \dots & \gamma_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}\right)$$

$$\in \Omega_W$$

Table 7

is depicted, via the correspondence g, at an (extended or not) supervisory perception of V in the system of nodes V and W at the next time moment $t^{'} := t + \Delta t$, of the form:

$$\gamma_V^{'}(t^{'}) = \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,1}^{(V\rightsquigarrow V)} & \dots & \beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \dots & \beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}, \begin{pmatrix} \gamma_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{1,1}^{(V\rightsquigarrow V)} & \dots & \gamma_{1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \gamma_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \gamma_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \gamma_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \gamma_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \gamma_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \dots & \gamma_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}\right)$$

$$\in \Omega_V.$$

Table 8

Similarly, if the momentary homomorphism $g\colon W \to V$ acts only on all the resources of W by transforming and transferring fractions of the available resources of W at a time t into the node resource standard $\left(r_1^{(V)}, \dots, r_{\mathcal{L}_V}^{(V)}\right)$ of V at a next time $t^{'} = t + \Delta t$, then the cyber-effect g is a mapping of the form $g = g_t: \mathcal{Q}_9^{(V)}(W)(t) \to \mathcal{P}_3^{(W)}(V)(t^{'})$. Here, as usually, the combinatorial triplet

$\mathcal{Q}_9^{(V)}(W) = \mathcal{Q}_9^{(V)}(W)(t^{'}) = (\mathfrak{R}_{available}(W), \mathcal{S}_V \mathfrak{R}_{available}(W), \mathcal{U}_V \mathfrak{R}_{available}(W)\ )$ represents a set of available resources of node W, at the time moment t, as evaluated in terms of their valuations and their vulnerabilities by the users of node V:

$$\mathfrak{R}_{available}(W) = \left\{\left(res_1^{(W)}, \dots, res_{\ell_W}^{(W)}\right)^T : res_k^{(W)}\ possible\ resource\ of\ W, k = 1,2,..,\ell_W\ , \ell_W \in \mathbb{N}\right\}:$$

the set of all ordered columns of available resources of W,

$$\mathcal{S}_V \mathfrak{R}_{available}(W) = \left\{\left(\ S_V[x_1, x_2, x_3, t\ ]\left(res_1^{(W)}\right), \dots, S_V[x_1, x_2, x_3, t\ ]\left(res_{\ell_W}^{(W)}\right)\right)^T : S_V[x_1, x_2, x_3, t\ ]\left(res_\xi^{(W)}\right)\ is\ valuation\ of\ possible\ resource\ in\ W\ subject\ to\ V, \xi = 1,2, \dots, \ell_W\ with\ \ \ell_W \in \mathbb{N}, at\ the\ spatiotemporal\ point\ (x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0,1]\right\}:$$

the set of all ordered columns of relative valuations of available constituents in W, from the viewpoint of the (user(s) of) node V, over $\mathbb{R}^3 \times [0,1]$,

$$\mathcal{U}_V \mathfrak{R}_{available}(W) = \left\{\left(\ U_V[x_1, x_2, x_3, t\ ]\left(res_1^{(W)}\right), \dots, U_V[x_1, x_2, x_3, t\ ]\left(res_{\ell_W}^{(W)}\right)\right)^T : U_V[x_1, x_2, x_3, t\ ]\left(res_\xi^{(W)}\right)\ is\ vulnerability\ of\ possible\ resource\ in\ W\ subject\ to\ V, \xi = 1,2, \dots, \ell_W\ with\ \ \ell_W \in \mathbb{N}, at\ the\ spatiotemporal\ point\ (x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0,1]\right\}:$$

the set of all ordered columns of relative vulnerabilities of available constituents in W, from the viewpoint of the (user(s) of) node V, over $\mathbb{R}^3 \times [0,1]$.

Similarly, the combinatorial triplet

$$\mathcal{P}_3^{(W)}(V) = \mathcal{P}_3^{(W)}(V)(t^{'}) = \left(\mathfrak{R}(V), \mathcal{S}_W \mathfrak{R}(V), \mathcal{U}_W \mathfrak{R}(V)\right)$$

represents a set of resources of node V, at the next time moment $t^{'}$, as evaluated in terms of their valuations and their vulnerabilities by the users of node W. In view of Definition 6.1, the correspondence $\mathscr{g} = \mathscr{g}_t$ can be seen as a mapping between (extended or not) supervisory perceptions $\mathscr{g} = \mathscr{g}_t : \gamma_W(t) \longmapsto \gamma'_V(t')$, in such a way that each (extended or not) supervisory perception of W in the system of nodes V and W at time moment t

$$\gamma_W(t) = \left(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\right) =$$

$$\left(\begin{pmatrix} 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \\ \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{pmatrix}, \begin{pmatrix} 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \\ \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{pmatrix}\right) \in \Omega_W$$

Table 9

is depicted, via the correspondence g, at an (extended or not) supervisory perception of V in the system of nodes V and W at the moment $t^{'} := t + \Delta t$ of the form

$$\gamma^{'}_V(t^{'}) = \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) =$$

$$\left(\begin{pmatrix} 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{n}} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{n}} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{m}} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{m}} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{pmatrix}\right) \in \Omega_V.$$

Table 10

Although the concept of cyber-effect at a time moment t seems to be rather sufficient, sometimes we care to describe the interaction that has one cyber-node on each other, as well as the mutual effects resulting at a later time $t^{'} = t + \Delta t$. In this case, the putative mutuality directly is influenced by the subjectivity of the users of the two cyber nodes. So, frequently, instead of the concept of a momentary cyber-effect, we are forced to consider mappings describing mutual influences between cyber-nodes.

8. Cyber-Interactions

As in Definition 5, let us consider the sets

$\Omega_V = \Omega_{[(V,W)\rightsquigarrow V]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}$ and

$$\Omega_W = \Omega_{[(V,W)\rightsquigarrow W]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}$$

of supervisory perception curves of V and W in the node system (V,W).

Definition 7 *If* $]\alpha,\beta[\subset\subset [0,1]$, *an interplay of the ordered cyber pair* (V,W) *over the time* $t \in]\alpha,\beta[$ *or, simply, a cyber-interplay, is an open*[2] *shift curve*

$$\mathscr{g}:]\alpha,\beta[\to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$

$$t \mapsto \mathscr{g}(t) := \big(\gamma_W(t), \gamma_V(t), \gamma_W(t+\Delta t), \gamma_V(t+\Delta t)\big).$$

If the cyber-interplay $\mathscr{g}$ *is composition of several separate interplays, we say that the cyber-interplay* $\mathscr{g}$ *is sequential; otherwise is called elementary*.

It is more appropriate to represent a cyber-interplay as a collection of point-wise correspondences

$$\left(\mathscr{g}_t: \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V: (\gamma_W(t), \gamma_V(t)) \mapsto \left(\gamma'_W(t'), \gamma'_V(t')\right)\right)_{t\in]\alpha,\beta[}$$

$$(t' := t + \Delta t),$$

where, as usually, we denote by $\gamma_X(t)$ and $\gamma'_X(t')$ the curves $\gamma_{[(V,W)\rightsquigarrow X]}(t)$ and $\gamma_{[(V,W)\rightsquigarrow X]}(t+\Delta t)$, respectively (with $X = V, W$) and we say that the interplay is a cyber- activity of W on V over the time $t \in]\alpha,\beta[$. If the cyber-interplay is sequential, we say that the cyber-activity of W on V is sequential; otherwise the cyber-activity is called elementary.

Definition 8 *A cyber-interaction or simply interaction between* W *and* V *at a given time moment* $t_0 \in]\alpha,\beta[$ *is a tetrad*

[2] Open intervals are used for so called open curves (line, parabola, hyperbola...). Closed intervals are used for closed curves (circles, ellipse...). The reason for use of open intervals for open curves and closed intervals for closed curves is that parameterization is a homeomorphism between to "shapes". Circle is not homeomorphic to the line, for example. But it is to any closed loop (http://math.stackexchange.com/questions/209309/open-interval-in-definition-of-curve).

$$Z = Z_{(W,V)}(t_0) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}})^4$$

for which there is an associated cyber-activity of W on V:

$$\left(g_t = g_t^{(Z)}: \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : (\gamma_W(t),\gamma_V(t)\,) \longmapsto \left(\gamma'_W(t'),\gamma'_V(t')\right)\right)_{t\in]\alpha,\beta[}$$

$$(t' := t + \Delta t),$$

such that

$$(z_1,\zeta_1) = \gamma_W(t_0) = \left(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$(z_2,\zeta_2) = \gamma_V(t_0) = \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$(z_3,\zeta_3) = \gamma'_W(t'_0) = \left(\mathbb{S}'_{V\to W} + i\widehat{\mathbb{S}}'_{W\to W}, \mathbb{U}'_{V\to W} + i\widehat{\mathbb{U}}'_{W\to W}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$(z_4,\zeta_4) = \gamma'_V(t'_0) = \left(\mathbb{S}'_{W\to V} + i\widehat{\mathbb{S}}'_{V\to V}, \mathbb{U}'_{W\to V} + i\widehat{\mathbb{U}}'_{V\to V}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}.$$

If the corresponding interplay

$$g = g^{(Z)} :]\alpha,\beta[\to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V :$$

$$t \mapsto g(t) := \left(\gamma_W(t),\gamma_V(t),\gamma'_W(t'),\gamma'_V(t')\right)$$

is sequential, we say that the cyber-interaction is sequential; otherwise the cyber-interaction is called elementary.

Obviously, in Definition 7, keeping a fixed supervisory perception $\gamma_V(t_0)$ in the archetype component Ω_V and a fixed supervisory perception $\gamma_W(t+\Delta t)$ in the component image Ω_W, the corresponding cyber-interaction becomes a cyber-effect in the sense of Definition 6. And, as we shall see, proper management of cyber-effects is enough to study cyber navigations ([2). However, in most cases, as in the case of cyber attacks (see again [2]), it is necessary to consider cyber-interactions. So, because cyber-effects are a partial case of cyber-interactions, we will give a slight priority in the most general context of cyber-interactions.

It is easily verified that the most detailed general form of a cyber-interaction is as follows.

$$Z = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big)(t_0)$$

$$= \Bigg(\underbrace{\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}}_{\gamma_W(t_0)}, \underbrace{\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}}_{\gamma_V(t_0)},$$

$$\underbrace{\mathbb{S}'_{V\to W} + i\widehat{\mathbb{S}}'_{W\to W}, \mathbb{U}'_{V\to W} + i\widehat{\mathbb{U}}'_{W\to W}}_{\gamma'_W(t'_0)}, \underbrace{\mathbb{S}'_{W\to V} + i\widehat{\mathbb{S}}'_{V\to V}, \mathbb{U}'_{W\to V} + i\widehat{\mathbb{U}}'_{V\to V}}_{\gamma'_V(t'_0)}\Bigg)$$

$$
= \left(\left(\underbrace{\begin{pmatrix}
\beta^{(V\to W)}_{1,1} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{1,1} & \cdots & \beta^{(V\to W)}_{1,\mathfrak{n}} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(V\to W)}_{m_W,1} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{m_W,1} & \cdots & \beta^{(V\to W)}_{m_W,\mathfrak{n}} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{m_W,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(V\to W)}_{\mathcal{M}_W,1} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \cdots & \beta^{(V\to W)}_{\mathcal{M}_W,\mathfrak{n}} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{n}} \\
\beta^{(V\to W)}_{\mathcal{M}_W+1,1} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \beta^{(V\to W)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(V\to W)}_{\mathcal{M}_W+\ell_W,1} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta^{(V\to W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,1} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \cdots & \beta^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}} + i\;\hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}}
\end{pmatrix}}_{z_1=\mathbb{S}_{V\to W}+i\hat{\mathbb{S}}_{W\to W}}, \underbrace{\begin{pmatrix}
\phi^{(V\to W)}_{1,1} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{1,1} & \cdots & \phi^{(V\to W)}_{1,\mathfrak{m}} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(V\to W)}_{m_W,1} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{m_W,1} & \cdots & \phi^{(V\to W)}_{m_W,\mathfrak{m}} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{m_W,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(V\to W)}_{\mathcal{M}_W,1} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \cdots & \phi^{(V\to W)}_{\mathcal{M}_W,\mathfrak{m}} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{m}} \\
\phi^{(V\to W)}_{\mathcal{M}_W+1,1} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \phi^{(V\to W)}_{\mathcal{M}_W+1,\mathfrak{m}} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(V\to W)}_{\mathcal{M}_W+\ell_W,1} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \phi^{(V\to W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,1} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \cdots & \phi^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}} + i\;\hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}
\end{pmatrix}}_{\zeta_1=\mathbb{U}_{V\to W}+i\hat{\mathbb{U}}_{W\to W}} \right),
$$

$$
\left(\underbrace{\begin{pmatrix}
\beta^{(W\rightsquigarrow V)}_{1,1} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{1,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{1,\mathfrak{n}} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(W\rightsquigarrow V)}_{m_V,1} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}} \\
\beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}} + i\;\hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}
\end{pmatrix}}_{z_2=\mathbb{S}_{W\to V}+i\hat{\mathbb{S}}_{V\to V}}, \underbrace{\begin{pmatrix}
\phi^{(W\rightsquigarrow V)}_{1,1} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{1,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{1,\mathfrak{m}} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(W\rightsquigarrow V)}_{m_V,1} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{m_V,\mathfrak{m}} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}} \\
\phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}} + i\;\hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}
\end{pmatrix}}_{\zeta_2=\mathbb{U}_{W\to V}+i\hat{\mathbb{U}}_{V\to V}} \right),
$$

$$
\left(\underbrace{\begin{pmatrix}
\beta'^{(V\to W)}_{1,1} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{1,1} & \cdots & \beta'^{(V\to W)}_{1,\mathfrak{n}} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(V\to W)}_{m_W,1} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{m_W,1} & \cdots & \beta'^{(V\to W)}_{m_W,\mathfrak{n}} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{m_W,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(V\to W)}_{\mathcal{M}_W,1} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W,\mathfrak{n}} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{n}} \\
\beta'^{(V\to W)}_{\mathcal{M}_W+1,1} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(V\to W)}_{\mathcal{M}_W+\ell_W,1} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,1} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}} + i\;\hat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}}
\end{pmatrix}}_{z_3=\mathbb{S}'_{V\to W}+i\hat{\mathbb{S}}'_{W\to W}}, \underbrace{\begin{pmatrix}
\phi'^{(V\to W)}_{1,1} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{1,1} & \cdots & \phi'^{(V\to W)}_{1,\mathfrak{m}} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(V\to W)}_{m_W,1} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{m_W,1} & \cdots & \phi'^{(V\to W)}_{m_W,\mathfrak{m}} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{m_W,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(V\to W)}_{\mathcal{M}_W,1} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W,\mathfrak{m}} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{m}} \\
\phi'^{(V\to W)}_{\mathcal{M}_W+1,1} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W+1,\mathfrak{m}} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(V\to W)}_{\mathcal{M}_W+\ell_W,1} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,1} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}} + i\;\hat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}
\end{pmatrix}}_{\zeta_3=\mathbb{U}'_{V\to W}+i\hat{\mathbb{U}}'_{W\to W}} \right),
$$

$$
\left(\underbrace{\begin{pmatrix}
\beta'^{(W\rightsquigarrow V)}_{1,1} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{1,\mathfrak{n}} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(W\rightsquigarrow V)}_{m_V,1} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}} \\
\beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}} + i\;\hat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}
\end{pmatrix}}_{z_4=\mathbb{S}'_{W\to V}+i\hat{\mathbb{S}}'_{V\to V}}, \underbrace{\begin{pmatrix}
\phi'^{(W\rightsquigarrow V)}_{1,1} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{1,\mathfrak{m}} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(W\rightsquigarrow V)}_{m_V,1} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{m_V,\mathfrak{m}} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}} \\
\phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}} + i\;\hat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}
\end{pmatrix}}_{\zeta_4=\mathbb{U}'_{W\to V}+i\hat{\mathbb{U}}'_{V\to V}} \right) \right).
$$

Table 11

Remark 5 The key sets

$$\Omega_V = \Omega_{[(V,W)\rightsquigarrow V]}([0,1]) \text{ and } \Omega_W = \Omega_{[(V,W)\rightsquigarrow W]}([0,1])$$

of (extended or not) supervisory perceptions of two cyber nodes V and W into the system of themselves, that are used in critical definitions given up to now, are subsets of the product spaces

$$\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \text{ and } (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} \times (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}} = (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}.$$

The spaces $\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}$ and $\mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$ will be called complex multi-coordinate spaces. Each element of a complex multi-coordinate space $\mathbb{C}^{\mathcal{N}\times\nu}$ is of the form

$$\left(z^{(1)},\dots,z^{(\nu)}\right)$$

with $z^{(r)}=\left(z_1^{(r)},\dots,z_{\mathcal{N}}^{(r)}\right)^T\in\mathbb{C}^{\mathcal{N}}$. Similarly, the spaces $(\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}}=(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}}$ and $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}=(\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}}$ are called complex multi-projective spaces. Each element of a complex multi-projective space $(\mathbb{CP}^1)^{\mathcal{N}\times\nu}=(\mathbb{CP}^{\mathcal{N}})^{\nu}$ has the form

$$\left(\zeta^{(1)},\dots,\zeta^{(\nu)}\right)$$

with $\zeta^{(r)}=\left(\zeta_1^{(r)},\dots,\zeta_{\mathcal{N}}^{(r)}\right)^T\in\mathbb{CP}^{\mathcal{N}}$.

Below, for terminology consolidation purposes, we will prefer not make any distinction between the spaces $\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}$ and $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}}$, and we will call them using the common name complex multi-spaces. As usually, if there is no risk of confusion, the complex multi-spaces may also be represented using the common notation

$$\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}.$$

On the other hand, by Definition, we are also interested for the twofold Cartesian products of complex multi spaces. In fact, each momentary cyber interaction $\mathscr{g}$ can be considered as a correspondence derived from a map transforming a subset $\mathcal{D}$ of the twofold Cartesian product $\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}$ of complex multi-spaces within its own self:

$$\mathscr{g}\colon\mathcal{D}(\subset\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}})\ \to\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}:$$

$$\left(\begin{pmatrix} z_1^{(1)} & \cdots & z_1^{(\mathfrak{n})}\\ \vdots & \cdots & \vdots\\ z_{\mathcal{N}}^{(1)} & \cdots & z_{\mathcal{N}}^{(\mathfrak{n})}\end{pmatrix},\begin{pmatrix} \zeta_1^{(1)} & \cdots & \zeta_1^{(\mathfrak{m})}\\ \vdots & \cdots & \vdots\\ \zeta_{\mathcal{N}}^{(1)} & \cdots & \zeta_{\mathcal{N}}^{(\mathfrak{m})}\end{pmatrix}\right)\longmapsto$$

$$\mathscr{g}\left(\begin{pmatrix} z_1^{(1)} & \cdots & z_1^{(\mathfrak{n})}\\ \vdots & \cdots & \vdots\\ z_{\mathcal{N}}^{(1)} & \cdots & z_{\mathcal{N}}^{(\mathfrak{n})}\end{pmatrix},\begin{pmatrix} \zeta_1^{(1)} & \cdots & \zeta_1^{(\mathfrak{m})}\\ \vdots & \cdots & \vdots\\ \zeta_{\mathcal{N}}^{(1)} & \cdots & \zeta_{\mathcal{N}}^{(\mathfrak{m})}\end{pmatrix}\right)=$$

$$\left(\begin{pmatrix} {z'}_1^{(1)} & \cdots & {z'}_1^{(\mathfrak{n})}\\ \vdots & \cdots & \vdots\\ {z'}_{\mathcal{N}}^{(1)} & \cdots & {z'}_{\mathcal{N}}^{(\mathfrak{n})}\end{pmatrix},\begin{pmatrix} {w'}_1^{(1)} & \cdots & {w'}_1^{(\mathfrak{m})}\\ \vdots & \cdots & \vdots\\ {w'}_{\mathcal{N}}^{(1)} & \cdots & {w'}_{\mathcal{N}}^{\mu}\end{pmatrix}\right).$$

Such a mapping will be called (complex) twofold multi-mapping. In particular, a cyber-navigation is a chain of twofold multi-mappings ([10]).

9. Coherent Interactive Families

We now intend to look at the areas in which occurs an increase or decrease in cyber-valuations and/or cyber-vulnerabilities during a interplay of the cyber pair (V, W) over the time $t \in]\alpha, \beta[\subset\subset [0,1]$. Under this approach, we will see when an interaction is evolving into an attack.

For simplification purposes, we will limit ourselves only to the case where $\mathbb{CM} = \mathbb{C}$. A study of the general case will remain open.

In the finite case, we will distinguish two cases. The first case deals with interactions occurring in parts of interacting nodes, while the second case refers to interactions that are assumed throughout entire nodes To this end, suppose $X, Y \in \{V, W\}$ and $\mathscr{r} > 0$. Let $fr\left(dev_{\mu_1}^{(X)}\right),\ldots, fr\left(dev_{\mu_\nu}^{(X)}\right)$ be given $(\mu_1, \ldots, \mu_\nu)$ – device parts in X. Let also $fr\left(res_{\kappa_1}^{(X)}\right),,\ldots, fr\left(res_{\kappa_\lambda}^{(X)}\right)$ be given $(\kappa_1, \ldots, \kappa_\lambda)$ – resource parts in X. Let finally $\mathbb{I}$ be a given set into the time subinterval $]\alpha, \beta[\subset\subset [0,1]$. We need to introduce a certain terminology. A family of interactions $\mathcal{F} = \{Z = Z_{(Y,X)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in$
$(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4, t \in \mathbb{I}\}$, with associated family of cyber-interplays of the ordered cyber pair (Y, X) over the time $t \in]\alpha, \beta[$

$$\mathcal{D}_{\mathcal{F}} = \Big\{\mathscr{g} = \mathscr{g}^{(Z)} : \mathbb{I} \to \Omega_Y \times \Omega_X \times \Omega_Y \times \Omega_X : \\ t \mapsto \mathscr{g}^{(Z)}(t) := \left(\gamma_Y^{(Z)}(t), \gamma_X^{(Z)}(t), \gamma_Y^{(Z)}(t+\Delta t), \gamma_X^{(Z)}(t+\Delta t)\right) : Z \in \mathcal{F}\Big\},$$

is called coherent interactive family in $\mathbb{I}$, if there is a homotopy

$$H: \mathbb{I} \times [0,1] \to \Omega_Y \times \Omega_X \times \Omega_Y \times \Omega_X$$

such that, for each cyber-interplay $\mathscr{g} = \mathscr{g}^{(Z)} \in \mathcal{D}_{\mathcal{F}}$ there is a $p \in [0,1]$ satisfying $H(t, p) = \mathscr{g}(t)$ at any moment time $t \in \mathbb{I}$ on which the cyber-interplay $\mathscr{g} = \mathscr{g}^{(Z)}$ implements the interaction Z. Recall that, in topology, two continuous functions from one topological space to another are called homotopic (Greek ὁμός (homós) = same, similar, and τόπος (tópos) = place) if one can be "continuously deformed" into the other, such a deformation being called a homotopy between the two functions. Formally, a homotopy between two continuous functions f and g from a topological space U to a topological space V is defined to be a continuous function $H : U \times [0,1] \to V$ from the product of the space U with the unit interval $[0,1]$ to V such that, if $x \in U$ then

$$H(x, 0) = f(x) \text{ and } H(x, 1) = g(x).$$

10. Subjectivity in Interactive Variations Germs of Cyber Attacks

10.1 Germs of Correlated Cyber-Attacks

Often, outside the objectivity of evaluating cyber-attacks, there is also a subjective approach which sometimes can give very strong arguments in assessing the reality. In this direction, in this section, we will propose several definitions and cases for an alternate consideration based on the *subjectivity* of the users of the involved nodes. We point out that, in the following definitions, the foundation adopted was based exclusively on the Euclidean norms. However, this is not restrictive, and we can consider any other norm in place in $\mathbb{R}^{\mathfrak{n}}$ and $\mathbb{R}^{\mathfrak{m}}$.

Let us begin with the case of valuation variations relative to the norm valuation and the subjectivity of user(s) of another or same node.

Definition 9 *Let again $\mathbb{I}$ be any given set in the time subinterval $]\alpha,\beta[\ \subset\subset [0,1]$. Let also $X,Y \in \{V,W\}$.*

i. *The area $[\mathcal{A}_Y^-(X)](\mathbb{I})$ of correlated reduction of total valuation for node X as evaluated subjectively from the user(s) of Y over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\|Rez_4\| = \|\,\beta'^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ of the resulting overall valuation in the node X as evaluated from the viewpoint of the user(s) of Y at the next moment t' is less than the (Euclidean) norm $\|Rez_2\| = \left\|\beta^{(Y\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ of the initial overall valuation in the node X as evaluated from the viewpoint of the user(s) of Y at the preceding moment t:*

$$\|Rez_4\| = \|\,\beta'^{(Y\rightsquigarrow X)}\| < \left\|\beta^{(Y\rightsquigarrow X)}\right\| = \|Rez_2\|.$$

If the difference $\|Rez_2\| - \|Rez_4\|$ exceeds a given valuation danger threshold for node X as evaluated by the user(s) of Y, we say that the interactions $Z = Z_{(Y,X)}(t)$ of $[\mathcal{A}_Y^-(X)](\mathbb{I})$ are evaluated as subjectively damaging for X from the viewpoint of Y.

ii. *The area $[\mathcal{A}_X^-(X)](\mathbb{I})$ of correlated reduction of total valuation for node X as assessed subjectively by themselves the user(s) of node X over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) =$*

$\big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|Imz_4\| = \left\| \hat{\beta}'^{(X \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \hat{\beta}'^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall valuation in the node* X *as assessed by themselves the user(s) of* X *at the next moment* t' *is less than the (Euclidean) norm* $\|Imz_2\| = \left\|\hat{\beta}^{(X \rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \hat{\beta}^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the initial overall valuation in the node* V *as assessed by themselves the user(s) of* V *at the preceding moment* t*:*

$$\|Imz_4\| = \left\| \hat{\beta}'^{(X \rightsquigarrow X)} \right\| < \left\|\hat{\beta}^{(X \rightsquigarrow X)}\right\| = \|Imz_2\|.$$

If the difference $\|Imz_2\| - \|Imz_4\|$ *exceeds a given valuation danger threshold for node* X *as assessed by themselves the user(s) of* X*, we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_X^-(X)](\mathbb{I})$ *are evaluated as reflexively damaging from the viewpoint of* X*.*

iii. *The area* $[\mathcal{A}_Y^+(X)](\mathbb{I})$ *of correlated growth of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|Rez_4\| = \left\| \beta'^{(Y \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \beta'^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is greater than the (Euclidean) norm* $\|Rez_2\| = \left\|\beta^{(Y \rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \beta^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the initial overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Rez_4\| = \left\| \beta'^{(Y \rightsquigarrow X)} \right\| > \left\|\beta^{(Y \rightsquigarrow X)}\right\| = \|Rez_2\|.$$

If the difference $\|Rez_4\| - \|Rez_2\|$ *exceeds a given valuation benefit limit for node* X *as evaluated by the user(s) of* Y*, we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_Y^+(X)](\mathbb{I})$ *are evaluated as subjectively advantageous for* X *from the viewpoint of* Y*.*

iv. *The area* $[\mathcal{A}_X^+(X)](\mathbb{I})$ *of correlated growth of total valuation for node* X *as evaluated subjectively from the user(s) of* X *over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|Imz_4\| =$

$\left\| \hat{\beta}'^{(X \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \hat{\beta}'^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is greater than the (Euclidean) norm* $\|Imz_2\| = \left\|\hat{\beta}^{(X \rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \hat{\beta}^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the initial overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Imz_4\| \left\| \hat{\beta}'^{(X \rightsquigarrow X)} \right\| > \left\|\hat{\beta}^{(X \rightsquigarrow X)}\right\| = \|Imz_2\|.$$

If the difference $\|Imz_4\| - \|Imz_2\|$ *exceeds a given valuation danger threshold for node* V *as assessed by themselves the user(s) of* V*, we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_X^{+}(X)](\mathbb{I})$ *are evaluated as reflexively advantageous from the viewpoint of* X*.*

Similar considerations apply to the vulnerability variations relative only to the user(s) of another or the same node.

