

# Highway Traffic State Estimation with Mixed Connected and Conventional Vehicles

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**Abstract:** A macroscopic model-based approach for estimation of the traffic state, specifically of the (total) density and flow of vehicles, is developed for the case of “mixed” traffic, i.e., traffic comprising both ordinary and connected vehicles. The development relies on the following realistic assumptions: (i) The density and flow of connected vehicles are known at the (local or central) traffic monitoring and control unit on the basis of their regularly reported positions; and (ii) the average speed of conventional vehicles is roughly equal to the average speed of connected vehicles. Thus, complete traffic state estimation (for arbitrarily selected segments in the network) may be achieved by merely estimating the percentage of connected vehicles with respect to the total number of vehicles. A model is derived, which describes the dynamics of the percentage of connected vehicles, utilizing only well-known conservation law equations that describe the dynamics of the density of connected vehicles and of the total density of all vehicles. Based on this model, which is a linear parameter-varying system, an estimation algorithm for the percentage of connected vehicles is developed employing a Kalman filter. The estimation methodology is validated through simulations, using a second-order macroscopic traffic flow model as ground truth for the traffic state, as well as using real microscopic traffic data collected within the Next Generation SIMulation (NGSIM) program.

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## 1. INTRODUCTION

A number of novel Vehicle Automation and Communication Systems (VACS) have already been introduced, and many more are expected to appear in the next years. These systems are mainly aimed to improve driving safety and convenience, but are also believed to have great potential in mitigating traffic congestion, if appropriately exploited for innovative traffic management and control, see, for example, Diakaki et al. (2015). To attain related traffic flow efficiency improvements on highways, it is of paramount importance to develop novel methodologies for modeling, estimation and control of traffic in presence of VACS. Several papers are providing useful results related to modeling and control of traffic flow in presence of VACS, employing either microscopic or macroscopic approaches, see, for example, Bose & Ioannou (2003), Davis (2007), Ge & Orosz (2014), Kesting et al. (2008), Ngoduy et al. (2009), Roncoli et al. (2015), Shladover et al. (2012), Varaiya (1993), Wang et al. (2014).

The availability of reliable real-time measurements or estimates of the traffic state is a prerequisite for successful highway traffic control. In conventional traffic, the necessary measurements are provided by spot sensors, which are placed at specific highway locations. If the sensor density is sufficiently high (e.g., every 500 m), then the collected measurements are usually sufficient for traffic surveillance and control; else, appropriate estimation schemes need to be employed in order to produce traffic state estimates at the required space resolution (typically 500 m); see, for instance, Alvarez-Icaza et al. (2004), Hegyi et al. (2006), Mihaylova et al. (2007), Wang & Papageorgiou (2005), among many other works addressing highway traffic estimation by use of conventional detector data. However, the implementation

and maintenance of road-side detectors entail considerable cost; hence various research works attempt to exploit different, less costly data sources, such as mobile phone, or GPS (Global Positioning System), or even vehicle speed data for travel time or highway state estimation; see, e.g., Astarita et al. (2006), Fabritiis et al. (2007), Deng et al. (2013), Herrera & Bayen (2010), Seo et al. (2015), Work et al. (2008), Yuan et al. (2014); employing various kinds of traffic or statistic models.

In fact, with the introduction of VACS of various kinds, an increasing number of vehicles become “connected”, i.e., enabled to send (and receive) real-time information to a local or central monitoring and control unit (MCU). Thus, connected vehicles may communicate their position, speed and other relevant information, i.e., they can act as mobile sensors. This will potentially allow for a sensible reduction (and, potentially, elimination) of the necessary number of spot sensors, which would lead to sensible reduction of the purchase, installation, and maintenance cost for traffic monitoring. This paper concerns the development of reliable and robust traffic state estimation methods and tools, which exploit information provided by connected vehicles and reduces the need for spot sensor measurements under all penetration rates of connected vehicles, i.e., for a mixed traffic flow that includes both conventional and connected vehicles.

Specifically, we address the problem of estimating the (total) density and flow of vehicles in highway segments of arbitrary length (typically 500 m) in presence of connected vehicles. The developments rely on the following realistic assumptions:

- The density and flow of connected vehicles may be readily obtained at the local or central MCU on the basis of their regularly reported positions.

- The average speed of conventional vehicles is roughly equal to the average speed of connected vehicles. This assumption relies on the fact that, even at very low densities, there is no reason for connected vehicles to feature a systematically different mean speed than conventional vehicles; while at higher densities, the assumption is further reinforced due to increasing difficulty of overtaking.

As a consequence of these assumptions, complete traffic state estimation (of the total density and flow in arbitrarily selected highway segments) may be achieved by merely estimating the percentage of connected vehicles with respect to the total number of vehicles. For the latter, a minimum amount of conventional measurements of traffic volumes, e.g., at all highway entries and exits, is also required. Thus, the problem of traffic estimation is recast in the problem of estimating the percentage of connected vehicles at the selected highway segments.

