TECHNICAL UNIVERSITY OF CRETE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

Synchronization, Channel Estimation & Detection of RFID Signals with Miller Coding

by

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Abstract

This work studies all necessary signal processing steps in a full-duplex Gen2 reader for RFID tags with Miller-2, 4 and 8 line coding. Relevant prior art focused on FM0 line coding. Synchronization, channel estimation and coherent detection techniques are presented. Symbol-by-symbol detection is studied and respective BER in AWGN is theoretically calculated, showing its equivalence with BER-optimal FM0 coherent detection. Since Miller line coding has memory, maximum likelihood coherent sequence detection is studied, using the Viterbi algorithm. Simulations in Rayleigh fading show that 16-bit sequence detection offers approximately only 1.2 dB advantage compared to symbol-by-symbol at 2 10^{-3} BER. Furthermore, higher lengths offer rather diminishing returns. This finding corroborates famous M. Simon's and D. Divsalar's relevant 2006 conjecture, stating that Miller's sequence detection with "Viterbi would not asymptotically recover a 3 dB advantage". Future work will study other fading channels, including Rice.

Acknowledgements

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Chapter 1

Introduction And Prior Art

1.1 Introduction

Miller - M can be described as follows:

1. There are two symbols that encode each bit, if for instance the bits 0, 0, 0 are sent then the symbols that will encode these bits are S_1, S_2, S_1. There cannot be sent consecutive S_1 due to the concept of memory. In the following scheme this example is shown and afterwards these symbols will fully be analyzed. Furthermore, this coding is quite different from the usuals. The transitions are not done at the end of each symbol that encodes each bit but at the beginning. For example, when bit 1 follows bit 0 then the waveform at its beginning remains at -1. This happens only in this case (from bit 0 to bit 1). In any other case for instance if bit 0 follows bit 0, meaning from symbol S_1 to S_2 or from S_2 to S_1, a transition is observed at the beginning of the waveform from 1 to -1 or vice versa which make sence as symbol $S_2 = -S_1$ and it will be shown later.

2. Symbols S_1 and S_2 are used to encode bit 0. In the middle of each waveform of symbol S_1 or S_2 a transition from -1 to 1 or vise versa is occurred. On the contrary, symbols S_3 and S_4 are used to encode bit 1 and in the middle of its waveform level -1 remains -1 for symbol S_3 while level 1 remains 1 for symbol S_4.

These are the basic rules used to construct Miller coding.

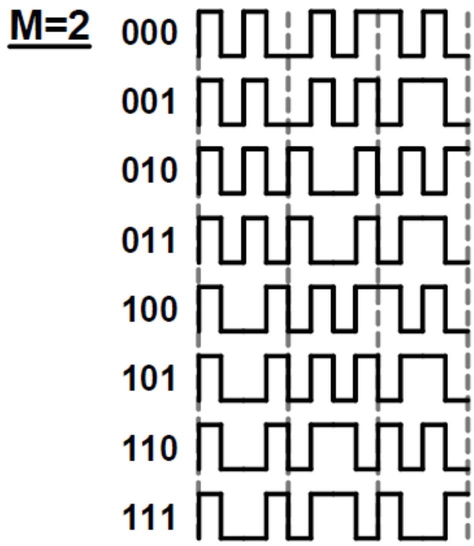

Figure 1.1: Miller-2 from [4]

For Miller line coding the symbols one can observe are the following:

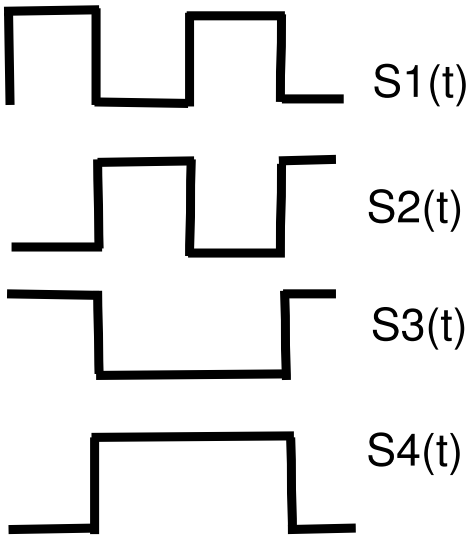

Figure 1.2: Symbols For Miller-2

Symbols $S1(t)$, $S2(t)$ are used to encode bit '0'.