Definition 10 *Let again* $\mathbb{I}$ *be any given subset of the time interval* $]\alpha, \beta[\subset\subset [0,1]$*. Let also* $X, Y \in \{V, W\}$*.*

i *The area* $[\mathcal{B}_Y^{-}(X)](\mathbb{I})$ *of correlated reduction of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* Y *over the time set* $\mathbb{I}$ *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|Re\zeta_4\| = \left\| \phi'^{(Y \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \phi'^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is less than the (Euclidean) norm* $\|Re\zeta_2\| = \left\|\phi^{(Y \rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \phi^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the initial overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Re\zeta_4\| = \left\| \phi'^{(Y \rightsquigarrow X)} \right\| < \left\|\phi^{(Y \rightsquigarrow X)}\right\| = \|Re\zeta_2\|.$$

If the difference $\|Re\zeta_2\| - \|Re\zeta_4\|$ *exceeds a given vulnerability danger threshold for node* X *as evaluated by the user(s) of* Y*, we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_Y^{-}(X)](\mathbb{I})$ *are evaluated as subjectively painless for* X *from the viewpoint of* Y*.*

ii *The area* $[\mathcal{B}_Y^+(X)](\mathbb{I})$ *of correlated growth of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* Y *over the time set* $\mathbb{I}$ *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Re\zeta_4\| = \|\phi'^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\phi'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is greater than the (Euclidean) norm* $\|Re\zeta_2\| = \|\phi^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\phi^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Re\zeta_4\| = \|\phi'^{(Y\rightsquigarrow X)}\| > \|\phi^{(Y\rightsquigarrow X)}\| = \|Re\zeta_2\|.$$

If the difference $\|Re\zeta_4\| - \|Re\zeta_2\|$ *exceeds a given vulnerability benefit limit for node* X *as evaluated by the user(s) of* Y, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_Y^+(X)](\mathbb{I})$ *are evaluated as subjectively painful for* X *from the viewpoint of* Y.

iii *The area* $[\mathcal{B}_X^-(X)](\mathbb{I})$ *of correlated reduction of total vulnerability for node* X *as assessed subjectively by themselves the user(s) of node* X *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Im\zeta_4\| = \left\|\hat{\phi}'^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\phi}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as assessed by themselves the user(s) of* X *at the next moment* t' *is less than the (Euclidean) norm* $\|Im\zeta_2\| = \|\hat{\phi}^{(X\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\phi}^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as assessed by themselves the user(s) of* X *at the preceding moment* t*:*

$$\|Im\zeta_4\| = \left\|\hat{\phi}'^{(X\rightsquigarrow X)}\right\| < \|\hat{\phi}^{(X\rightsquigarrow X)}\| = \|Im\zeta_2\|.$$

If the difference $\|Im\zeta_2\| - \|Im\zeta_4\|$ *exceeds a given vulnerability danger threshold for node* X *as assessed by themselves the user(s) of* X, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_X^-(X)](\mathbb{I})$ *are evaluated as subjectively painless for* X *from the viewpoint of* X *itself.*

iv *The area* $[\mathcal{B}_X^+(X)](\mathbb{I})$ *of correlated growth of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* X *over the time set* $\mathbb{I}$, *is* *the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|Im\zeta_4\| = \left\| \hat{\phi}'^{(X\rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\hat{\phi}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is greater than the (Euclidean) norm* $\|Im\zeta_2\| = \left\|\hat{\phi}^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\hat{\phi}^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Im\zeta_4\| \left\| \hat{\phi}'^{(X\rightsquigarrow X)} \right\| > \left\|\hat{\phi}^{(X\rightsquigarrow X)}\right\| = \|Im\zeta_2\|.$$

If the difference $\|Imz_4\| - \|Imz_2\|$ *exceeds a given vulnerability danger threshold for node* X *as assessed by themselves the user(s) of* X*, we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_X^+(X)](\mathbb{I})$ *are evaluated as subjectively painful for* X *from the viewpoint of* X *itself.*

Definition 11 *A germ of correlated cyber-attack from* W *against* V*, during a given time set* $\mathbb{I}$ *in* $]\alpha,\beta[\subset\subset [0,1]$*, is a family of coherent interactions* $Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$*,* $t \in \mathbb{I}$*, lying in the so called correlated danger sector* $\mathfrak{X} = \mathfrak{X}_{W\to V}(\mathbb{I})$ *to the node* V *from the node* W *during the entire time set* $\mathbb{I}$*, defined by intersection*

$$\mathfrak{X} = \big\{\big([\mathcal{A}_W^-(V)](\mathbb{I}) \cap [\mathcal{A}_V^-(V)](\mathbb{I})\big) \cap \big([\mathcal{A}_V^+(W)](\mathbb{I}) \cap [\mathcal{A}_W^+(W)](\mathbb{I})\big) \cap \big([\mathcal{B}_V^-(W)](\mathbb{I}) \cap [\mathcal{B}_W^-(W)](\mathbb{I})\big) \cap \big([\mathcal{B}_V^+(V)](\mathbb{I}) \cap [\mathcal{B}_W^+(V)](\mathbb{I})\big)\big\},$$

provided, of course, that $\mathfrak{X} \neq \emptyset$*. If each one of the coherent interactions* $Z_{(W,V)}(t)$ *is elementary, we say that the germ is elementary; otherwise, it is called sequential or complex. If* $\mathbb{I} = \{t_0\}$ *for some* $t_0 \in]0,1[$*, the germ is called momentary.*

Definition 12 *The node* V *is said to be affine secure from attacks of* W *during the time set* $\mathbb{I}$ *if* $\mathfrak{X} = \emptyset$*.*

Definition 13 *More generally, an affine secure area of* V *from the correlated cyber attacks of* W *during the time set* $\mathbb{I}$ *is any set in the complementary* $\mathfrak{X}^C$ *in* $(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *of* $\mathfrak{X}$*.*

10.2 Germs of Absolute Cyber-Attacks

Next, we consider the case of valuation variations relative only to the user(s) of another or the same node and independently of the valuation variations of this node.

Definition 14 *Let* $\mathbb{I}$ *be any given set in the time subinterval* $]\alpha,\beta[\subset\subset[0,1]$. *Let also* $X,Y\in\{V,W\}$.

i *The area* $[\tilde{A}_Y^-(X)](\mathbb{I})$ *of absolute reduction of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z=Z_{(Y,X)}(t)\in(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|\beta'^{(Y\rightsquigarrow X)}\|:=\left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at a next moment* t' *is less than a given threshold* $\mathcal{C}$*:*

$$\|\beta'^{(Y\rightsquigarrow X)}\|<\mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of total valuation reduction in* X *from the viewpoint of* Y. *If the extensibility radius* $\mathcal{C}$ *is less than a given valuation damage threshold* $\mathcal{V}a\ell_Y(X)$, *we say that* $[\tilde{A}_Y^-(X)](\mathbb{I})$ *is an area of absolute danger for* X *as evaluated subjectively by the user(s) of* Y.

ii *The area* $[\tilde{A}_X^-(X)](\mathbb{I})$ *of absolute reduction of total valuation for node* X *as assessed subjectively by themselves the user(s) of node* X *themselves over the time set* $\mathbb{I}$, *s the family of coherent interactions* $Z=Z_{(Y,X)}(t)\in(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\left\|\hat{\beta}'^{(X\rightsquigarrow X)}\right\|:=\left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\beta}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is less than a threshold* $\mathcal{C}$*:*

$$\left\|\hat{\beta}'^{(X\rightsquigarrow X)}\right\|<\mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total valuation reduction in* X *from the viewpoint of* X *itself. If this extensibility radius* $\mathcal{C}$ *is less than a given valuation damage threshold* $\mathcal{V}a\ell_X(X)$, *we say that* $[\tilde{A}_X^-(X)](\mathbb{I})$ *is an area of absolute danger of node* X *as evaluated subjectively by the user(s) of* X.

iii *The area* $[\tilde{A}_Y^+(X)](\mathbb{I})$ *of absolute growth of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$, *is the family of* *coherent interactions $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\| \beta'^{(Y \rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\beta'^{(Y \rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ of the resulting overall valuation in the node X as evaluated from the viewpoint of the user(s) of Y at the next moment t' is greater that a threshold $\mathcal{C}$:*

$$\| \beta'^{(Y \rightsquigarrow X)}\| > \mathcal{C}.$$

The number $\mathcal{C}$ is called extensibility radius of the total valuation growth in X from the viewpoint of Y. If this extensibility radius $\mathcal{C}$ is greater than a given valuation benefit limit $\mathcal{BenLim}_Y(X)$, we say that $\left[\tilde{A}_Y^+(X)\right](\mathbb{I})$ is an area of absolute security of node X as evaluated subjectively by the user(s) of Y.

iv *The area $\left[\tilde{A}_X^+(X)\right](\mathbb{I})$ of absolute growth of total valuation for node X as evaluated subjectively from the user(s) of X themselves over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\left\| \hat{\beta}'^{(X \rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\hat{\beta}'^{(X \rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ of the resulting overall valuation in node X as evaluated from the viewpoint of the user(s) of X at the next moment t' is greater that a threshold $\mathcal{C}$:*

$$\left\| \hat{\beta}'^{(X \rightsquigarrow X)}\right\| > \mathcal{C}.$$

The number $\mathcal{C}$ is called extensibility radius of the total valuation growth in X from the viewpoint of X itself. If this extensibility radius $\mathcal{C}$ is greater than a given valuation benefit limit $\mathcal{BenLim}_X(X)$, we say that $\left[\tilde{A}_X^+(X)\right](\mathbb{I})$ is an area of absolute security of node X as evaluated subjectively from the viewpoint of X itself.

Next, we consider the case of valuation variations relative only to the user(s) of another or the same node and independently of the valuation variations of this node.

Definition 15 *Let again $\mathbb{I}$ be a given set in the time subinterval $]\alpha,\beta[\subset\subset [0,1]$. Let also $X, Y \in \{V, W\}$.*

i *The area $[\tilde{B}_Y^-(X)](\mathbb{I})$ of absolute reduction of total vulnerability for node X as evaluated subjectively from the viewpoint of the user(s) of Y over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\| \phi'^{(Y \rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_Y+\mathcal{L}_X} \left|\phi'^{(Y \rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ of the resulting overall vulnerability in the node X as evaluated from the viewpoint of* *the user(s) of Y at the next moment t' is less than a given threshold $\mathcal{C}$:*

$$\| \phi'^{(Y \rightsquigarrow X)} \| < C.$$

The number $\mathcal{C}$ is called extensibility radius of the total vulnerability reduction in X from the viewpoint of Y. If this extensibility radius $\mathcal{C}$ is less than a given vulnerability benefit limit $\widetilde{\mathcal{BenLim}}_Y(X)$, we say that $[\tilde{B}_Y^-(X)](\mathbb{I})$ is a secure area for node X as evaluated subjectively from the user(s) of Y.

ii *The area $[\tilde{B}_Y^+(X)](\mathbb{I})$ of absolute growth of total vulnerability for node X as evaluated subjectively from the viewpoint of the user(s) of Y over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\| \phi'^{(Y \rightsquigarrow X)} \| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X + \mathcal{L}_X} \left| \phi'^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ of the resulting overall vulnerability in the node X as evaluated from the viewpoint of the user(s) of Y at the next moment t' is greater that a threshold $\mathcal{C}$:*

$$\| \phi'^{(Y \rightsquigarrow X)} \| > C.$$

The number $\mathcal{C}$ is called extensibility radius of the total vulnerability growth in $X$from the viewpoint of Y. If this extensibility radius $\mathcal{C}$ is greater than a given vulnerability damaging threshold $\mathcal{Vul}_Y(X)$ for node X as evaluated by the user(s) of Y, we say that $[\tilde{B}_Y^+(X)](\mathbb{I})$ is a damaging area for X from the viewpoint of Y.

iii *The area $[\tilde{B}_X^-(X)](\mathbb{I})$ of absolute reduction of total vulnerability for node X as assessed subjectively by the user(s) of node X themselves over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\left\| \hat{\phi}'^{(X \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X + \mathcal{L}_X} \left| \hat{\phi}'^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ of the resulting overall vulnerability in the node X as evaluated from the viewpoint of the user(s) of X at the next moment t' is less than a threshold $\mathcal{C}$:*

$$\left\| \hat{\phi}'^{(X \rightsquigarrow X)} \right\| < C.$$

The number $\mathcal{C}$ is called extensibility radius of total vulnerability reduction in X from the viewpoint of X itself. If this extensibility radius $\mathcal{C}$ is less than a given vulnerability benefit $\widetilde{\mathcal{BenLim}}_X(X)$, we say that $[\tilde{B}_Y^-(X)](\mathbb{I})$ is a subjectively secure area for X.

iv *The area $[\tilde{B}_X^+(X)](\mathbb{I})$ of absolute growth of total vulnerability for node X as evaluated subjectively from the viewpoint of the user(s) of V themselves over the time set $\mathbb{I}$, is the family of coherent interactions $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for which the (Euclidean) norm $\left\| \hat{\phi}'^{(X \rightsquigarrow X)} \right\| :=$*

$\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\phi}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node X as evaluated from the viewpoint of the user(s) of X at the next moment t' is greater that a threshold $\mathcal{C}$:*

$$\left\| \hat{\phi}'^{(X\rightsquigarrow X)} \right\| > \mathcal{C}.$$

The number $\mathcal{C}$ is called extensibility radius of the total vulnerability growth in V from the viewpoint of X itself. If this extensibility radius $\mathcal{C}$ is greater than a given vulnerability damaging threshold $\mathcal{V}u\ell_X(X)$ for node X as evaluated by the user(s) of X themselves, we say that $[\tilde{B}_X^+(X)](\mathbb{I})$ is a subjectively damaging area for X.

Definition 16 *A germ of absolute cyber attack from W against V, during a given time set $\mathbb{I}$ in the subinterval $]\alpha,\beta[\ \subset\subset [0,1]$, is a family of <u>coherent</u> interactions $Z = Z_{(W,V)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$, $t \in \mathbb{I}$, lying in the so called absolute danger sector $\tilde{\mathfrak{X}} = \tilde{\mathfrak{X}}_{W\to V}$ to the node V from the node W during the entire time set $\mathbb{I}$, defined by intersection*

$$\tilde{\mathfrak{X}} = \Big\{\Big([\tilde{A}_W^-(V)](\mathbb{I})\cap[\tilde{A}_V^-(V)](\mathbb{I})\Big)\cap\Big([\tilde{A}_V^+(W)](\mathbb{I})\cap[\tilde{A}_W^+(W)](\mathbb{I})\Big)\cap$$
$$\big(\{[\tilde{B}_V^-(W)](\mathbb{I})\cap[\tilde{B}_W^-(W)](\mathbb{I})\}\big)\cap\Big([\tilde{B}_V^+(V)](\mathbb{I})\cap[\tilde{B}_W^+(V)](\mathbb{I})\Big)\Big\}.$$

provided, of course, that $\tilde{\mathfrak{X}} \neq \emptyset$. If each one of the coherent interactions $Z_{(W,V)}(t)$ is elementary, we say that the germ is said to be elementary; otherwise, it is called sequential or complex. If $\mathbb{I} = \{t_0\}$ for some $t_0 \in\ ]\alpha,\beta[$, the germ is called momentary.

Definition 17 *The node V is absolutely secure from cyber attacks of W during the time set $\mathbb{I}$ if $\tilde{\mathfrak{X}} = \emptyset$.*

Definition 18. *And, more generally, an absolutely secure area for node V from cyber attacks of W during the time set $\mathbb{I}$ is any set in the complementary $\tilde{\mathfrak{X}}^C$ in $(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ of $\tilde{\mathfrak{X}}$.*

10.3 Germs of partial cyber-attacks

It is known that cyber-attacks carried out in a targeted or oriented manner against specific parts of particular devices or against specific parts of particular resources. So, in this section, we will consider the case of partial interactions, i.e., of cyber interactions between parts of some devices or resources cyber two nodes. To do this, let's again

$$X, Y \in \{V, W\} \text{ and } r > 0.$$

Let also $(\mu_1, \dots, \mu_\nu)$ – device parts, say

$$fr\left(dev_{\mu_1}^{(X)}\right), fr\left(dev_{\mu_2}^{(X)}\right), \dots, fr\left(dev_{\mu_\nu}^{(X)}\right)$$

of X, and $(\kappa_1, \dots, \kappa_\lambda)$ – resource parts, say

$$fr\left(res_{\kappa_1}^{(X)}\right), fr\left(res_{\kappa_2}^{(X)}\right), \dots, fr\left(res_{\kappa_\lambda}^{(X)}\right)$$

of X. Let finally $\mathbb{I}$ be a given set in the time subinterval $]\alpha, \beta[\subset\subset [0,1]$.

Definition 19 *Let $\mathbb{I}$ be a given set in the time subinterval $]\alpha, \beta[\subset\subset [0,1]$. Let also $X, Y \in \{V, W\}$.*

i. *The region $[\mathcal{R}_{\mathbb{S}-}(X)](\mathbb{I})$ (or simply denoted by $[\mathcal{R}_1(X)](\mathbb{I})$) of partial valuation reduction of node V as evaluated subjectively from the viewpoint of the user(s) of W over the time set $\mathbb{I}$, with extensiveness radius $r > 0$, in the $(\mu_1, \dots, \mu_\nu)$ – device parts $fr\left(dev_{\mu_1}^{(X)}\right)$, $fr\left(dev_{\mu_2}^{(X)}\right), \dots, fr\left(dev_{\mu_\nu}^{(X)}\right)$ of X and the $(\kappa_1, \dots, \kappa_\lambda)$ – resource parts $fr\left(res_{\kappa_1}^{(X)}\right)$, $fr\left(res_{\kappa_2}^{(X)}\right), \dots, fr\left(res_{\kappa_\lambda}^{(X)}\right)$ of X is the set of all coherent interactions $Z = Z_{(W,V)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), \quad (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4$ between Y and X in $\mathbb{I}$, for each of which the corresponding index- set:*

$$\Big\{ j \in \{\mu_1, \dots, \mu_\nu, \kappa_1, \dots, \kappa_\lambda\} : \sum_{k=1}^{\mathfrak{n}} \left| \beta_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\beta}_{j,k}^{(Y \rightsquigarrow X)} \right|^2 > \sum_{k=1}^{\mathfrak{n}} \left| \beta'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y \rightsquigarrow X)}_{j,k} \right|^2 \text{ with at least one}$$

$$\text{index } k \in \{1, 2, \dots, \mathfrak{n}\} \text{ being such that } \left| \beta_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\beta}_{j,k}^{(Y \rightsquigarrow X)} \right| - \left| \beta'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y \rightsquigarrow X)}_{j,k} \right| > r \Big\},$$

whenever $Y = V, W$. If the extensiveness radius r of $[\mathcal{R}_{\mathbb{S}-}(X)](\mathbb{I})$ is greater that a given valuation damage threshold, the interactions $Z = Z_{(Y,X)}(t)$ of $[\mathcal{R}_{\mathbb{S}-}(X)](\mathbb{I})$ are said to be damaging in X.

ii *The region $[\mathcal{R}_{\mathbb{S}+}(X)](\mathbb{I})$ (or simply denoted by $[\mathcal{R}_2(X)](\mathbb{I})$) of partial valuation growth of node V as evaluated subjectively from the viewpoint of the user(s) of W over the time set $\mathbb{I}$, with extensiveness radius r, in the $(\mu_1, \dots, \mu_\nu)$ – device parts $fr\left(dev_{\mu_1}^{(X)}\right)$, $fr\left(dev_{\mu_2}^{(X)}\right), \dots, fr\left(dev_{\mu_\nu}^{(X)}\right)$ of X and the $(\kappa_1, \dots, \kappa_\lambda)$ – resource parts $fr\left(res_{\kappa_1}^{(X)}\right)$, $fr\left(res_{\kappa_2}^{(X)}\right), \dots, fr\left(res_{\kappa_\lambda}^{(X)}\right)$ of X is the set of all coherent* *interactions* $Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for each of which the corresponding index- set:*

$$\Big\{j \in \{\mu_1,\dots,\mu_\nu,\kappa_1,\dots,\kappa_\lambda\} \colon \sum_{k=1}^{\mathfrak{n}} \Big|\beta'^{(Y\rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y\rightsquigarrow X)}_{j,k}\Big|^2 > \sum_{k=1}^{\mathfrak{n}} \Big|\beta^{(Y\rightsquigarrow X)}_{j,k} + i\ \hat{\beta}^{(Y\rightsquigarrow X)}_{j,k}\Big|^2 \text{ with at least one index } k \in \{1,2,\dots,\mathfrak{n}\} \text{ being such that } \Big|\beta'^{(Y\rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y\rightsquigarrow X)}_{j,k}\Big| - \Big|\beta^{(Y\rightsquigarrow X)}_{j,k} + i\ \hat{\beta}^{(Y\rightsquigarrow X)}_{j,k}\Big| > r\Big\},$$

whenever $Y = V, W$*. If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{S}+}(X)](\mathbb{I})$ *is greater that a given valuation benefit limit, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{S}+}(X)](\mathbb{I})$ *are said to be advantageous in* X*.*

iii. *The region* $[\mathcal{R}_{\mathbb{U}-}(X)](\mathbb{I})$ *(or simply denoted by* $[\mathcal{R}_3(X)](\mathbb{I})$*) of partial vulnerability reduction of node* V *as evaluated subjectively from the viewpoint of the user(s) of* W *over the time set* $\mathbb{I}$*, with extensiveness radius* r*, in the* $(\mu_1,\dots,\mu_\nu)-$ *device parts* $fr\big(dev^{(V)}_{\mu_1}\big)$*,* $fr\big(dev^{(V)}_{\mu_2}\big)$*,…,* $fr\big(dev^{(V)}_{\mu_\nu}\big)$ *of* V *and the* $(\kappa_1,\dots,\kappa_\lambda)-$ *resource parts* $fr\big(res^{(V)}_{\kappa_1}\big)$*,* $fr\big(res^{(V)}_{\kappa_2}\big)$*,…,* $fr\big(res^{(V)}_{\kappa_\lambda}\big)$ *of* V *is the set of all coherent interactions* $Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for each of which the corresponding index- set:*

$$\Big\{j \in \{\mu_1,\dots,\mu_\nu,\kappa_1,\dots,\kappa_\lambda\} \colon \sum_{k=1}^{\mathfrak{m}} \Big|\phi^{(Y\rightsquigarrow X)}_{j,k} + i\ \hat{\phi}^{(Y\rightsquigarrow X)}_{j,k}\Big|^2 > \sum_{k=1}^{\mathfrak{m}} \Big|\phi'^{(Y\rightsquigarrow X)}_{j,k} + i\ \widehat{\phi}'^{(Y\rightsquigarrow X)}_{j,k}\Big|^2 \text{ with at least one index } k \in \{1,2,\dots,\mathfrak{m}\} \text{ being such that } \Big|\phi^{(Y\rightsquigarrow X)}_{j,k} + i\ \hat{\phi}^{(Y\rightsquigarrow X)}_{j,k}\Big| - \Big|\phi'^{(Y\rightsquigarrow X)}_{j,k} + i\ \widehat{\phi}'^{(Y\rightsquigarrow X)}_{j,k}\Big| > r\Big\},$$

whenever $Y = V, W$*. If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{U}-}(X)](\mathbb{I})$ *is less than a given vulnerability damage threshold, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{U}-}(X)](\mathbb{I})$ *are said to be advantageous in* X*.*

iv. *The region* $[\mathcal{R}_{\mathbb{U}+}(X)](\mathbb{I})$ *(or simply denoted by* $[\mathcal{R}_4(X)](\mathbb{I})$*) of partial vulnerability growth of node* V *as evaluated subjectively from the viewpoint of the user(s) of* W *over the time set* $\mathbb{I}$*, with extensiveness radius* r*, in the* $(\mu_1,\dots,\mu_\nu)-$ *device parts* $fr\big(dev^{(X)}_{\mu_1}\big)$*,* $fr\big(dev^{(X)}_{\mu_2}\big)$*,…,* $fr\big(dev^{(X)}_{\mu_\nu}\big)$ *of* X *and the* $(\kappa_1,\dots,\kappa_\lambda)-$ *resource parts* $fr\big(res^{(X)}_{\kappa_1}\big)$*,* $fr\big(res^{(X)}_{\kappa_2}\big)$*,…,* $fr\big(res^{(X)}_{\kappa_\lambda}\big)$ *of* X *is the set of all coherent interactions* $Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for each of which the corresponding index- set:*

$\Big\{j \in \{\mu_1, \dots, \mu_\nu, \kappa_1, \dots, \kappa_\lambda\}: \sum_{k=1}^{\mathfrak{m}} \left|\phi'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\phi}'^{(Y \rightsquigarrow X)}_{j,k}\right|^2 > \sum_{k=1}^{\mathfrak{m}} \left|\phi^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\phi}^{(Y \rightsquigarrow X)}_{j,k}\right|^2$ *with at least one index* $k \in \{1,2,\dots,\mathfrak{m}\}$ *being such that* $\left|\phi'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\phi}'^{(Y \rightsquigarrow X)}_{j,k}\right| - \left|\phi^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\phi}^{(Y \rightsquigarrow X)}_{j,k}\right| > r\Big\}$,

whenever $Y = V, W$. *If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{U}+}(X)](\mathbb{I})$ *is greater than a given vulnerability damage threshold, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{U}+}(X)](\mathbb{I})$ *are said to be damaging in* X.

Based on this preliminary material, we are now able to give the following general definition.

Definition 20 *A germ of partial cyber-attack from* W *against the* $(\mu_1, \dots, \mu_\nu) -$ *device parts* $fr\left(dev^{(V)}_{\mu_1}\right)$, $fr\left(dev^{(V)}_{\mu_2}\right), \dots, fr\left(dev^{(V)}_{\mu_\nu}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda) -$ *resource parts* $fr\left(res^{(V)}_{\kappa_1}\right)$, $fr\left(res^{(V)}_{\kappa_2}\right), \dots, fr\left(res^{(V)}_{\kappa_\lambda}\right)$ *of* V, *during a given time subset* $\mathbb{I}$ *of a subinterval* $[\alpha, \beta] \subset\subset [0,1]$, *is a family of coherent interactions* $Z = Z_{(W,V)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4$, $t \in \mathbb{I}$, *lying in the so called partial danger sector* $\mathcal{E} = \mathcal{E}_{W \to V}$ *to the node* V *from the node* W *during the entire time set* $\mathbb{I}$, *defined by intersection*

$$\mathcal{E} := [\mathcal{R}_{\mathbb{S}-}(V)](\mathbb{I}) \cap [\mathcal{R}_{\mathbb{S}+}(W)](\mathbb{I}) \cap [\mathcal{R}_{\mathbb{U}-}(W)](\mathbb{I}) \cap [\mathcal{R}_{\mathbb{U}+}(V)](\mathbb{I}).$$

If a coherent interaction is elementary, we say that the continuous cyber-attack is elementary; otherwise, it is called sequential or complex. If $\mathbb{I} = \{t_0\}$ *for some* $t_0 \in]0,1[$, *the germ is called momentary.*

Definition 21 *The node* V *is partially secure from cyber attacks of* W *against the* $(\mu_1, \dots, \mu_\nu) -$ *device parts* $fr\left(dev^{(V)}_{\mu_1}\right)$, $fr\left(dev^{(V)}_{\mu_2}\right), \dots, fr\left(dev^{(V)}_{\mu_\nu}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda) -$ *resource parts* $fr\left(res^{(V)}_{\kappa_1}\right)$, $fr\left(res^{(V)}_{\kappa_2}\right), \dots, fr\left(res^{(V)}_{\kappa_\lambda}\right)$ *of* V, *during a given closed time subinterval* $\mathbb{I} \subset\subset]0,1[$, *if* $\mathcal{E} = \emptyset$.

Definition 22 *And, more generally, a partially secure area for node* V *from cyber attacks of* W *against the* $(\mu_1, \dots, \mu_\nu) -$ *device parts* $fr\left(dev^{(V)}_{\mu_1}\right)$, $fr\left(dev^{(V)}_{\mu_2}\right), \dots, fr\left(dev^{(V)}_{\mu_\nu}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda) -$ *resource parts* $fr\left(res^{(V)}_{\kappa_1}\right)$, $fr\left(res^{(V)}_{\kappa_2}\right), \dots, fr\left(res^{(V)}_{\kappa_\lambda}\right)$ *of* V, *during a given closed time subinterval* $\mathbb{I} \subset\subset]0,1[$, *is any set in the complementary* $\mathcal{E}^C$ *in* $(\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4$ *of* $\mathcal{E}$.

11. Description of Cyber Navigations and Protection from Unplanned Attacks

11.1 Cyber navigations

Cyber navigation refers to the process of navigating a network of information resources in cyberspace, which is organized as hypertext or hypermedia. The mathematical modeling of cyber-navigation and its risks, as well as protection against such risks will be the main theme of this session. To this direction, let us begin with the following definition.

Definition 23 Suppose $t = t_0 < t_1 < \cdots < t_k = t'$ is a partition of the interval $[t, t'] \subset]0, 1[$.

i. The corresponding **cyber walk** with start node $V_{(x_1,x_2,x_3,t_0)}$ in the source $ob\big(cy(t_0)\big)$ and final node $V_{(x_1,x_2,x_3,t_k)}$ in the ending $ob\big(cy(t_k)\big)$ is an ordered node quote

$$V_0 V_1 \dots V_k = \underbrace{V_{(x_1,x_2,x_3,t_0)}}_{\in ob(cy(t_0))} \underbrace{V_{(x_1,x_2,x_3,t_1)}}_{\in ob(F_1[cy(t_0)])} \cdots \underbrace{V_{(x_1,x_2,x_3,t_k)}}_{\in ob([F_k \circ \dots \circ F_1][cy(t_0)])},$$

defined by given mappings

$$F_i: \underbrace{\{cy: \mathbb{I} \to ([ob(W_e)], d_{W_e})\}}_{T} \to \underbrace{\{cy: \mathbb{I} \to ([ob(W_e)], d_{W_e})\}}_{T},\ i = 1, 2, \dots, k$$

with the following three properties

1) $cy(t_\nu) = [F_\nu \circ \dots \circ F_1][cy(t_0)], \nu = 1, 2, \dots, k$

2) $V_0, V_1 \in ob(F_1[cy(t_0)]), V_1, V_2 \in ob([F_2 \circ F_1][cy(t_0)]), \dots$

$\dots, V_{k-1}, V_k \in ob([F_k \circ \dots \circ F_1][cy(t_0)])$

3) $h_1 = [V_0, V_1] \in hom(F_1[cy(t_0)]), \dots, h_k = [V_{k-1}, V_k] \in hom([F_k \circ \dots \circ F_1][cy(t_0)])$.