In more technical terms, we derive a linear parameter-varying model, which describes the dynamics of the percentage (Section 2), utilizing merely the (time-discrete) conservation law equations for the density of connected vehicles and for the total density of vehicles (no traffic modelling of speed, such as the fundamental diagram, is required). We show that the system is observable (Section 3.1) and employ a Kalman filter (Section 3.2) for the estimation of the percentage of connected vehicles. We demonstrate our estimation design with a numerical example employing a second-order macroscopic traffic flow model as ground truth for the traffic state dynamics (Section 4.1). We also present a case study in which the proposed estimation scheme is tested using NGSIM microscopic data (Section 4.2).

## 2. MODEL DERIVATION FOR THE PERCENTAGE OF CONNECTED VEHICLES

We consider the following discrete-time equations that describe the dynamics of the total density  $\rho$  of the vehicles on a highway and the density  $\rho^a$  of the connected vehicles (see, for example, Papageorgiou & Messmer (1990); see the upper part of Fig. 1)

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\Delta_i} (q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)) \quad (1)$$

$$\rho_i^a(k+1) = \rho_i^a(k) + \frac{T}{\Delta_i} (q_{i-1}^a(k) - q_i^a(k) + r_i^a(k) - s_i^a(k)), \quad (2)$$

where  $i = 1, \dots, N$  is the index of the specific segment at the highway,  $N$  being the number of discrete cells on the highway; for all traffic variables, we denote by index sub- $i$  its value at the segment  $i$  of the highway;  $q_i$  and  $q_i^a$  are the total flow and the flow of the connected vehicles, respectively, at segment  $i$ ;  $T$  is the time-discretization step,  $\Delta_i$  is the length of the discrete segments of the highway, and  $k = 0, 1, \dots$  is the discrete time index. The variables  $r_i$  and  $s_i$  denote the inflow and outflow of vehicles at on-ramps and off-ramps, respectively, at segment  $i$ , whereas  $r_i^a$  and  $s_i^a$  are the corresponding inflow and outflow of connected vehicles. Define the inverse of the percentage of the connected vehicles at segment  $i$  of the highway as  $\bar{p}_i$ , i.e.,

$$\bar{p}_i = \frac{\rho_i}{\rho_i^a}. \quad (3)$$

Assuming that the average speed of conventional vehicles at a segment  $i$  equals the average speed of connected vehicles in the same segment, namely  $v_i$ , we conclude that the following holds

$$\bar{p}_i = \frac{\rho_i}{\rho_i^a} = \frac{q_i}{q_i^a}, \quad (4)$$

where we used the known relations

$$q_i = \rho_i v_i \quad (5)$$

$$q_i^a = \rho_i^a v_i. \quad (6)$$

Using (1), (2), and (4) we get from (3) that

$$\begin{aligned} \bar{p}_i(k+1) = & \frac{\left(\rho_i^a(k) - \frac{T}{\Delta_i} q_i^a(k)\right) \bar{p}_i(k) + \frac{T}{\Delta_i} q_{i-1}^a(k) \bar{p}_{i-1}(k)}{g_i^a(k)} \\ & + \frac{T}{\Delta_i} \frac{(r_i(k) - s_i(k))}{g_i^a(k)} \end{aligned} \quad (7)$$

$$g_i^a(k) = \rho_i^a(k) + \frac{T}{\Delta_i} (q_{i-1}^a(k) - q_i^a(k) + r_i^a(k) - s_i^a(k)), \quad (8)$$

$i = 1, \dots, N$ . Defining the state

$$x = (\bar{p}_1, \dots, \bar{p}_N)^T, \quad (9)$$

we re-write (7) as

$$x(k+1) = A(q^a(k), \rho^a(k), r^a(k), s^a(k)) x(k) + B(q^a(k), \rho^a(k), r^a(k), s^a(k)) u(k) \quad (10)$$

$$y(k) = Cx(k), \quad (11)$$

where

$$A = \left\{ \begin{array}{ll} a_{ij} = \frac{T}{\Delta_i} \frac{q_{i-1}^a(k)}{g_i^a(k)}, & \text{if } i - j = 1 \text{ and } i \geq 2 \\ a_{ij} = \frac{\rho_i^a(k) - \frac{T}{\Delta_i} q_i^a(k)}{g_i^a(k)}, & \text{if } i = j \\ a_{ij} = 0, & \text{otherwise} \end{array} \right\} \quad (12)$$

$$B = \left\{ \begin{array}{ll} b_{ij} = \frac{T}{\Delta_i} \frac{1}{g_1^a(k)}, & \text{if } i = 1 \text{ and } j = 1, 2 \\ b_{ij} = \frac{T}{\Delta_i} \frac{1}{g_i^a(k)}, & \text{if } j - i = 1 \\ b_{ij} = 0, & \text{otherwise} \end{array} \right\} \quad (13)$$

