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PARAMETRIC FINITE ELEMENT ANALYSIS OF MASONRY STRUCTURES USING DIFFERENT CONSTITUTIVE MODELS

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Abstract. *The masonry of old structures varies in a wide range: geometry of walls and columns, type and dressing of stones, joint constructions and materials, and others. Also repair work and strengthening techniques of old masonry have a major influence on the mechanical behavior so additionally material models are needed to describe the seismic behavior of strengthened masonry.*

The masonry material constituting the structures of monumental and old constructions is often characterized by very low tensile strength with respect to the compression strength. In parallel, masonry compression behaviour is of crucial importance for design and safety assessment purposes, since masonry structures are primarily stressed in compression. However, the compression failure mechanism of quasi-brittle materials is rather complex, especially when compared with tensile failure.

The finite element method is usually adopted to achieve sophisticated simulations of the structural behaviour. A mathematical description of the material behaviour, which yields the relation between the stress and strain tensor in a material point of the body, is necessary for this purpose. This constitutive model must be capable of predicting the behaviour of the structure from the linear elastic stage, through cracking and degradation until total loss of strength.

Masonry is a composite material made of bricks and mortar, which exhibits distinct directional properties due to the mortar joints which act as planes of weakness. In our research, on the numerical representation the macromodeling of masonry as a composite is used, which is applicable when the structure is composed of solid walls with sufficiently large dimensions so that the stresses across or along a macro-length will be essentially uniform and also is more practice oriented.

Nonlinear behaviour of both components should be considered to obtain a realistic model able to describe cracking, slip, and crushing of the material. Its yield behaviour is a strong function of hydrostatic pressure and tensile yield stress and compressive yield stress, under uniaxial loading are different. In order to obtain a better representation, individual yield criteria must be considered, according to different failure mechanisms, one in tension and the other in compression. Something which is not so easy in many yield and failure criteria which are already programmed to finite elements programs.

In this paper, some results of a parametric investigation about the applicability of widely used criteria like the Drucker and Prager, the Parabolic Mohr- Coulomb and the Buyukozturk, in the dynamic analysis of masonry wall are presented. The analysis was done, considering various dynamic loads in order to study the influence of the selected criterion on the dynamic behaviour locally or globally of the structure. The correlation of the results is concentrated on the effectiveness of the examined criteria to represent the real mechanical behavior and the estimation of critical areas. Also the differences which are presented seems to be remarkable when complicated dynamic loads are applied.

1 INTRODUCTION

The finite element method is usually adopted to achieve sophisticated simulations of the structural behaviour. A mathematical description of the material behaviour, which is named a constitute model, is necessary for this purpose. An important objective of today's research is to obtain robust numerical tools, capable of predicting the behaviour of the structure from the linear elastic stage, through cracking and degradation until total loss of strength. Also an important parameter is the type of finite element which will be used. Numerical simulations are fundamental to provide insight into the structural behaviour and to assess/retrofit existing masonry structures [1].

Masonry is a composite material made of units which are such as stones, bricks and others, and joints which can be clay, lime/cement based mortar or other mortar. Due to the mortar joints which act as planes of weakness, the masonry exhibits distinct directional properties. In general, the approach towards its numerical representation can focus on the micro-modelling of the individual components, or the macro-modelling of masonry as a composite. Depending on the level of accuracy and the simplicity desired, one modelling strategy can be preferred over the other. Micro-modelling studies are necessary to give a better understanding about the local behaviour of masonry structures. Macro-models are applicable when the structure is composed of solid walls with sufficiently large dimensions so that the stresses across or along a macro-length will be essentially uniform. Clearly, macro-modelling is more practice oriented.

So in large structures, the knowledge of the behaviour of the interaction between units and joints does not usually determine the global behaviour of the structure. In this case, it is more adequate to resort to continuum models, which establish the relation between average stresses and average strains in masonry [2, 3]. In parallel various cracks are developed due to low tension strength. Energy dissipation mechanisms arising due to contact and friction along these contact interfaces are certainly responsible for the beneficial aseismic behavior [4, 5]. The nonlinear behavior of both components should be considered to obtain a realistic model able to describe cracking, slip, and crushing of the material. Its is therefore of relevant importance that, for each type of masonry, experiments to correlate the strength characteristics of constituent materials with the characteristics of masonry must be carried out.

In this study a recoverable, nonlinear elastic behaviour and a plastic irrecoverable behaviour were considered for the masonry. In both cases, the relationship of stress-strain is nonlinear, however, in the case of nonlinear elastic analysis the unloading follows the curve of stress-strain, while in the plastic analysis take place elastic unloading. The elastic-plastic results can be considered as reliable only when instability phenomena can be ruled out [6]. The material was considered as homogenous and it was modeled by elastoplastic theory, using the simple forms of yield surfaces written in terms of the first and the second deviatoric stress invariants.