Also, $S3(t)$ and $S4(t)$ are used to encode bit '1'.
These symbols will be used in the Gram-Schmidt process to find out how many bases vectors exist. These bases are used to determine the matched filters in the detector as will be shown later. In this thesis, a receiver was applied based on the EPC protocol for RFID systems. Using Rayleigh Fading channel with ML symbol by symbol detector for Miller-2 line coding it was observed that the results are exactly the same as those from optimal FMO line coding. Furthermore, Marvin Simon postulated that applying an algorithm with memory, like Viterbi, an improvement more than 3-dB cannot be achieved. Indeed, in this thesis applying the Viterbi Algorithm, an improvement of 1.2-dB was achieved.

1.2 Prior Art on FM0 and Miller-M line coding

Work in level [11] explored the noncoherent reception and signal processing schemes for the EPC Gen2 Protocol. Three synchronization schemes were compared for two different encodings and they concluded that the cross-correlation synchronization method worked better than the energy synchronization for all SNRs.

Work in level [1] focused on a complete, fully-coherent, full-duplex Gen2 reader for RFID tags with FMO line coding, utilizing a single transceiver board on a commodity USRP2 (N200) software-defined radio; this work targeted coherent, linear processing at the reader for RFID tags with optimal exploitation at the detection level of line coding-induced memory and channel estimation for coherent processing.

Also, work in level [10] proved the theoretical BER for Miller-2 line coding which has the same BER with coherent 2T FMO line coding. Furthermore, it postulated that this BER could be further improved with a Viterbi algorithm.

1.3 Thesis Outline

Chapter 2: In this chapter, the continuous time model of the received signal at the reader is described. Then, the discrete time signal processing is made.
Chapter 3: Channel estimation is described and the necessary proofs are given. Maximum Likelihood Estimation (MLE) was used in the calculations which is a method of estimating the parameters of a statistical model given observations, by finding the parameters values that maximize the likelihood of making the observations given the parameters. Furthermore, the least-squares method was used to find the optimal parameter values by minimizing the sum of squared residuals.
Chapter 4: The ML symbol by symbol detector as well as the Viterbi algorithm are described. Also, the Bit Error Rate (BER) for Additive White Gaussian Noise (AWGN) channel is proved.
Finally, in Chapter 5 the numerical results of this work are provided.

Chapter 2

System Model, QPSK modulation for Miller-M

2.1 System Model

During uplink (tag to reader) communication the reader transmits a carrier wave (CW) and the RFID tag modulates its information by switching its antenna load between two stages. In that way tag information is binary modulated on the reflection coefficient changes. The reader receives the superposition of its own transmitted signal and the tag's backscattered signal. The complex baseband equivalent of the received signal at the reader is given by [2, Eqs. (26), (33)]:

$$y(t) = [m_{dc} + m_{mod}\, x(t)]e^{j2\pi\Delta_f t} + n(t) \tag{2.1}$$

where the DC component $m_{dc} \in \mathcal{C}$ is due to the CW and an unmodulated component scattered back by the tag; the modulated $m_{mod} \in \mathcal{C}$ component depends on the channel coefficients of the reader transmitting antenna-to-tag and tag-to-antenna receiving antenna links, the tag antenna reflection coefficients, the tag scattering efficiency and the carrier transmitting power; $x(t) \in (\pm 1)$ is a binary real valued tag scattered waveform and Δ_f is the carrier frequency offset (CFO) between CW transmission and the reader reception chain(e.g. CW transmitter and receiver could be dislocated or they could employ different oscillators);
Finally, $n(t)$ is the circularly symmetric white Gaussian noise ($n(t_j) \perp n(t_i)$ for $t_j \neq t_i$.)