$$u^T = [q_0(k) \ r_1(k) - s_1(k) \ \dots \ r_N(k) - s_N(k)] \quad (14)$$

$$C = [0 \ \dots \ 0 \ 1], \quad (15)$$

$q^a = (q_0^a, \dots, q_N^a)^T$ ,  $\rho^a = (\rho_1^a, \dots, \rho_N^a)^T$ ,  $r^a = (r_1^a, \dots, r_N^a)^T$ ,  $s^a = (s_1^a, \dots, s_N^a)^T$ ,  $g_i^a$ ,  $i = 1, \dots, N$ , are defined in (8),  $A \in \mathbb{R}^{N \times N}$ ,  $B \in \mathbb{R}^{N \times (N+1)}$ , and  $q_0$  denotes the total flow of vehicles at the entry of the highway and acts as an input to system (10), along with  $r_i$  and  $s_i$ ; while  $r_i^a$ ,  $s_i^a$ ,  $\rho_i^a$ , and  $q_i^a$  are viewed as time-varying parameters of system (10). Finally, the variable  $\bar{p}_N$  is viewed as the output of the system and may be obtained, using total flow measurements  $q_N$  at the highway exit, via

$$\bar{p}_N = \frac{q_N}{q_N^a}. \quad (16)$$

Before studying the observability of system (10)–(15), we summarize the assumptions that guarantee that the matrix  $A$  is known, and that the input  $u$  and output  $y$  are measured.

- The average speed of the connected vehicles at a segment of the highway equals the average speed of all vehicles at the same segment, i.e.,  $v_i^a = v_i$ .

- The segment flows and densities of connected vehicles,  $q_i^a$ ,  $i = 0, \dots, N$ , and  $\rho_i^a$ ,  $i = 1, \dots, N$ , respectively, as well as the flows of connected vehicles at on-ramps and off-ramps,  $r_i^a$  and  $s_i^a$ ,  $i = 1, \dots, N$ , respectively, may be obtained from regularly received messages by the connected vehicles.
- The total flow of vehicles at the entry and exit of the highway,  $q_0$  and  $q_N$ , respectively, are measured via conventional detectors.
- The total flow of vehicles at on-ramps and off-ramps,  $r_i$  and  $s_i$ ,  $i = 1, \dots, N$ , respectively, are measured via conventional detectors.

The above formulation may be modified to include additional mainstream total flow measurements (using conventional detectors) to replace a corresponding number of total flows at on-ramps or off-ramps, without affecting the observability of the system, see Bekiaris-Liberis et al. (2015).

### 3. PERCENTAGE ESTIMATION USING A KALMAN FILTER

#### 3.1 Observability of the System

System (10) is viewed as a linear time-varying system. As it is stated in Section 2, it is assumed that  $q_0$ ,  $\bar{p}_N$ ,  $q_i^a$ ,  $\rho_i^a$ ,  $r_i^a$ ,  $s_i^a$ ,  $r_i$ , and  $s_i$ , for all  $i$ , are available, which implies that the matrices  $A$ ,  $B$  and the input  $u$  in (10) may be calculated in real time. We show next that system (10)–(15) is observable at  $k = k_0 + N - 1$ , for any initial time  $k_0 \geq 0$ . We construct the observability matrix

$$O(k_0, k_0 + N) = \begin{bmatrix} C \\ CA(k_0) \\ CA(k_0 + 1)A(k_0) \\ \vdots \\ CA(k_0 + N - 2) \cdots A(k_0) \end{bmatrix}. \quad (17)$$

Since  $O$  is square, the system is observable at  $k = k_0 + N - 1$  if  $\det(O) \neq 0$ . Since from (12) it is evident that  $A$  is a lower triangular matrix, it follows from (15) that  $O$  is an anti-lower triangular matrix, namely, a matrix with zero elements above the anti-diagonal. Therefore, relation  $\det(O) \neq 0$  holds if the anti-diagonal elements of  $O$  are non-zero. The anti-diagonal elements of  $O$  are given by  $1, a_{NN-1}(k_0), a_{NN-1}(k_0 + 1)a_{N-1N-2}(k_0), \dots, a_{NN-1}(k_0 + N - 2) \cdots a_{21}(k_0)$ . Since  $q_i^a$ ,  $\rho_i^a$ ,  $i = 1, \dots, N$ , are positive, as well as lower<sup>1</sup> and upper bounded, it follows from (12) that  $a_{ij}(k)$ , for all  $k = k_0, \dots, k_0 + N - 2$  and any  $k_0 \geq 0$ , and for all  $i, j$  such that  $i - j = 1$  and  $i \geq 2$ , are lower and upper bounded (and positive). Therefore, the matrix  $O$  is invertible, and hence, system (10)–(15) is completely observable. Note that the measurement of  $\bar{p}_N$  (or, equivalently, of  $q_N$ ), rather than any other intermediate percentage, is necessary for system (10)–(15) to be observable. To see this note that if  $C = \begin{cases} c_{ij} = 1, & \text{if } i = 1 \text{ and } j = J \\ c_{ij} = 0, & \text{otherwise} \end{cases}$  with  $J < N$ , then the  $J + 1, \dots, N$  columns of  $O(k_0, k_0 + \bar{N})$  are zero for all  $k_0 \geq 0$  and  $\bar{N} \geq N$ . Thus, the system cannot be observable. In other words, a fixed flow sensor should necessarily be placed at the last segment of the highway in order to guarantee percentage observability based on model (10)–(15).

<sup>1</sup> This is true if at each time instant at least one connected vehicle is present within each segment and, during an interval  $(k, k + 1]$ , for each segment there is at least one connected vehicle moving into the next one.