In particularly, some results of a parametric investigation are presented about the applicability of widely used criteria like the Drucker and Prager, the Parabolic Mohr- Coulomb and the Buyukozturk, in the dynamic analysis of a masonry wall. The analysis of a masonry wall with openings, part of typical masonry building, was done, considering various dynamic loads in order to study the influence of the every time selected criterion on the dynamic behaviour locally or globally of the structure. The correlation of the results is concentrated on the effectiveness of the examined criteria to represent the real mechanical behavior, relates with the type of applied loadings.

2 MATERIAL MODEL

2.1 Continuum models

In order to consider a composite material as homogenous, an homogenization technique must be applied either with experimental tests or with analytical and computational methods [7, 8, 9, 10]. The application of computational homogenization techniques for structural masonry computations, is an alternative to the formulation of complex closed-form macroscopic constitutive laws. Due to the difference existing between brick units and mortar, a complex interaction between the two masonry components occurs with masonry deformation. In general, non-linear behaviour of masonry unit is dominated by mortar joint. In order to derive the homogenized inelastic material properties of masonry basic cell, a reliable material model for masonry components (brick and mortar) is important [11].

The well-known failure criteria namely Mohr-Coulomb, Saint Venant and Navier, can be successfully used in order to predict the stress state (biaxial and shear stresses) at which the three fundamental failure modes can be expected; i.e. slipping of the mortar joints, cracking of bricks and splitting of joints, and spalling in the middle plane. The Mohr-Coulomb frictional law can be slightly modified to take into account the nonlinear dependence of shear strength on normal stress at high compression levels [12].

In reality, the material in non-homogeneous and a close material representation is only possible if the units and joints are modelled separately [13]. In case of modeling large structures, subjected to loads and boundary conditions such that the state of stress and strain across a macro-length can be assumed to be uniform, a continuum model can be used. A macro-modelling strategy represents a compromise between efficiency and accuracy. The model introduced in Lourenço et al [14] combines the advantages of modern plasticity concepts with a powerful representation of anisotropic material behaviour, which includes different hardening/softening behaviour along each material axis. The model includes the combination of a Rankine-like yield surface for tension and a Hill-like yield surface in compression. This composite surface permits to reproduce the results obtained in uniaxial tests, in which different behaviour are obtained along different directions.

One serious problem associated with smooth criteria is the poor representation of materials with a large difference between uniaxial compressive strength and uniaxial tensile strength, which leads to unacceptable overestimation of strength in the tension-compression regime. To obtain a better representation, individual yield criteria must be considered, according to different failure mechanisms., one in tension and the other in compression. The former is associated with a localised fracture process, denoted by cracking of the material, and, the latter, is associated with a more distributed fracture process which is usually termed crushing of the material. Several models have been tested for these [3, 15].

Softening is a gradual decrease of mechanical resistance under a continuous increase of deformation forced upon a material specimen or structure. It is a salient feature of quasi-brittle materials like clay brick, mortar, ceramics, rock or concrete, which fail due to a process of progressive internal crack growth. Such mechanical behaviour is commonly attributed to the heterogeneity of the material, due to the presence of different phases and material defects, like flaws and voids. A model which is formulated on the basis of softening plasticity for tension, shear, and compression, was presented by Lourenco and Rots [13]. Numerical implementation is based on modern algorithmic concepts such as implicit integration of the rate equations and consistent tangent stiffness matrices. The approach used in this work is based on idea of concentrating all the damage in the relatively weak joints and, if necessary, in potential tension cracks in the bricks.

2.2 Generalized yield failure criteria

A fundamental notion in the plasticity theory is the existence of a yield function that bounds the elastic domain. According to Coulomb-Navier theory a ductile material such as soil, rocks, concrete and masonries failure under a multiaxial stress loading system when the effective shear stress in a specific plane get over from a critical value which is usually a function of shear strength and hydrostatic pressure. The general mathematical formulation of this type yield criteria is:

$$\left.\begin{array}{l}\tau \geq \mathrm{f}(\sigma) \\ \sigma = \dfrac{\mathrm{I}_1}{3}, \tau = c\sqrt{\mathrm{J}_2}\cos(\boldsymbol{\theta})\end{array}\right\} \Rightarrow F(\mathrm{I}_1, \mathrm{J}_2, \mathrm{c}_1, \mathrm{c}_2, \cdots) \leq 0 \tag{1}$$

where:

I_1	Stress tensor first invariant (hydrostatic stress contribution in yielding or failure)
J_2	Deviatoric stress tensor second invariant (shear stress contribution in yielding or failure, shear internal forces work)
θ	Lode angle $(-\pi/6 \leq \theta \leq \pi/6)$
c	Coefficient which vary from 0 to 1
c_1, c_2	material strength, cohesion and internal friction parameters