2.2 Reader SDR Processing

Assuming coherent detection and symbol synchronization, reader can perfectly estimate and subtract m_{dc} from the received waveform (in Eq. (2.1) m_{dc} is removed, assuming estimation of Δ_f, multiply with $e^{-j2\pi\Delta_f t}$ and set $m_{mod} = h$), then the received digitized signal from Eq (2.1) is expressed as:

$$y[k] = y(kT_s) = hx[k] + n[k], x[k] = \sum_{n=0}^{N-1} S_{d(n)}[k - nL - \tau] \tag{2.2}$$

where $n[k] = n(kT_s) \sim \mathcal{CN}(0, 2\sigma_n^2)$, τ is the synchronization delay due to tag internal delay and tag-reader propagation delay, T denotes the nominal bit duration, T_s sampling period, N is the total number of the bits, $L = \frac{T}{T_s}$ the oversampling factor which shows samples per bit and $S_{d(n)}$ can be selected between the following waveforms where $d(n) \in \{1, 2, 3, 4\}$:

$$S_1[k] = \begin{cases} 1 & \text{when } 0 \leq k < \frac{L}{4} \\ \\ -1 & \text{when } \frac{L}{4} \leq k < \frac{L}{2} \\ \\ 1 & \text{when } \frac{L}{2} \leq k < \frac{3L}{4} \\ \\ -1 & \text{when } \frac{3L}{4} \leq k < L \end{cases} \quad S_2[k] = \begin{cases} -1 & \text{when } 0 \leq k < \frac{L}{4} \\ \\ 1 & \text{when } \frac{L}{4} \leq k < \frac{L}{2} \\ \\ -1 & \text{when } \frac{L}{2} \leq k < \frac{3L}{4} \\ \\ 1 & \text{when } \frac{3L}{4} \leq k < L \end{cases} \tag{2.3}$$

$$S_3[k] = \begin{cases} 1 & \text{when } 0 \leq k < \frac{L}{4} \\ \\ -1 & \text{when } \frac{L}{4} \leq k < \frac{L}{2} \\ \\ -1 & \text{when } \frac{L}{2} \leq k < \frac{3L}{4} \\ \\ 1 & \text{when } \frac{3L}{4} \leq k < L \end{cases} \qquad S_4[k] = \begin{cases} -1 & \text{when } 0 \leq k < \frac{L}{4} \\ \\ 1 & \text{when } \frac{L}{4} \leq k < \frac{L}{2} \\ \\ 1 & \text{when } \frac{L}{2} \leq k < \frac{3L}{4} \\ \\ -1 & \text{when } \frac{3L}{4} \leq k < L \end{cases} \tag{2.4}$$

Definition of $\mathcal{CN}(0, \sigma^2)$
If $\mathcal{X}$ is circular gaussian with variance σ^2 then:

$$\begin{aligned} f_{\mathcal{X}}(x) &= f_{\mathcal{X}_r,\mathcal{X}_i}(x_r, x_i) \\ &= f_{\mathcal{X}_r}(x_r) f_{\mathcal{X}_i}(x_i) \\ &= \frac{1}{\sqrt{2\pi\frac{\sigma^2}{2}}} e^{-\frac{x_r^2}{\sigma^2}} \frac{1}{\sqrt{2\pi\frac{\sigma^2}{2}}} e^{-\frac{x_i^2}{\sigma^2}} \\ &= \frac{1}{\pi\sigma^2} e^{-\frac{x_r^2 + x_i^2}{\sigma^2}} \\ &= \frac{1}{\pi\sigma^2} e^{-\frac{|x|^2}{\sigma^2}} \end{aligned}$$

where $x = x_r + jx_i$.
Consequently, the received signal after matched filtering with a square pulse of length $\frac{L}{4}$ samples and from Eq. (2.2) where $n = 0$ and $\tau = 0$ can be written as:

$$y_0 = \sum_{k=0}^{\frac{L}{4}-1} y[k] = \sum_{k=0}^{\frac{L}{4}-1} hx[k] + \sum_{k=0}^{\frac{L}{4}-1} n[k] = h'x_0 + n'_0 \tag{2.5}$$

$$y_1 = \sum_{k=\frac{L}{4}}^{\frac{L}{2}-1} y[k] = \sum_{k=\frac{L}{4}}^{\frac{L}{2}-1} hx[k] + \sum_{k=\frac{L}{4}}^{\frac{L}{2}-1} n[k] = h'x_1 + n'_1$$

$$y_2 = \sum_{k=\frac{L}{2}}^{\frac{3L}{4}-1} y[k] = \sum_{k=\frac{L}{2}}^{\frac{3L}{4}-1} hx[k] + \sum_{k=\frac{L}{2}}^{\frac{3L}{4}-1} n[k] = h'x_2 + n'_2$$

$$y_3 = \sum_{k=\frac{3L}{4}}^{L-1} y[k] = \sum_{k=\frac{3L}{4}}^{L-1} hx[k] + \sum_{k=\frac{3L}{4}}^{L-1} n[k] = h'x_3 + n'_3$$

$$\bar{y} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} h' + \begin{bmatrix} n'_0 \\ n'_1 \\ n'_2 \\ n'_3 \end{bmatrix} \tag{2.6}$$

where $n'_0 \perp n'_1 \perp n'_2 \perp n'_3$, $n'_i \sim CN(0, \frac{L}{2}\sigma_n^2)$, $h' = h\frac{L}{4}$ and $x_i \in \{\pm 1\}$, $i \in \{0, 1, 2, 3\}$.
Definition vectors:

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right\}$$

Gram-Schmidt process was used to determine the bases vectors in the signal space of Miller-2. Given a set of signals of limited energy $\{\tilde{S}_1(t), \tilde{S}_2(t), \tilde{S}_3(t), \tilde{S}_4(t)\}$, an orthonormal set of bases vectors $\bar{\phi}_1(t), \bar{\phi}_2(t)$ is produced as follows:

1. Set $g_1(t) = \tilde{S}_1(t)$ and the base vector $\bar{\phi}_1(t) = g_1(t)/||g_1(t)||$
2. Set $g_2(t) = \tilde{S}_3(t) - < \tilde{S}_3(t), \bar{\phi}_1(t) > \bar{\phi}_1(t)$ and the base vector $\bar{\phi}_2(t) = g_2(t)/||g_2(t)||$
3. Set $g_3(t) = \tilde{S}_4(t) - < \tilde{S}_4(t), \bar{\phi}_1(t) > \bar{\phi}_1(t) - < \tilde{S}_4(t), \bar{\phi}_2(t) > \bar{\phi}_2(t)$ and this result will be equal to 0.
4. Set $g_4(t) = \tilde{S}_2(t) - < \tilde{S}_2(t), \bar{\phi}_1(t) > \bar{\phi}_1(t) - < \tilde{S}_2(t), \bar{\phi}_2(t) > \bar{\phi}_2(t)$ and

this result will be equal to 0.

5. If there was a third base vector one of $g_3(t), g_4(t)$ would not be equal to 0. So, it is determined that there are 2 bases vectors $\bar{\phi}_1(t), \bar{\phi}_2(t)$.

Applying the above process in this thesis, these results follow:

$$\bar{S}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\bar{\phi}_1 = \frac{\bar{S}_1}{|\bar{S}_1|} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\bar{S}_2 = -\bar{S}_1 \ ,$$

$$\bar{S}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\bar{\phi}_2 = \bar{S}_3 - < \bar{S}_3, \bar{\phi}_1 > \bar{\phi}_1$$

$$< \bar{S}_3, \bar{\phi}_1 > = [1 - 1 - 1 \ 1]\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = 0$$

$$\bar{\phi}_2 = \frac{\bar{S}_3}{|\bar{S}_3|} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\bar{S}_4 = -\bar{S}_3$$

From the above, it is understood that $\bar{S}_1$ and $\bar{S}_2$ are parallel vectors so they have the same base vector $\bar{\phi}_1$. Likewise, for $\bar{S}_3$ and $\bar{S}_4$ which have the same base vector $\bar{\phi}_2$.

The modulation is Quadrature Phase Shift Keying (QPSK). In this modulation, the information is carried in the phase of sinusoidal waveforms of the same frequency and amplitude thus same energy. For the transmission four waveforms are used, each one carries two bits modulating the phase of a carrier with frequency f_c and amplitude $\sqrt{\frac{2E_s}{T}}$ where E_s is the energy of the waveforms and T is the symbol duration. The signal space is two dimensional and each symbol is described by the following coordinates $\{\pm\sqrt{E_s}, \pm\sqrt{E_s}\}$.