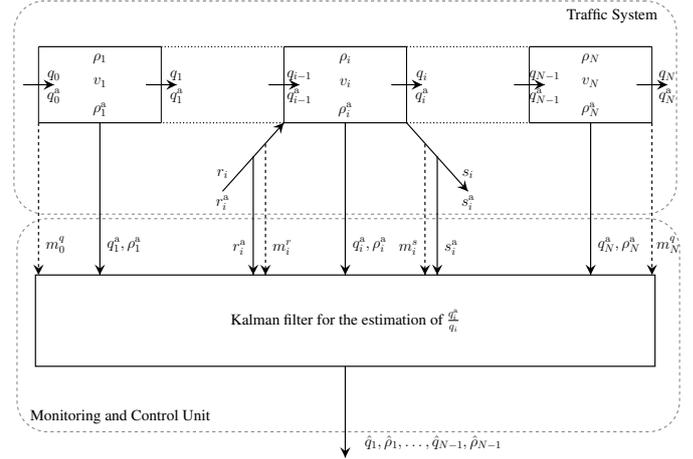


Fig. 1. The traffic system under consideration and the Kalman filter implemented at the MCU. The data used to operate the Kalman filter are either coming from connected vehicles (solid lines) or fixed sensors (dashed lines). The variable  $m_i^w$  denotes the measurement of quantity  $w$  at segment  $i$ , which might be different than the actual quantity  $w$ , due to, for example, the presence of measurement noise.

#### 3.2 Kalman Filter

We implement a Kalman filter for the estimation of the percentage of connected vehicles on a highway (see Fig. 1). Defining  $\hat{x} = (\hat{p}_1, \dots, \hat{p}_N)^T$ , the equations for the Kalman filter are given by (see, for example, Anderson & Moore (1979))

$$\hat{x}(k+1) = A(k)\hat{x}(k) + B(k)u(k) + A(k)K(k)(z(k) - C\hat{x}(k)) \quad (18)$$

$$K(k) = P(k)C^T (CP(k)C^T + R)^{-1} \quad (19)$$

$$P(k+1) = A(k)(I - K(k)C)P(k)A(k)^T + Q, \quad (20)$$

where  $z$  is a noisy version of the measurement  $y$ ,  $R > 0$  and  $Q = Q^T > 0$  are tuning parameters. The initial conditions of the estimator (18)–(20) are given by

$$\hat{x}(k_0) = \mu \quad (21)$$

$$P(k_0) = H, \quad (22)$$

$H = H^T > 0$ . Exponential stability of (18)–(22), (12)–(15) follows using similar arguments to Bekiaris-Liberis et al. (2015).

The Kalman filter (18)–(22) delivers estimates of the inverse percentages  $\hat{p}_i$ ; using (4) and the available data for  $q_i^a$ ,  $\rho_i^a$ , we can obtain estimates for all segment (total) flows and densities  $\hat{q}_i, \hat{\rho}_i$  as indicated at the output of the Kalman filter in Fig. 1.

## 4. PERFORMANCE EVALUATION

#### 4.1 Performance Evaluation Using METANET Model

For preliminary assessment of the developed estimation scheme, we test in this section the performance of the Kalman filter employing the second-order METANET model Papageorgiou & Messmer (1990) (i.e., a model in which the average speed of the vehicles at the highway has its own dynamics) as ground truth. We employ equations (1) and (2) for the total density of the vehicles and the density of connected vehicles, respectively,

Table 1. Parameters of model (1), (2), (5), (6), (23).

$T$	$\frac{1}{360}$ (h)	$\delta$	1.4	$\Delta_i$	0.5 (km)	$N$	20
$v_f$	$120$ ( $\frac{\text{km}}{\text{h}}$ )	$\tau$	$\frac{1}{180}$ (h)	$\rho_{cr}$	$33.5$ ( $\frac{\text{veh}}{\text{km}}$ )		
$v$	$35$ ( $\frac{\text{km}^2}{\text{h}}$ )	$\alpha$	1.4324	$\kappa$	$13$ ( $\frac{\text{veh}}{\text{km}}$ )		

Table 2. The measurement noise  $\gamma_i^w$  and the process noise  $\xi_i^w$ ,  $i = 0, \dots, N$  affecting the  $w$  variable at segment  $i$ . The variable  $w$  can represent a flow (i.e.,  $w = q$ ,  $w = r$ , or  $w = s$ ) or speed (i.e.,  $w = v$ ).

	$\gamma_i^q$	$\gamma_i^r$	$\gamma_i^s$	$\xi_i^{q^a}$	$\xi_i^v$	$\xi_i^q$
SD	$25$ ( $\frac{\text{veh}}{\text{h}}$ )	$10$ ( $\frac{\text{veh}}{\text{h}}$ )	$5$ ( $\frac{\text{veh}}{\text{h}}$ )	$15$ ( $\frac{\text{veh}}{\text{h}}$ )	$5$ ( $\frac{\text{km}}{\text{h}}$ )	$25$ ( $\frac{\text{veh}}{\text{h}}$ )

Table 3. Parameters of the Kalman filter (18)–(22) and (12)–(15).