2.3 Mohr Coulomb linear or Drucker Prager

The Mohr-Coulomb criterion is a first two-parametric yield surface, for the maximum compression and tension. The model is the first one that takes shearing into account. it should be noted that the criterion considers the maximum difference between the major and the minor principal stresses only, and does not take the intermediate principal stress in the strength criterion. The Mohr-Coulomb strength criterion can be represented graphically, by Mohr's circle. Most of the classical engineering materials, including rock materials, somehow follow this rule in at least a portion of their shear failure envelope. The Generalized Mohr-Coulomb linear or Drucker-Prager yield-failure criteria can be mathematical expressed by the equation 2:

$$\mathrm{F}(\mathrm{I}_1, \mathrm{J}_2, \mathrm{a}, \mathrm{k}) = \mathrm{aI}_1 + \sqrt{\mathrm{J}_2} - \mathrm{k} = 0 \tag{2}$$

where:

$$\mathrm{a} = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)} = \frac{\sigma_{\mathrm{Yc}} - \sigma_{\mathrm{Yt}}}{\sqrt{3}(\sigma_{\mathrm{Yc}} + \sigma_{\mathrm{Yt}})} = \frac{\mathrm{m}-1}{\sqrt{3}(\mathrm{m}+1)}$$

$$k = \frac{6\mathrm{S}_0 \cos\phi}{\sqrt{3}(3-\sin\varphi)} = \frac{2\sigma_{\mathrm{Yc}}\sigma_{\mathrm{Yt}}}{\sqrt{3}(\sigma_{\mathrm{Yc}} + \sigma_{\mathrm{Yt}})} = \frac{2\sigma_{\mathrm{Yc}}}{\sqrt{3}(\mathrm{m}+1)} = \frac{\sigma_{\mathrm{Y}}}{\sqrt{3}}$$

$$\mathrm{m} = \frac{\sigma_{\mathrm{Yc}}}{\sigma_{\mathrm{Yt}}}$$

σ_{Yt}	Material tensile yield stress or tensile strength
σ_{Yc}	Material compression yield stress or tensile strength

S_0	Material shear yield stress or shear strength or cohesion
φ	Material internal friction angle
σ_Y	initial equivalent yield stress

2.4 Mohr CoulombParabolic

The Mohr-Coulomb parabolic yield-failure criterion mathematical can be expressed by the equation 3.

$$F(I_1, J_2, \beta, \sigma_Y) = \sqrt{3}\beta\sigma_Y I_1 + 3J_2 - \sigma_Y^2 = 0 \tag{3}$$

Where:

$$\beta = \frac{(m-1)}{\sqrt{3m}}$$

$\sigma_Y = \dfrac{\sigma_{Yc}}{\sqrt{m}}$ initial equivalent yield stress

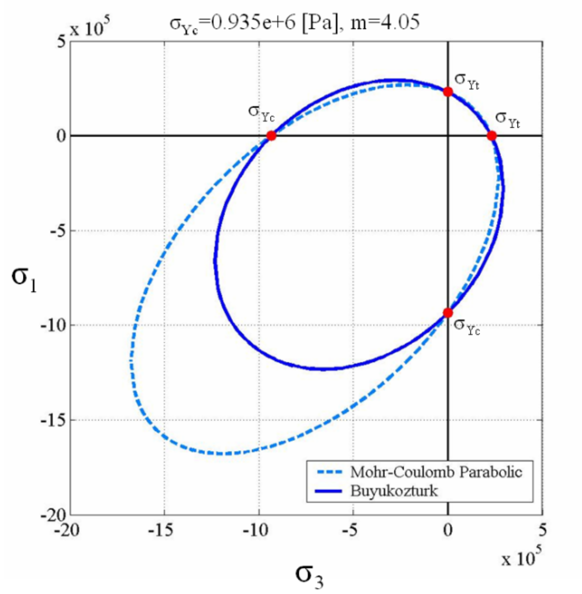


Figure 1:Yield surface in 2D of the Mohr-Coulomb Parabolic and the Buyukozturk models.

2.5 Buyukozturk's Modified Model

Oral Buyukozturk formulate a modified version of Mohr-Coulomb parabolic yield-failure criterion (1975), adding in the equation (3) an extra term which is a function of first stress tensor invariant square. The generalized yield and failure criteria are developed to account for the two major sources of nonlinearity: the progressive cracking of concrete in tension, and the nonlinear response of concrete under multiaxial compression. Using these criteria, incremental stress-strain relationships are established in suitable form for the nonlinear finite element analysis [16]. The Buyukozturk yield function is given by the equation 4.

$$F(I_1, J_2, \alpha, \beta, \sigma_Y) = \sqrt{3}\beta\sigma_Y I_1 + \alpha I_1^2 + 3J_2 - \sigma_Y^2 = 0 \tag{4}$$

where:

α, a function's shape parameter. The optimum value for α parameter is 0.2.

$$\beta = \frac{(m-1)\sqrt{1+\alpha}}{\sqrt{3m}}$$

$$\sigma_Y = \sigma_{Yc}\sqrt{\frac{1+\alpha}{m}}$$ Initial equivalent yield stress

In figure 1 the yield surfaces in two dimensions of the Mohr-Coulomb Parabolic and the Buyukozturk models are presented.