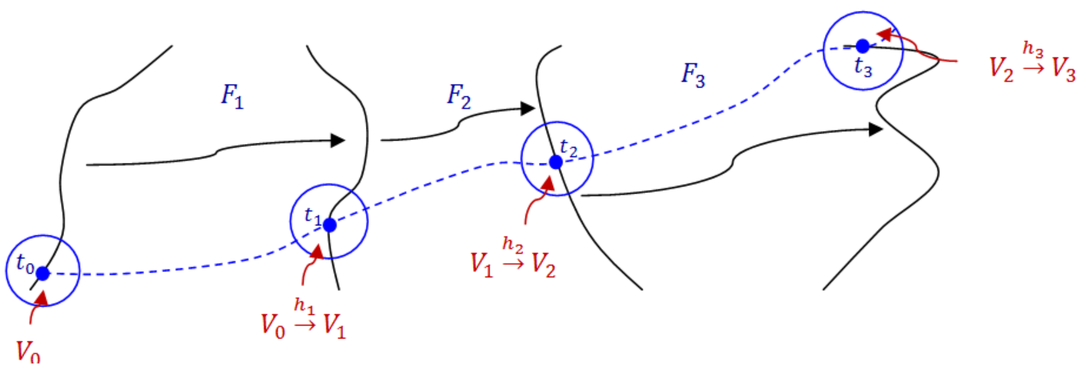

Figure 3 (Cyber Walk)

ii. A **cyber navigation** of the cyber node $U = U_{(x_1,x_2,x_3,t)} \in \cap_{\alpha=1}^{k} ob\big(cy(t_\alpha)\big)$ (over a cyber walk from the node V_0 up to the node V_k) is a finite sequence of reflexive cyber-effects

$$\aleph = \Big(\mathscr{g}_0 \equiv \mathscr{g}_{t_0}: \mathbb{G}_t^{(U)} \to \mathbb{G}_{t+\Delta t}^{(V_0)} \forall t \in [t_0, t_1[,$$
$$\mathscr{g}_1 \equiv \mathscr{g}_{t_1}: \mathbb{G}_t^{(U)} \to \mathbb{G}_{t+\Delta t}^{(V_1)} \forall t \in [t_1, t_2[,$$
$$\ldots$$
$$\mathscr{g}_{k-1} \equiv \mathscr{g}_{t_{k-1}}: \mathbb{G}_t^{(U)} \to \mathbb{G}_{t+\Delta t}^{(V_{k-1})} \forall t \in [t_{k-1}, t_k[,$$
$$\mathscr{g}_k \equiv \mathscr{g}_{t_k}: \mathbb{G}_{t_k}^{(U)} \to \mathbb{G}_{t_k}^{(V_k)}\Big)$$

such that the ordered node quote $V_0 V_1 \ldots V_k$ is a cyber walk and the diagrams below commute

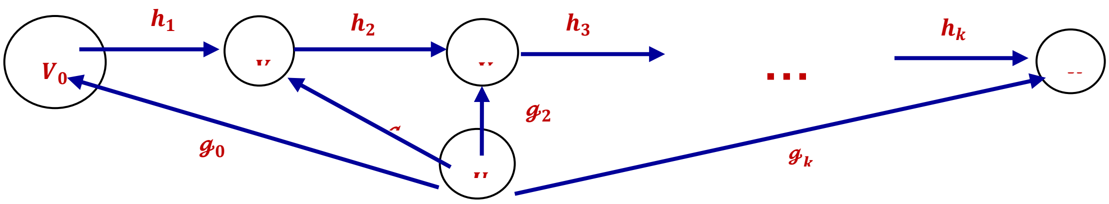

Figure 4 (Cyber Navigation)

in the sense that $\mathscr{g}_1 = h_1 \circ \mathscr{g}_0, \mathscr{g}_2 = h_2 \circ \mathscr{g}_1, \ldots, \mathscr{g}_k = h_k \circ \mathscr{g}_{k-1}$ It is clear that

$$\mathscr{g}_k = h_k \circ h_{k-1} \circ \ldots \circ h_2 \circ h_1 \circ \mathscr{g}_0 = h \circ \mathscr{g}_0 \text{where } h := h_k \circ \ldots \circ h_1. \blacksquare$$

11.2 Inadequacy of Cyber Nodes

Suppose $t = t_0 < t_1 < \cdots < t_k = t'$ is a partition of the interval $[t, t'] \subset]0, 1[$. Let

$$V_0 V_1 \ldots V_k = \underbrace{V_{(x_1, x_2, x_3)(t_0)}}_{\in ob(cy(t_0))} \underbrace{V_{(x_1, x_2, x_3)(t_1)}}_{\in ob(F_1[cy(t_0)])} \cdots \underbrace{V_{(x_1, x_2, x_3)(t_n)}}_{\in ob([F_k \circ \ldots \circ F_1][cy(t_0)])}$$

be corresponding walk with starting node $V_0 = V_{(x_1, x_2, x_3)(t_0)}$ in the source $ob\big(cy(t_0)\big)$ and defined by the mappings

$$F_i: \underbrace{\{cy: \mathbb{I} \to (\lceil ob(W_e) \rceil, d_{W_e})\}}_{T} \to \underbrace{\{cy: \mathbb{I} \to (\lceil ob(W_e) \rceil, d_{W_e})\}}_{T}, \; i = 1, 2, \ldots, k.$$

Let also a cyber-navigation $\aleph = (\mathscr{g}_0, \mathscr{g}_1, \ldots \mathscr{g}_{k-1}, \mathscr{g}_k)$ of a cyber node $U = U_{(x_1, x_2, x_3, t)} \in \cap_{\alpha=1}^{k} ob\big(cy(t_\alpha)\big)$ over a cyber walk from the node V_0 up to the node V_k.

Definition 24 To each part $E = fr\big(\mathcal{K}^{(U)}\big)$in the$\sigma$ −algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of available (or not)constituents in the node U:

$$\mathcal{K} = \begin{cases} \boldsymbol{dev}, if\ the\ constituent\ is\ a\ device, \\ \boldsymbol{res}, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

the users of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate an **efficiency threshold vector**

$$\mathcal{T}(\boldsymbol{E}) = \left(\mathcal{T}_{\mathbf{1}}(\boldsymbol{E}), \dots, \mathcal{T}_{\mathfrak{n}}(\boldsymbol{E})\right) \in [\mathbf{0}, +\infty[^{\mathfrak{n}}.$$

i. The cyber node $\boldsymbol{U}$ is said to be **partially inadequate** in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, if there is a variant node $\boldsymbol{U'} = \boldsymbol{U_{t_\lambda}}$ and a valuation

$$\boldsymbol{A}^{(Z \rightsquigarrow U')}(\boldsymbol{E}) = \left(\boldsymbol{a}_{\mathbf{1}}^{(Z \rightsquigarrow U')}(\boldsymbol{E}), \boldsymbol{a}_{\mathbf{2}}^{(Z \rightsquigarrow U')}(\boldsymbol{E}), \dots, \boldsymbol{a}_{\mathfrak{n}}^{(Z \rightsquigarrow U)}(\boldsymbol{E})\right)^{\boldsymbol{T}}$$

of $\mathcal{K}^{(\boldsymbol{U})}$ in $\boldsymbol{U'}$from the viewpoint of the user(s) of $\boldsymbol{Z}$, with some coordinates **less** than the corresponding coordinates of the efficiency threshold vector:

$$\boldsymbol{a}_{i_j}^{(Z \rightsquigarrow U')}(\boldsymbol{E}) < \mathcal{T}_{i_j}(\boldsymbol{E}),\ \mathbf{1} \leq \boldsymbol{j} \leq \mathfrak{n}.$$

The number

$$\varrho := \boldsymbol{max}_{1 \leq j \leq \mathfrak{n}}\left(\mathcal{T}_{i_j}(\boldsymbol{E}) - \boldsymbol{a}_{i_j}^{(Z \rightsquigarrow U')}(\boldsymbol{E})\right)$$

is called the **degree of partial inadequacy** of part $\boldsymbol{E}$ in the cyber node $\boldsymbol{U}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$. In the particular case where $\boldsymbol{a}_{\boldsymbol{j}}^{(Z \rightsquigarrow U')}(\boldsymbol{E}) < \mathcal{T}_{\boldsymbol{j}}(\boldsymbol{E})$ whenever $\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots, \mathfrak{n}$, we say that $\boldsymbol{U}$ is **completely inadequate** in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$from the viewpoint of the user(s) of $\boldsymbol{Z}$.

ii. The cyber node $\boldsymbol{U}$ is said to be **totally inadequate** in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, if there is a variant node $\boldsymbol{U'} = \boldsymbol{U_{t_\lambda}}$ and a valuation

$$\boldsymbol{A}^{(Z \rightsquigarrow U')}(\boldsymbol{E}) = \left(\boldsymbol{a}_{\mathbf{1}}^{(Z \rightsquigarrow U')}(\boldsymbol{E}), \boldsymbol{a}_{\mathbf{2}}^{(Z \rightsquigarrow U')}(\boldsymbol{E}), \dots, \boldsymbol{a}_{\mathfrak{n}}^{(Z \rightsquigarrow U')}(\boldsymbol{E})\right)^{\boldsymbol{T}}$$

of $\mathcal{K}^{(\boldsymbol{U})}$ in $\boldsymbol{U'}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with (Euclidean or not) norm **less** than the (corresponding Euclidean or not) norm of the efficiency threshold vector:

$$\left\|\boldsymbol{A}^{(Z \rightsquigarrow U)}(\boldsymbol{E})\right\| < \|\mathcal{T}(\boldsymbol{E})\|.$$

The number

$$\varrho^{(\infty)} := \|\boldsymbol{B}(\boldsymbol{E})\| - \left\|\boldsymbol{A}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U}')}(\boldsymbol{E})\right\|$$

is the **degree of total inadequacy** of part $\boldsymbol{E}$ in the cyber node $\boldsymbol{U}$ over the cyber walk $\boldsymbol{V_0}\boldsymbol{V_1}\ldots\boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$. In the contrary case, where $\boldsymbol{U}$ is **not partially inadequate** and **not totally inadequate** in its part $\boldsymbol{E}$over the cyber walk $\boldsymbol{V_0}\boldsymbol{V_1}\ldots\boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, the node $\boldsymbol{U}$ is said to be **adequate** in its part $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\right)$over the cyber walk $\boldsymbol{V_0}\boldsymbol{V_1}\ldots\boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$. ■

11.3 Infected Cyber Nodes

Suppose $\boldsymbol{t} = \boldsymbol{t_0} < \boldsymbol{t_1} < \cdots < \boldsymbol{t_k} = \boldsymbol{t}'$ is a partition of the interval $[\boldsymbol{t}, \boldsymbol{t}'] \subset]\boldsymbol{0}, \boldsymbol{1}[$. Let

$$\boldsymbol{V_0}\boldsymbol{V_1}\ldots\boldsymbol{V_k} = \underbrace{\boldsymbol{V}_{(x_1,x_2,x_3)(\boldsymbol{t_0})}}_{\in \boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t_0}))} \underbrace{\boldsymbol{V}_{(x_1,x_2,x_3)(\boldsymbol{t_1})}}_{\in \boldsymbol{ob}(\boldsymbol{F_1}[\boldsymbol{cy}(\boldsymbol{t_0})])} \cdots \underbrace{\boldsymbol{V}_{(x_1,x_2,x_3)(\boldsymbol{t_n})}}_{\in \boldsymbol{ob}([\boldsymbol{F_k}\circ\ldots\circ\boldsymbol{F_1}][\boldsymbol{cy}(\boldsymbol{t_0})])}$$

be corresponding walk with starting node $\boldsymbol{V_0} = \boldsymbol{V}_{(x_1,x_2,x_3)(\boldsymbol{t_0})}$ in the source $\boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t_0}))$ and defined by the mappings

$$\boldsymbol{F_i}: \underbrace{\left\{\boldsymbol{cy}: \mathbb{I} \to \left(\lceil \boldsymbol{ob}(\boldsymbol{W_e})\rceil, \boldsymbol{d_{W_e}}\right)\right\}}_{\boldsymbol{T}} \to \underbrace{\left\{\boldsymbol{cy}: \mathbb{I} \to \left(\lceil \boldsymbol{ob}(\boldsymbol{W_e})\rceil, \boldsymbol{d_{W_e}}\right)\right\}}_{\boldsymbol{T}}, \ \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{k}.$$

Let also a cyber-navigation $\aleph = (\boldsymbol{g_0},\ \boldsymbol{g_1},\ \ldots\ \boldsymbol{g_{k-1}},\ \boldsymbol{g_k})$of a cyber node $\boldsymbol{U} = \boldsymbol{U}_{(x_1,x_2,x_3,\boldsymbol{t})} \in \bigcap_{\alpha=1}^{k} \boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t_\alpha}))$ over a cyber walk from the node $\boldsymbol{V_0}$ up to the node $\boldsymbol{V_k}$.

To each part $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}\right)$ in the $\boldsymbol{\sigma}$ –algebra$\boldsymbol{\mathfrak{U}_{\mathcal{P}}}$ of subsets of available (or not) constituents of the node $\boldsymbol{U}$:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{dev}, if\ the\ constituent\ is\ a\ device, \\ \boldsymbol{res}, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

the user(s) of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate a **health tolerance vector**

$$\boldsymbol{\mathfrak{T}}(\boldsymbol{E}) = \left(\boldsymbol{\mathfrak{T}_1}(\boldsymbol{E}), \ldots, \boldsymbol{\mathfrak{T}_{\mathfrak{m}}}(\boldsymbol{E})\right) \in [\boldsymbol{0}, +\infty[^{\mathfrak{m}}.$$

Definition 25 The cyber node $\boldsymbol{U}$ is said to be **partially infected** in its part $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\right)$over the cyber walk $\boldsymbol{V_0}\boldsymbol{V_1}\ldots\boldsymbol{V_k}$from the viewpoint of the user(s) of $\boldsymbol{Z}$, if there is a variant node $\boldsymbol{U}' = \boldsymbol{U_{t_\lambda}}$ and a vulnerability

$$\boldsymbol{B}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U}')}(\boldsymbol{E}) = \left(\boldsymbol{b}_{\boldsymbol{1}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U}')}(\boldsymbol{E}), \boldsymbol{b}_{\boldsymbol{2}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U}')}(\boldsymbol{E}), \ldots, \boldsymbol{b}_{\mathfrak{m}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U}')}(\boldsymbol{E})\right)^{\boldsymbol{T}}$$

of $\mathcal{K}^{(U)}$ in U' from the viewpoint of the user(s) of Z, with some coordinates **greater** than the corresponding coordinates of the **health tolerance vector**:

$$b_{i_j}^{(Z \rightsquigarrow U')}(E) > \mathfrak{T}_{i_j}(E),\ 1 \le j \le \mathfrak{m}.$$

The number

$$\delta := min_{1 \le j \le \mathfrak{m}} \left(\mathfrak{T}_{i_j}(E) - b_{i_j}^{(Z \rightsquigarrow U')}(E) \right)$$

is the **degree of partial infection** of part E in the cyber node U over the cyber walk $V_0 V_1 \dots V_k$from the viewpoint of the user(s) of Z. In the particular case where $b_j^{(Z \rightsquigarrow U')}(E) > \mathfrak{T}_j(E)$ whenever $j = 1, 2, \dots, \mathfrak{m}$, we say that U is **completely infected** in its part Eover the cyber walk $V_0 V_1 \dots V_k$from the viewpoint of the user(s) of Z.

ii. The cyber node U is said to be **totally infected** (or **totally compromised**) in its part $E = fr(\mathcal{A}^{(U)})$over the cyber walk $V_0 V_1 \dots V_k$ from the viewpoint of the user(s) of Z, if there is a variant node $U' = U_{t_\lambda}$ and a valuation

$$B^{(Z \rightsquigarrow U')}(E) = \left(b_1^{(Z \rightsquigarrow U')}(E), b_2^{(Z \rightsquigarrow U')}(E), \dots, b_{\mathfrak{m}}^{(Z \rightsquigarrow U)}(E) \right)^T$$

of $\mathcal{K}^{(U)}$ in U' from the viewpoint of the user(s) of Z, with (Euclidean or not) norm **greater** than the (corresponding Euclidean or not) norm of the health tolerance vector:

$$\left\| B^{(Z \rightsquigarrow U)}(E) \right\| > \| \mathfrak{T}(E) \|.$$

The number

$$\delta^{(\infty)} := \left\| B^{(Z \rightsquigarrow U')}(E) \right\| - \| \mathfrak{T}(E) \|$$

is the **degree of the total infection** of part E in the cyber node U over the cyber walk $V_0 V_1 \dots V_k$ from the viewpoint of the user(s) of Z. In the contrary case, where U is **not partially infected** and **not totally infected** in its part E over the cyber walk $V_0 V_1 \dots V_k$ from the viewpoint of the user(s) of Z, the node U is said to be **healthy** in its part E over the cyber walk $V_0 V_1 \dots V_k$ from the viewpoint of the user(s) of Z.■

11.4 Dangerous Navigations

Let again $E = fr(\mathcal{K}^{(U)})$be a set in the $\sigma-$algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of available (or not) constituents of the cyber node U:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{dev}, if\ the\ constituent\ is\ a\ device, \\ \boldsymbol{res}, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

Suppose the user(s) of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate an efficiency threshold vector

$$\boldsymbol{\mathcal{T}}(\boldsymbol{E}) = \left(\boldsymbol{\mathcal{T}}_{\boldsymbol{1}}(\boldsymbol{E}), \dots, \boldsymbol{\mathcal{T}}_{\mathfrak{n}}(\boldsymbol{E})\right) \in [\boldsymbol{0}, +\infty[^{\mathfrak{n}},$$

as well as a health tolerance vector

$$\boldsymbol{\mathfrak{T}}(\boldsymbol{E}) = \left(\boldsymbol{\mathfrak{T}}_{\boldsymbol{1}}(\boldsymbol{E}), \dots, \boldsymbol{\mathfrak{T}}_{\mathfrak{m}}(\boldsymbol{E})\right) \in [\boldsymbol{0}, +\infty[^{\mathfrak{m}}.$$

Definition 26 The navigation $\aleph = (\boldsymbol{g}_{\boldsymbol{0}}, \boldsymbol{g}_{\boldsymbol{1}}, \dots \boldsymbol{g}_{\boldsymbol{k-1}}, \boldsymbol{g}_{\boldsymbol{k}})$ of an adequate and healthy cyber node $\boldsymbol{U} = \boldsymbol{U}_{(x_1, x_2, x_3, t)} \in \bigcap_{\alpha=1}^{k} \boldsymbol{ob}\big(\boldsymbol{cy}(\boldsymbol{t}_{\alpha})\big)$ (over a cyber node homomorphism from a node $\boldsymbol{V}_{\boldsymbol{0}}$ up to an infected node $\boldsymbol{V}_{\boldsymbol{k}}$) is said to be a **dangerous navigation** or an **unplanned attack with degree of danger** $\boldsymbol{d} := \boldsymbol{max}\{\boldsymbol{\varrho}, \boldsymbol{\varrho}^{(\infty)}\} + \boldsymbol{max}\{\boldsymbol{\delta}, \boldsymbol{\delta}^{(\infty)}\}$ in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V}_{\boldsymbol{0}}\boldsymbol{V}_{\boldsymbol{1}} \dots \boldsymbol{V}_{\boldsymbol{k}}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, if the node $\boldsymbol{U}$ becomes

- inadequate in its part $\boldsymbol{E} = \boldsymbol{fr}\big(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\big)$over the cyber walk $\boldsymbol{V}_{\boldsymbol{0}}\boldsymbol{V}_{\boldsymbol{1}} \dots \boldsymbol{V}_{\boldsymbol{k}}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with degree of partial inadequacy equal to $\boldsymbol{\varrho}$ and degree of total inadequacy equal to $\boldsymbol{\varrho}^{(\infty)}$ and
- infected in its part $\boldsymbol{E} = \boldsymbol{fr}\big(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\big)$ over the cyber walk $\boldsymbol{V}_{\boldsymbol{0}}\boldsymbol{V}_{\boldsymbol{1}} \dots \boldsymbol{V}_{\boldsymbol{k}}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with degree of partial infection equal to $\boldsymbol{\delta}$ and degree of total infection equal to $\boldsymbol{\delta}^{(\infty)}$.

11.5 Protection of cyber nodes from unplanned attacks

Let again $\boldsymbol{E} = \boldsymbol{fr}\big(\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}\big)$ in the $\boldsymbol{\sigma}$ –algebra $\boldsymbol{\mathfrak{U}}_{\boldsymbol{\mathcal{P}}}$ of subsets of an available or not constituent $\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}$ in node $\boldsymbol{U}$:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{dev}, if\ the\ constituent\ is\ a\ device, \\ \boldsymbol{res}, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

Suppose the user(s) of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate an efficiency threshold vector $\boldsymbol{\mathcal{T}}(\boldsymbol{E}) \in [\boldsymbol{0}, +\infty[^{\mathfrak{n}}$, as well as a **health tolerance vector** $\boldsymbol{\mathfrak{T}}(\boldsymbol{E}) \in [\boldsymbol{0}, +\infty[^{\mathfrak{m}}$.

Definition 27 At a given time, the constituent part $\boldsymbol{E}$ of node $\boldsymbol{U}$ is said to be **protected from unplanned attacks, with degree of protection** $\boldsymbol{p} \in]\boldsymbol{0}, \boldsymbol{1}]$, if, at this time, there is a nodal fixed filter system $\overline{\boldsymbol{U}}^{(\boldsymbol{E})}$in part $\boldsymbol{E}$ that allows every **self-**

inflicted parallactic cyber-effect $\mathcal{g}'_j \circ \mathcal{g}_j$ in any cyber-navigation of degree of danger $\boldsymbol{d} \leq -\boldsymbol{logp}$ to reach only constituent parts of the initial target $\boldsymbol{U}$ that are different from part $\boldsymbol{E}$ of $\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}$.

ii. At a given time, the node $\boldsymbol{U}$ is said to be **completely protected from unplanned attacks of danger** degree $\boldsymbol{d}$, if, at this time, any part of every constituent of $\boldsymbol{U}$ is protected from unplanned attacks with degree of protection $\boldsymbol{p} \leq \boldsymbol{e}^{-\boldsymbol{d}}$. The node $\boldsymbol{U}$ is said to be **completely protected from unplanned attacks** at a given time, if, at this time, any constituent part of $\boldsymbol{U}$ is protected from unplanned attacks with degree of protection $\boldsymbol{p} = \boldsymbol{1}$.

12. Description of Various Types of Cyber Attacks and Protection

12.1 Passive cyber-attacks

A passive attack is a network attack in which a system is monitored and sometimes scanned for open ports and vulnerabilities. The purpose is solely to gain information about the target and no data is changed on the target. So, a passive attack contrasts with an active attack, in which an intruder attempts to alter data on the target system or data en route for the target system.

Let $\boldsymbol{U}, \boldsymbol{V} \in \boldsymbol{ob}\big(\boldsymbol{cy}(\boldsymbol{t})\big)$, whenever $\boldsymbol{t}$ is in an arbitrary subset $\mathbb{I} =]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset [\boldsymbol{0}, \boldsymbol{1}]$. Let also

$$\boldsymbol{\delta}_{\boldsymbol{U}}: [\boldsymbol{0}, \boldsymbol{1}] \to \mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\delta}_{\boldsymbol{W}}(\boldsymbol{t}) = (\mathbb{z}_{\boldsymbol{1}}, \mathbb{w}_{\boldsymbol{1}})(\boldsymbol{t}) \text{ and}$$

$$\boldsymbol{\gamma}_{\boldsymbol{V}}: [\boldsymbol{0}, \boldsymbol{1}] \to \mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\gamma}_{\boldsymbol{V}}(\boldsymbol{t}) = (\mathbb{z}_{\boldsymbol{2}}, \mathbb{w}_{\boldsymbol{2}})(\boldsymbol{t})$$

be two supervisory perception curves of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the node system $(\boldsymbol{U}, \boldsymbol{V})$.

A family of interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_{\boldsymbol{1}}, \mathbb{w}_{\boldsymbol{1}}), (\mathbb{z}_{\boldsymbol{2}}, \mathbb{w}_{\boldsymbol{2}}), (\mathbb{z}'_{\boldsymbol{1}}, \mathbb{w}'_{\boldsymbol{1}}), (\mathbb{z}'_{\boldsymbol{2}}, \mathbb{w}'_{\boldsymbol{2}})\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}\big)^{\boldsymbol{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

$\boldsymbol{X}, \boldsymbol{Y} \in \{\boldsymbol{U}, \boldsymbol{V}\}$, with associated family of cyber-interplays

$$\boldsymbol{\mathcal{D}}_{\boldsymbol{\mathcal{F}}} = \Big\{\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}^{(\boldsymbol{\mathcal{Z}})}: \mathbb{I} \to \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{Y})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{Y})}:$$
$$\boldsymbol{t} \mapsto \boldsymbol{\mathcal{G}}(\boldsymbol{t}) = \Big(\boldsymbol{\delta}_{\boldsymbol{Y}}^{(\boldsymbol{\mathcal{Z}})}(\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{X}}^{(\boldsymbol{\mathcal{Z}})}(\boldsymbol{t}), \boldsymbol{\delta}_{\boldsymbol{Y}}^{(\boldsymbol{\mathcal{Z}})}(\boldsymbol{t} + \Delta\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{X}}^{(\boldsymbol{\mathcal{Z}})}(\boldsymbol{t} + \Delta\boldsymbol{t})\Big): \boldsymbol{t} + \Delta\boldsymbol{t} \in \mathbb{I}, \boldsymbol{\mathcal{Z}} \in \boldsymbol{\mathcal{F}}\Big\}$$

of the ordered cyber pair $(\boldsymbol{Y}, \boldsymbol{X})$ over the time $\boldsymbol{t} \in \mathbb{I}$**,** is called **coherent interactive family** in $\mathbb{I}$, if there is a homotopy

$$\boldsymbol{H}: \mathbb{I} \times [\mathbf{0}, \mathbf{1}] \to \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{Y})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{Y})}$$

such that, for each cyber-interplay $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}^{(\mathcal{Z})} \in \boldsymbol{\mathcal{D}}_{\boldsymbol{\mathcal{F}}}$ there is a $\boldsymbol{p} \in [\mathbf{0}, \mathbf{1}]$ satisfying $\boldsymbol{H}(\boldsymbol{t}, \boldsymbol{p}) = \boldsymbol{\mathcal{G}}(\boldsymbol{t})$ at any moment time $\boldsymbol{t} \in \mathbb{I}$ on which the cyber-interplay $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}^{(\mathcal{Z})}$ implements the interaction $\boldsymbol{\mathcal{Z}}$.

Definition 28 A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ of) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ is a **germ of (partial) passive attack from $\boldsymbol{U}$ against the** $(\boldsymbol{\kappa}_1, \dots, \boldsymbol{\kappa}_\lambda)$ **– resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_2}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_\lambda}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ during an entire time interval $\mathbb{I}(=]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset \subset [\mathbf{0}, \mathbf{1}])$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, there is an integer $\boldsymbol{\nu} = \boldsymbol{\nu}(\boldsymbol{t}) > \mathbf{0}$ such that the pair $\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) =$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a}_{\boldsymbol{\mathcal{M}}_V+1,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}}_V+1,1}^{(V\rightsquigarrow V)} & & \boldsymbol{a}_{\boldsymbol{\mathcal{M}}_V+1,\mathfrak{n}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \vdots & & \vdots \\ \boldsymbol{a}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,1}^{(V\rightsquigarrow V)} & & \boldsymbol{a}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{n}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b}_{\boldsymbol{\mathcal{M}}_V+1,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}}_V+1,1}^{(V\rightsquigarrow V)} & & \boldsymbol{b}_{\boldsymbol{\mathcal{M}}_V+1,\mathfrak{m}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \vdots & & \vdots \\ \boldsymbol{b}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,1}^{(V\rightsquigarrow V)} & & \boldsymbol{b}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{m}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right),\right.$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)\Bigg)$$

Table 12

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, via the associated family of cyber-activities

$$\boldsymbol{\mathcal{D}}_{\boldsymbol{\mathcal{F}}} = \Big(\boldsymbol{g}_{\boldsymbol{t}} = \boldsymbol{g}_{\boldsymbol{t}}^{(\boldsymbol{\mathcal{Z}})} \colon \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\colon$$

$$\big(\boldsymbol{\delta}_{\boldsymbol{U}}(\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{V}}(\boldsymbol{t})\big) \longmapsto \big(\boldsymbol{\delta}'_{\boldsymbol{U}}(\boldsymbol{t}'), \boldsymbol{\gamma}'_{\boldsymbol{V}}(\boldsymbol{t}')\big)\Big)_{\boldsymbol{t}\in\mathbb{I}}$$

over the time $\boldsymbol{t} \in \mathbb{I}$, at $\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) \in \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) =$

$$\left(\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \vdots & & \vdots \\ \boldsymbol{b'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \vdots & & \vdots \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_{\backslash U}+1,\mathfrak{n}} \\ \vdots & \dots\dots\dots & \vdots \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{n}}+\boldsymbol{i}\;\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right.\right.,$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \vdots & & \vdots \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} \\ \vdots & \dots\dots\dots & \vdots \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{m}}+\boldsymbol{i}\;\widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right).$$

Table 13

It is easy to prove/verify the next two results.