$Q$	$R$	$\mu$	$H$
$I_{N \times N}$	100	$(10, \dots, 10)^T$	$I_{N \times N}$

together with relations (5) and (6) for the total flow and the flow of connected vehicles, respectively. The equation for the average speed at segment  $i$  is given by

$$v_i(k+1) = v_i(k) + \frac{T}{\tau} (V(\rho_i(k)) - v_i(k)) + \frac{T}{\Delta_i} v_i(k) \times (v_{i-1}(k) - v_i(k)) - \frac{vT}{\tau \Delta_i} \frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \kappa} - \frac{\delta T}{\Delta_i} \frac{r_i(k) v_i(k)}{\rho_i(k) + \kappa}, \quad i = 1, \dots, N, \quad (23)$$

with  $v_0 = v_1$ ,  $\rho_N = \rho_{N+1}$ , where the nominal average speed is  $V(\rho) = v_f e^{-\frac{1}{\alpha} (\frac{\rho}{\rho_{cr}})^\alpha}$ , and  $\tau$ ,  $v$ ,  $\kappa$ ,  $\delta$ ,  $v_f$ ,  $\rho_{cr}$ , and  $\alpha$  are positive model parameters. In particular,  $v_f$  denotes the free speed,  $\rho_{cr}$  the critical density, and  $\alpha$  the exponent of the stationary speed equation. The model parameters, which are taken from Wang & Papageorgiou (2005), are shown in Table 1.

Although the measurements  $q_i^a$ ,  $\rho_i^a$ ,  $r_i^a$ ,  $s_i^a$ , for all  $i$ , stemming from connected vehicle data, which are utilized by the estimator (18)–(20), (12)–(15), is likely to be associated with error or noise, we assume, for this preliminary assessment, that they are accurate. In contrast, the measurements of the total flow of the vehicles at the entry and exit of the highway are subject to additive noise  $\gamma_0^q \sim N(0, D_q^2)$  and  $\gamma_N^q \sim N(0, D_q^2)$ , respectively. Furthermore, the measurements of the total flow at the on-ramps and off-ramps might be subject to additive measurement noise say  $\gamma_i^r \sim N(0, D_r^2)$  and  $\gamma_i^s \sim N(0, D_s^2)$ , respectively. In addition, there is additive process noise  $\xi_i^v \sim N(0, D_v^2)$ ,  $\xi_i^{q^a} \sim N(0, D_{q^a}^2)$ , and  $\xi_i^q \sim N(0, D_q^2)$ ,  $i = 0, \dots, N$ , affecting the speed and flow equations, namely, (23), and (5), (6), respectively. The noise statistics are summarized in Table 2.

The parameters of the Kalman filter (18)–(22), (12)–(15) are shown in Table 3. In Fig. 2 we show the employed scenario of input flow of connected vehicles and total input flow at the entry of the highway for our simulation investigation. We assume that there are three on-ramps at segments 2, 6, 10. The total flow and the flow of connected vehicles at the on-ramps are  $r_i = r_i^a = 150 \frac{\text{veh}}{\text{h}}$ ,  $i = 2, 6, 10$ . Four off-ramps are supposedly present on the highway under study, specifically at segments 4, 8, 12. It is assumed that  $s_i = 0.1q_{i-1}$  and  $s_i^a = 0.1q_{i-1}^a$ ,  $i = 4, 8, 12$ .

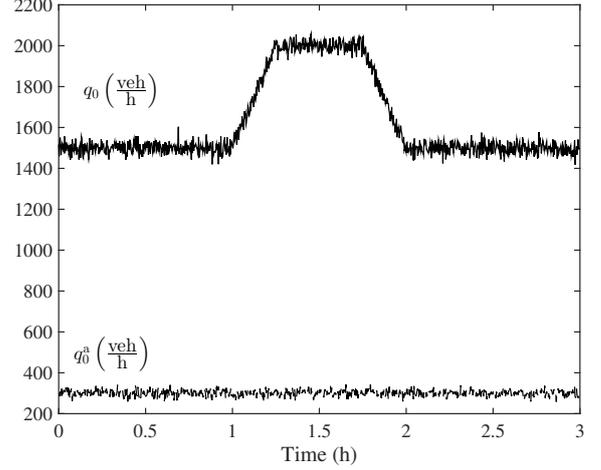


Fig. 2. The total flow of vehicles  $q_0$  and the flow of connected vehicles  $q_0^a$  at the entry of the highway.

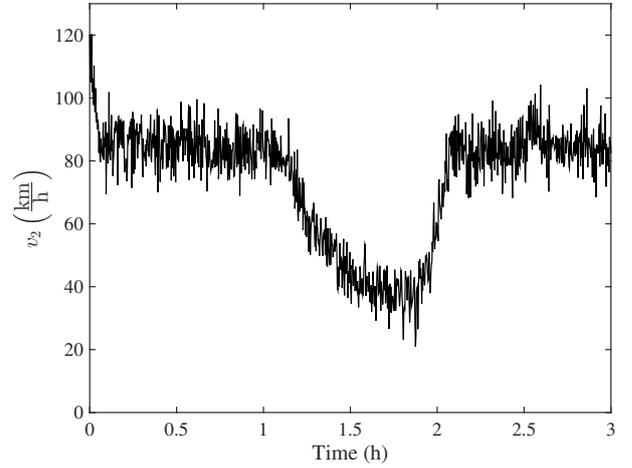


Fig. 3. The average speed  $v_2$  of segment 2 as it is produced by the METANET model (1), (5), (23) with parameters as in Table 1 and process noise as in Table 2.