3 FINITE ELEMENT MODELING

3.1 Geometry and loads of the models

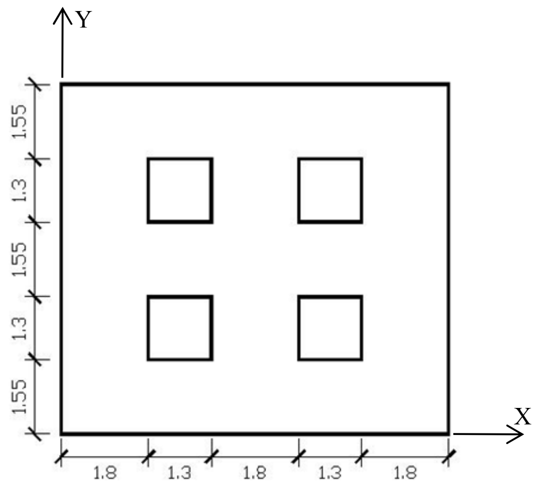


Figure 2: Geometry of the masonry wall.

In order to investigate the response of a masonry wall with openings (see fig. 2) under typical conditions, like in plane and out of plane buckling, in plane compression, and dynamic behaviour under base excitation, different time varied loads were considered. The loading histories arising from the multiplication of a scale factor with a linear function $f(t)=(1/t_{max})t$, or with a sinusoidal function $f(t)=sin(2\pi t/T)$, $t_{max}=3T/2$, and t_{max}=3T/2.

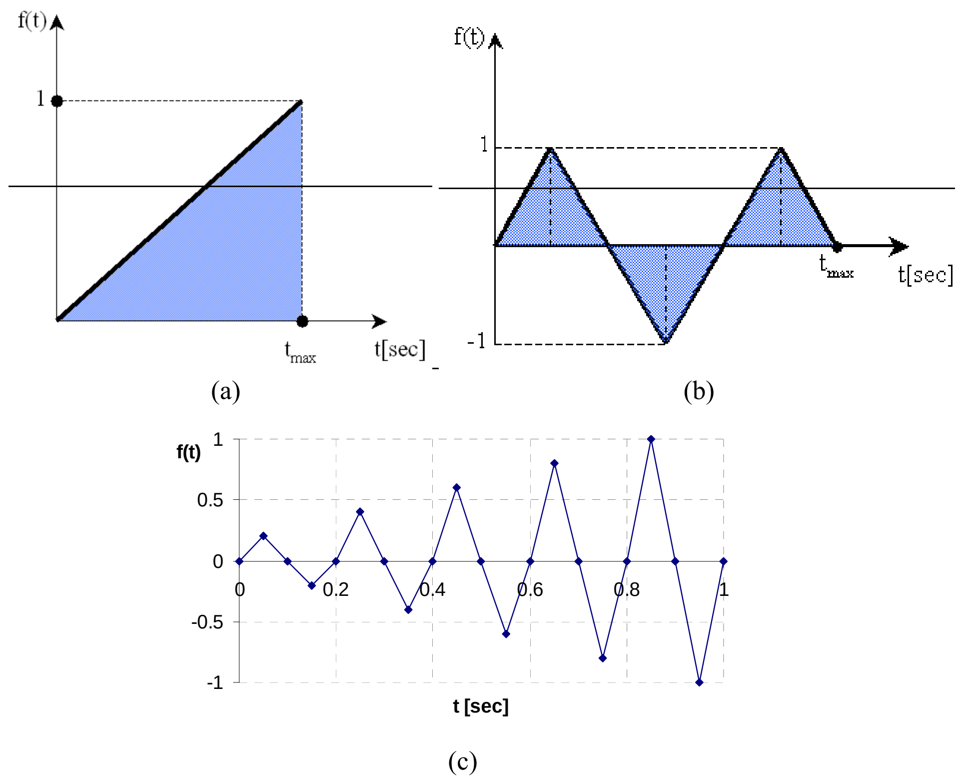


Figure 3: Displacement histories for a) load case 1, 2 and 3, b) load case 4 and 6 and c) load case 5.

The following load cases were considered:

Load case 1(Lc1): A linear time varied horizontal displacement u_x (in plane direction) (Fig. 3a) which are applied at the top of the wall with a maximum value equal to 0.001m. Fixed condition at the base are assumed.

Load case 2 (Lc2): A linear time varied horizontal displacement, u_z (out of plane direction) (Fig. 3a) which are applied at the top of the wall with maximum value equal to -0.005m. Fixed condition at the base are assumed.