Proposition 1 In a passive attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against $\boldsymbol{V}$, the number of resource parts in $\boldsymbol{U}$ at a moment $\boldsymbol{t'}=\boldsymbol{t}+\Delta\boldsymbol{t}$ has increased by at least $\boldsymbol{\lambda}$ new resource parts, say $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\mathcal{M}_U+\ell_U+1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\mathcal{M}_U+\ell_U+2}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\mathcal{M}_U+\ell_U+\lambda}\right)$, derived from the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_2}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$ that existed in the node $\boldsymbol{V}$ the previous moment $\boldsymbol{t}$, in such a way that the following elementary properties hold.

i. If the relative valuations of $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_V+\ell_V+1}^{(U)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_V+\ell_V+2}^{(U)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_V+\ell_V+\lambda}^{(U)}\right)$ from the viewpoint of the (user(s) of) node $\boldsymbol{U}$ at the previous moment $\boldsymbol{t}$ are $\left(\boldsymbol{a}_{\mathcal{M}_V+\mu_1,1}^{(U\rightsquigarrow V)},\dots,\boldsymbol{a}_{\mathcal{M}_V+\mu_1,\mathfrak{n}}^{(U\rightsquigarrow V)}\right)$,…,$\left(\boldsymbol{a}_{\mathcal{M}_V+\mu_\lambda,1}^{(U\rightsquigarrow V)},\dots,\boldsymbol{a}_{\mathcal{M}_V+\mu_\lambda,\mathfrak{n}}^{(U\rightsquigarrow V)}\right)$ respectively, with $\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_\lambda}\in\{\mathbf{1},\mathbf{2},\dots,\boldsymbol{\ell_V}\}$, then the resulting valuation vectors $\left(\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1},\dots,\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}}\right)$,…,$\left(\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\lambda,1},\dots,\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\lambda,\mathfrak{n}}\right)$ of the **new** resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_U+\ell_U+1}^{(U)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_U+\ell_U+2}^{(U)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_U+\ell_U+\lambda}^{(U)}\right)$ in $\boldsymbol{U}$, as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at a next moment $\boldsymbol{t'}=\boldsymbol{t}+\Delta\boldsymbol{t}$ are equal to $\left(\boldsymbol{a}_{\mathcal{M}_V+\mu_1,1}^{(U\rightsquigarrow V)},\dots,\boldsymbol{a}_{\mathcal{M}_V+\mu_1,\mathfrak{n}}^{(U\rightsquigarrow V)}\right)$,…,$\left(\boldsymbol{a}_{\mathcal{M}_V+\mu_\lambda,1}^{(U\rightsquigarrow V)},\dots,\boldsymbol{a}_{\mathcal{M}_V+\mu_\lambda,\mathfrak{n}}^{(U\rightsquigarrow V)}\right)$**:**

$$\left(\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\alpha,1},\dots,\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\alpha,\mathfrak{n}}\right)=\left(\boldsymbol{a}_{\mathcal{M}_V+\mu_\alpha,1}^{(U\rightsquigarrow V)},\dots,\boldsymbol{a}_{\mathcal{M}_V+\mu_\alpha,\mathfrak{n}}^{(U\rightsquigarrow V)}\right),\ \forall\boldsymbol{\alpha}\in\{\mathbf{1},\mathbf{2},\dots,\boldsymbol{\lambda}\}.$$

ii. All resulting valuations and vulnerabilities of **new** resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_U+\ell_U+1}^{(U)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mathcal{M}_U+\ell_U+\lambda}^{(U)}\right)$ in $\boldsymbol{U}$ from the viewpoint of the user(s) of $\boldsymbol{V}$ remain equal to $\mathbf{0}$**:**

$$\forall\boldsymbol{j}\in\{\mathbf{1},\mathbf{2},\dots,\mathfrak{n}\}\ \boldsymbol{and}\ \forall\boldsymbol{\alpha}\in\{\mathbf{1},\mathbf{2},\dots,\boldsymbol{\lambda}\}\Longrightarrow\boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\alpha,j}=\mathbf{0},$$

$$\forall\boldsymbol{k}\in\{\mathbf{1},\mathbf{2},\dots,\mathfrak{m}\}\ \boldsymbol{and}\ \forall\boldsymbol{\alpha}\in\{\mathbf{1},\mathbf{2},\dots,\boldsymbol{\lambda}\}\Longrightarrow\boldsymbol{b}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\alpha,k}=\mathbf{0}.$$

iii. There is at least one resulting valuation $\boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda_\alpha,j}$ of a part $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\alpha}^{(V)}\right)$ in $\boldsymbol{V}$ from the viewpoint of the user(s) of $\boldsymbol{U}$ which decreases:

$$\exists\boldsymbol{j}\in\{\mathbf{1},\mathbf{2},\dots,\mathfrak{n}\}\ \boldsymbol{and}\ \exists\boldsymbol{\lambda_\alpha}\in\{\mathcal{M}_V+\mathbf{1},\dots,\mathcal{M}_V+\boldsymbol{\ell_V}\}\colon\boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda_\alpha,j}<\boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda_\alpha,j};$$

similarly, there is at least one vulnerability $\boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\rho_\alpha,k}$ of part $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\alpha}^{(V)}\right)$ in $\boldsymbol{V}$ from the viewpoint of the user(s) of $\boldsymbol{U}$ which increases

$$\exists\boldsymbol{k}\in\{\mathbf{1},\mathbf{2},\dots,\mathfrak{m}\}\ \boldsymbol{and}\ \exists\boldsymbol{\rho_\alpha}\in\{\mathcal{M}_V+\mathbf{1},\dots,\mathcal{M}_V+\boldsymbol{\ell_V}\}\colon$$

$$\boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\rho_\alpha,k}>\boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\rho_\alpha,k}.$$

iv. The valuations and vulnerabilities of each part $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\alpha}^{(V)}\right)$ in $\boldsymbol{V}$ from the viewpoint of the user(s) of $\boldsymbol{V}$ remain unchanged:

$$\forall \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \dots, \mathfrak{n}\} \; \boldsymbol{and} \; \forall \boldsymbol{\lambda_\alpha} \in \{\boldsymbol{\mathcal{M}_V} + \mathbf{1}, \dots, \boldsymbol{\mathcal{M}_V} + \boldsymbol{\ell_V}\} \Longrightarrow$$
$$\widehat{\boldsymbol{a}}'^{(\boldsymbol{V} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\lambda_\alpha},\boldsymbol{j}} = \widehat{\boldsymbol{a}}^{(\boldsymbol{V} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\lambda_\alpha},\boldsymbol{j}},$$
$$\forall \boldsymbol{k} \in \{\mathbf{1}, \mathbf{2}, \dots, \mathfrak{m}\} \; \boldsymbol{and} \; \forall \boldsymbol{\mu_\alpha} \in \{\boldsymbol{\mathcal{M}_V} + \mathbf{1}, \dots, \boldsymbol{\mathcal{M}_V} + \boldsymbol{\ell_V}\} \Longrightarrow$$
$$\widehat{\boldsymbol{b}}'^{(\boldsymbol{V} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V}+\boldsymbol{\mu_\alpha},\boldsymbol{k}} = \widehat{\boldsymbol{b}}^{(\boldsymbol{V} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V}+\boldsymbol{\mu_\alpha},\boldsymbol{k}}.$$

Proposition 2 In a passive attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against $\boldsymbol{V}$, the number of resource parts in $\boldsymbol{U}$ at a moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$ has increased by at least $\boldsymbol{\lambda}$ new resource parts, say $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U}+\mathbf{1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U}+\mathbf{2}}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U}+\boldsymbol{\lambda}}\right)$, derived from the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_2}}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_\lambda}}\right)$ that existed in the node $\boldsymbol{V}$ the previous moment $\boldsymbol{t}$, in such a way that the following elementary properties hold.

i. The (Euclidean) norm $\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell_U}+\boldsymbol{\lambda}} \left|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{\mathbf{2}}\right)^{1/2}$ of the resulting overall valuation in the variant node $\boldsymbol{U}'$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is **greater** than the (Euclidean) norms

$\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell_U}} \left|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{\mathbf{2}}\right)^{\mathbf{1/2}}$ and

$\left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{\mathbf{2}}\right)^{\mathbf{1/2}}$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\| > max\left\{\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\|, \left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\|\right\}.$$

ii. The norm $\left\|\boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{\mathbf{2}}\right)^{1/2}$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{W}$ at the next moment $\boldsymbol{t}'$ is **less** than the norm $\left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{\mathbf{2}}\right)^{\mathbf{1/2}}$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$at the preceding moment $\boldsymbol{t}$:

$$\left\|\boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| < \left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\|.$$

iii. The norm $\left\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\ell_U+\nu}\left|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\mathcal{M}_U+\lambda,j}\right|^{2}\right)^{1/2}$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is **less or equal** than the norms

$$\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\nu=1}^{\ell_W}\left|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\mathcal{M}_U+\nu,j}\right|^{2}\right)^{1/2} \text{ and}$$

$$\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\nu=1}^{\ell_V}\left|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\mathcal{M}_V+\nu,j}\right|^{2}\right)^{1/2}$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|\leq \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|,\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}.$$

iv. The norm $\left\|\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\nu=1}^{\ell_V}\left|\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\mathcal{M}_V+\nu,j}\right|^{2}\right)^{1/2}$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$at the next moment $\boldsymbol{t}'$ is **greater** than the norm $\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\nu=1}^{\ell_V}\left|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\mathcal{M}_U+\nu,j}\right|^{2}\right)^{1/2}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|>\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|.\blacksquare$$

The **degree $\boldsymbol{d}=\boldsymbol{d}_{\kappa_1,\dots,\kappa_\nu}$ of the passive attack $\boldsymbol{f}$ against the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_2}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$** of node $\boldsymbol{V}$ from the offensive node $\boldsymbol{U}$ at time moment $\boldsymbol{t}\in\mathbb{I}$ is the maximum of the two quotients

$$\boldsymbol{d}_1:=\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|/\left\|\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| \text{ and } \boldsymbol{d}_2:=\left(\left\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|/\left\|\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right)^{-1}.$$

Thus

$$\boldsymbol{d}=\boldsymbol{d}_{\kappa_1,\dots,\kappa_\lambda}:=\boldsymbol{max}\{\boldsymbol{d}_1,\boldsymbol{d}_2\}.$$

If the degree $\boldsymbol{d}$ surpasses a given threshold $\boldsymbol{\mathcal{S}}_{\kappa_1,\dots,\kappa_\lambda}^{(W,V)}\in[\mathbf{0},\infty[$, called the **passive attack threshold in the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_2}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$** of $\boldsymbol{V}$ at time moment$\boldsymbol{t}\in\mathbb{I}$,we say that the passive attack $\boldsymbol{f}$ is **dangerous with degree**

of danger $\boldsymbol{d}$ in theresource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$.

12.2 Protected cyber nodes from passive attacks

Definition 29.

i. The node $\boldsymbol{V}$ is said to be **protected from passive attacks, with degree of protection** $\boldsymbol{p} \in]\boldsymbol{0}, \boldsymbol{1}]$ over the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\nu}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ over a time period $\mathbb{I}$, if, during this time period, there is a nodal fixed filter system $\overline{\boldsymbol{V}}^{(\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\nu})}$in the union $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right) \cup \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\right) \cup \dots \cup \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\nu}}^{(\boldsymbol{V})}\right)$that allow every parallactic cyber passive attack against the resource parts (from any offensive node $\boldsymbol{U}$)with degree of danger $\boldsymbol{d} \leq -\boldsymbol{logp}$ to reach only resource parts $\boldsymbol{K}$ of the initial target $\boldsymbol{V}$ that are disjoint from $\boldsymbol{E}$.

ii. During the time period $\mathbb{I}$, the node $\boldsymbol{V}$ is said to be **completely protected from passive attacks of danger degree**$\boldsymbol{d}$, if, at this time period, any resource part in $\boldsymbol{V}$ is protected from passive attacks against $\boldsymbol{V}$, with degree of protection $\boldsymbol{p} \leq \boldsymbol{e}^{-\boldsymbol{d}}$. The node $\boldsymbol{V}$ is said to be **completely protected from passive attacks at a given time period**, if, during this time period, any resource part of $\boldsymbol{V}$ is protected from active attacks against $\boldsymbol{V}$ with degree of protection $\boldsymbol{p} = \boldsymbol{1}$.■

12.3 Active cyber-attacks

An attack is active if it is an attack with data transmission to all parties thereby acting as a liaison enabling severe compromise. The purpose is to alter system resources or affect their operation. So, in an active attack, an intruder attempts to alter data on the target system or data “en route” for the target system.

Let $\boldsymbol{U}, \boldsymbol{V} \in \boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t}))$, whenever $\boldsymbol{t}$ is in an arbitrary interval $\mathbb{I} =]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset [\boldsymbol{0}, \boldsymbol{1}]$. Let also

$$\boldsymbol{\delta_U}: [\boldsymbol{0}, \boldsymbol{1}] \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\delta_U}(\boldsymbol{t}) = (\mathbb{z}_{\boldsymbol{1}}, \mathbb{w}_{\boldsymbol{1}})(\boldsymbol{t}) \text{ and}$$

$$\boldsymbol{\gamma_V}: [\boldsymbol{0}, \boldsymbol{1}] \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\gamma_V}(\boldsymbol{t}) = (\mathbb{z}_{\boldsymbol{2}}, \mathbb{w}_{\boldsymbol{2}})(\boldsymbol{t})$$

be two supervisory perception curves of $\boldsymbol{V}$ and $\boldsymbol{U}$ in the node system $(\boldsymbol{V}, \boldsymbol{U})$.

Definition 30 A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times \boldsymbol{\mathscr{k}}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{\mathscr{k}}}\big)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

lying in (the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) active attack** against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ –device parts $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\Big)$, $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_2}}^{(\boldsymbol{V})}\Big)$,…,$\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\Big)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda})$ – resource parts $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\Big)$, $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\Big)$,…,$\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\Big)$ of $\boldsymbol{V}$, during the time interval $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, there is an integer $\boldsymbol{N} = \boldsymbol{N}(\boldsymbol{t}) > \mathbf{0}$ such that the pair $\big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\big) \in \big(\mathbb{C}^{\mathfrak{n}\times \boldsymbol{\mathscr{k}}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{\mathscr{k}}}\big)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\big) =$$

$$\left(\left(\begin{pmatrix}
\boldsymbol{a}_{\mathbf{1,1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1,1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}_{\boldsymbol{m_V},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m_V},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & & \boldsymbol{a}_{\boldsymbol{m_V},\mathfrak{n}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m_V},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{a}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{n}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{n}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix}
\boldsymbol{b}_{\mathbf{1,1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1,1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \dots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}_{\boldsymbol{m_V},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{m_V},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\boldsymbol{m_V},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{m_V},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{b}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\cdots & & \cdots \\
\boldsymbol{b}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right),$$

$$
\left(\left(\begin{pmatrix}
\boldsymbol{a}_{\mathbf{1,1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1,1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(U\rightsquigarrow U)} \\
\dots & \dots & \dots \\
\boldsymbol{a}_{m_U,\mathbf{1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_U,\mathbf{1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{a}_{m_U,\mathfrak{n}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_U,\mathfrak{n}}^{(U\rightsquigarrow U)} \\
\mathbf{0} & & \mathbf{0} \\
\dots & \dots & \dots \\
\mathbf{0} & \dots & \mathbf{0} \\
\boldsymbol{a}_{\mathcal{M}_U+\mathbf{1},\mathbf{1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\mathbf{1},\mathbf{1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{a}_{\mathcal{M}_U+\mathbf{1},\mathfrak{n}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\mathbf{1},\mathfrak{n}}^{(U\rightsquigarrow U)} \\
\dots & & \dots \\
\boldsymbol{a}_{\mathcal{M}_U+\ell_U,\mathbf{1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\ell_U,\mathbf{1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{a}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}^{(U\rightsquigarrow U)} \\
\mathbf{0} & \dots & \mathbf{0} \\
\dots & \dots & \dots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
\boldsymbol{b}_{\mathbf{1,1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1,1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(U\rightsquigarrow U)} \\
\dots & \dots & \dots \\
\boldsymbol{b}_{m_U,\mathbf{1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_U,\mathbf{1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{b}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow W)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_V,\mathfrak{m}}^{(W\rightsquigarrow W)} \\
\mathbf{0} & & \mathbf{0} \\
\dots & \dots & \dots \\
\mathbf{0} & \dots & \mathbf{0} \\
\boldsymbol{b}_{\mathcal{M}_U+\mathbf{1},\mathbf{1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+\mathbf{1},\mathbf{1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{b}_{\mathcal{M}_U+\mathbf{1},\mathfrak{m}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+\mathbf{1},\mathfrak{m}}^{(U\rightsquigarrow U)} \\
\dots & & \dots \\
\boldsymbol{b}_{\mathcal{M}_U+\ell_U,\mathbf{1}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+\ell_U,\mathbf{1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{b}_{\mathcal{M}_U+\ell_U,\mathfrak{m}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+\ell_U,\mathfrak{m}}^{(U\rightsquigarrow U)} \\
\mathbf{0} & \dots & \mathbf{0} \\
\dots & \dots & \dots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right)
$$

Table 14

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, via the associated family of cyber-activities

$$\boldsymbol{\mathcal{D}}_{\boldsymbol{\mathcal{F}}} = \Big(\boldsymbol{g}_{\boldsymbol{t}} = \boldsymbol{g}_{\boldsymbol{t}}^{(\mathcal{Z})} : \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} :$$

$$\big(\boldsymbol{\delta}_{\boldsymbol{U}}(\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{V}}(\boldsymbol{t})\big) \longmapsto \big(\boldsymbol{\delta}'_{\boldsymbol{U}}(\boldsymbol{t}'), \boldsymbol{\gamma}'_{\boldsymbol{V}}(\boldsymbol{t}')\big)\Big)_{\boldsymbol{t}\in\mathbb{I}}$$

over the time $\boldsymbol{t} \in \mathbb{I}$, at $\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) \in \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) =$

$$
\left(\left(\begin{pmatrix}
a'^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{1,1} & \cdots & a'^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
a'^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{m_V,1} & & a'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\cdots & & \cdots \\
a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
b'^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{1,1} & \cdots & b'^{(U\rightsquigarrow V)}_{1,\mathfrak{m}} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
b'^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & b'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{m}} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & & \cdots \\
b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right),
$$

$$
\left(\begin{pmatrix}
a'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{1,1} & \cdots & a'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
a'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{m_U,1} & \cdots & a'^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & \cdots & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} & \cdots & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right.,
$$

$$\left.\left.\begin{pmatrix} b'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{1,1} & \cdots & b'^{(V\rightsquigarrow U)}_{1,\mathfrak{m}} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ b'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{m_U,1} & \cdots & b'^{(V\rightsquigarrow U)}_{m_U,\mathfrak{m}} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{m_U,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & \cdots & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} & \cdots & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{m}} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right). \blacksquare$$

Table 15

It is easy to prove and/or verify the next two results.

Proposition 3 In an active attack $\mathcal{F}$ from U against the $(\mu_1, \dots, \mu_\nu)$ −device parts $fr\left(dev^{(V)}_{\mu_1}\right), \dots, fr\left(dev^{(V)}_{\mu_\nu}\right)$ of V and the $(\kappa_1, \dots, \kappa_\lambda)$ − resource parts $fr\left(res^{(V)}_{\kappa_1}\right), \dots, fr\left(res^{(V)}_{\kappa_\lambda}\right)$ of V, the following elementary properties hold.

i. All **new** resource valuations of the offensive node U are derived from the set of all initial resource valuations of V, i.e., for any $j \in \{\mathcal{M}_U + \ell_U + 1, \dots, \mathcal{M}_U + \ell_U + N\}$ and any $k \in \{1, 2, \dots, \mathfrak{n}\}$, the new valuations

$$a'^{(V\rightsquigarrow U)}_{j,k} + i\widehat{a}'^{(U\rightsquigarrow U)}_{j,k}$$

are obtained as functions of the initial valuations

$$a^{(U\rightsquigarrow V)}_{p,l} + i\widehat{a}^{(V\rightsquigarrow V)}_{p,l},\ p \in \{1, 2, \dots, m_V, \mathcal{M}_V + 1, \dots, \mathcal{M}_V + \ell_V\},\ l \in \{1, 2, \dots, \mathfrak{n}\}.$$

ii. Similarly, all **new** resource vulnerabilities of the offensive node U are derived from the set of all initial resource vulnerabilities of V, i.e., for any $j \in \{\mathcal{M}_U + \ell_U + 1, \dots, \mathcal{M}_U + \ell_U + N\}$ and any $k \in \{1, 2, \dots, \mathfrak{n}\}$, the new vulnerabilities

$$b'^{(V\rightsquigarrow U)}_{j,k} + i\widehat{b}'^{(U\rightsquigarrow U)}_{j,k}$$

are obtained as functions of the initial vulnerabilities

$$b_{p,l}^{(U\rightsquigarrow V)} + i\widehat{b}_{p,l}^{(V\rightsquigarrow V)}, p \in \{1,2,\dots,m_V, \mathcal{M}_V+1,\dots,\mathcal{M}_V+\ell_V\}, k \in \{1,2,\dots,\mathfrak{m}\}.$$

iii. Finally, from the viewpoint of the (user(s) of) node V, all valuations of U remain unchanged, i.e., if $j \in \{1,2,\dots,m_U, \mathcal{M}_U+1,\dots,\mathcal{M}_U+\ell_U\}$, then $a_{j,k}^{(V\rightsquigarrow U)} = a'^{(V\rightsquigarrow U)}_{j,k}$ for any $k \in \{1,2,\dots,\mathfrak{n}\}$ and $b_{j,k}^{(V\rightsquigarrow U)} = b'^{(V\rightsquigarrow U)}_{j,k}$ for any $k \in \{1,2,\dots,\mathfrak{m}\}$.

Proposition 4 In an active attack $\mathcal{F}$ from U against the $(\mu_1,\dots,\mu_\nu)$ −device parts $fr\left(dev_{\mu_1}^{(V)}\right),\dots,fr\left(dev_{\mu_\nu}^{(V)}\right)$ of V and the $(\kappa_1,\dots,\kappa_\lambda)$− resource parts $fr\left(res_{\kappa_1}^{(V)}\right),\dots,fr\left(res_{\kappa_\lambda}^{(V)}\right)$ of V, the following elementary properties hold.

i. The (Euclidean) norm $\left\|a'^{(U\rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\ell_V} \left|a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda,j}\right|^2\right)^{1/2}$ of the resulting overall valuation in node V as evaluated from the viewpoint of the user(s) of U at the next moment t' is *less* than the (Euclidean) norm $\left\|a^{(U\rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\ell_V} \left|a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda,j}\right|^2\right)^{1/2}$ of the initial overall valuation in V as evaluated from the viewpoint of the user(s) of U at the preceding moment t:

$$\left\|a'^{(U\rightsquigarrow V)}\right\| < \left\|a^{(U\rightsquigarrow V)}\right\|.$$

ii. The (Euclidean) norm $\left\|b'^{(U\rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\ell_V} \left|b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda,j}\right|^2\right)^{1/2}$ of the resulting overall vulnerability in the node V as evaluated from the viewpoint of the user(s) of U at the next moment t' is greater than the (Euclidean) norm $\left\|b^{(U\rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\ell_V} \left|b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\lambda,j}\right|^2\right)^{1/2}$ of the initial overall vulnerability in the node V as evaluated from the viewpoint of the user(s) of U at the preceding moment t:

$$\left\|b'^{(U\rightsquigarrow V)}\right\| > \left\|b^{(U\rightsquigarrow V)}\right\|.$$

iii. The (Euclidean) norm

$$\left\|\widehat{a}'^{(U\rightsquigarrow U)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \left\{\sum_{\lambda=1}^{m_U} \left|\widehat{a}'^{(U\rightsquigarrow U)}_{\lambda,j}\right|^2 + \sum_{\lambda=1}^{\ell_U+N} \left|\widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\lambda,j}\right|^2\right\}\right)^{1/2}$$

of the resulting overall valuation in the variant node U as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is greater than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}}\left\{\sum_{\lambda=1}^{\boldsymbol{m}_U}\left|\widehat{\boldsymbol{a}}_{\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right|^{2}+\sum_{\lambda=1}^{\boldsymbol{\ell}_W}\left|\widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right|^{2}\right\}\right)^{1/2} \text{ and}$$

$$\left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}}\left\{\sum_{\lambda=1}^{\boldsymbol{m}_V}\left|\boldsymbol{a}_{\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right|^{2}+\sum_{\lambda=1}^{\boldsymbol{\ell}_V}\left|\boldsymbol{a}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right|^{2}\right\}\right)^{1/2}$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| > \boldsymbol{max}\left\{\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|,\left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}.$$

iv. The (Euclidean) norm $\left\|\widehat{\boldsymbol{b}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\boldsymbol{\ell}_U+\boldsymbol{N}}\left|\widehat{\boldsymbol{b}'}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right|^{2}\right)^{1/2}$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is *less or equal* than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\boldsymbol{\ell}_U}\left|\widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right|^{2}\right)^{1/2} \text{ and}$$

$$\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\boldsymbol{\ell}_V}\left|\boldsymbol{b}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\lambda},\boldsymbol{j}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right|^{2}\right)^{1/2}$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| \le \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|,\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}.$$

The **degree** $\boldsymbol{d} = \boldsymbol{d}_{\{\boldsymbol{\mu}_1,\dots,\boldsymbol{\mu}_\nu\}\cup\{\boldsymbol{\kappa}_1,\dots,\boldsymbol{\kappa}_\lambda\}}$ **of the active attack** $\boldsymbol{f}$ against the $(\boldsymbol{\mu}_1,\dots,\boldsymbol{\mu}_\nu)$ –device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_2}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_\nu}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa}_1,\dots,\boldsymbol{\kappa}_\lambda)$ – resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_2}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_\lambda}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ from the offensive node $\boldsymbol{U}$ at time moment $\boldsymbol{t} \in \mathbb{I}$ is defined to be the maximum of the two quotients

$$\boldsymbol{d}_1 := \left\|\widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| / \left\|\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| \text{ and } \boldsymbol{d}_2 := \left(\left\|\widehat{\boldsymbol{b}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| / \left\|\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right)^{-1}.$$

Thus, $\boldsymbol{d} = \boldsymbol{d}_{\{\boldsymbol{\mu}_1,\dots,\boldsymbol{\mu}_\nu\}\cup\{\boldsymbol{\kappa}_1,\dots,\boldsymbol{\kappa}_\lambda\}} := \boldsymbol{max}\{\boldsymbol{d}_1,\boldsymbol{d}_2\}$. If the degree $\boldsymbol{d}$ surpasses a given threshold $\boldsymbol{\mathcal{T}}_{\{\boldsymbol{\mu}_1,\dots,\boldsymbol{\mu}_\nu\}\cup\{\boldsymbol{\kappa}_1,\dots,\boldsymbol{\kappa}_\lambda\}}^{(\boldsymbol{U},\boldsymbol{V})} \in [\boldsymbol{0},\infty[$, called **threshold of active attack** from $\boldsymbol{U}$ against the $(\boldsymbol{\mu}_1,\dots,\boldsymbol{\mu}_\nu)$ –device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_\nu}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the$(\boldsymbol{\kappa}_1,\dots,\boldsymbol{\kappa}_\lambda)$ – resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right)$,,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_\lambda}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$at time moment $\boldsymbol{t} \in \mathbb{I}$, we say that **the passive attack $\boldsymbol{f}$ is dangerous with degree of danger $\boldsymbol{d}$ in the**

$(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ **–device parts** $\boldsymbol{fr\left(dev_{\mu_1}^{(V)}\right), \dots, fr\left(dev_{\mu_\nu}^{(V)}\right)}$ **of** $\boldsymbol{V}$ **and the** $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda})$ **– resource parts** $\boldsymbol{fr\left(res_{\kappa_1}^{(V)}\right), fr\left(res_{\kappa_2}^{(V)}\right), \dots, fr\left(res_{\kappa_\lambda}^{(V)}\right)}$ **of** $\boldsymbol{V}$**.■**

Remark 6 It is easy to verify that the following conditions 1 to 4 can be considered as stronger forms of the corresponding conditions in previous Proposition.

i. 1st Condition: From the point of view of users of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$, every attacked device part, as well as any attacked resource part, acquire new valuation measures that are smaller than the original corresponding valuations in node $\boldsymbol{V}$, with (at least) one such a valuation measure very reduced, i.e., for any $\boldsymbol{j} \in \{\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}\} \cup \{\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}\}$, it holds

$$\sum_{k=1}^{\mathfrak{n}} \left| \boldsymbol{a}_{j,k}^{(X \rightsquigarrow V)} + \boldsymbol{i}\widehat{\boldsymbol{a}}_{j,k}^{(X \rightsquigarrow V)} \right|^2 > \sum_{k=1}^{\mathfrak{n}} \left| \boldsymbol{a}'^{(X \rightsquigarrow V)}_{j,k} + \boldsymbol{i}\widehat{\boldsymbol{a}'}^{(X \rightsquigarrow V)}_{j,k} \right|^2$$

with at least one index $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \mathfrak{n}\}$ being such that

$$\left| \boldsymbol{a}_{j,k}^{(X \rightsquigarrow V)} + \boldsymbol{i}\widehat{\boldsymbol{a}}_{j,k}^{(X \rightsquigarrow V)} \right| \gg \left| \boldsymbol{a}'^{(X \rightsquigarrow V)}_{j,k} + \boldsymbol{i}\widehat{\boldsymbol{a}'}^{(X \rightsquigarrow V)}_{j,k} \right|$$

whenever $\boldsymbol{X} = \boldsymbol{V}, \boldsymbol{U}$.

ii. 2nd Condition: Similarly, from the point of view of users of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$, every attacked device part, as well as any attacked resource part, acquire new vulnerability measures that are smaller than the original corresponding vulnerabilities in node $\boldsymbol{V}$, with (at least) one such a vulnerability measure very reduced, i.e., for any $\boldsymbol{j} \in \{\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}\} \cup \{\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}\}$, it holds

$$\sum_{k=1}^{\mathfrak{m}} \left| \boldsymbol{b}_{j,k}^{(X \rightsquigarrow V)} + \boldsymbol{i}\widehat{\boldsymbol{b}}_{j,k}^{(X \rightsquigarrow V)} \right|^2 \le \sum_{k=1}^{\mathfrak{m}} \left| \boldsymbol{b}_{j,k}^{(X \rightsquigarrow V)} + \boldsymbol{i}\widehat{\boldsymbol{b}}_{j,k}^{(X \rightsquigarrow V)} \right|^2$$

with at least one index $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \mathfrak{m}\}$ being such that

$$\left| \boldsymbol{b}_{j,k}^{(X \rightsquigarrow V)} + \boldsymbol{i}\widehat{\boldsymbol{b}}_{j,k}^{(X \rightsquigarrow V)} \right| < \left| \boldsymbol{b}'^{(X \rightsquigarrow V)}_{j,k} + \boldsymbol{i}\widehat{\boldsymbol{b}'}^{(X \rightsquigarrow V)}_{j,k} \right|$$

whenever $\boldsymbol{X} = \boldsymbol{V}, \boldsymbol{U}$.

iii. 3rd Condition: From the viewpoint of the (user(s) of) node $\boldsymbol{U}$, in the offensive node $\boldsymbol{U}$ there are strongly growing valuations, i.e., there are $\boldsymbol{j} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m_U}, \boldsymbol{\mathcal{M}_U} + \boldsymbol{1}, \dots, \boldsymbol{\mathcal{M}_U} + \boldsymbol{\ell_U}\}$ and $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \mathfrak{n}\}$, such that

$$\left| \widehat{\boldsymbol{a}}_{j,k}^{(U \rightsquigarrow U)} \right| \ll \left| \widehat{\boldsymbol{a}'}^{(U \rightsquigarrow U)}_{j,k} \right|.$$

iv. 4th Condition: From the viewpoint of the (user(s) of) node $\boldsymbol{U}$, in the offensive node $\boldsymbol{U}$ there is no growing vulnerability, i.e., for any $\boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{m}_{\boldsymbol{U}}, \boldsymbol{\mathcal{M}}_{\boldsymbol{U}} + \mathbf{1}, \dots, \boldsymbol{\mathcal{M}}_{\boldsymbol{U}} + \boldsymbol{\ell}_{\boldsymbol{U}}\}$ and any $\boldsymbol{k} \in \{\mathbf{1}, \mathbf{2}, \dots, \mathfrak{m}\}$, it holds

$$\left|\widehat{\boldsymbol{b}}_{\boldsymbol{j},\boldsymbol{k}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right| \geq \left|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{j},\boldsymbol{k}}\right|.$$

12.4 Protected cyber nodes from active attacks

Finally, let's see how we could define the concept of protection from active cyber-attacks.