The average speed at segment 2 (where the first on-ramp is located) is shown in Fig. 3. It is evident from Fig. 3 that a congestion is created between the first and second hour of our test, whereas, free-flow conditions are reported for the first and last hour. Congestion starts approximately at the location of the second on-ramp, i.e., at the sixth segment of the highway, and propagates backwards all the way to the input of the highway.

In both traffic conditions, our estimator successfully estimates the percentage of connected vehicles on the highway, as it is evident from Fig. 4, which displays the actual percentage and its estimate at segment 8. Note the very fast convergence of the produced percentage estimates, starting from remote initial values. Fig. 5 displays the resulting estimation of the total density of vehicles at segment 2 using relation (4).

#### 4.2 Performance Evaluation Using NGSIM Data

We present here another test using real microscopic traffic data collected within the Next Generation Simulation program US DoT (2005). Since these data incorporate non-negligible errors in the position of individual vehicles (see, e.g., Punzo

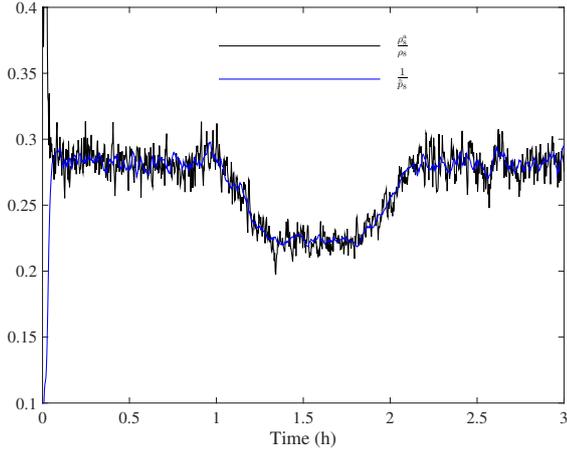


Fig. 4. The percentage of connected vehicles  $\frac{\rho_8^a}{\rho_8}$  of segment 8 (black line) and its estimate  $\frac{1}{\hat{\rho}_8}$  (blue line) as it is produced by the Kalman filter with parameters given in Table 3.

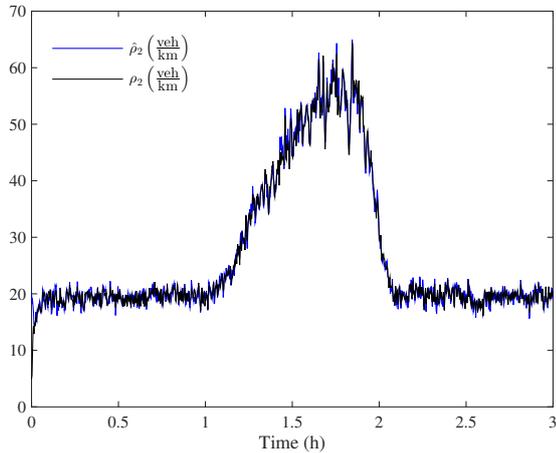


Fig. 5. Total density  $\rho_2$  of segment 2 (black line) and its estimate (blue line)  $\hat{\rho}_2 = \rho_2^a \hat{\rho}_2$  produced by the Kalman filter (18)–(22), (12)–(15) with parameters as in Table 3.

et al. (2011)), correction methodologies are proposed in the literature to improve their reliability; in this work we utilize the data processed by Montanino & Punzo (2013a), Montanino & Punzo (2013b), which include the trajectories of all vehicles travelling along a stretch in the northbound direction of I-80 in Emeryville, California, recorded from 4:00 PM to 4:15 PM on April 13, 2005. The considered stretch (sketched in Fig. 6) is 400 m long and an on-ramp is entering the mainstream, where the merge nose is located 175 m after the network origin. The particular, high-occupancy vehicle (HOV) lane 1 is excluded.

A massive congestion is present within the stretch, where congestion waves are coming from downstream and crossing the entire stretch. In order to perform macroscopic evaluations, the stretch is divided into  $N = 8$  homogeneous segments of 50 m in length (see Fig. 6); the on-ramp is placed within segment 4; while a discrete time step  $T = 5$  s is used.