Load case 3 (Lc3): A linear time varied vertical displacement, u_y (Fig. 3a) which are applied at the top of the wall with a maximum value equal to -0.001m. Fixed condition at the base are assumed.

Load case 4 (Lc4): A horizontal sinusoidal displacement uz (Fig. 3b) at the base of the wall are applied in parallel with the weight of the mass, a vertical pressure at the level of the first floor (simulating the loads which are transferred to the wall from the horizontal slab) and a vertical pressure at the top level (simulating the loads of the roof). Maximum value of displacement equal to -0.01m.

Load case 5 (Lc5): An amplifying sinusoidal horizontal displacement uz (Fig. 3c) at the base of the wall are applied in parallel with the weight of the mass, a vertical pressure at the level of the first floor (simulating the loads which are transferred to the wall from the horizontal slab) and a vertical pressure at the top level (simulating the loads of the roof). Maximum value of displacement equal to -0.005m.

Load case 6 (Lc6): A horizontal sinusoidal displacement uz (Fig. 3b) at the base of the wall are applied in parallel with the weight of the mass, a vertical pressure at the level of the first floor (simulating the loads which are transferred to the wall from the horizontal slab) and a vertical pressure at the top level (simulating the loads of the roof). Maximum value of displacement equal to -0.1m.

3.2 Material model

In case of an earthquake, the structure will be subjected to a series of cyclic horizontal actions, which will often cause high additional bending and shear stresses in structural walls, exceeding the range of the elastic behaviour. The nonlinearity of the material appears for example if the stress-strain relationship or constitutive equation is nonlinear. Thus, for the nonlinear analysis of the examined models, in addition to the elastic material constants (Young's modulus and Poisson's ratio), the yield stress and yield function must be determined in order to describe the inelastic (plastic) material behavior by the definition of a stress-strain curve which is described from two branches, the first one which corresponds to the elastic region of the material and the second one to the plastic region. The magnitude of the yield stress is generally obtained from a uniaxial test but since the stresses in a structure are usually multiaxial, a yield condition must be used for measurement of yielding of the mutliaxial state of stress. The yield condition can be dependent on all stress components, on shear components only, or on hydrostatic stresses.

In our applications the general purpose finite element program MARC, was used in which several elasto-plastic models can be used [17]. Special the generalized Mohr-Coulomb model developed by Drucker and Prager, the Mohr-Coulomb Parabolic and the Buyukozturk model were selected in order to use in our applications.

The material data of the masonry is given to Table 1 and the values of the material models parameters as were described in precious section, are given in Table 2. The material has been considered as homogeneous and isotropic, the numerical values have been chosen on the basis of compression tests performed on specimens.

	Masonry
E[Pa]	8.82e+9
v	0.15
ρ[Kg/m³]	1700
φ	44.1
τ_Y[Pa]	0.198e+6
σ_{Yt}[Pa]	0.231e+6
σ_{Yc}[Pa]	0.935e+6
m	4.05

Table 1: Mechanical properties of masonry

The finite element method was used on a three - dimensional, solid model of the wall. Solid finite elements have been used for the analysis.

The following three models, with different material models were examined:

Model 1 : Masonry wall with the Drucker and Prager material model
Model 2 : Masonry wall with the Parabolic Mohr-Coulomb material model
Model 3 : Masonry wall with the Buyukozturk material model

	Drucker and Prager	Mohr-Coulomb Parabolic	Buyukozturk
σ_Y (Pa)	370000	464619	508965
Alpha	0.35	-	-
Beta	-	0.88	0.96

Table 2: Parameters of yield functions for the material models.

4 RESULTS

In the case with the elasto-plastic material model, the estimation of the region with plastic strain is an indication of failure and crack development. Some specific nodes were selected in order to study their response during the time and at the final time step (see Fig. 4).

From the deformation of the examined models for the load case 1where tension is developed across the left side and compression across the right side and the final equivalent plastic strains (see Fig. 5a) the node 1227 was selected in order to see its behaviour to tension (Fig. 5b). The models 2 and 3 give the same results and small differences to plastic stresses are presented for model 1. The same indication is given form the diagram of the equivalent plastic strains across the section 1 of models for the load case 2 (Fig. 6). Comparing the final contours of the plastic strains for the examined models neglected differences are presented for both load case 1 and 2 where we have out of plane and in plane buckling of wall. In opposite overestimation of the plastic strains is happened for Model 1 from the comparison of the results for the load case 3 (Fig. 7a, b). The same conclusion arising for the diagram of tension and compression response of the center point 3090, as they are shown in figures 8 and 9.