Definition 31

i. The node $\boldsymbol{V}$ is said to be **protected from active attacks, with degree of protection** $\boldsymbol{p} \in]\mathbf{0}, \mathbf{1}]$ over the $(\boldsymbol{\mu}_{\mathbf{1}}, \dots, \boldsymbol{\mu}_{\boldsymbol{\nu}})$ –device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_\nu}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa}_{\mathbf{1}}, \dots, \boldsymbol{\kappa}_{\boldsymbol{\lambda}})$ – resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right)$, …,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_\lambda}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ over a time period $\mathbb{I}$, if, during this time period, there is a nodal fixed filter system $\overline{\boldsymbol{V}}^{\{\boldsymbol{\mu}_1,\dots,\boldsymbol{\mu}_\nu\} \cup \{\boldsymbol{\kappa}_1,\dots,\boldsymbol{\kappa}_\lambda\}}$ in the union $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right) \cup \dots \cup \boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_\nu}^{(\boldsymbol{V})}\right) \cup \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right) \cup \dots \cup \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_\nu}^{(\boldsymbol{V})}\right)$ that allow every parallactic cyber active attack against the $(\boldsymbol{\mu}_{\mathbf{1}}, \dots, \boldsymbol{\mu}_{\boldsymbol{\nu}})$ –device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_\nu}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa}_{\mathbf{1}}, \dots, \boldsymbol{\kappa}_{\boldsymbol{\lambda}})$ – resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_\lambda}^{(\boldsymbol{V})}\right)$ of node $\boldsymbol{V}$ (from any offensive node $\boldsymbol{U}$) with degree of danger $\boldsymbol{d} \leq -\boldsymbol{logp}$ to reach only resource parts $\boldsymbol{K}$ of the initial target $\boldsymbol{V}$ that are disjoint from $\boldsymbol{E}$.

ii. During the time period $\mathbb{I}$, the node $\boldsymbol{V}$ is said to be **completely protected from active attacks of danger degree $\boldsymbol{d}$**, if, at this time period, any resource part in $\boldsymbol{V}$ is protected from active attacks against $\boldsymbol{V}$, with degree of protection $\boldsymbol{p} \leq \boldsymbol{e}^{-\boldsymbol{d}}$. The node $\boldsymbol{V}$ is said to be **completely protected from active attacks at a given time period**, if, during this time period, any resource part of $\boldsymbol{V}$ is protected from active attacks against $\boldsymbol{V}$ with degree of protection $\boldsymbol{p} = \mathbf{1}$.

13. Proactive Cyber Defense Against Cyber Attacks

13.1 Proactive Correlated Cyber Defense against Germs of Correlated Cyber-Attacks

Let $\mathcal{F} = \mathcal{F}_{W\to V}^{(correlated)}[\mathbb{I}]$ be a germ of correlated cyber attack from W against V, during a given time set $\mathbb{I}$ in a subinterval $]\alpha,\beta[\subset\subset [0,1]$. Recall that $\mathcal{F}$ is a family $\mathcal{F} = \{Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4, t \in \mathbb{I}\}$ of <u>coherent</u> interactions lying in the correlated danger sector $\mathfrak{X} = \mathfrak{X}_{W\to V}$ to the node V from the node W during the entire time set $\mathbb{I}$, provided, of course, that $\mathfrak{X} \neq \emptyset$. Denote by

$$\mathcal{D}_{\mathcal{F}} = \{\mathscr{g} = \mathscr{g}^{(Z)}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$
$$t \mapsto \mathscr{g}^{(Z)}(t) := \left(\gamma_W^{(Z)}(t), \gamma_V^{(Z)}(t), \gamma_W^{(Z)}(t+\Delta t), \gamma_V^{(Z)}(t+\Delta t)\right) : Z \in \mathcal{F}\Big\},$$

the associated coherent interactive family, a proactive correlated cyber-defense $\mathscr{f}$ against the cyber attack $\mathcal{F}$ during $\mathbb{I}$ is a map defined on the space of all cyber-interplays of the ordered cyber pair (V,W) over the entire time set $\mathbb{I}$,such that the image of $\mathfrak{X}$ via any member of the coherent interactive family $\mathcal{D}_{\mathcal{F}}$ in $\mathbb{I}$ is sent, through $\mathscr{f}$ in the complement $\mathfrak{X}^c = (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4 \setminus \mathfrak{X}$ of $\mathfrak{X}$. Specifically,

Definition 32 *Let X be the space of cyber activities $\mathscr{g}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V$ from the node W to the node V during the entire time set $\mathbb{I}$. A mapping $\mathscr{f}: X \to X$ is called proactive correlated cyber defense against the germ of attack $\mathcal{F}$ during $\mathbb{I}$, if $\mathscr{f}\big(\mathscr{g}(\mathfrak{X})\big) \subset \mathfrak{X}^c$, whenever $\mathscr{g} \in \mathcal{D}_{\mathcal{F}}$. The method of constructing and organizing a proactive correlated cyber defense, together with the way of processing and integrating the method in the node system, is called proactive correlated protection against the germ of attack $\mathcal{F}$. We will deal later with the question of such a protection.*

13.2 Proactive Absolute Cyber Defense against Germs of Absolute Cyber-Attacks

Let $\mathcal{F} = \mathcal{F}_{W\to V}^{(absolute)}[\mathbb{I}]$ be a germ of absolute cyber attack from W against V, during a given time set $\mathbb{I}$ in a subinterval $]\alpha,\beta[\subset\subset [0,1]$. Recall that $\mathcal{F}$ is a family $\mathcal{F} = \{Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4, t \in \mathbb{I}\}$ of <u>coherent</u> interactions lying in the absolute danger sector $\widetilde{\mathfrak{X}} = \widetilde{\mathfrak{X}}_{W\to V}$ to the node V from the node W during the entire time set $\mathbb{I}$, provided, of course, that $\widetilde{\mathfrak{X}} \neq \emptyset$. Denote by

$$\mathcal{D}_{\mathcal{F}} = \{g = g^{(Z)}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$
$$t \mapsto g^{(Z)}(t) := \left(\gamma_W^{(Z)}(t), \gamma_V^{(Z)}(t), \gamma_W^{(Z)}(t+\Delta t), \gamma_V^{(Z)}(t+\Delta t)\right): Z \in \mathcal{F}\Big\},$$

the associated coherent interactive family, a proactive absolute cyber-defense f against the cyber attack $\mathcal{F}$ during $\mathbb{I}$ is a map defined on the space of all cyber-interplays of the ordered cyber pair (V, W) over the entire time set $\mathbb{I}$,such that the image of $\widetilde{\mathfrak{X}}$ via any member of the coherent interactive family $\mathcal{D}_{\mathcal{F}}$ in $\mathbb{I}$ is sent, through f in the complement $\widetilde{\mathfrak{X}}^c = (\mathbb{C}^{\mathcal{N}\times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times \mathfrak{m}})^4 \setminus \widetilde{\mathfrak{X}}$ of $\widetilde{\mathfrak{X}}$. Specifically,

Definition 33 *Let again X be the space of cyber activities $g: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V$ from the node W to the node V during the entire time set $\mathbb{I}$. A mapping $f: X \to X$ is called proactive absolute cyber defense against the germ of attack $\mathcal{F}$ during $\mathbb{I}$, if $f\big(g(\mathfrak{X})\big) \subset \mathfrak{X}^c$, whenever $g \in \mathcal{D}_{\mathcal{F}}$. The method of constructing and organizing a proactive absolute cyber defense, together with the way of processing and integrating the method in the node system, is called proactive absolute protection against the germ of attack $\mathcal{F}$. We will deal later with the question of such a protection.*

13.3 Proactive partial cyber defense against germs of partial cyber-attacks

Let $\mathcal{F} = \mathcal{F}_{W\to V}^{(partial)}[\mathbb{I}]$ be a germ of partial cyber-attack from W against V, during a given time set $\mathbb{I}$ in a subinterval $]\alpha, \beta[\subset\subset [0,1]$. Recall that $\mathcal{F}$ is a family $\mathcal{F} = \{Z = Z_{(W,V)}(t) = \big((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times \mathfrak{m}})^4, t \in \mathbb{I}\}$ of <u>coherent</u> interactions lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{W\to V}$ to the node V from the node W during the entire time set $\mathbb{I}$, of course, that $\mathcal{E} \neq \emptyset$. Denote by

$$\mathcal{D}_{\mathcal{F}} = \{g = g^{(Z)}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$
$$t \mapsto g^{(Z)}(t) := \left(\gamma_W^{(Z)}(t), \gamma_V^{(Z)}(t), \gamma_W^{(Z)}(t+\Delta t), \gamma_V^{(Z)}(t+\Delta t)\right): Z \in \mathcal{F}\Big\},$$

the associated coherent interactive family, a proactive partial cyber-defense f against the cyber attack $\mathcal{F}$ during $\mathbb{I}$ is a map defined on the space of all cyber-interplays of the ordered cyber pair (V, W) over the entire time set $\mathbb{I}$,such that the image of $\mathcal{E}$ via any member of the coherent interactive family $\mathcal{D}_{\mathcal{F}}$ in $\mathbb{I}$ is sent, through f in the complement $\mathcal{E}^c = (\mathbb{C}^{\mathcal{N}\times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times \mathfrak{m}})^4 \setminus \mathcal{E}$ of $\mathfrak{X}$. Specifically,

Definition 34 *Let X be the space of cyber activities $g: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V$ from the node W to the node V during the entire time set $\mathbb{I}$. A mapping $f: X \to X$ is* *called proactive partial cyber defense against the germ of attack $\mathcal{F}$ during $\mathbb{I}$, if $\oint(\mathscr{g}(\mathfrak{X})) \subset \mathfrak{X}^c$, whenever $\mathscr{g} \in \mathcal{D}_{\mathcal{F}}$. The method of constructing and organizing a proactive partial cyber defense, together with the way of processing and integrating the method in the node system, is called proactive partial protection against the germ of attack $\mathcal{F}$. We will deal later with the question of such a protection.*

13.4 Proactive Protection against Germs of Partial Cyber-Attacks

Let us finally see how to illustrate such a proactive cyber defense.

Definition 35 *Suppose*

$$Z = Z_{(W,V)}(t_0) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),\,(z_4,\zeta_4)\big)(t_0) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

is a cyber-interaction between W and V at a fixed time moment $t_0 \in\,]\alpha,\beta[\, \subset\subset [0,1]$ ($W,V \in ob(cy(t))$), with corresponding cyber- interplay

$$\mathscr{g}\colon]\alpha,\beta[\, \mapsto \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V : t \mapsto \mathscr{g}(t) := \left(\gamma_W(t),\gamma_V(t),\gamma_W^{'}(t^{'}),\gamma_V^{'}(t^{'})\right)$$

and cyber-activity

$$\left(\mathscr{g}_t\colon \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : (\gamma_W(t),\gamma_V(t)\,) \longmapsto \left(\gamma_W^{'}(t^{'}),\gamma_V^{'}(t^{'})\right)\right)_{t\in]\alpha,\beta[} \quad (t^{'} := t+\Delta t).$$

A forced cyber-reflection of Z is another cyber-interaction

$$Z^{'} = Z_{(W,V)}^{'}(t_0) = \left((z_1^{'},\zeta_1^{'}),(z_2^{'},\zeta_2^{'}),(z_3^{'},\zeta_3^{'}),(z_4^{'},\zeta_4^{'})\right) = \left((z_1^{'},\zeta_1^{'}),(z_2^{'},\zeta_2^{'}),(z_3^{'},\zeta_3^{'}),(z_4^{'},\zeta_4^{'})\right)(t_0^{'}) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

between W and V at a next time moment $t_0^{'} = t_0 + \Delta t_0 \in\,]\alpha,\beta[$ with corresponding forced cyber- interplay

$$\mathscr{g}^{'}\colon]\alpha,\beta[\, \mapsto \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V : t \mapsto \mathscr{g}^{'}(t) := \left(\gamma_W^{'}(t^{'}),\gamma_V^{'}(t^{'}),\gamma_W^{''}(t^{''}),\gamma_V^{''}(t^{''})\right)$$

and associated forced cyber-activity:

$$\left(\mathscr{g}_{t^{'}}^{'}\colon \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : \left(\gamma_W^{'}(t^{'}),\gamma_V^{'}(t^{'})\,\right) \longmapsto \left(\gamma_W^{''}(t^{''}),\gamma_V^{''}(t^{''})\right)\right)_{t^{'}\in]0,1[} \quad (t^{''} := t^{'} + \Delta t^{'})$$

that satisfies the following property: into an open neighborhood $]t_0-\varepsilon, t_0+\varepsilon[$ of t_0, forces activity $\mathscr{g}$ to push forward its composition with activity $\mathscr{g}^{'}$, in such a way that the occurrence of $\mathscr{g}$ guarantees the appearance of the composition $\mathscr{g}^{'} \circ \mathscr{g}$.

Obviously, the matrices of the tetrad

$$Z^{'} = Z_{(W,V)}^{'}(t_0^{'}) = \left((z_1^{'},\zeta_1^{'}),(z_2^{'},\zeta_2^{'}),(z_3^{'},\zeta_3^{'}),(z_4^{'},\zeta_4^{'})\right) = \left((z_1^{'},\zeta_1^{'}),(z_2^{'},\zeta_2^{'}),(z_3^{'},\zeta_3^{'}),(z_4^{'},\zeta_4^{'})\right)(t_0^{'}) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

are of the form

$$\left(z_1',\zeta_1'\right)=\gamma_W'\left(t_0'\right)=\left(\mathbb{S}_{V\to W}'+i\widehat{\mathbb{S}}_{W\to W}',\mathbb{U}_{V\to W}'+i\widehat{\mathbb{U}}_{W\to W}'\right)\in\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$
$$\left(z_2',\zeta_2'\right)=\gamma_V'\left(t_0'\right)=\left(\mathbb{S}_{W\to V}'+i\widehat{\mathbb{S}}_{V\to V}',\mathbb{U}_{W\to V}'+i\widehat{\mathbb{U}}_{V\to V}'\right)\in\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$
$$\left(z_3',\zeta_3'\right)=\gamma_W''\left(t_0''\right)=\left(\mathbb{S}_{V\to W}''+i\widehat{\mathbb{S}}_{W\to W}'',\mathbb{U}_{V\to W}''+i\widehat{\mathbb{U}}_{W\to W}''\right)\in\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$
$$\left(z_4',\zeta_4'\right)=\gamma_V''\left(t_0''\right)=\left(\mathbb{S}_{W\to V}''+i\widehat{\mathbb{S}}_{V\to V}'',\mathbb{U}_{W\to V}''+i\widehat{\mathbb{U}}_{V\to V}''\right)\in\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}.$$

Definition 36 *The cyber-activity*

$$g\equiv g_t:\Omega_W\times\Omega_V\to\Omega_W\times\Omega_V:(\gamma_W(t),\gamma_V(t))\longmapsto\left(\gamma_W'(t'),\gamma_V'(t')\right)$$

together with its forced cyber-activity

$$g'=g_{t'}':\Omega_W\times\Omega_V\to\Omega_W\times\Omega_V:\left(\gamma_W'(t'),\gamma_V'(t')\right)\longmapsto\left(\gamma_W''(t''),\gamma_V''(t'')\right)$$

is called a reflexive cyber-activity between W and V during the time interval $]\alpha,\beta[$. Their composition

$$g'\circ g:\Omega_W\times\Omega_V\to\Omega_W\times\Omega_V:(\gamma_W(t),\gamma_V(t))\longmapsto\left(\gamma_W''(t'+\Delta t),\gamma_V''(t'+\Delta t)\right)$$

is said to be a self-inflicted cyber-activity between W and V during the time interval $]\alpha,\beta[$. In particular, the interaction $Z'=Z_{(W,V)}'\left(t_0'\right)$ is called forced cyber-reflection of $Z=Z_{(W,V)}(t_0)$ at time moment t_0. A mapping

$$\Phi:(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^2\to(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^2$$

which maps the cyber-interaction $Z=Z_{(W,V)}(t_0)$ to its forced cyber-reflection $Z'=Z_{(W,V)}'\left(t_0'\right)$ is called reflexive cyber-interaction mapping at time moment t_0.

Remark 7 It is frequent that, under a self-inflicted cyber-activity

$$g'\circ g:\Omega_W\times\Omega_V\to\Omega_W\times\Omega_V:(\gamma_W(t),\gamma_V(t))\longmapsto\left(\gamma_W''(t'+\Delta t),\gamma_V''(t'+\Delta t)\right)$$

between W and V during the time interval $]\alpha,\beta[$, some valuations and vulnerabilities of the initial node W change at a moment $t_0\in]\alpha,\beta[$, in such a way to get new constituent valuations and new constituent vulnerabilities for the node W. *For emphasis*, this “new” node is called variant node of W and is denoted by W', or sometimes, without any risk of confusion, again by W. In such a case, the forced cyber-reflection $Z'=Z_{(W,V)}'\left(t_0'\right)$ is called cyber parallax of the cyber-interaction $Z=Z_{(W,V)}(t_0)$ at t_0 and the forced cyber-activity $g'=g_{t'}':\Omega_W\times\Omega_V\to\Omega_W\times\Omega_V:\left(\gamma_W'(t'),\gamma_V'(t')\right)\longmapsto\left(\gamma_W''(t''),\gamma_V''(t'')\right)$ is called parallactic cyber-activity. Finally, we say that *the self-inflicted parallactic cyber-activity $g'\circ g$: $\Omega_W(t)\times\Omega_V(t)\to\Omega_W(t'+\Delta t)\times\Omega_V(t'+\Delta t)$ between W and V at t_0 gives rise to a parallactic cyber-*

interaction at t_0.

Let us give a schematic representation.

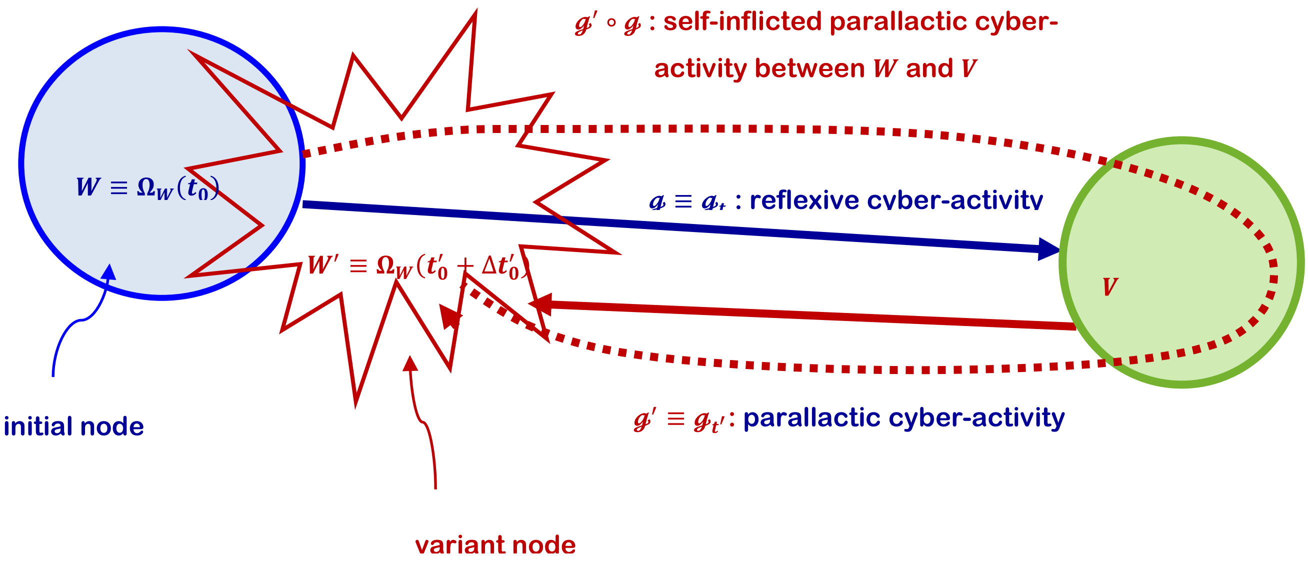

Figure 5 (Cyber Parallax)

Definition 37 *Let* $E = fr\left(\mathcal{A}^{(W)}\right)$ *be a set in the* σ *–algebra* $\mathfrak{U}_{\mathcal{P}}$ *of subsets of available or not constituents of node* W*:*

$$\mathcal{A} = \begin{cases} dev, if\ the\ constituent\ is\ a\ device\ , \\ res, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

i *A shield of* E *in the node* W *(or a node shield containing* E*) at time* t *is an intermediate fixed node* $\overline{\overline{W}} = \overline{\overline{W}}_t$ *which, at this time, is interposed in each cyber parallax* g' *that aims at* E *in the node* W*, so that the self-inflicted parallactic cyber-activity* $g' \circ g$ *between* W *and* V *at moment time* t *ends up in the intermediate node* $\overline{\overline{W}}$*, and never can reach part* E *of the initial target* W*. The detailed process by which the node shield* $\overline{\overline{W}}$ *of a node* W *blocks the self-inflicted parallactic cyber-activity* $g' \circ g$ *and never ends up in the initial target* W*, is being analyzed in a forthcoming paper.*

ii. *Given a node* W*, a node filter in part* E *of the constituent* $\mathcal{A}^{(W)}$ *in* W *at a time moment* t *is an intermediate fixed node* $\overline{W}^{(E)}$ *which, at this time moment, is interposed in each parallactic cyber-activity* g' *that aims at part* E *of node* W*, so that the filter* $\overline{W}^{(E)}$ *allows the self-inflicted parallactic cyber-activity* $g' \circ g$ *at* t *to* *reach only constituent parts of the initial target W that are different from part E of the constituent $\mathcal{A}^{(W)}$ of W.*

14. Elements of Proactive Cyber Defense

Having rigorously determined the concept of **proactive cyber defense** and **proactive cyber protection**, as well as the 4 types of proactive cyber defenses, we take the initiative to present some applicable elements of this defense.

14.1 General Remarks

Given that the potential number of all possible devices of a node $\boldsymbol{V}$ is equal to $\boldsymbol{\mathcal{M}_V}$, while the number of $\boldsymbol{V}$'s available devices in a specific time t is only $\boldsymbol{m_V} = \boldsymbol{m_V(t)}$, with $\boldsymbol{m_V} < \boldsymbol{\mathcal{M}_V}$ and in addition the potential quantity (or number) of all possible resource elements of $\boldsymbol{V}$ is equal to $\boldsymbol{\mathcal{L}_V}$, while the quantity (or number) of $\boldsymbol{V}$'s available resource elements in a specific time $\boldsymbol{t}$ is only $\boldsymbol{\ell_V} = \boldsymbol{\ell_V(t)}$, in the sense that $\boldsymbol{\ell_V} < \boldsymbol{\mathcal{L}_V}$, we can say that **if we increase $\boldsymbol{m_V}$ and $\boldsymbol{\ell_V}$ in such a level that availability of devices $\boldsymbol{m_V} = \boldsymbol{m_V}(t)$, and resources $\boldsymbol{\ell_V} = \boldsymbol{\ell_V}(t)$ are comparable to $\boldsymbol{\mathcal{M}_V}$ and $\boldsymbol{\mathcal{L}_V}$, we can succeed the redundancy of both devices and resources that is vital in a cyber-domain**.

In addition, increasing $\boldsymbol{m_V}$ and $\boldsymbol{\ell}_V$ we can also harden the success of correlation between devices and resources, in other words reduce the probability of success of a multitude of attack (for example, reconnaissance attack) since the process is more difficult.

In any case, when an attacker node $\boldsymbol{U}$ decides to make an enumeration of available devices $\boldsymbol{m_V} = \boldsymbol{m_V(t)}$ and available resources $\boldsymbol{\ell_V} = \boldsymbol{\ell_V(t)}$ of a target node $\boldsymbol{V}$, in order to check potential vulnerabilities or/and prepare the ground for a more active and malicious attack, this task is being more and more tough proportional to the number of $\boldsymbol{m_V}$ and $\boldsymbol{\ell_V}$.

As it is more than obvious that after an enumeration or reconnaissance attack it is probable a more sophisticated and malicious attack to take place, a proactive measure could be to deactivate/disable the devices and resources that are not vital and critical for the functionality of the node. Thus, the valuation of constituents of the node $\boldsymbol{V}$, from the point of user/s of node $\boldsymbol{V}$, in the case of deactivation should be

$$\boldsymbol{a_i}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})} = \boldsymbol{0}, \qquad \boldsymbol{i} = \boldsymbol{1}, \dots, \boldsymbol{m_V}, \boldsymbol{m_V} + \boldsymbol{1}, \dots, \boldsymbol{m_V} + \boldsymbol{\ell_V}$$

for $\boldsymbol{dev_i^{(V)}}$and $\boldsymbol{res_i^{(V)}}$ that are not critical and vital for the functionality of the node $\boldsymbol{V}$.

On the other hand, the concept of node filter $\overline{\boldsymbol{U}}^{(\boldsymbol{\kappa}_\circ)}$ in the $\boldsymbol{\kappa}_\circ$ −constituent of a given node $\boldsymbol{U}$, that has already given in [2], is quite critical for a robust proactive defense. It can be said that the $|\boldsymbol{m_U} + \boldsymbol{\ell_U}|$ impose the level of quality of services, satisfaction, independency of cyber users in a node, thus

$$|\boldsymbol{m_U} + \boldsymbol{\ell_U}| \propto \boldsymbol{Q_t}$$

where $\boldsymbol{Q_t} \in [\boldsymbol{0}, \infty)$ is a metric that overall depicts the level of user satisfaction in a specific node $\boldsymbol{U}$ from the perspective of the available to the users services.

It is well known that if we want to be in the edge of the art we cannot be completely protected. We cannot provide “everything” to our node users assuring also them a complete secure cyber environment.

It is obvious that this level contradicts the security of the node since it is well known that the higher $|\boldsymbol{m_V} + \boldsymbol{\ell_V}|$, that is connected to the availability of services and multimedia, the independency and the “loose” safety (access control etc.) regulations, the less security applied. For that reason, **if we want to proactively safeguard a node we have to “wider” the node filter $\overline{\boldsymbol{U}}^{(\boldsymbol{\kappa}_\circ)}$ to the extent of multi $\boldsymbol{\kappa}_\circ$ −constituents, in other words to make it more “broadband”.**

Remark 8 The presence of a kind of node filters $\overline{\boldsymbol{U}}^{(\boldsymbol{\kappa}_\circ)}$ in the $\boldsymbol{\kappa}_\circ$ −constituent as well as node shields $\overline{\overline{\boldsymbol{U}}}$ of a node $\boldsymbol{U}$ in **e-flows** (between nodes) can proactively solve a multitude of problems. This is a critical procedure that can be applied in main backbones of a cyber-domain.

Apart from the above, it should be noted that there are some straight-forward characteristics of a node $\boldsymbol{V}$ that suggest, under some circumstances, a suspicious cyber-activity.

- **Geographical coordinates** $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}$ of the node $\boldsymbol{V}$: Without doubt, there are potential “malicious” areas in the cyber-domain.
- **Time** $\boldsymbol{t}$ is critical for an attack outburst.
- The **knowledge of previous malicious activity** of the node $\boldsymbol{V}$.

14.2 Proactive Defense in a Cyber-Walk

If in a cyber-walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_n}$, we have $\boldsymbol{n} \gg \boldsymbol{cyber\ geodesic}$, that means that the walk is much more complex of a normal and anticipated one, either something wrong evolved or something malicious is ongoing. Without doubt, **lengthy cyber-walks imply abnormality** and can be a critical indication that something malicious "is coming close".

Moreover, introducing new concepts, we can add an innovative approach to proactive defense that is, without any doubt the main goal of this paper. For example, considering the Euclidean distances

$$\left|\boldsymbol{\mathcal{A}}^{(U,V)}\right| \coloneqq \left\|\boldsymbol{A}^{(U\rightsquigarrow U)}(\boldsymbol{U}) - \boldsymbol{A}^{(U\rightsquigarrow V)}(\boldsymbol{V})\right\|_2 \equiv \left(\sum_{i=1}^{\boldsymbol{k}} \left|\widehat{\boldsymbol{a}}_i^{(U\rightsquigarrow U)} - \boldsymbol{a}_i^{(U\rightsquigarrow V)}\right|^2\right)^{1/2} \text{ and}$$

$$\left|\boldsymbol{\mathcal{B}}^{(U,V)}\right| \coloneqq \left\|\boldsymbol{B}^{(U\rightsquigarrow U)}(\boldsymbol{U}) - \boldsymbol{B}^{(U\rightsquigarrow V)}(\boldsymbol{V})\right\|_2 \equiv \left(\sum_{i=1}^{\boldsymbol{k}} \left|\widehat{\boldsymbol{b}}_i^{(U\rightsquigarrow U)} - \boldsymbol{b}_i^{(U\rightsquigarrow V)}\right|^2\right)^{1/2}$$

($\boldsymbol{k} \coloneqq \boldsymbol{\mathcal{M}}_V + \boldsymbol{\mathcal{L}}_V = \boldsymbol{\mathcal{M}}_U + \boldsymbol{\mathcal{L}}_U$) between valuations and vulnerabilities in two nodes $\boldsymbol{U}$ and $\boldsymbol{V}$, one could investigate the validity of the next two assertions:

- *If there is an option for a node $\boldsymbol{U}$ to choose, in a cyber-walk, the very next step $\boldsymbol{V_i}$, this should be (healthier) to choose the node with $\boldsymbol{max}\left|\boldsymbol{\mathcal{A}}^{(U,V_i)}\right|$ and/or $\boldsymbol{min}\left|\boldsymbol{\mathcal{B}}^{(U,V_i)}\right|$.*
- *The probability of a momentary "malicious" homomorphism $\boldsymbol{g_t} : \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \to \mathbb{G}_{\boldsymbol{t}+\Delta \boldsymbol{t}}^{(\boldsymbol{V})}$ between the two cyber nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ at a moment time $\boldsymbol{t}$ is higher as the Euclidean distances $\left|\boldsymbol{\mathcal{A}}^{(U,V)}\right|$ and $\left|\boldsymbol{\mathcal{B}}^{(U,V)}\right|$ are higher.*

In any case, defenders have to strive in order to prevent or at least to early recognize a reconnaissance activity, since as it is already mentioned that this is the prerequisite and at the same time the preparation of a passive or/and an active attack.