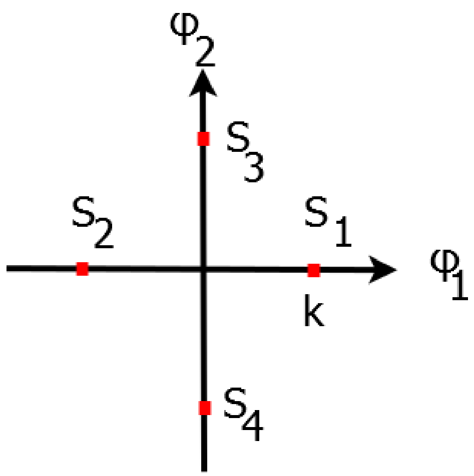

Figure 2.1: Miller-M Constellation Diagram

From these 2 bases vectors, 2 matched filters will be produced:

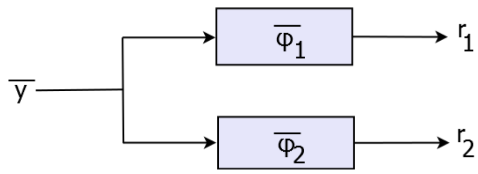

Figure 2.2: Matched Filters

The r_1,r_2 from the Matched filters are used in the ML symbol by symbol detector. The r_1 is produced from the inner product $< \bar{y}, \bar{\phi}_1 >$ where $\bar{y}$ is defined in Eq. (2.5). Likewise, for r_2 which is produced from the inner product $< \bar{y}, \bar{\phi}_2 >$.

$$
\begin{aligned}
< \bar{y}, \bar{\phi}_1 > &= \bar{\phi}_1{}^T \bar{y} \\
&= \frac{1}{2}[1 \ -1 \ \ 1 \ -1] \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \\
&= \frac{y_0 - y_1 + y_2 - y_3}{2} \\
&= r_1
\end{aligned}
$$

$$\begin{aligned}
<\bar{y}, \bar{\phi_2}> &= \bar{\phi_2}^T \bar{y} \\
&= \frac{1}{2}[1 \ -1 \ -1 \ \ 1] \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \\
&= \frac{y_0 - y_1 - y_2 + y_3}{2} \\
&= r_2
\end{aligned}$$

Furthermore, $r_1 = \bar{\phi}_1^T \bar{y}$ with $\bar{y}$ from Eq. (2.6):

$$\begin{aligned}
r_1 &= \bar{\phi}_1^T \bar{y} \\
&= \bar{\phi}_1^T \left(h' \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} n'_0 \\ n'_1 \\ n'_2 \\ n'_3 \end{bmatrix} \right) \\
&= h' \bar{\phi}_1^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \bar{\phi}_1^T \begin{bmatrix} n'_0 \\ n'_1 \\ n'_2 \\ n'_3 \end{bmatrix} \\
&= h' q_1 + w_1
\end{aligned}$$

The calculation for q_1 for each Definition vector is:

$$\bar{\phi}_1^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \bar{\phi}_1^T \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \frac{1}{2}[1 \ -1 \ \ 1 \ -1] \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = 2$$

$$\bar{\phi}_1^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \bar{\phi}_1^T \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{2}[1 \ -1 \ \ 1 \ -1] \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = 0$$

$$\bar{\phi}_1^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \bar{\phi}_1^T \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{2}[1 \ -1 \ \ 1 \ -1] \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = -2$$

$$\bar{\phi}_1^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \bar{\phi}_1^T \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \frac{1}{2}[1 \ -1 \ \ 1 \ -1] \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = 0$$

For the specific $\bar{\phi}_1$, $q_1 \in \{0, \pm 2\}$.

Also, $w_1 \sim \mathcal{CN}(0, \sigma_1^2)$ with variance equal to:

$$\begin{aligned}
\sigma_1^2 &= E[w_1^* w_1] = E[w_1 w_1^*] \\
&= \bar{\phi}_1^T E[\begin{bmatrix} n'_0 \\ n'_1 \\ n'_2 \\ n'_3 \end{bmatrix} [n_0^* n_1^* n_2^* n_3^*]]\bar{\phi}_1 \\
&= \bar{\phi}_1^T \frac{L}{2}\sigma_n^2 I_4 \bar{\phi}_1 \\
&= \frac{L}{2}\sigma_n^2 \bar{\phi}_1^T \bar{\phi}_1 \\
&= \frac{L}{2}\sigma_n^2 |\phi_1|^2 \\
&= \frac{L}{2}\sigma_n^2
\end{aligned}$$

For $r_2 = \bar{\phi}_2^T \bar{y}$ the calculations with the above procedure end up in:

$$r_2 = h' q_2 + w_2$$

with $q_2 \in \{0, \pm 2\}$, $w_2 \sim \mathcal{CN}(0, \sigma_2^2)$ and $\sigma_2^2 = \frac{L}{2}\sigma_n^2$.