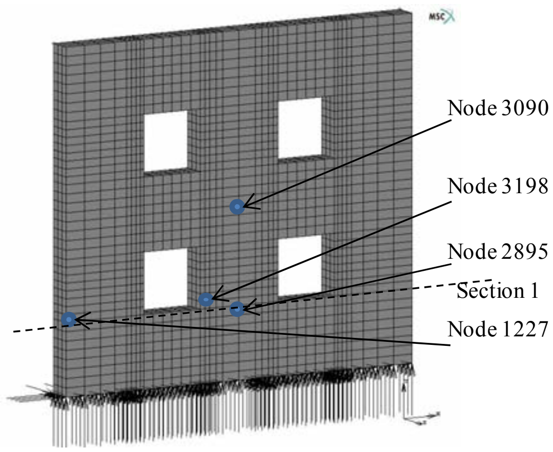
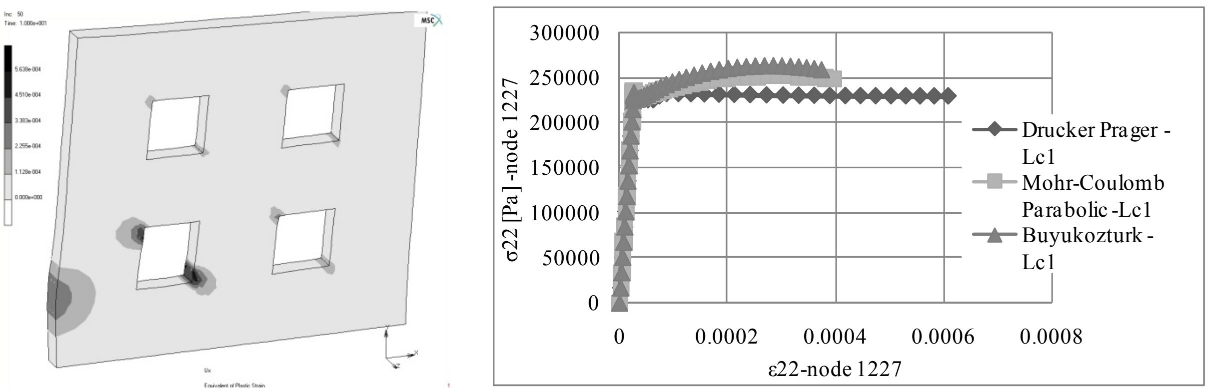
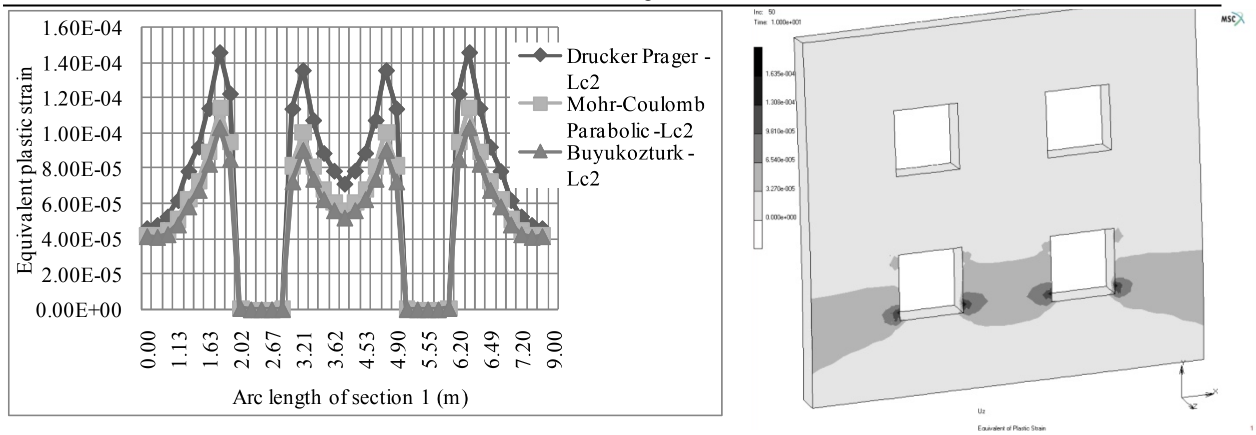


Figure 4: Finite element model and specific nodes for results presentation.