14.3 The Correlation Indicator

Both a proactive defense measure and a counter action that can be taken against a cyber-espionage attack or/and in an embedding of malicious software (worms, Trojans etc.) in a node, is a frequent **Correlation Indicator (CI)** application.

To do so, assume a node $\boldsymbol{V}$ and the $[\boldsymbol{\mathcal{C}_{available}}(\boldsymbol{V})](\boldsymbol{t_i})]$ that is the set of ordered columns of all available constituents $\left(\boldsymbol{dev}_{\boldsymbol{1}}^{(\boldsymbol{V})}, \ldots, \boldsymbol{dev}_{\boldsymbol{m_V}}^{(\boldsymbol{V})}, \boldsymbol{res}_{\boldsymbol{1}}^{(\boldsymbol{V})}, \ldots, \boldsymbol{res}_{\boldsymbol{\ell_V}}^{(\boldsymbol{V})}\right)^{\boldsymbol{T}}$ of $\boldsymbol{V}$, over the time $\boldsymbol{t_i} \in [\boldsymbol{0}, \boldsymbol{1}]$ and $[\boldsymbol{\mathcal{C}_{available}}(\boldsymbol{V})](\boldsymbol{t_j})]$ that is the set of ordered columns of all available constituents $\left(dev_1^{(V)},\ldots,dev_{m'_V}^{(V)},res_1^{(V)},\ldots,res_{\ell'_V}^{(V)}\right)^T$ of V, over the time $t_j \in [0,1]$. We use CI in the following way in order to find any *difference between the random (in time) vectors* $[\mathcal{C}_{available}(V)](t_i)]$ and $[\mathcal{C}_{available}(V)](t_j)]$:

$$CI \begin{cases} = 0 \; when \; m_V = m'_V \; and \; \ell_V = \ell'_V \\ \neq 0 \; when \; m_V \neq m'_V \; and/or \; \ell_V \neq \ell'_V. \end{cases}$$

Thus, when $CI = 0$, the possibility of a cyber espionage attack or node infection is very low and similarly, when $CI \neq 0$ a cyber espionage attack is high, given that the changes on $\ell'_V \; and/or \; m'_V$ are not intentionally from the node V itself. In other words, if in a node there is an internal process of a frequent application of the **Correlation Indicator (CI)** any abnormality of an embedment of malicious resources or/and devices can be potentially detected.

It is worth mentioning that systems/nodes, if the detection process is not effective, may stay infected forever, putting at stake the whole system/node.

14.4 Proactiveness against Smooth Cyber-Attacks

Most of the times, the majority of cyber-attacks are being developed gradually and smoothly. So, if we notice one or a combination of the following "behaviours", an indication of an abnormality can be submerged and thus an on-going attack or a preparation of an attack:

- Sudden degradation of our neighbor node, thus when $max\{\varphi^{(U \rightsquigarrow V)}(t), \hat{\varphi}^{(V \rightsquigarrow V)}(t)\} \ll 0$ (:decreasing suddenly, see below §7.1 for the definition of $\varphi^{(U \rightsquigarrow V)}(t)$ and $\hat{\varphi}^{(V \rightsquigarrow V)}(t)$) and without a warning or reasonable (power fail, schedule maintenance etc.) cause.
- Frequent reconnaissance attacks against our node.

14.5 Proactiveness in Fractal Cyber-Space

If we start thinking the cyber space as a **fractal**, we can easily simplify the processes and give more proactiveness to our policy and approach. So, if we make the assumption that all processes can also apply to a "multi-node" area or even bigger, we can predict any trend of malicious behaviour in advance.

Accordingly, we can assume that every wide cyber area at any moment t, is a hyper node

$$\boldsymbol{V_h = V_{h\left(x_{1_h}, x_{2_h}, x_{3_h}, t\right)}}$$

in location $\left(\boldsymbol{x_{1_h}, x_{2_h}, x_{3_h}}\right)$, which is the cyber-center of gravity of all nodes of the hyper node $\boldsymbol{V_1^{(V_h)}}$, $\boldsymbol{V_2^{(V_h)}} \dots \boldsymbol{V_n^{(V_h)}}$. It is reasonable and clearly understood that if a node or a group of nodes of this hyper node moved the cyber-center of gravity changes coordinates thus the hyper node alters its characteristics. The valuation of a hyper node it depends on $\left(\boldsymbol{x_{1_h}, x_{2_h}, x_{3_h}, t}\right)$ and actually is

$$\boldsymbol{\beta^{(h)} = \left(\beta^{(1)} + \beta^{(2)} + \cdots + \beta^{(n)}\right)/n}.$$

It is assumed that this valuation depicts the overall cyber energy of the area. This valuation of the hyper node $\boldsymbol{\beta^{(h)}}$ is supposed to lie in the unit interval $[\boldsymbol{0}, \boldsymbol{1}]$. It is clear here that in a case of a physical (intentionally or not) destruction of a node or a hyper node, the valuation $\boldsymbol{\beta^{(h)}} := \boldsymbol{0}$.

14.6 Node Sourcing

Having in mind a relatively new concept of **Crowd Sourcing** that is the process of obtaining information or input into a particular task by enlisting the services of a number of people, typically via the Internet, we can introduce here an identical concept of **Node Sourcing** in the context of valuation and vulnerability.

Imagine that in any circumstance a node $\boldsymbol{U}$ can ask the Node Sourcing from its neighbor's node user/s in order to assess the valuation and vulnerability of a target node $\boldsymbol{V}$. For example, if a node $\boldsymbol{U}$ needs the **Node Sourcing of its neighbor**s nodes $\boldsymbol{A, B, E, H}$ to assess the valuation of available resources of $\boldsymbol{V}$, then the result that potentially is more accurate than the

$$\boldsymbol{\mathcal{S}_U \mathfrak{R}_{available}(V)} = \left\{\left(\boldsymbol{\mathcal{S}_U[x_1, x_2, x_3, t]}\left(\boldsymbol{res_1^{(V)}}\right), \dots, \boldsymbol{\mathcal{S}_U[x_1, x_2, x_3, t]}\left(\boldsymbol{res_{\ell_V}^{(V)}}\right)\right)^{\boldsymbol{T}}\right.$$

is the following:

$$\boldsymbol{\mathcal{S}_{U\ Node\ Sourcing}\ \mathfrak{R}_{available}(V)} =$$

$$\frac{\boldsymbol{\mathcal{S}_W \mathfrak{R}_{available}(V) + \mathcal{S}_A \mathfrak{R}_{available}(V) + \mathcal{S}_B \mathfrak{R}_{available}(V) + \mathcal{S}_E \mathfrak{R}_{available}(V) + \mathcal{S}_H \mathfrak{R}_{available}(V)}}{\boldsymbol{5}}.$$

The same holds for any assessment of valuation and vulnerabilities of any node.

15. Mathematical Description of Representative Cyber Attacks

So, having consistently examined the more general cases of a passive and active attacks, we will try to focus on some indicative, yet quite important, cases, namely the cyber espionage attack, the access attack, the reconnaissance attack, the denial of service attack, and the distributed denial of service attack.

In order to go further and get the full description of these indicative cyber-attacks, it would be wise to mathematically orient and define some further concepts. The sophistication of development of any cyber-attack is a critical issue and can be described as follows.

15.1 Sophistication of Cyber Attacks

The term “sophisticated” is often used inconsistently or incorrectly by the cyber community. Seldom will the victim of a cyber-attack disclose that they have been targeted without characterising either the attack or assailant as “sophisticated”. But the label is often applied inconsistently, either inadvertently or deliberately. The term, even though it is highly important and critical, loses its value when overused, and should instead be employed to differentiate exceptional attacks or attackers from the norm.

Victims of cyber-attacks are not necessarily best placed to identify how exceptional their compromise is compared with other incidents. There may also be reasons for the victim to exaggerate the complexity of the attack, or the perpetrator’s ability. In doing so they imply the breach was unavoidable, absolving them of responsibility in the eyes of potentially litigious customers or shareholders. Wrongly characterizing an attack, however, is not without consequence. If simple, preventable attacks are labeled as sophisticated and inevitable, rather than a product of rectifiable vulnerabilities or security lapses, then those vulnerabilities may be allowed to fester.

It’s obvious that the most sophisticated cyber-attacks have not yet been detected. While sophisticated attacks are often effective, attacks need not be sophisticated to be effective. In that direction, and in order to establish a concrete behavior against sophisticated cyber-attacks, we will try to define the term “sophistication” of a cyber-attack in accordance to the whole concept of this dissertation. We earnestly believe that prescriptive definitions are problematic because there will inevitably be exceptions and the criteria will have to be dynamic enough to reflect the unrelenting pace of cyber capability development and proliferation.

The “sophistication” of a cyber-attack concept is a puzzle of definitions that form the big picture. To enter the structural operational status of such a **“sophisticated” attack puzzle**, suppose *the derivatives*

$$\boldsymbol{\varphi}^{(U \rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\boldsymbol{a}^{(U \rightsquigarrow V)}\}}{\partial t}(\boldsymbol{t}) =$$

$$\frac{\partial\{\left(\boldsymbol{a}_1^{(U \rightsquigarrow V)}, \ldots \boldsymbol{a}_{m_V}^{(U \rightsquigarrow V)}, \boldsymbol{a}_{m_V+1}^{(U \rightsquigarrow V)}, \ldots, \boldsymbol{a}_{\mathcal{M}_V}^{(U \rightsquigarrow V)}, \boldsymbol{a}_{\mathcal{M}_V+1}^{(U \rightsquigarrow V)}, \ldots, \boldsymbol{a}_{\mathcal{M}_V+\ell_V+1}^{(U \rightsquigarrow V)}, \boldsymbol{a}_{\mathcal{M}_V+\ell_V+1}^{(U \rightsquigarrow V)}, \ldots, \boldsymbol{a}_{\mathcal{M}_V+\mathcal{L}_V}^{(U \rightsquigarrow V)}\right)^T\}}{\partial t}(\boldsymbol{t})$$

and

$$\widehat{\boldsymbol{\varphi}}^{(V \rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\}}{\partial t}(\boldsymbol{t}) =$$

$$\frac{\partial\{\left(\widehat{\boldsymbol{a}}_1^{(V \rightsquigarrow V)}, \ldots \widehat{\boldsymbol{a}}_{m_V}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{a}}_{m_V+1}^{(V \rightsquigarrow V)}, \ldots, \widehat{\boldsymbol{a}}_{\mathcal{M}_V}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1}^{(V \rightsquigarrow V)}, \ldots, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V+1}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V+1}^{(V \rightsquigarrow V)}, \ldots, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\mathcal{L}_V}^{(V \rightsquigarrow V)}\right)^T\}}{\partial t}(\boldsymbol{t})$$

exist in a time interval $\mathbb{I} = \,]\boldsymbol{\alpha}, \boldsymbol{\beta}[$ *in the sense of distributions*. In such a case, we say that the relative effectiveness states $\boldsymbol{a}^{(U \rightsquigarrow V)} = \boldsymbol{a}^{(U \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,] \in \mathbb{R}^{\mathcal{k}}$ and $\widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)} = \widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,] \in \mathbb{R}^{\mathcal{k}}$ are two **smooth node valuations** and the distributional derivatives $\boldsymbol{\varphi}^{(U \rightsquigarrow V)}(\boldsymbol{t})$ and $\widehat{\boldsymbol{\varphi}}^{(V \rightsquigarrow V)}(\boldsymbol{t})$ are the **rate changes/slopes of the valuations** $\boldsymbol{a}^{(U \rightsquigarrow V)}$ and $\widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)}$ respectively, at a point $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\,)$ of a part $\boldsymbol{E}$ into the node $\boldsymbol{V}$ from the viewpoint of the (user(s) of) node $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively, over the time interval $\mathbb{I}$. Here, as usually, $\mathcal{k} := \mathcal{M}_V + \mathcal{L}_V$.

For $\boldsymbol{\Phi} = \boldsymbol{\varphi}, \widehat{\boldsymbol{\varphi}}$ and $\boldsymbol{X}, \boldsymbol{Y} \in \{\boldsymbol{U}, \boldsymbol{V}\}$, it is obvious that

1. If $\boldsymbol{\Phi}^{(X \rightsquigarrow Y)}(\boldsymbol{t}) > \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{A}}_X^+(\boldsymbol{Y})](\mathbb{I})$ of correlated growth for the total valuation of the node $\boldsymbol{Y}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
2. If $\boldsymbol{\Phi}^{(X \rightsquigarrow Y)}(\boldsymbol{t}) < \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{A}}_X^-(\boldsymbol{Y})](\mathbb{I})$ of correlated reduction for the total valuation of the node $\boldsymbol{Y}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
3. If $\boldsymbol{\Phi}^{(X \rightsquigarrow Y)}(\boldsymbol{t}) = \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$, there is no correlated growth or reduction for the total valuation of the node $\boldsymbol{Y}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$, due to a multitude of potential reasons.

By analogy, suppose *the derivatives*

$$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\boldsymbol{b}^{(U\rightsquigarrow V)}[\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3},\boldsymbol{t}\,]\}}{\partial \boldsymbol{t}}(\boldsymbol{t}) =$$

$$\frac{\partial\{\left(\boldsymbol{b}_1^{(U\rightsquigarrow V)},\ldots,\boldsymbol{b}_{m_V}^{(U\rightsquigarrow V)},\boldsymbol{b}_{m_V+1}^{(U\rightsquigarrow V)},\ldots,\boldsymbol{b}_{\mathcal{M}_V}^{(U\rightsquigarrow V)},\boldsymbol{b}_{\mathcal{M}_V+1}^{(U\rightsquigarrow V)},\ldots,\boldsymbol{b}_{\mathcal{M}_V+\ell_V+1}^{(U\rightsquigarrow V)},\boldsymbol{b}_{\mathcal{M}_V+\ell_V+1}^{(U\rightsquigarrow V)},\ldots,\boldsymbol{b}_{\mathcal{M}_V+\mathcal{L}_V}^{(U\rightsquigarrow V)}\right)^{\boldsymbol{T}}\}}{\partial \boldsymbol{t}}(\boldsymbol{t})$$

and

$$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}[\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3},\boldsymbol{t}\,]\}}{\partial \boldsymbol{t}}(\boldsymbol{t}) =$$

$$\frac{\partial\{\left(\widehat{\boldsymbol{b}}_1^{(V\rightsquigarrow V)},\ldots,\widehat{\boldsymbol{b}}_{m_V}^{(V\rightsquigarrow V)},\widehat{\boldsymbol{b}}_{m_V+1}^{(V\rightsquigarrow V)},\ldots,\widehat{\boldsymbol{b}}_{\mathcal{M}_V}^{(V\rightsquigarrow V)},\widehat{\boldsymbol{b}}_{\mathcal{M}_V+1}^{(V\rightsquigarrow V)},\ldots,\widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V+1}^{(V\rightsquigarrow V)},\widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V+1}^{(V\rightsquigarrow V)},\ldots,\widehat{\boldsymbol{b}}_{\mathcal{M}_V+\mathcal{L}_V}^{(V\rightsquigarrow V)}\right)^{\boldsymbol{T}}\}}{\partial \boldsymbol{t}}(\boldsymbol{t})$$

exist in a time interval $\mathbb{I} =]\boldsymbol{\alpha},\boldsymbol{\beta}[$ *in the sense of distributions*. In such a case, we say that the relative effectiveness states $\boldsymbol{b}^{(U\rightsquigarrow V)} = \boldsymbol{b}^{(U\rightsquigarrow V)}[\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3},\boldsymbol{t}\,] \in \mathbb{R}^{\mathscr{k}}$ and $\widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)} = \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}[\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3},\boldsymbol{t}\,] \in \mathbb{R}^{\mathscr{k}}$ are two **smooth node vulnerabilities** and the distributional derivatives $\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t})$ and $\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$ are the **rate changes/slopes of the vulnerabilities** $\boldsymbol{b}^{(U\rightsquigarrow V)}$ and $\widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}$ respectively, at a point $(\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3}\,)$ of a part $\boldsymbol{E}$ into the node $\boldsymbol{V}$ from the viewpoint of the (user(s) of) node $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively, over the time interval $\mathbb{I}$.

As above, for $\boldsymbol{\Psi} = \boldsymbol{\psi},\widehat{\boldsymbol{\psi}}$ and $\boldsymbol{X},\boldsymbol{\Upsilon} \in \{\boldsymbol{U},\boldsymbol{V}\}$, it is obvious that:

1. If $\boldsymbol{\Psi}^{(X\rightsquigarrow \Upsilon)}(\boldsymbol{t}) > \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{B}}_{\boldsymbol{X}}^{+}(\boldsymbol{\Upsilon})](\mathbb{I})$ of correlated growth for the total vulnerability of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
2. If $\boldsymbol{\Psi}^{(X\rightsquigarrow \Upsilon)}(\boldsymbol{t}) < \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{B}}_{\boldsymbol{X}}^{-}(\boldsymbol{\Upsilon})](\mathbb{I})$ of correlated reduction for the total vulnerability of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
3. If $\boldsymbol{\Psi}^{(X\rightsquigarrow \Upsilon)}(\boldsymbol{t}) = \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$, there is no correlated growth or reduction of the total vulnerability for node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$, due to a multitude of potential reasons.

Remark 9 Having defined the rate change of valuations and vulnerabilities we can proceed to orientation of sophistication in cyber-attacks, definition which will support our further posture in this paper. So, if we have one or combination of the following states that declare a slow infection (constituents' degradation) we assume that there should be a **suspicion of sophistication** $\widehat{\boldsymbol{\varphi}}^{(V \rightsquigarrow V)} \cong \mathbf{0}^-$ and $\widehat{\boldsymbol{\psi}}^{(V \rightsquigarrow V)} \cong \mathbf{0}^+$.

15.2 Man in the Middle Vs Wiretapping Cyber Attacks

It would be very helpful and constructive, for the sake of the smooth development of this dissertation, to mathematically define on parallel the aforementioned attacks. **Man in the Middle attack**, where the attacker secretly relays and possibly alters the communication between two parties who believe they are directly communicating with each other, belongs to active cyber-attacks, and on the other hand, **wiretapping attack** which is a passive attack that consists in the monitoring of cyber activity, often by covert means.

In the **Man in the Middle (MiTM) attack of a node $\boldsymbol{Z}$** in the cyber-interaction between nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ we have the "active" intersection of node $\boldsymbol{Z}$. Actually in this "active" intersection (MitM) attack, instead of this "normal" interaction we experience an active attack from node $\boldsymbol{Z}$ to either or/and both of other nodes **using some resources of the other interacted node**. In such a case, a family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{Z\to V}$ to the node $\boldsymbol{V}$ from the node Z during the entire time set $\mathbb{I}$, is a **germ of (partial) active attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ – **device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_2}^{(V)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_\nu}^{(V)}\right)$ **of $\boldsymbol{V}$ and the** $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda})$ – **resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_2}^{(V)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$ **of $\boldsymbol{V}$**, during a given time set $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{Z}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) =$$

$$
\left(\left(\begin{pmatrix}
a_{1,1}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{1,1}^{(V\rightsquigarrow V)} & \cdots & a_{1,\mathfrak{n}}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\cdots & \cdots & \cdots \\
a_{m_V,1}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{m_V,1}^{(V\rightsquigarrow V)} & & a_{m_V,\mathfrak{n}}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
a_{\mathcal{M}_V+1,1}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & a_{\mathcal{M}_V+1,\mathfrak{n}}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\cdots & \cdots & \cdots \\
a_{\mathcal{M}_V+\ell_V,1}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & a_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
b_{1,1}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{1,1}^{(V\rightsquigarrow V)} & \cdots & b_{1,\mathfrak{m}}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\cdots & \cdots & \cdots \\
b_{m_V,1}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & b_{m_V,\mathfrak{m}}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
b_{\mathcal{M}_V+1,1}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & b_{\mathcal{M}_V+1,\mathfrak{m}}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\cdots & & \cdots \\
b_{\mathcal{M}_V+\ell_V,1}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & b_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right),
$$

$$
\left(\left(\begin{pmatrix}
a_{1,1}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{1,1}^{(Z\rightsquigarrow Z)} & \cdots & a_{1,\mathfrak{n}}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{1,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\
\cdots & \cdots & \cdots \\
a_{m_Z,1}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{m_Z,1}^{(Z\rightsquigarrow Z)} & \cdots & a_{m_Z,\mathfrak{n}}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{m_Z,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
a_{\mathcal{M}_Z+1,1}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+1,1}^{(Z\rightsquigarrow Z)} & \cdots & a_{\mathcal{M}_Z+1,\mathfrak{n}}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+1,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\
\cdots & & \cdots \\
a_{\mathcal{M}_Z+\ell_Z,1}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+\ell_Z,1}^{(Z\rightsquigarrow Z)} & \cdots & a_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$\begin{pmatrix} \boldsymbol{b}_{\mathbf{1,1}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{\mathbf{1,1}}^{(Z \rightsquigarrow Z)} & \cdots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(Z \rightsquigarrow Z)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{m_Z,\mathbf{1}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{m_Z,\mathbf{1}}^{(Z \rightsquigarrow Z)} & \cdots & \boldsymbol{b}_{m_Z,\mathfrak{m}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{m_Z,\mathfrak{m}}^{(Z \rightsquigarrow Z)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_Z+\mathbf{1},\mathbf{1}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+\mathbf{1},\mathbf{1}}^{(Z \rightsquigarrow Z)} & \cdots & \boldsymbol{b}_{\mathcal{M}_Z+\mathbf{1},\mathfrak{m}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+\mathbf{1},\mathfrak{m}}^{(Z \rightsquigarrow Z)} \\ \cdots & & \cdots \\ \boldsymbol{b}_{\mathcal{M}_Z+\ell_Z,\mathbf{1}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+\ell_Z,\mathbf{1}}^{(Z \rightsquigarrow Z)} & \cdots & \boldsymbol{b}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}}^{(V \rightsquigarrow Z)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}}^{(Z \rightsquigarrow Z)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix})))$$

Table 16

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta \boldsymbol{t}$, at a pair $\left((\mathbb{z}_{\mathbf{1}}', \mathbb{w}_{\mathbf{1}}'), (\mathbb{z}_{\mathbf{2}}', \mathbb{w}_{\mathbf{2}}')\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ having the form $\left((\mathbb{z}_{\mathbf{1}}', \mathbb{w}_{\mathbf{1}}'), (\mathbb{z}_{\mathbf{2}}', \mathbb{w}_{\mathbf{2}}')\right) =$

$$((\begin{pmatrix} \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathbf{1,1}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathbf{1,1}} & \cdots & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathbf{1},\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathbf{1},\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{m_V,\mathbf{1}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{m_V,\mathbf{1}} & & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}} & \cdots & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathbf{1}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathbf{1}} & \cdots & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathbf{1,1}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathbf{1,1}} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathbf{1},\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathbf{1},\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{m_V,\mathbf{1}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{m_V,\mathbf{1}} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{m_V,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{m_V,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\mathbf{1},\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathbf{1}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathbf{1}} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}),$$

$$\Bigg(\begin{pmatrix}
a'^{(V\rightsquigarrow Z)}_{1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{1,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
a'^{(V\rightsquigarrow Z)}_{m_Z,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{m_Z,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{m_Z,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{m_Z,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{Z+1,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\
\cdots & & \cdots \\
a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\
a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+1,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{n}} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},$$

$$\begin{pmatrix}
b'^{(V\rightsquigarrow Z)}_{1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
b'^{(V\rightsquigarrow Z)}_{m_V,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{m_V,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{m_V,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{m_V,\mathfrak{m}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & & \cdots \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,1} & & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{m}} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{m}} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\Bigg)\Bigg).$$

Table 17

With exactly the same way, a MitM attack can be conducted against $\boldsymbol{U}$ node without the knowledge of node $\boldsymbol{V}$. Most of the times the sophistication of this attack is low to medium due to active orientation of this attack.

It is obvious that if the nodes have smooth valuations and smooth vulnerabilities, the following states applied during this attack:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) > 0$

$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) < 0$
$\widehat{\boldsymbol{\varphi}}^{(Z\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\widehat{\boldsymbol{\psi}}^{(Z\rightsquigarrow Z)}(\boldsymbol{t}) < 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(U\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow Z)}(\boldsymbol{t}) < 0$

On the other hand, **wiretapping** attack which is, as mentioned, a passive attack that consists in the monitoring of Cyber activity, often by covert means, escalates as follows. A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{Z}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) passive attack from an intermediate node $\boldsymbol{Z}$ against the $(\boldsymbol{\kappa}_{\mathbf{1}}, \dots, \boldsymbol{\kappa}_{\lambda}) -$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_{\mathbf{1}}}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_{\mathbf{2}}}^{(V)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_{\lambda}}^{(V)}\right)$** of $\boldsymbol{V}$, during a given time subset $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\big) =$$

$$\left(\left(\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+\mathbf{1},\mathfrak{n}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\mathbf{1},\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ \boldsymbol{a}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathbf{1}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{n}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.\right.$$

$$\left.\left.\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\mathbf{1},\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+\mathbf{1},\mathfrak{m}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\mathbf{1},\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ \boldsymbol{b}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathbf{1}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{m}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right)\right)\right),$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & \cdots & \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & \cdots & \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & \cdots & \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & \cdots & \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)$$

Table 18

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ having the form

$$\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) =$$

$$\left(\left(\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{\beta}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{a'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{a'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{b'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{b'}^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{matrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} \\ \cdots & \cdots\cdots\cdots & \cdots \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.$$

$$\left.\left(\begin{matrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} \\ \cdots & \cdots\cdots\cdots & \cdots \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right)\right)\right)\right).$$

Table 19

With exactly the same way, a wiretapping attack can be conducted against $\boldsymbol{U}$ node without the knowledge of node $\boldsymbol{V}$. Most of the times the sophistication of this attack is medium to high due to “passive” orientation of this.

Specifically, during Wiretapping attack the following states applied:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t})\boldsymbol{\psi}^{\varphi}, \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})\ \boldsymbol{\psi}^{c}$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) = \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) = 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) = 0$

$\widehat{\boldsymbol{\varphi}}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{Z})}(\boldsymbol{t}) > 0$	$\widehat{\boldsymbol{\psi}}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{Z})}(\boldsymbol{t}) < 0$
$\boldsymbol{\varphi}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U})}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U})}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{Z})}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{Z})}(\boldsymbol{t}) = 0$

15.3 Access Attack

An **access attack** is actually an attack where intruder gains **access** to a device/system to which he has no right for access. Thus, during this attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g_t} \colon \boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) \to \boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$$

where $\boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t})$ and $\boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) = \left(\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U}), \boldsymbol{\mathcal{S}_V}\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U}), \boldsymbol{\mathcal{U}_V}\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U})\right)$ and

$\boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}') = \left(\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U}\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U}\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V})\right)$,

respectively ([5]).