Now each Miller symbol can be written as 2x1 complex vector:

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = h' k \bar{q}' + \bar{w} = h' k \begin{bmatrix} q'_1 \\ q'_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{2.7}$$

where $h' = h\frac{L}{4}$, $k = 2$ for Miller-2, $\bar{w} \sim \mathcal{CN}(\bar{0}, \frac{L}{2}\sigma_n^2 I_2)$, $q_i = k q'_i, (i \in \{1, 2\})$ and $\bar{q}'$ is one of the following:

$$\bar{q}' \in \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\} \tag{2.8}$$

$$\bar{q}' \in \left\{ \bar{q}_H^{(0)}, \bar{q}_L^{(0)}, \bar{q}_H^{(1)}, \bar{q}_L^{(1)} \right\} \tag{2.9}$$

The tag signal to noise ratio (SNR) for Miller-2 is defined as:

$$\begin{aligned} SNR &= \frac{|h'k|^2|\bar{q'}|^2}{\frac{L}{2}\sigma_n^2} \\ &= \frac{|h\frac{L}{4}2|^2}{\frac{L}{2}\sigma_n^2} \\ &= \frac{|h|^2\frac{L^2}{4}}{\frac{L}{2}\sigma_n^2} \\ &= \frac{|h|^2 L}{2\sigma_n^2} \end{aligned}$$

For Miller-4 the equations are the following:

$$\bar{y} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} h' + \begin{bmatrix} n'_0 \\ n'_1 \\ n'_2 \\ n'_3 \\ n'_4 \\ n'_5 \\ n'_6 \\ n'_7 \end{bmatrix} \tag{2.10}$$

Definition vectors:

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \in \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right\}$$

with $n'_i \sim \mathcal{CN}(0, \frac{L}{4}\sigma_n^2)$ and $h' = h\frac{L}{8}$.
From Gram-Schmidt process the bases are:

$$\bar{\phi}_1 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}^T$$
$$\bar{\phi}_2 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \end{bmatrix}^T$$

The scheme for Matched filters remains the same. Now, the filters are equal to:

$$r_1 = \frac{1}{2\sqrt{2}}[y_0 - y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7]$$
$$r_2 = \frac{1}{2\sqrt{2}}[y_0 - y_1 + y_2 - y_3 - y_4 + y_5 - y_6 + y_7]$$

Also, either in Miller-2 or Miller-4 the Eq. (2.7) remains the same but, for Miller-4 $k = 2\sqrt{2}$, $\bar{w} \sim \mathcal{CN}(\bar{0}, \frac{L}{4}\sigma_n^2 I_2)$.

The tag signal to noise ratio (SNR) for Miller-4 is defined as:

$$\begin{aligned} SNR &= \frac{|h'\bar{q}'k|^2}{\frac{L}{4}\sigma_n^2} \\ &= \frac{|h|^2(\frac{L}{8})^2 8}{\frac{L}{4}\sigma_n^2} \\ &= \frac{|h|^2 L}{2\sigma_n^2} \end{aligned}$$

For Miller-8 the equations are the following:

$$\bar{y} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix} h' + \begin{bmatrix} n'_0 \\ n'_1 \\ n'_2 \\ n'_3 \\ n'_4 \\ n'_5 \\ n'_6 \\ n'_7 \\ n'_8 \\ n'_9 \\ n'_{10} \\ n'_{11} \\ n'_{12} \\ n'_{13} \\ n'_{14} \\ n'_{15} \end{bmatrix} \tag{2.11}$$

Definition vectors:

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix} \in \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right\}$$

with $n'_i \sim \mathcal{CN}(0, \frac{L}{8}\sigma_n^2)$ and $h' = h\frac{L}{16}$.
From Gram-Schmidt process the bases are:

$$\bar{\phi}_1 = \frac{1}{4}\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}^T$$

$$\bar{\phi}_2 = \frac{1}{4}\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}^T$$

The scheme for Matched filters remains the same. Now, the filters are equal to:

$$r_1 = \frac{1}{4}[y_0 - y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7 + y_8 - y_9 + y_{10} - y_{11} + y_{12} - y_{13} + y_{14} - y_{15}]$$

$$r_2 = \frac{1}{4}[y_0 - y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7 - y_8 + y_9 - y_{10} + y_{11} - y_{12} + y_{13} - y_{14} + y_{15}]$$

Also, either in Miller-2 or Miller-8 the Eq. (2.7) remains the same but, for Miller-8 $k = 4$, $\bar{w} \sim \mathcal{CN}(\bar{0}, \frac{L}{8}\sigma_n^2 I_2)$.