The NGSIM data are processed defining a mixed traffic scenario, where the 20% (on average) of vehicles are assumed to be connected. Vehicles entering the network stretch are ran-

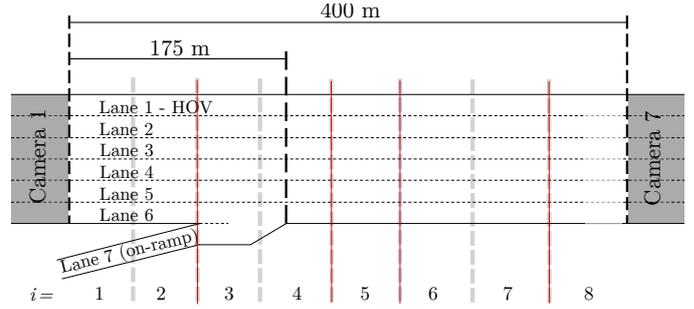


Fig. 6. A graphical representation of the stretch of the highway I-80 in Emeryville, California, related to the NGSIM data.

domly tagged as connected according to a uniform distribution, therefore the percentage effectively varies in time and space. All necessary information to perform estimation is extracted from the available trajectory data. In particular, the density of connected vehicles  $\rho_i^a(k)$  is computed by counting the number of connected vehicles that are present within segment  $i$  at time instant  $kT$  divided by the segment length (0.05 km); all flow measurements are computed via virtual spot detectors placed at the network entrance (providing  $q_0$  and  $q_0^a$ ), at the network exit (providing  $q_N$  and  $q_N^a$ ), as well as at the boundaries between each pair of adjacent segments (providing  $q_i^a$ ); also, the on-ramp flows  $r_4(k)$  and  $r_4^a(k)$ , are computed similarly to the computation of flow by a virtual detector, i.e., by counting the number of vehicles leaving the on-ramp (Lane 7 in Fig. 6) and entering the mainstream during time interval  $(k, k + 1]$ . The ground truth, used for evaluating the estimation results, is represented by the total density in each segment  $\rho_i(k)$ , which is computed analogously to density of connected vehicles  $\rho_i^a(k)$ .

Due to the low and time-varying penetration rates, at some time instants some segments may not contain any connected vehicle. This may cause observability (e.g., if  $q_i^a = 0$ ) or numerical (e.g., if  $g_i^a = 0$ ) issues. In order to overcome these potential issues, we feed the filter with an average of all measurements related to connected vehicles and the ramp inflow  $r_4(k)$ ; in particular, a moving average using a time window of 60 s is employed.

The parameters used for the Kalman filter are the following:  $Q = I_{N \times N}$ ;  $R = 1$ ;  $\mu = (1, \dots, 1)$ ;  $H = I_{N \times N}$ .

In Fig. 7, the total density  $\rho_i(k)$  is compared with the estimated density computed as  $\hat{\rho}_i(k) = \hat{\rho}(k) \hat{\rho}_i^a(k)$ , where  $\hat{\rho}_i^a(k)$  is the smoothed density of connected vehicles, obtained according to the previously described moving average. We can see that the produced estimates largely capture the dynamic variations of total densities, in particular reproducing the congestion pattern caused by back-spilling shockwaves.

## 5. CONCLUSIONS

A topic of ongoing research is the validation, as well as the performance comparison, of the presented traffic estimation methodology and the alternative traffic estimation approach developed in Bekiaris-Liberis et al. (2015), by use of a detailed microscopic simulation platform; considering various scenarios for all involved real-time measurements.

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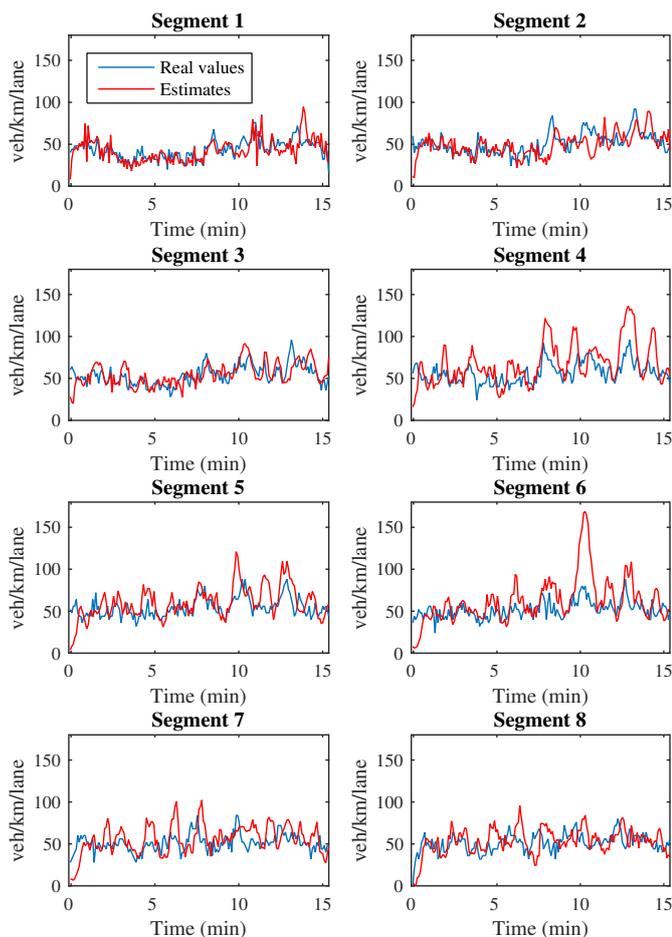


Fig. 7. The trajectories of real and estimated densities for the experiments based on NGSIM data, assuming a 20% penetration rate of connected vehicles.