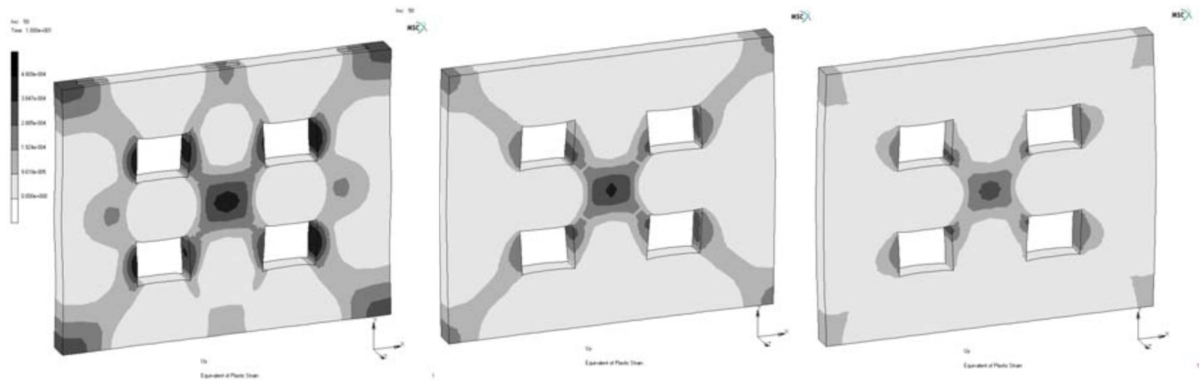
(a) (b)

Figure 5: Stress-strain curve in y direction of node 1227 and countour plot of equivalent plastic strains (Model 3) for load case 1.



(a) (b)

Figure 6: Equivalent plastic strain across section 1 and countour plot of equivalent plastic strains (Model 3) for load case 2.



Model 1 Model 2 Model 3

Figure 7: Contours plot of equivalent plastic strain at the final time step of load case 3.

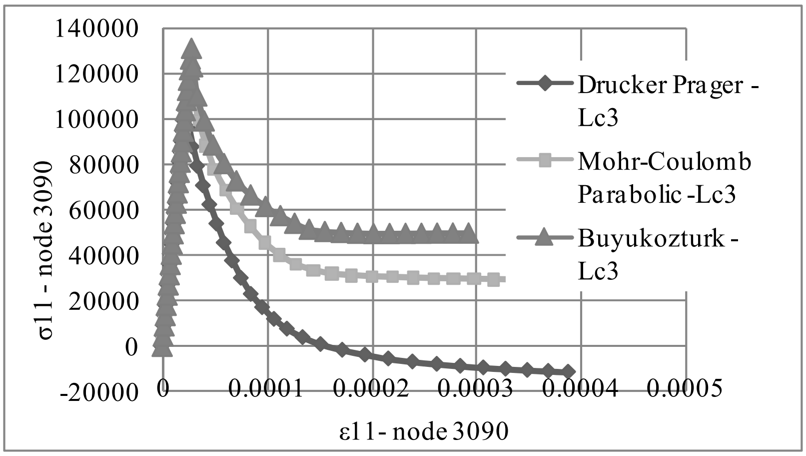


Figure 8: Stress-strain curve in x direction, of node 3090 and for load case 3.

The estimation of critical areas where possible cracks may be appeared due to this loads compare well with corresponding exprerimenat results and pictures from real structures. Since more complicated dynamic loads llike seismic loads, are usually applied to real structures wich arise more complicated phenomena, other cyclic dynamic loads were considered in our study in order to investigate the selected material models. For this reason the load cases 4 and 5were considered and the response of selected points were also examined. From calculated equivalent plastic strains the critical areas around the lower conrners of the lower openings and the bottom of the wall were estimated (Fig. 10a). The differenses of the examined model are higher at higher time steps as it is shown to the history plot of equivalent plastic strain of node 3198 and for load case 4 (Fig. 10b). The same conclusion is given if the same load history is applied with a higher value of scaling factor (load case 6).

In case of the amplifying sinusoidal horizontal displacement uz (load case 5) the stress strain curve of the node 2895 (between the lower two openings) describe the dynamic phenomena which are arising during the time and the difference between the examined models. The different response of model 1 in comparison the models 2 and 3 seems to be remarkable at the higher time steps.