In such a case, a family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}_{\mathbf{1}}', \mathbb{w}_{\mathbf{1}}'), (\mathbb{z}_{\mathbf{2}}', \mathbb{w}_{\mathbf{2}}')\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\mathscr{k}}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\mathscr{k}}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) access attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_v})$ **– device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_1}^{(V)}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_2}^{(V)}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_v}^{(V)}\right)$ **of** $\boldsymbol{V}$ during a given time subset $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\mathscr{k}}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\mathscr{k}}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) =$$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\boldsymbol{m_V},\mathbf{1}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m_V},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & & \boldsymbol{a}_{\boldsymbol{m_V},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m_V},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\begin{pmatrix} \boldsymbol{b}^{(U\rightsquigarrow V)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{b}^{(U\rightsquigarrow V)}_{1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}^{(U\rightsquigarrow V)}_{m_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \boldsymbol{b}^{(U\rightsquigarrow V)}_{m_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}^{(V\rightsquigarrow U)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{1,1} & \dots & \boldsymbol{a}^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}^{(V\rightsquigarrow U)}_{m_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{m_U,1} & \cdots & \boldsymbol{a}^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(W\rightsquigarrow W)}_{m_U,\mathfrak{n}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \boldsymbol{b}^{(V\rightsquigarrow U)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{1,1} & \dots & \boldsymbol{b}^{(V\rightsquigarrow U)}_{1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{m_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{m_U,1} & \cdots & \boldsymbol{b}^{(V\rightsquigarrow U)}_{m_U,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{m}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right)\right)\right)$$

Table 20

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form $\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) =$

$$\left(\left(\begin{pmatrix} \boldsymbol{a'}^{(U\rightsquigarrow V)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a'}}^{(V\rightsquigarrow V)}_{1,1} & \cdots & \boldsymbol{a'}^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a'}}^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a'}}^{(V\rightsquigarrow V)}_{m_V,1} & & \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a'}}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \mathbf{0} & \dots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\begin{pmatrix} \boldsymbol{b'}^{(U\rightsquigarrow V)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b'}}^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b'}}^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b'}}^{(V\rightsquigarrow V)}_{m_U,1} & \cdots & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_U,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b'}}^{(V\rightsquigarrow V)}_{m_U,\mathfrak{m}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right),$$

$$
\left(\left(\begin{array}{ccc}
a'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{1,1} = a'^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & a'^{(V\rightsquigarrow U)}_{1,n} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{1,n} = a'^{(U\rightsquigarrow V)}_{1,n} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{1,n} \\
\cdots & \cdots & \cdots \\
a'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{m_U,1} = a'^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & a'^{(V\rightsquigarrow U)}_{m_U,n} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{m_U,n} = a'^{(U\rightsquigarrow V)}_{m_V,n} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{m_V,n} \\
a'^{(U\rightsquigarrow V)}_{m_V+1,1} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{m_V+1,1} & & a'^{(U\rightsquigarrow V)}_{m_V+1,n} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{m_V+1,n} \\
\cdots & \cdots & \cdots \\
a'^{(U\rightsquigarrow V)}_{m_V+\lambda,1} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{m_V+\lambda,1} & \cdots & a'^{(U\rightsquigarrow V)}_{m_V+\lambda,n} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{m_V+\lambda,n} \\
 & \cdots & \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.
$$

$$
\left.\left(\begin{array}{ccc}
b'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{1,1} = b'^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & b'^{(V\rightsquigarrow U)}_{1,n} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{1,n} = b'^{(U\rightsquigarrow V)}_{1,n} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{1,n} \\
\cdots & \cdots & \cdots \\
b'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{m_U,1} = b'^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & b'^{(V\rightsquigarrow U)}_{m_U,n} + i\ \widehat{b}'^{(U\rightsquigarrow U)}_{m_U,n} = b'^{(U\rightsquigarrow V)}_{m_V,n} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{m_V,n} \\
b'^{(U\rightsquigarrow V)}_{m_V+1,1} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{m_V+1,1} & & b'^{(U\rightsquigarrow V)}_{m_V+1,n} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{m_V+1,n} \\
\cdots & \cdots & \cdots \\
b'^{(U\rightsquigarrow V)}_{m_V+\lambda,1} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{m_V+\lambda,1} & \cdots & b'^{(U\rightsquigarrow V)}_{m_V+\lambda,n} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{m_V+\lambda,n} \\
 & \cdots & \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\right)\right).
$$

Table 21

Most of the times the sophistication of this attack is medium to high. Specifically, during Access attack the following states applied:

$\varphi^{(U\rightsquigarrow V)}(t), \widehat{\varphi}^{(V\rightsquigarrow V)}(t)$	$\psi^{(U\rightsquigarrow V)}(t), \widehat{\psi}^{(V\rightsquigarrow V)}(t)$
$\varphi^{(U\rightsquigarrow V)}(t) < 0$	$\psi^{(U\rightsquigarrow V)}(t) > 0$
$\widehat{\varphi}^{(V\rightsquigarrow V)}(t) = 0$	$\widehat{\psi}^{(V\rightsquigarrow V)}(t) = 0$
$\varphi^{(V\rightsquigarrow U)}(t) = 0$	$\psi^{(V\rightsquigarrow U)}(t) = 0$
$\widehat{\varphi}^{(U\rightsquigarrow U)}(t) > 0$	$\widehat{\psi}^{(U\rightsquigarrow U)}(t) < 0$

Proposition 5 It is clear that during an access attack $\mathcal{F}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \ldots, \boldsymbol{\mu_\nu})-$ device parts $fr\left(dev^{(V)}_{\mu_1}\right)$, $fr\left(dev^{(V)}_{\mu_2}\right)$,…, $fr\left(dev^{(V)}_{\mu_\nu}\right)$ of $\boldsymbol{V}$, the following elementary properties hold.

i. The (Euclidean) norm $\| \boldsymbol{a}'^{(U\rightsquigarrow V)} \|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less than the (Euclidean) norm $\|\boldsymbol{a}^{(U\rightsquigarrow V)}\|$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of$\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \| < \| \boldsymbol{a}^{(U \rightsquigarrow V)} \|.$$

ii. The (Euclidean) norm $\| \boldsymbol{b}'^{(U \rightsquigarrow V)} \|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norm $\| \boldsymbol{b}^{(U \rightsquigarrow V)} \| := \left(\sum_{j=1}^{\boldsymbol{m}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left| \boldsymbol{b}_{\mathcal{M}_U+\lambda,j}^{(U \rightsquigarrow V)} \right|^2 \right)^{1/2}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{b}'^{(U \rightsquigarrow V)} \| > \| \boldsymbol{b}^{(U \rightsquigarrow V)} \|.$$

iii. The (Euclidean) norm $\| \widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)} \|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norms

$$\| \widehat{\boldsymbol{a}}^{(U \rightsquigarrow U)} \| \text{ and } \| \boldsymbol{a}^{(U \rightsquigarrow V)} \|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \widehat{\boldsymbol{\beta}}'^{(U \rightsquigarrow U)} \| > \boldsymbol{max}\{ \| \widehat{\boldsymbol{\beta}}^{(U \rightsquigarrow U)} \|, \| \boldsymbol{\beta}^{(U \rightsquigarrow V)} \| \}.$$

iv. The (Euclidean) norm $\| \widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)} \|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less or equal than the (Euclidean) norms

$$\| \widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)} \| \text{ and } \| \boldsymbol{b}^{(U \rightsquigarrow V)} \|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)} \| \leq \boldsymbol{min}\{ \| \widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)} \|, \| \boldsymbol{b}^{(U \rightsquigarrow V)} \| \}. \blacksquare$$

Remark 10 Of course, in the special case where there is a fully successful access attack the following hold:

$$\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \| \approx \boldsymbol{0}, \; \| \boldsymbol{a}'^{(U \rightsquigarrow U)} \| = \sqrt{\boldsymbol{m_U}}, \; \| \boldsymbol{b}'^{(U \rightsquigarrow V)} \| = \sqrt{\boldsymbol{m_U}}.$$

An access attack, besides a reflexive homomorphism, can take place **physically** when an attacker $\boldsymbol{U}$, physically gains access of victim node devices $\boldsymbol{V}$.

15.4 Reconnaissance Attack

A **reconnaissance attack** is actually an attack which involves unauthorized detection system mapping and services to steal data. This attack can potentially take place both actively and passively. Specifically, in passive reconnaissance, an intruder monitors systems for vulnerabilities without interaction, through methods like session capture. In active reconnaissance, the intruder engages with the target system through methods like port scans.

Thus, during this attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g_t}: \boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U})(\boldsymbol{t}) \to \boldsymbol{\mathcal{P}_7^{(U)}}(\boldsymbol{V})(\boldsymbol{t'})$$

where $\boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U})(\boldsymbol{t'})$ and $\boldsymbol{\mathcal{P}_7^{(U)}}(\boldsymbol{V})(\boldsymbol{t'})$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U}) = \boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U})(\boldsymbol{t'}) = (\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U}\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U}\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V})\ )$ and

$\boldsymbol{\mathcal{P}_7^{(U)}}(\boldsymbol{V})(\boldsymbol{t'}) = (\boldsymbol{\mathfrak{C}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U}\boldsymbol{\mathfrak{C}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U}\boldsymbol{\mathfrak{C}_{available}}(\boldsymbol{V})\ )$

respectively ([5]).

It is obvious that the purpose of this attack is for node $\boldsymbol{U}$ to uncover all constituents' vulnerabilities of node $\boldsymbol{V}$.

A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{\boldsymbol{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

lying in (the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is **a germ of reconnaissance attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})-$ **device parts** $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\Big)$, $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_2}}^{(\boldsymbol{V})}\Big)$,…, $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\Big)$ **and the** $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda})$ **– resource parts** $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\Big)$, $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\Big)$,…, $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\Big)$ **of** $\boldsymbol{V}$ during a given time set $\mathbb{I} \subset\subset [\boldsymbol{0}, \boldsymbol{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{\boldsymbol{2}}$ of supervisory constituents perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) =$

$$\left(\left(\left(\begin{matrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.\right.$$

$$\left.\left.\left(\begin{matrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right)\right)\right.,$$

$$\left(\left(\begin{matrix} a^{(V\rightsquigarrow U)}_{1,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{1,1} & & a^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\ \cdots & \cdots\cdots\cdots & \cdots \\ a^{(V\rightsquigarrow U)}_{m_U,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{m_U,1} & & a^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots\cdots\cdots & \cdots \\ a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.$$

$$\left.\left(\begin{matrix} b^{(V\rightsquigarrow U)}_{1,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{1,1} & & b^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\ \cdots & \cdots\cdots\cdots & \cdots \\ b^{(V\rightsquigarrow U)}_{m_U,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{m_U,1} & & b^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots\cdots\cdots & \cdots \\ b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & & \cdots \\ b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right)\right)\right)$$

Table 22

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta \boldsymbol{t}$, at a pair $\left((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$\left((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right) =$

$$
\Bigg(\Bigg(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},
\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathbf{1}} & & \boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\mathbf{1},\mathfrak{m}} \\
\cdots & & \cdots \\
\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathbf{1}} & & \boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V},\mathfrak{m}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\Bigg),
\Bigg(\begin{pmatrix}
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1},\mathfrak{n}} \\
\cdots & \cdots\cdots\cdots & \cdots \\
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathfrak{n}} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots\cdots\cdots & \cdots \\
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\mathbf{1},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\mathbf{1},\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U}+\boldsymbol{\ell_U},\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},
$$

$$
\begin{pmatrix}
b'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{1,1} & & b'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\
\cdots & \cdots\cdots\cdots & \cdots \\
b'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{m_U,1} & & b'^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\
b'^{(V\rightsquigarrow U)}_{m_U+1,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{m_U+1,1} & & b'^{(V\rightsquigarrow U)}_{m_U+1,\mathfrak{n}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{m_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
b'^{(V\rightsquigarrow U)}_{m_U+\ell_V,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{m_U+\ell_V,1} & & b'^{(V\rightsquigarrow U)}_{m_U+\ell_V,n} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{m_U+\ell_V,n} \\
0 & \cdots\cdots\cdots & 0 \\
\cdots & & \cdots \\
0 & & 0 \\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,n} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,n} \\
\cdots & \cdots\cdots\cdots & \cdots \\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V 1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,n} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,n} \\
0 & & 0 \\
\cdots & & \cdots \\
0 & & 0
\end{pmatrix}\Bigg)\Bigg).
$$

Table 23

Most of the times the sophistication of this attack is very low and highly "transparent" to attacked node. Frequently, after this attack a more sophisticated attack is expected. Specifically, during Reconnaissance attack the following states applied:

$\varphi^{(U\rightsquigarrow V)}(t), \widehat{\varphi}^{(V\rightsquigarrow V)}(t)$	$\psi^{(U\rightsquigarrow V)}(t), \widehat{\psi}^{(V\rightsquigarrow V)}(t)$
$\varphi^{(U\rightsquigarrow V)}(t) < 0$	$\psi^{(U\rightsquigarrow V)}(t) > 0$
$\widehat{\varphi}^{(V\rightsquigarrow V)}(t) = 0$	$\widehat{\psi}^{(V\rightsquigarrow V)}(t) = 0$
$\varphi^{(V\rightsquigarrow U)}(t) = 0$	$\psi^{(V\rightsquigarrow U)}(t) = 0$
$\widehat{\varphi}^{(U\rightsquigarrow U)}(t) > 0$	$\widehat{\psi}^{(U\rightsquigarrow U)}(t) < 0$

Proposition 6 It is obvious that during a reconnaissance attack $\mathcal{F}$ from U against the $(\mu_1, \dots, \mu_\nu)-$ resource parts $fr\left(res^{(V)}_{\mu_1}\right)$, $fr\left(res^{(V)}_{\mu_2}\right)$,…, $fr\left(res^{(V)}_{\mu_\nu}\right)$ of V, the following elementary properties hold:

i. The (Euclidean) norm $\| a'^{(U\rightsquigarrow V)} \|$ of the resulting overall valuation in the node V as evaluated from the viewpoint of the user(s) of U at the next moment t' is less than the (Euclidean) norm $\left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\| \boldsymbol{a}'^{(U \rightsquigarrow V)}\right\| < \left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|.$$

ii. The (Euclidean) norm $\left\| \boldsymbol{b}'^{(U \rightsquigarrow V)}\right\|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norm $\left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{\boldsymbol{m}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{b}_{\boldsymbol{\mathcal{M}_U+\lambda,j}}^{(U \rightsquigarrow V)}\right|^2\right)^{1/2}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\| \boldsymbol{b}'^{(U \rightsquigarrow V)}\right\| > \left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\|.$$

iii. The (Euclidean) norm $\left\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\right\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(U \rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\right\| > \boldsymbol{max}\left\{\left\|\widehat{\boldsymbol{a}}^{(U \rightsquigarrow U)}\right\|, \left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|\right\}.$$

iv. The (Euclidean) norm $\left\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\right\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less or equal than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\right\| \le \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\right\|, \left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\|\right\}.$$

The criticality of this attack is high since most of times it is the omen of a more severe or more sophisticated attack.

15.5 Denial of Service (DoS) attack and Distributed Denial of Service (DDoS) attack

Both attacks intent to deny services and generally resources to authorized users. The attacker makes a computing or memory resource too busy or too full to handle legitimate requests, thus denying legitimate user access to a machine. The difference between a **Denial of Service (DoS) attack** and a **Distributed Denial of Service (DDoS) attack** is the source of attack. In the first attack (DoS) the attack initiated by only one node. On the other hand, in DDoS attack there is the engagement of a multitude of nodes (intentionally or not, e.g. via Botnets).

Thus, during this kind of attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g_t}: \boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U})(\boldsymbol{t}) \to \boldsymbol{\mathcal{P}_9^{(U)}}(\boldsymbol{V})(\boldsymbol{t'})$$

where $\boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U})(\boldsymbol{t'})$ **and** $\boldsymbol{\mathcal{P}_9^{(U)}}(\boldsymbol{V})(\boldsymbol{t'})$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U}) = \boldsymbol{\mathcal{Q}_9^{(V)}}(\boldsymbol{U})(\boldsymbol{t'}) = (\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U \mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U \mathfrak{R}_{available}}(\boldsymbol{V})\ )$ and

$\boldsymbol{\mathcal{P}_9^{(U)}}(\boldsymbol{V})(\boldsymbol{t'}) = (\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U \mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U \mathfrak{R}_{available}}(\boldsymbol{V})\ )$

respectively ([5]).

It is obvious that the purpose of this attack is for node $\boldsymbol{U}$ to keep all resources/services of node $\boldsymbol{V}$ busy in order to make them unavailable to all users that really need them.

A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}_{(Y,X)}}(\boldsymbol{t}) = \left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathbb{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathbb{m}\times\boldsymbol{k}}\right)^{\boldsymbol{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of DoS attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ – $\boldsymbol{fr}\left(\boldsymbol{dev_{\mu_2}^{(V)}}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{dev_{\mu_\nu}^{(V)}}\right)$ **resource parts** $\boldsymbol{fr}\left(\boldsymbol{res_{\kappa_1}^{(V)}}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{res_{\kappa_2}^{(V)}}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{res_{\kappa_\lambda}^{(V)}}\right)$ **of** $\boldsymbol{V}$ during a given time set $\mathbb{I} \subset\subset [\boldsymbol{0}, \boldsymbol{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) \in \left(\mathbb{C}^{\mathbb{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathbb{m}\times\boldsymbol{k}}\right)^{\boldsymbol{2}}$ of supervisory constituents perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) =$$

$$
\left(\left(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\vdots & & \vdots \\
\boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\vdots & & \vdots \\
\boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right),
$$

$$
\left(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\vdots & & \vdots \\
\boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.
$$

$$
\left.\left.\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\
\vdots & & \vdots \\
\boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right)
$$

Table 24

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\left((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$\left((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right) =$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} = \mathbf{0} & & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} = \mathbf{0} \\ \cdots & & \cdots \\ a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} = \mathbf{0} & & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} = \mathbf{0} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}, \right.\right.$$

$$\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} = \mathbf{1} & & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} = \mathbf{1} \\ \cdots & & \cdots \\ b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} = \mathbf{1} & & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} = \mathbf{1} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right),$$

$$\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{a'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}, \right.$$

$$\left.\left.\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \widehat{b'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)\right).$$

Table 25

During this attack the results depicted in previous matrices are usually temporary and only strictly during the application of the attack. Most of the times the sophistication of this attack is very low and highly “transparent” to attacked node since the lack of resources is more than obvious. Frequently, after or during this attack a more sophisticated attack is expected. Specifically, during DoS and DDoS attacks the following states applied:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) < \mathbf{0}$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) > \mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) < \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) > \mathbf{0}$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) > \mathbf{0}$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) > \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$

Proposition 7 It is obvious that during a DoS and DDoS attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})-$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\mu_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mu_2}^{(V)}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\mu_\nu}^{(V)}\right)$ of $\boldsymbol{V}$, the following elementary properties hold:

i. The (Euclidean) norm $\left\| \boldsymbol{a}'^{(U\rightsquigarrow V)} \right\|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is temporary $\mathbf{0}$:

$$\left\| \boldsymbol{a}'^{(U\rightsquigarrow V)} \right\| = \mathbf{0}.$$

ii. The (Euclidean) norm $\left\| \boldsymbol{b}'^{(U\rightsquigarrow V)} \right\|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is temporary $\mathbf{1}$:

$$\left\| \boldsymbol{b}'^{(U\rightsquigarrow V)} \right\| = \mathbf{1}.$$

iii. The (Euclidean) norm $\left\|\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}\right\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{a}^{(U\rightsquigarrow V)}\right\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{\beta}}'^{(U\rightsquigarrow U)}\right\| > \boldsymbol{max}\left\{\left\|\widehat{\boldsymbol{\beta}}^{(U\rightsquigarrow U)}\right\|, \left\|\boldsymbol{\beta}^{(U\rightsquigarrow V)}\right\|\right\}.$$

iv. The (Euclidean) norm $\left\|\widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}\right\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less or equal than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{b}^{(U\rightsquigarrow V'')}\right\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}\right\| \leq \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}\right\|, \left\|\boldsymbol{b}^{(U\rightsquigarrow V)}\right\|\right\}. \blacksquare$$

The importance of this attack is high since most of the time, especially during DDoS attack, the nodes that participate are already compromised via Access attack that has already discussed.

Accordingly, in DDoS attack, since the attack is being generated by a multitude of already compromised nodes

$$\boldsymbol{U_1, U_2, U_3, \dots, U_n}$$

that compose a botnet, the visualization of this attack can be the following:

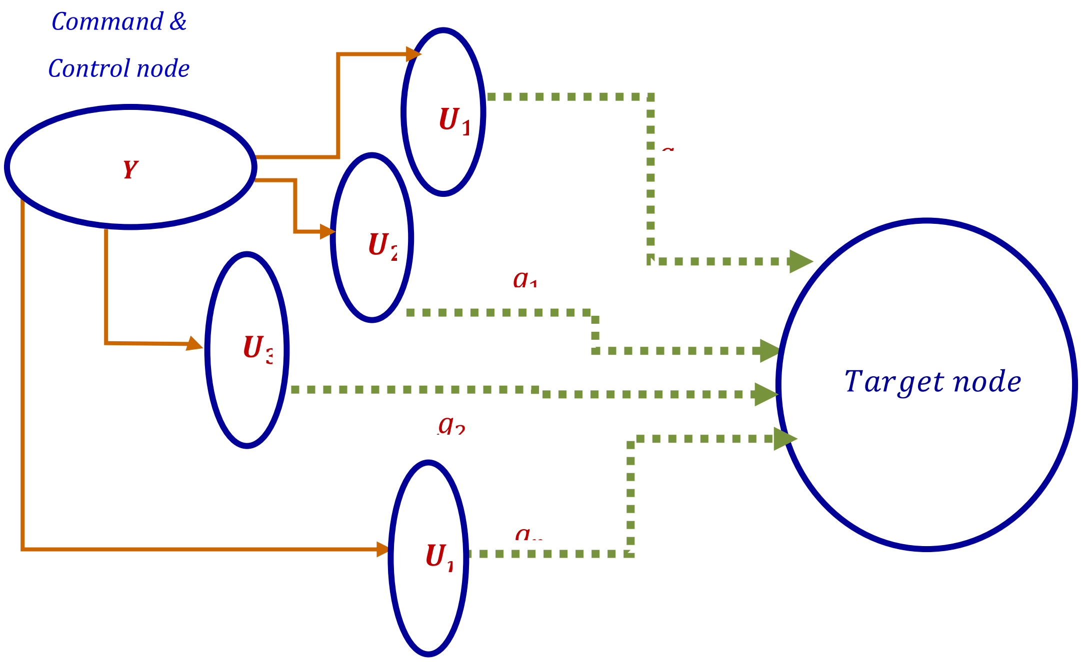

Figure 6 (Botnet)

In addition and actually in reality, the geographical distribution of $\boldsymbol{U_1, U_2, U_3, \dots}$ $\dots, \boldsymbol{U_n}$ is spread evenly. The controller of a botnet (Command and Control node) is able to direct the activities of these compromised computers through e-flows in order to conduct a DDoS attack.

16. Mathematical Description of Representative Cyber Attacks

16.1

Having already approached in this dissertation a consistent mathematical study of cyber-attack techniques/vectors and relevant defenses procedures we may describe a rigorous description of Advanced Persistent Threat (APT) actors' modus operandi through scenarios and various Cyber Kill Chain stages. To this end, we describe the means to detect the modus operandi and some TTPs (Tactics, Techniques and Procedures) through 5 scenarios that the most sophisticated cyber actors (APTs) use to evolve cyber complex attacks [34]. Identifying these vectors through the Cyber Kill Chain the defenses are straight forward and no value would be added enumerating them.

16.2 APT Hunting Scenario 1

The APT actor, that in this section will be depicted as $\boldsymbol{Z_{APT}}$**,** clandestinely relays and possibly modifies the communication between two nodes who suppose that they are directly exchange info with each other.

In this scenario the **node** $\boldsymbol{Z_{APT}}$**,** that is the APT actor, cyber-interacts between nodes $\boldsymbol{U}$ and $\boldsymbol{V}$. Actually in this "active" intersection attack, instead of this "normal" interaction we experience an active attack from node $\boldsymbol{Z_{APT}}$ to either or/and both of other nodes **using some resources of the other interacted node**. In such a case, a family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{Z_{APT}} = \boldsymbol{Z_{APT(Y,X)}}(\boldsymbol{t}) = \left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\boldsymbol{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{Z_{APT}}\to\boldsymbol{V}}$ to the node $\boldsymbol{V}$ from the node $\boldsymbol{Z_{APT}}$ during the entire time set $\mathbb{I}$, is a **germ [6] of (partial) active attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}) -$ **device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}^{(\boldsymbol{V})}_{\boldsymbol{\mu_1}}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{dev}^{(\boldsymbol{V})}_{\boldsymbol{\mu_2}}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{dev}^{(\boldsymbol{V})}_{\boldsymbol{\mu_\nu}}\right)$ **of** $\boldsymbol{V}$ **and the** $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}) -$ **resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_1}}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_2}}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_\lambda}}\right)$ **of** $\boldsymbol{V}$, during a given time set $\mathbb{I} \subset\subset [\boldsymbol{0}, \boldsymbol{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\boldsymbol{2}}$ of supervisory resource perceptions of $\boldsymbol{Z_{APT}}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{Z_{APT}}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) =$$

$$
\left(\left(\begin{pmatrix}
\boldsymbol{a}_{1,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{1,\mathfrak{n}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}_{m_V,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{m_V,1}^{(V\rightsquigarrow V)} & & \boldsymbol{a}_{m_V,\mathfrak{n}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{a}_{\mathcal{M}_V+1,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\cdots & & \cdots \\
\boldsymbol{a}_{\mathcal{M}_V+\ell_V,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
\boldsymbol{b}_{1,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{1,\mathfrak{m}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}_{m_V,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{m_V,\mathfrak{m}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{b}_{\mathcal{M}_V+1,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\cdots & & \cdots \\
\boldsymbol{b}_{\mathcal{M}_V+\ell_V,1}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(Z_{APT}\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right),
$$

$$
\left(\left(\begin{matrix}
\boldsymbol{a}_{1,1}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{1,1}^{(Z_{APT}\rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{a}_{1,\mathfrak{n}}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{1,\mathfrak{n}}^{(Z_{APT}\rightsquigarrow Z_{APT})} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}_{m_{Z_{APT}},1}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{m_{Z_{APT}},1}^{(Z_{APT}\rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{a}_{m_{Z_{APT}},\mathfrak{n}}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{m_{Z_{APT}},\mathfrak{n}}^{(Z_{APT}\rightsquigarrow Z_{APT})} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{a}_{\mathcal{M}_{Z_{APT}}+1,1}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_{Z_{APT}}+1,1}^{(Z_{APT}\rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{a}_{\mathcal{M}_{Z_{APT}}+1,\mathfrak{n}}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_{Z_{APT}}+1,\mathfrak{n}}^{(Z_{APT}\rightsquigarrow Z_{APT})} \\
\cdots & & \cdots \\
\boldsymbol{a}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}},1}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}Z},1}^{(Z_{APT}\rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{a}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}},\mathfrak{n}}^{(V\rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}},\mathfrak{n}}^{(Z_{APT}\rightsquigarrow Z_{APT})} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{matrix}\right),\right.
$$

$$
\left.\left.\left.\begin{pmatrix}
\boldsymbol{b}_{1,1}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{1,1}^{(Z_{APT} \rightsquigarrow Z_{APT})} & \dots & \boldsymbol{b}_{1,\mathfrak{m}}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{1,\mathfrak{m}}^{(Z_{APT} \rightsquigarrow Z_{APT})} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}_{m_Z,1}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{m_{Z_{APT}},1}^{(Z_{APT} \rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{b}_{m_{Z_{APT}},\mathfrak{m}}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{m_{Z_{APT}},\mathfrak{m}}^{(Z_{APT} \rightsquigarrow Z_{APT})} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{b}_{\mathcal{M}_{Z_{APT}}+1,1}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_{Z_{APT}}+1,1}^{(Z_{APT} \rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{b}_{\mathcal{M}_{Z_{APT}}+1,\mathfrak{m}}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_{Z_{APT}}+1,\mathfrak{m}}^{(Z_{APT} \rightsquigarrow Z_{APT})} \\
\cdots & & \cdots \\
\boldsymbol{b}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}},1}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}}Z,1}^{(Z_{APT} \rightsquigarrow Z_{APT})} & \cdots & \boldsymbol{b}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}},\mathfrak{m}}^{(V \rightsquigarrow Z_{APT})} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_{Z_{APT}}+\ell_{Z_{APT}},\mathfrak{m}}^{(Z_{APT} \rightsquigarrow Z_{APT})} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right)\right)
$$

Table 26

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\left((\mathbb{z}_1', \mathbb{w}_1'), (\mathbb{z}_2', \mathbb{w}_2')\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{Z_{APT}} = \boldsymbol{Z}$ and $\boldsymbol{V}$ having the form

$$
\left((\mathbb{z}_1', \mathbb{w}_1'), (\mathbb{z}_2', \mathbb{w}_2')\right) =
$$

$$
\left(\left(\left(\begin{matrix}
\boldsymbol{a}'^{(Z \rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{1,1} & \cdots & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}'^{(Z \rightsquigarrow V)}_{m_V,1} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{m_V,1} & & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \boldsymbol{a}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{matrix}\right),\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
\boldsymbol{b}'^{(Z \rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}'^{(Z \rightsquigarrow V)}_{m_V,1} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{m_V,1} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{m_V,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{m_V,\mathfrak{m}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & & \cdots \\
\boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \boldsymbol{b}'^{(Z \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right),
$$

$$\left(\begin{pmatrix} a'^{(V\rightsquigarrow Z)}_{1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{1,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ a'^{(V\rightsquigarrow Z)}_{m_Z,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{m_Z,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{m_Z,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{m_Z,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{Z+1,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+1,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+1,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,1} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,1} & \cdots & a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{n}} = a'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\,\widehat{a'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right.,$$

$$\left.\begin{pmatrix} b'^{(V\rightsquigarrow Z)}_{1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ b'^{(V\rightsquigarrow Z)}_{m_V,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{m_V,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{m_V,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{m_V,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,1} & & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{m}} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{m}} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right).$$

Table 27

Following the same process, the identical attack may be conducted against $\boldsymbol{U}$ node without the knowledge of node $\boldsymbol{V}$. The sophistication of this attack is low to medium.

Given that involved nodes have smooth valuations and smooth vulnerabilities, the following status applies during this scenario:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) > 0$

$\boldsymbol{\varphi}^{(Z_{APT} \rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z_{APT} \rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V \rightsquigarrow Z_{APT})}(\boldsymbol{t}) > 0$	$\boldsymbol{\psi}^{(V \rightsquigarrow Z_{APT})}(\boldsymbol{t}) < 0$
$\widehat{\boldsymbol{\varphi}}^{(Z_{APT} \rightsquigarrow Z_{APT})}(\boldsymbol{t}) > 0$	$\widehat{\boldsymbol{\psi}}^{(Z_{APT} \rightsquigarrow Z_{APT})}(\boldsymbol{t}) < 0$
$\boldsymbol{\varphi}^{(Z_{APT} \rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z_{APT} \rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(U \rightsquigarrow Z_{APT})}(\boldsymbol{t}) > 0$	$\boldsymbol{\psi}^{(U \rightsquigarrow Z_{APT})}(\boldsymbol{t}) < 0$

16.3 APT Hunting Scenario 2

In second scenario, APT activity is actually a passive attack and the hunting comprises of the monitoring of Cyber activity. A group of coherent interactions

$$\mathcal{F} = \left\{\mathcal{Z} = \mathcal{Z}_{(Y,X)}(\boldsymbol{t}) = \left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to the node $\boldsymbol{V}$ from the node $\boldsymbol{Z_{APT}} = \boldsymbol{Z}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) passive attack from an intermediate node $\boldsymbol{Z}$ against the $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}) -$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)$**, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$, during a given time subset $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) =$$

$$\left(\left(\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}_{\boldsymbol{\mathcal{M}_V+1,1}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V+1,1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{n}}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{n}}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{n}}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{n}}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right),\right.\right.$$

$$\left.\left.\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}_{\boldsymbol{\mathcal{M}_V+1,1}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V+1,1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{m}}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{m}}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{m}}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{m}}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right)\right)\right),$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & \cdots & \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \vdots & \cdots & \vdots \\ \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & \cdots & \boldsymbol{a}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & \cdots & \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{m}} \\ \vdots & \cdots & \vdots \\ \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & \cdots & \boldsymbol{b}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)$$

Table 28

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\big((\mathbb{z}_1', \mathbb{w}_1'), (\mathbb{z}_2', \mathbb{w}_2')\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^2$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ having the form

$$\big((\mathbb{z}_1', \mathbb{w}_1'), (\mathbb{z}_2', \mathbb{w}_2')\big) =$$

$$\left(\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{\beta}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \vdots & & \vdots \\ \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \vdots & & \vdots \\ \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{matrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & \dots\dots\dots & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} \\ \cdots & \dots\dots\dots & \cdots \\ \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} & & \boldsymbol{a'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.$$

$$\left.\left(\begin{matrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1} & \dots\dots\dots & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}} \\ \cdots & \dots\dots\dots & \cdots \\ \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,1} & & \boldsymbol{b'}^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right)\right)\Bigg).$$

Table 29

It is possible an identical attack to be conducted against $\boldsymbol{U}$ node without the knowledge of $\boldsymbol{V}$. Most of the times, the sophistication of this attack is medium to high due to “passive” orientation of this.