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#### REFERENCES

- L. Alvarez-Icaza, L. Munoz, X. Sun, and R. Horowitz. Adaptive observer for traffic density estimation. *ACC*, 2004.
- B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, Prentice-Hall, NJ, 1979.
- V. Astarita, R. L. Bertini, S. d' Elia, and G. Guido. Motorway traffic parameter estimation from mobile phone counts. *European J. of Operational Research*, 175: 1435–1446, 2006.
- N. Bekiaris-Liberis, C. Roncoli, & M. Papageorgiou. Highway traffic state estimation with mixed connected and conventional vehicles using speed measurements. *IEEE ITSC*, 2015.
- A. Bose & P. Ioannou. Mixed manual/semi-automated traffic: a macroscopic analysis. *Transp. Res. C*, 11: 439–462, 2003.
- L. C. Davis. Effect of adaptive cruise control systems on mixed traffic flow near an on-ramp. *Physica A*, 379: 274–290, 2007.
- C. de Fabritiis, R. Ragona, and G. Valenti. Traffic estimation and prediction based on real time floating car data. *IEEE ITSC*, China, 2008.
- W. Deng, H. Lei, & X. Zhou. Traffic state estimation and uncertainty quantification based on heterogeneous data sources: A three detector approach. *Transp. Res. B*, 57: 132–157, 2013.
- C. Diakaki, M. Papageorgiou, I. Papamichail, and I. K. Nikolos. Overview and analysis of vehicle automation and communication systems from a motorway traffic management perspective. *Transp. Res. Part A*, 75: 147–165, 2015.
- J. I. Ge and G. Orosz. Dynamics of connected vehicle systems with delayed acceleration feedback. *Transportation Research Part C*, 46: 46–64, 2014.
- J. C. Herrera and A. M. Bayen. Incorporation of Lagrangian measurements in freeway traffic state estimation. *Transportation Research Part B*, 44: 460–481, 2010.
- A. Hegyi, D. Girimonte, R. Babuska, and B. De Schutter. A comparison of filter configurations for freeway traffic state estimation. *IEEE Conference on ITS*, Toronto, Canada, 2006.
- A. Kesting, M. Treiber, M. Schonhof, and D. Helbing. Adaptive cruise control design for active congestion avoidance. *Transportation Research Part C*, 16: 668–683, 2008.
- L. Mihaylova, R. Boel, and A. Hegyi. Freeway traffic estimation within particle filtering framework. *Automatica*, 43: 290–300, 2007.
- M. Montanino & V. Punzo. Making NGSIM data usable for studies on traffic flow theory. *Transportation Research Record*, 2390: 99–111, 2013.
- M. Montanino & V. Punzo. Reconstructed NGSIM I80-1. *COST ACTION TU0903 - MULTITUDE*, www.multitude-project.eu/exchange/101.html, 2013.
- D. Ngoduy, S.P. Hoogendoorn, R. Liu. Continuum modeling of cooperative traffic flow dynamics. *Physica A: Statistical Mechanics and its Applications*, 388: 2705–2716, 2009.
- M. Papageorgiou and A. Messmer. METANET: A macroscopic simulation program for motorway networks. *Traffic Engineering & Control*, 31: 466–470, 1990.
- V. Punzo, M. T. Borzacchiello, and B. Ciuffo. On the assessment of vehicle trajectory data accuracy and application to the Next Generation Simulation (NGSIM) program data. *Transportation Research Part C*, 19: 1243–1262, 2011.
- C. Roncoli, M. Papageorgiou, and I. Papamichail. Traffic flow optimisation in presence of vehicle automation and communication systems - Part II: Optimal control for multi-lane motorways. *Transp. Res. Part C*, 57: 260–275, 2015.
- T. Seo, T. Kusakabe, & Y. Asakura. Estimation of flow and density using probe vehicles with spacing measurement equipment. *Transp. Res. Part C*, 53: 134–150, 2015.
- S. E. Shladover, D. Su, and X.-Y. Lu. Impacts of cooperative adaptive cruise control on freeway traffic flow. *Transportation Research Record*, 2324: 63–70, 2012.
- US DoT. Next Generation SIMulation (NGSIM). www.ngsim-community.org, 2005.
- P. Varaiya. Smart cars on smart roads: problems of control. *IEEE Trans. Automatic Control*, 38: 195–207, 1993.
- M. Wang, W. Daamen, S. P. Hoogendoorn, and B. van Arem. Rolling horizon control framework for driver assistance systems. part II: cooperative sensing and cooperative control. *Transportation Research Part C*, 40: 290–311, 2014.
- Y. Wang & M. Papageorgiou. Real-time freeway traffic state estimation based on extended Kalman filter: a general approach. *Transportation Research Part B*, 39: 141–167, 2005.
- D. B. Work, O.-P. Tossavainen, S. Blandin, A. M. Bayen, T. Iwuchukwu, and K. Tracton. An ensemble Kalman filtering approach to highway traffic estimation using GPS enabled mobile devices. *IEEE CDC*, Cancun, Mexico, 2008.
- Y. Yuan, J. W. C. van Lint, R. E. Wilson, F. van Wageningen-Kessels, and S. P. Hoogendoorn. Network-wide traffic state estimation using loop detector and floating car data. *Journal of Intelligent Transportation Systems*, 18: 41–50, 2014.