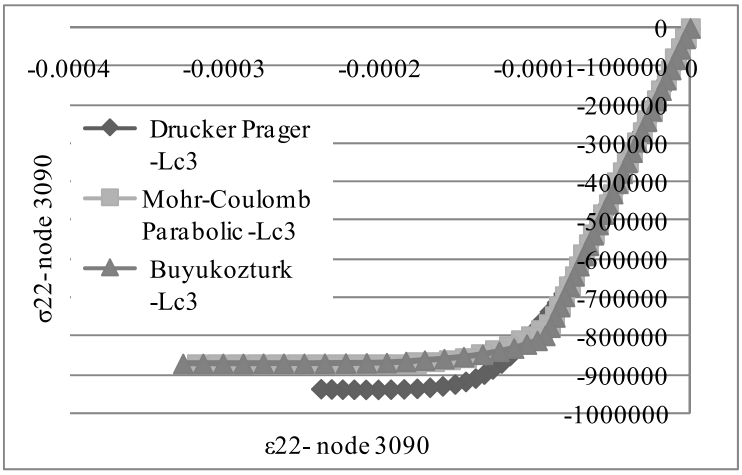
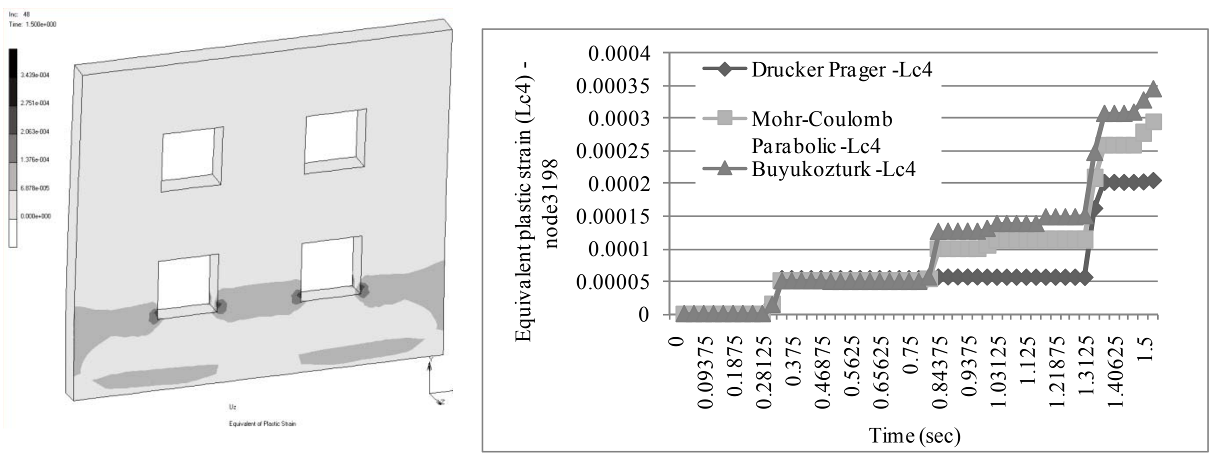


Figure 9: Stress-strain curve in y direction, of node 3090 and for load case 3.



(a) (b)

Figure 10:a)Countour plot of equivalent plastic strains (Model 2), b) History plot of equivalent plastic strain of node 3198 and for load case 4.

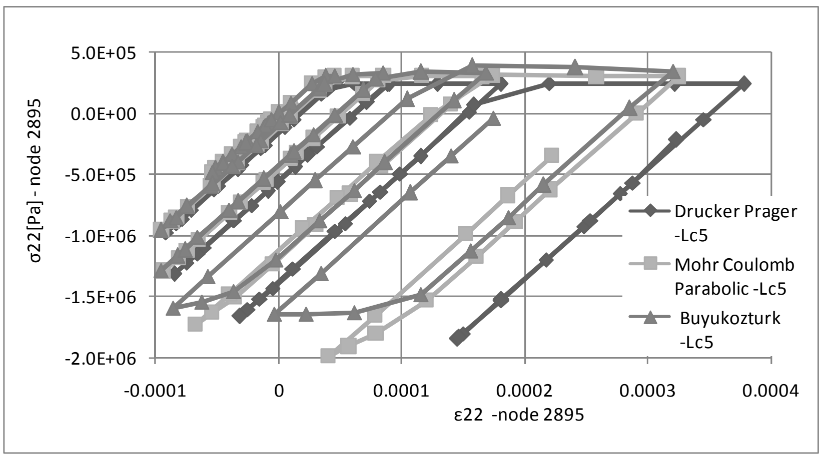


Figure 11: Stress-strain curve in y direction of node 3090 and for load case5.

5 CONCLUSIONS

In our applications widely used elasto-plastic material models like the generalized Mohr-Coulomb model which was developed by Drucker and Prager, the Mohr-Coulomb Parabolic and the Buyukozturk models were selected to be used in the dynamic analysis of a typical masonry wall, part of a real structure. Simple linear time varied, sinusoidal and amplifying sinusoidal dynamic loads were applied in order to examine the dynamic behaviour of the wall and to estimate the critical areas, areas where plastic strains are developed.

From the results the selected material models can simulate the failure mechanisms in tension and compression and the dynamic behaviour of the wall under dynamic loads. Significant differences presented to the overestimation of the plastic strains when the Drucker Prager model is used, which could give a picture of failure unrealistic and strong reinforcements could be selected. These differences are remarkable when the sinusoidal or the amplifying sinusoidal dynamic horizontal displacements at the base of the wall are applied. The Mohr-Coulomb Parabolic and the Buyukozturk models give almost the same results with small differences special to the amplifying sinusoidal dynamic loads.

General the applicability of these models depends on the level of accuracy, the simplicity desired, the loading conditions and the special interest on the local or global behaviour of a small or a large structure. Significant differences relates with the complication of the dynamic loads. Another factor is the estimation of the material parameters which relates with the nonlinearity of the masonry and the components of these composite material. So it is important, for each type of masonry, experimental results to correlate the strength characteristics of constituent materials with the characteristics of masonry.

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