Specifically, during this APT attack the following states applies:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t})\boldsymbol{\psi}^{\boldsymbol{\varphi}}, \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})\ \boldsymbol{\psi}^{\boldsymbol{c}}$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) = \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) = 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) = 0$

$\widehat{\boldsymbol{\varphi}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}(\boldsymbol{t}) > 0$	$\widehat{\boldsymbol{\psi}}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}(\boldsymbol{t}) < 0$
$\boldsymbol{\varphi}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U})}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{U})}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{Z})}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{Z})}(\boldsymbol{t}) = 0$

16.4 APT Hunting Scenario 3

According to this evolved scenario a highly sophisticated attack, where intruder gains **access** to a device/system and compromise it, takes place. Similarly here the node $\boldsymbol{U}$ is the APT actor that conducts the attack. During this attack the following general form of cyber-effect applies **[5]**:

$$\boldsymbol{g} = \boldsymbol{g}_{\boldsymbol{t}}\colon \boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) \to \boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$$

where $\boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t})$ and $\boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) = \left(\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U}), \boldsymbol{\mathcal{S}}_{\boldsymbol{V}}\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U}), \boldsymbol{\mathcal{U}}_{\boldsymbol{V}}\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U})\right)$ and

$\boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}') = \left(\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{S}}_{\boldsymbol{U}}\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{U}}_{\boldsymbol{U}}\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V})\right)$,

respectively **([5]).**

In such a case, a family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) access attack against the** $(\boldsymbol{\mu}_{\mathbf{1}}, \dots, \boldsymbol{\mu}_{\boldsymbol{\nu}})$ **– device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_2}^{(\boldsymbol{V})}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_{\boldsymbol{\nu}}}^{(\boldsymbol{V})}\right)$ **of** $\boldsymbol{V}$ during a given time subset $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) =$$

$$\left(\left(\left(\begin{matrix} \boldsymbol{a}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & & \boldsymbol{a}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.\right.$$

$$\left.\left.\begin{pmatrix} \boldsymbol{b}_{\mathbf{1,1}}^{(U\rightsquigarrow V)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1,1}}^{(V\rightsquigarrow V)} & \dots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(U\rightsquigarrow V)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{m_V,\mathbf{1}}^{(U\rightsquigarrow V)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_V,\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{m_V,\mathfrak{m}}^{(U\rightsquigarrow V)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}_{\mathbf{1,1}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1,1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{m_U,\mathbf{1}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_U,\mathbf{1}}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{a}_{m_U,\mathfrak{n}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_U,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\left.\begin{pmatrix} \boldsymbol{b}_{\mathbf{1,1}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1,1}}^{(U\rightsquigarrow U)} & \dots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(U\rightsquigarrow U)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{m_U,\mathbf{1}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_U,\mathbf{1}}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{b}_{m_U,\mathfrak{m}}^{(V\rightsquigarrow U)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_U,\mathfrak{m}}^{(U\rightsquigarrow U)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right)\right)\right)$$

Table 30

and is transformed, at a next moment $\boldsymbol{t}'=\boldsymbol{t}+\Delta\boldsymbol{t}$, at a pair $\left((\mathbb{z}_1',\mathbb{w}_1'),(\mathbb{z}_2',\mathbb{w}_2')\right)\in\left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}}\times\mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form $\left((\mathbb{z}_1',\mathbb{w}_1'),(\mathbb{z}_2',\mathbb{w}_2')\right)=$

$$\left(\left(\left(\begin{pmatrix} \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathbf{1,1}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathbf{1,1}} & \cdots & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathbf{1},\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathbf{1},\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V,\mathbf{1}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V,\mathbf{1}} & & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \mathbf{0} & \dots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix},\right.\right.\right.$$

$$\left.\left.\begin{pmatrix} \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathbf{1,1}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathbf{1,1}} & \dots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathbf{1},\mathfrak{m}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{\mathbf{1},\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_U,\mathbf{1}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_U,\mathbf{1}} & \cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_U,\mathfrak{m}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_U,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\begin{pmatrix}
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{1,1} = \boldsymbol{a}'^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} = \boldsymbol{a}'^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{m_U,1} = \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} = \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V+1,1} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V+1,1} & & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V+\lambda,1} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V+\lambda,1} & \cdots & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} \\
& \cdots & \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}
\boldsymbol{b}'^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{1,1} = \boldsymbol{b}'^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \boldsymbol{b}'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} = \boldsymbol{b}'^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}'^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U,1} = \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \boldsymbol{b}'^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} = \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V+1,1} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V+1,1} & & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V+\lambda,1} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V+\lambda,1} & \cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} \\
& \cdots & \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\Bigg).$$

Table 31

The sophistication of this vector is medium to high. During this scenario the following state applies:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) < \mathbf{0}$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) > \mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = \mathbf{0}$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = \mathbf{0}$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = \mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) > \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$

It is clear that during this scenario the attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ that plays the role of APT actor against the $(\boldsymbol{\mu_1}, \ldots, \boldsymbol{\mu_\nu})$ – device parts $\boldsymbol{fr}\left(\boldsymbol{dev}^{(\boldsymbol{V})}_{\boldsymbol{\mu_1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}^{(\boldsymbol{V})}_{\boldsymbol{\mu_2}}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}^{(\boldsymbol{V})}_{\boldsymbol{\mu_\nu}}\right)$ of $\boldsymbol{V}$, the following elementary properties hold.

v. The (Euclidean) norm $\left\| \boldsymbol{a}'^{(U\rightsquigarrow V)}\right\|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less than the (Euclidean) norm $\left\|\boldsymbol{a}^{(U\rightsquigarrow V)}\right\|$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of$\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{a}'^{(U \rightsquigarrow V)}\| < \|\boldsymbol{a}^{(U \rightsquigarrow V)}\|.$$

vi. The (Euclidean) norm $\| \boldsymbol{b}'^{(U \rightsquigarrow V)}\|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norm $\|\boldsymbol{b}^{(U \rightsquigarrow V)}\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\ell_V} \left|\boldsymbol{b}_{\mathcal{M}_U+\lambda,j}^{(U \rightsquigarrow V)}\right|^2\right)^{1/2}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{b}'^{(U \rightsquigarrow V)}\| > \|\boldsymbol{b}^{(U \rightsquigarrow V)}\|.$$

vii. The (Euclidean) norm $\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norms

$$\|\widehat{\boldsymbol{a}}^{(U \rightsquigarrow U)}\| \text{ and } \|\boldsymbol{a}^{(U \rightsquigarrow V)}\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{\beta}}'^{(U \rightsquigarrow U)}\| > \boldsymbol{max}\{\|\widehat{\boldsymbol{\beta}}^{(U \rightsquigarrow U)}\|, \|\boldsymbol{\beta}^{(U \rightsquigarrow V)}\|\}.$$

viii. The (Euclidean) norm $\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less than the (Euclidean) norms

$$\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\| \text{ and } \|\boldsymbol{b}^{(U \rightsquigarrow V)}\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\| < \boldsymbol{min}\{\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\|, \|\boldsymbol{b}^{(U \rightsquigarrow V)}\|\}. \blacksquare$$

In the special case where there is a fully successful access attack the following hold:

$$\| \boldsymbol{a}'^{(U \rightsquigarrow V)}\| \approx \mathbf{0}, \ \| \boldsymbol{a}'^{(U \rightsquigarrow U)}\| = \sqrt{\boldsymbol{m_U}}, \ \| \boldsymbol{b}'^{(U \rightsquigarrow V)}\| = \sqrt{\boldsymbol{m_U}}. \ \blacksquare$$

An access attack, besides a reflexive homomorphism, can take place **physically** when an attacker $\boldsymbol{U}$, physically gains access of victim node devices $\boldsymbol{V}$.

16.5 APT Hunting Scenario 4

In this scenario the actual attack vector which involves an unauthorized detection mapping and services to steal data. This attack may potentially take place both actively and passively. Specifically, in passive scenario 4, an intruder monitors system for vulnerabilities without interaction, through techniques like session capture. In active scenario, the intruder engages with the target system through techniques like port scans. Again, here the node that plays the role of the APT actor is the $\boldsymbol{U}$**.**

Thus, during this attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g}_{\boldsymbol{t}}: \boldsymbol{\mathcal{Q}}_{\mathbf{9}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) \rightarrow \boldsymbol{\mathcal{P}}_{\mathbf{7}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$$

where $\boldsymbol{\mathcal{Q}}_{\mathbf{9}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}')$ and $\boldsymbol{\mathcal{P}}_{\mathbf{7}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}}_{\mathbf{9}}^{(\boldsymbol{V})}(\boldsymbol{U}) = \boldsymbol{\mathcal{Q}}_{\mathbf{9}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}') = (\boldsymbol{\mathfrak{R}}_{\boldsymbol{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}}_{\boldsymbol{U}}\boldsymbol{\mathfrak{R}}_{\boldsymbol{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}}_{\boldsymbol{U}}\boldsymbol{\mathfrak{R}}_{\boldsymbol{available}}(\boldsymbol{V})\ )$ and

$\boldsymbol{\mathcal{P}}_{\mathbf{7}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}') = (\boldsymbol{\mathfrak{C}}_{\boldsymbol{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}}_{\boldsymbol{U}}\boldsymbol{\mathfrak{C}}_{\boldsymbol{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}}_{\boldsymbol{U}}\boldsymbol{\mathfrak{C}}_{\boldsymbol{available}}(\boldsymbol{V})\ )$

respectively **([5]).**

The scope of this attack is for node $\boldsymbol{U}$ to uncover all constituents' vulnerabilities of node $\boldsymbol{V}$.

A family of coherent interactions

$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}_{\mathbf{1}}', \mathbb{w}_{\mathbf{1}}'), (\mathbb{z}_{\mathbf{2}}', \mathbb{w}_{\mathbf{2}}')\big)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\}$,

lying in (the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\rightarrow\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is **a germ of scenario 4 attack against the** $(\boldsymbol{\mu}_{\mathbf{1}}, \dots, \boldsymbol{\mu}_{\boldsymbol{\nu}})$ **– device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_1}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_2}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_{\boldsymbol{\nu}}}^{(\boldsymbol{V})}\right)$ **and the** $(\boldsymbol{\kappa}_{\mathbf{1}}, \dots, \boldsymbol{\kappa}_{\boldsymbol{\lambda}})$ **– resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_1}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_2}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_{\boldsymbol{\lambda}}}^{(\boldsymbol{V})}\right)$ **of** $\boldsymbol{V}$ during a given time set $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory constituents perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$\big((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\big) =$

$$
\left(\left(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\cdots & & \cdots \\
a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & & \cdots \\
b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right),
$$

$$
\left(\left(\begin{pmatrix}
a^{(V\rightsquigarrow U)}_{1,1} + i\ \hat{a}^{(U\rightsquigarrow U)}_{1,1} & & a^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \hat{a}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\
\cdots & \cdots\cdots\cdots & \cdots \\
a^{(V\rightsquigarrow U)}_{m_U,1} + i\ \hat{a}^{(U\rightsquigarrow U)}_{m_U,1} & & a^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \hat{a}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots\cdots\cdots & \cdots \\
a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
b^{(V\rightsquigarrow U)}_{1,1} + i\ \hat{b}^{(U\rightsquigarrow U)}_{1,1} & & b^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \hat{b}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\
\cdots & \cdots\cdots\cdots & \cdots \\
b^{(V\rightsquigarrow U)}_{m_U,1} + i\ \hat{b}^{(U\rightsquigarrow U)}_{m_U,1} & & b^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \hat{b}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots\cdots\cdots & \cdots \\
b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right)\right)
$$

Table 32

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form $\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) =$

$$
\left(\left(\left(\begin{matrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} & & \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{n}}} \\
\vdots & & \vdots \\
\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} & & \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{n}}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{matrix}\right),\right.\right.
$$

$$
\left.\left.\left(\begin{matrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} & & \boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{m}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{m}}} \\
\vdots & & \vdots \\
\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} & & \boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{m}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{m}}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{matrix}\right)\right),\right.
$$

$$
\left(\left(\begin{matrix}
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1,1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\mathbf{1},\mathfrak{n}} \\
\vdots & \cdots\cdots\cdots & \vdots \\
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{m_U},\mathfrak{n}} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0} \\
\vdots & \cdots\cdots\cdots & \vdots \\
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+1,1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+1,\mathfrak{n}}} \\
\vdots & & \vdots \\
\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U,1}} & & \boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U,\mathfrak{n}}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{matrix}\right),\right.
$$

$$
\left(\begin{array}{ccc}
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\
\cdots & \dots\dots\dots & \cdots \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U+1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U+\ell_V,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U+\ell_V,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U+\ell_V,n}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{m_U+\ell_V,n} \\
\mathbf{0} & \dots\dots\dots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0} \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,n}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,n} \\
\cdots & \dots\dots\dots & \cdots \\
\boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V 1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,n}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,n} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\Bigg)\Bigg).
$$

Table 33

The sophistication, according to [6], of this attack is very low and highly "transparent" to attacked node. Most often after this attack a more sophisticated vector is planned. Specifically, during scenario 4 attack the following states applied:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}),\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}),\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t})<\mathbf{0}$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t})>\mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})=\mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})=\mathbf{0}$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t})=\mathbf{0}$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t})=\mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t})>\mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t})<\mathbf{0}$

It is obvious that during this attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_v})$ – resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\mu_1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\mu_2}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\mu_v}\right)$ of $\boldsymbol{V}$, the following elementary properties hold:

v. The (Euclidean) norm $\|\boldsymbol{a}'^{(U\rightsquigarrow V)}\|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is much less than the (Euclidean) norm $\|\boldsymbol{a}^{(U\rightsquigarrow V)}\|$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\| \ll \|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|.$$

vi. The (Euclidean) norm $\| \boldsymbol{b}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is much greater than the (Euclidean) norm $\|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\ell_V} \left|\boldsymbol{b}_{\mathcal{M}_U+\lambda,\boldsymbol{j}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right|^2\right)^{\boldsymbol{1/2}}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{b}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\| \gg \|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|.$$

vii. The (Euclidean) norm $\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is much greater than the (Euclidean) norms

$$\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\| \text{ and } \|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\| \gg \boldsymbol{max}\{\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\|, \|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|\}.$$

viii. The (Euclidean) norm $\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less than the (Euclidean) norms

$$\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\| \text{ and } \|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\| < \boldsymbol{min}\{\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\|, \|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\|\}. \blacksquare$$

The criticality of this attack is high since most of times it is the omen of a more severe or more sophisticated attack.

16.6 APT Hunting Scenario 5

In this scenario we orient 2 attack vectors that intent to sophisticatedly deny services and generally resources to authorized users. The attacker $\boldsymbol{U}$ that again plays the role of the APT actor makes a computing or memory resource too busy or too full to handle legitimate requests, thus denying legitimate user access to an asset. The difference between these 2 types of attacks is actually the source. In the first type the attack is initiated by only one node. On the other hand, the second vector has the engagement of a multitude of nodes (intentionally or not, e.g. via Botnets).

Thus, during this kind of attack the following general form of cyber-effect applies:

$$\boldsymbol{g = g_t}: \boldsymbol{\mathcal{Q}_9^{(V)}(U)(t) \to \mathcal{P}_9^{(U)}(V)(t')}$$

where $\boldsymbol{\mathcal{Q}_9^{(V)}(U)(t')}$ **and** $\boldsymbol{\mathcal{P}_9^{(U)}(V)(t')}$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}_9^{(V)}(U) = \mathcal{Q}_9^{(V)}(U)(t') = (\mathfrak{R}_{available}(V), \mathcal{S}_U\mathfrak{R}_{available}(V), \mathcal{U}_U\mathfrak{R}_{available}(V)}\ )$ and

$\boldsymbol{\mathcal{P}_9^{(U)}(V)(t') = (\mathfrak{R}_{available}(V), \mathcal{S}_U\mathfrak{R}_{available}(V), \mathcal{U}_U\mathfrak{R}_{available}(V)}\ )$

respectively ([5]).

It is obvious that the purpose of this attack is for node $\boldsymbol{U}$ to keep all resources/services of node $\boldsymbol{V}$ occupied in order to make them unavailable to all users when needed.

A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z} = \mathcal{Z}_{(Y,X)}(t)} = \big((\mathbb{z}_\mathbf{1}, \mathbb{w}_\mathbf{1}), (\mathbb{z}_\mathbf{2}, \mathbb{w}_\mathbf{2}), (\mathbb{z}'_\mathbf{1}, \mathbb{w}'_\mathbf{1}), (\mathbb{z}'_\mathbf{2}, \mathbb{w}'_\mathbf{2})\big)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^\mathbf{4}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U\to V}}$ to the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of scenario 5 attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ – $\boldsymbol{fr\left(dev_{\mu_2}^{(V)}\right), \dots, fr\left(dev_{\mu_\nu}^{(V)}\right)}$ **resource parts** $\boldsymbol{fr\left(res_{\kappa_1}^{(V)}\right), fr\left(res_{\kappa_2}^{(V)}\right), \dots, fr\left(res_{\kappa_\lambda}^{(V)}\right)}$ **of** $\boldsymbol{V}$ during a given time set $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_\mathbf{1}, \mathbb{w}_\mathbf{1}), (\mathbb{z}_\mathbf{2}, \mathbb{w}_\mathbf{2})\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^\mathbf{2}$ of supervisory constituents perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_\mathbf{1}, \mathbb{w}_\mathbf{1}), (\mathbb{z}_\mathbf{2}, \mathbb{w}_\mathbf{2})\big) =$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{a_{\mathcal{M}_V+1,1}^{(U\rightsquigarrow V)} + i\ \widehat{a}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)}} & & \boldsymbol{a_{\mathcal{M}_V+1,\mathfrak{n}}^{(U\rightsquigarrow V)} + i\ \widehat{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)}} \\ \cdots & & \cdots \\ \boldsymbol{a_{\mathcal{M}_V+\ell_V,1}^{(U\rightsquigarrow V)} + i\ \widehat{a}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)}} & & \boldsymbol{a_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(U\rightsquigarrow V)} + i\ \widehat{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & \boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & \boldsymbol{b}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{a}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right.\right.,$$

$$\left.\left.\left.\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)\right)$$

Table 34

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{2}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) =$

$$\left(\left(\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} = \mathbf{0} & & \boldsymbol{a'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} = \mathbf{0} \\ \cdots & & \cdots \\ \boldsymbol{a'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} = \mathbf{0} & & \boldsymbol{a'}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} = \mathbf{0} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right.\right.\right.,$$

$$
\left.\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
b'^{(U \rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{b}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+1,1} = 1 & & b'^{(U \rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \widehat{b}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} = 1 \\
\vdots & & \vdots \\
b'^{(U \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{b}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} = 1 & & b'^{(U \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{b}'^{(V \rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} = 1 \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\Bigg),
$$

$$
\Bigg(\Bigg(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
a'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{a}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{a}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\vdots & & \vdots \\
a'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{a}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{a}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},
$$

$$
\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \widehat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\
\vdots & & \vdots \\
b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \widehat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\vdots & & \vdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\Bigg)\Bigg)\Bigg).
$$

Table 35

During this scenario injects that reside in previous matrices are usually temporary and only strictly during the application of the attack. According to [6] the sophistication of this attack is low and highly “transparent” to attacked node since the lack of resources is more than obvious. Frequently, after or during this attack a more sophisticated attack is expected. Specifically, during these attacks the following states applied:

$\varphi^{(U \rightsquigarrow V)}(t), \widehat{\varphi}^{(V \rightsquigarrow V)}(t)$	$\psi^{(U \rightsquigarrow V)}(t), \widehat{\psi}^{(V \rightsquigarrow V)}(t)$
$\varphi^{(U \rightsquigarrow V)}(t) < 0$	$\psi^{(U \rightsquigarrow V)}(t) > 0$
$\widehat{\varphi}^{(V \rightsquigarrow V)}(t) < 0$	$\widehat{\psi}^{(V \rightsquigarrow V)}(t) > 0$
$\varphi^{(V \rightsquigarrow U)}(t) > 0$	$\psi^{(V \rightsquigarrow U)}(t) < 0$

$\widehat{\boldsymbol{\varphi}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}(\boldsymbol{t}) > \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}(\boldsymbol{t}) < \mathbf{0}$

It is obvious that during this scenario's attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})-$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\right), \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\mu_2}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$, the following elementary properties hold:

v. The (Euclidean) norm $\| \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is temporary $\mathbf{0}$:

$$\| \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\| = \mathbf{0}.$$

vi. The (Euclidean) norm $\| \boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is temporary $\mathbf{1}$:

$$\| \boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\| = \mathbf{1}.$$

vii. The (Euclidean) norm $\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is much greater than the (Euclidean) norms

$$\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\| \text{ and } \|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{\beta}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\| \geq \boldsymbol{max}\{\|\widehat{\boldsymbol{\beta}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\|, \|\boldsymbol{\beta}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\| \}.$$

viii. The (Euclidean) norm $\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less than the (Euclidean) norms

$$\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\| \text{ and } \|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V}''')}\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\| < \boldsymbol{min}\{\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\|, \|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\| \}. \blacksquare$$

The importance of this attack is high since most of the time, especially during distributed one, the nodes that participate are already compromised via Access attack that has already discussed.

17. CONCLUSIONS

It is obvious through the current dissertation that the approach presented and supported is a consistent one that may lead to further research endeavors. Building a comprehensive mathematic basis of cyberspace gives the momentum to understand clearly the activities in this ecosystem. The analytic description of several attack vectors and defensive measures to mitigate these attacks gives a clear view of the benefit of this research.

Performance and any deficiencies of mitigation defensive measures is now feasible to be evaluated and further ameliorated. Further attempts may focus on describing more behavior and abnormal based defensive approaches. It widely clear that having defined mathematically the key elements of cyberspace we may shift from signature based to more heuristic based approach on our defensive measure and this is one of the great contribution of the research presented in this dissertation.

ABBREVIATIONS - ACRONYMS

APT	Advanced Persistent Threat
DNS	Domain Name Systems

REFERENCES

1. Niall M. Adams and Nicholas A. Heard (Eds.), *Data Analysis for Network Cyber-Security*. World Scientific, 200 pages, 2014
2. Argyrios Alexopoulos, and Nicholas J. Daras, *A mathematical study of the proactive defense against different special types of germs of cyber attacks*, In preparation
3. Kenneth J. Arrow, and Frank Hahn, *General Competitive Analysis*. San Francisco: Holden-Day, and Edinburgh: Oliver & Boyd. Business & Economics, 452 pages, 1971
4. Kenneth J. Arrow, and Michael D. Intriligator (Eds.), *Handbook of Mathematical Economics. I*. North-Holland, New York, 378 pages, 1981
5. Kenneth J. Arrow, and Michael D. Intriligator (Eds.), *Handbook of Mathematical Economics. II*. North-Holland, New York. 688 pages, 1982
6. *William H. Batchelder: Mathematical Psychology. In Encyclopedia of Psychology, Alan E. Kazdin (Ed.), Washington/NY: APA/Oxford University Press, 4128 pages, 120-123, 2000*
7. Philip Bonacich, and Phillip Lu, *Introduction to mathematical sociology*. Princeton University Press. 240 pages, 2012
8. Richard Colbaugh, and Kristin Glass, *Proactive defense for evolving cyber threats*. SANDIA REPORT, SAND2012-10177, Unlimited Release.2012 http://fas.org/irp/eprint/proactive.pdf
9. Nicholas J. Daras, 2016. *On the mathematical definition of cyberspace*, The Journal of Nonlinear Science and Applications (JNSA), submitted
10. Nicholas J. Daras, and Argyrios Alexopoulos, *Modeling cyber-security*, Modern Discrete Mathematics and Analysis with Applications in Cryptography, Springer Optimization and its Applications, to appear
11. Nicholas J. Daras, and Argyrios Alexopoulos, Preferences and Rational Choices when Evaluating the Nodes in Cyberspace to be Attacked, submitted
12. Nicholas J. Daras, and John Th. Mazis, *Geopolitical systemic modeling, Part 1 Prediction of geopolitical events*, GeoJournal, DOI 10.1007/s10708-014-9569-3, Springer Science Verlag, Dordrecht, published online: 12 August 2014
13. Nicholas J. Daras, and John Th. Mazis, *Systemic geopolitical modeling. Part 2: Subjectivity in prediction of geopolitical events.* GeoJournal, **108**(81), (2017), 81-108, DOI 10.1007/s10708-015-9670-2, Springer Science Verlag, Dordrecht, published online: 04 September 2015
14. Gérard Debreu, *Theory of value. An axiomatic analysis of economic equilibrium* (4th Printing), Cowles Foundation for Research in Economics at Yale University. John

Wiley & Sons. 126 pages, 2016 http://cowles.yale.edu/sites/default/files/files/pub/mon/m17-all.pdf

15. Gérard Debreu, *Mathematical Economics, Twenty papers of Gerard Debreu*. Cambridge Cambridgeshire New York: Cambridge University Press. 264 pages, 1986
16. Daniel M. Dunlavy, Bruce Hendrickson, and Tamara G. Kolda, *Mathematical challenges in cyber-security*. Sandia National Laboratories. Printed February 2009 www.sandia.gov/~dmdunla/publications/SAND2009-0805.pdf
17. Andrew Gelman, *Mathematics, Statistics, and Political Science*. 31 July 2011. http://www.stat.columbia.edu/~gelman/presentations/mathstatpshandout.pdf
18. Joseph W. Houpt, Leslie M. Blaha (Eds.), *Mathematical Models of Perception and Cognition. I-II*. *A Festschrift for James T. Townsend*, Scientific Psychology Series. Psychology Press, Routledge. Taylor & Francis Group, New York and London, 554 pages, 2016
19. Robert Duncan Luce, Robert R. Bush, and Eugene Galanter (Eds.), *Handbook of mathematical psychology, Volumes I-III*. John Wiley & Sons. xiii + 491 pages, 1963
20. Fabrizio Luccio, Linda Pagli, and Graham Stee, *Mathematical and algorithmic foundations of the Internet*. CRC Press. July 6 2011, 221 pages
21. Joe Malkevitch, *Mathematics and Internet security*. December 2015 http://www.ams.org/samplings/feature-column/fcarc-internet
22. Andreu Mas-Colell, *The theory of general economic equilibrium: A differentiable approach*. Econometric Society Monographs 9, Cambridge University Press, Cambridge, U.K. 392 pages, 1985
23. Juan Meza, Scott Campbell, and David Bailey, *Mathematical and statistical opportunities in cyber security*, 2009 http://arxiv.org/pdf/0904.1616v1.pdf . Also http://www.davidhbailey.com/dhbpapers/CyberMath.pdf
24. Abraham Neyman, *Mathematical Economics, Lecture Notes*. Fall 2006, Jerusalem, https://scholars.huji.ac.il/sites/default/files/abrahamn/files/matheco0607.pdf
25. President's Information Technology Advisory Committee (PITAC), *Report to the President, cyber security: A crisis of prioritization*, February 2005, https://www.nitrd.gov/pitac/reports/20050301_cybersecurity/cybersecurity.pdf
26. Dwayne Ramsey, *Mathematical problems in cyber security*. In *DOE Workshop on Mathematical Research Challenges in Optimization of Complex Systems*, December 7-8, 2006. http://www.courant.nyu.edu/ComplexSystems/

27. Neil C. Rowe, E. John Custy, and Binh T. Duong, *Defending cyberspace with fake honeypots*, Journal of Computers, **2**(2), (April 2007), 25-36 http://www.academypublisher.com/jcp/vol02/no02/jcp02022536.pdf
28. Peter Y. A. Ryan, *Mathematical models of computer security.* In *Foundations of Security Analysis and Design: Tutorial Lectures (FOSAD 2000)*, Riccardo Focardi and Roberto Gorrieri (Eds.). LNCS 2171. Springer-Verlag Berlin Heidelberg 2001, 1-62. http://www.ece.cmu.edu/~ece732/readings/mathematical-models-of-computer-security.pdf
29. Dinesh K. Saini, *Cyber Defense: Mathematical modeling and simulation*, International Journal of Applied Physics and Mathematics, **2**(5), (2012), 312-315 http://www.ijapm.org/papers/121-P00009.pdf
30. Dan Shen, Genshe Chen, Jose B. Cruz, Jr., Erik Blasch, and Khanh Pham, *An adaptive Markov game model for cyber threat intent inference*. In *Theory and novel applications of machine learning*, Meng Joo Er and Yi Zhou Eds.), I-Tech, Vienna, Austria. 376 pages, February 2009 http://cdn.intechweb.org/pdfs/6200.pdf
31. Abraham Subil, and Nair Suku, *Cyber security analytics: A stochastic model for security quantification using absorbing Markov chains*, Journal of Communications **9**(12), (2014), 899-907 http://www.jocm.us/uploadfile/2014/1231/20141231022619157.pdf
32. **Andranik Tangian:** *Mathematical theory of democracy,* Studies in social choice and welfare, Springer, Berlin-Heidelberg. 615 pages, 2014
33. Joanne Wendelberger, Christopher Griffin, Louis Wilder, Yu Jiao. and Tamara Kolda, *A mathematical basis for science-based cyber-security* (with alternate title *Mathematical Underpinnings for Science-Based Cyber-Security*), February 2008, Sandia National Laboratories, Chad Scherrer, Pacific Northwest National Laboratory. https://wiki.cac.washington.edu/download/attachments/7479040/doecybermath25feb08.doc