The tag signal to noise ratio (SNR) for Miller-8 is defined as:

$$\begin{aligned} SNR &= \frac{|h'|^2 k^2 |q'|^2}{\frac{L}{8}\sigma_n^2} \\ &= \frac{|h|^2 (\frac{L}{16})^2 16}{\frac{L}{8}\sigma_n^2} \\ &= \frac{|h|^2 L}{2\sigma_n^2} \end{aligned}$$

It is concluded that signal noise ratio (SNR) is the same for all Miller codes, 2, 4, 8, as it should.

Chapter 3

Channel Estimation

3.1 Channel Estimation

A Gen2 tag that uses Miller line coding transmits a known (real) sequence (preamble) s_p before sending information bits. At first, frame synchronization is performed and the delay τ is estimated by correlating the received signal with the known preamble. The reader can search for a suitable τ in a small interval i.e.,$\{0, ..., L'\}$ where $L' = L = \frac{T}{T_s}$.

$$\tau^* = \underset{\tau \in \{0,...,L\}}{\operatorname{argmax}} \left| \sum_{n=0}^{N_p-1} s_p[n] y[\tau + n] \right| \tag{3.1}$$

Where N_p is the number of samples in the preamble. The unknown parameter h can then be estimated by solving a least squares problem:

$$h' = \frac{\sum_{k=\tau^*}^{\tau^*+N_p-1} y[k] s_p[k - \tau^*]}{|s_p|^2} \tag{3.2}$$

3.2 Proofs

The proof for Eq. (3.1) is:

$$
\begin{aligned}
y[k] &= hx[k] + n[k]\ ,\ n[k] \sim \mathcal{CN}(0, 2\sigma_n^2) \\
x[k] &= x(kT_s) = \sum_{n=0}^{N-1} S_{d(n)}(t - nT - \tilde{\tau})|_{t=kT_s} \\
&= \sum_{n=0}^{N-1} S_{d(n)}(kT_s - nT - \tau T_s) \\
&= \sum_{n=0}^{N-1} S_{d(n)}((k-\tau)T_s - nT) \\
&= S[k - \tau]
\end{aligned}
$$

Where $S[k] = \sum_{n=0}^{N-1} S_{d(n)}(t - nT)|_{t=kT_s}$

$$
y[k] = \begin{cases} n[k] & \text{when } 0 \leq k \leq k_0 - 1 \\ S[k - k_0] + n[k] & \text{when } k_0 \leq k \leq k_0 + M - 1 \end{cases}
$$

where M is the number of samples of the preamble.

$$
\begin{aligned}
&\prod_{k=0}^{k_0-1} \frac{1}{(\pi\sigma^2)^2} e^{-\frac{|y[k]|^2}{\sigma^2}} \prod_{k_0}^{k_0+M-1} \frac{1}{(\pi\sigma^2)^2} e^{-\frac{|y[k]-S[k-k_0]|^2}{\sigma^2}} \\
&= \frac{1}{((\pi\sigma^2)^2)^M} e^{-\frac{\sum_{k=0}^{N-1} |y[k]|^2}{\sigma^2}} \prod_{k_0}^{k_0+M-1} e^{-\frac{-y[k]S^*(k-k_0) - y^*[k]S[k-k_0] + |S(k-k_0)|^2}{\sigma^2}}
\end{aligned}
$$

with the following:

$$
\begin{aligned}
&(y[k] - S[k-k_0])(y[k]^* - S^*[k-k_0]) \\
&= |y[k]|^2 - y[k]S^*[k-k_0] - y[k]^*S[k-k_0] + |S[k-k_0]|^2
\end{aligned}
$$

with MLE for k_0 you either get the:

$$\underset{k_0}{\operatorname{argmax}} \, e^{-\frac{1}{\sigma^2}\sum_{k_0}^{k_0+M-1} -y[k]S^*(k-k_0)-y^*[k]S[k-k_0]+|S[k-k_0]|^2}$$

or minimize the:

$$\sum_{k=k_0}^{k_0+M-1} -y[k]S^*[k-k_0] - y^*[k]S[k-k_0] + |S[k-k_0]|^2 \ , \ k_0 \in \{0,1,...,L-1\}$$

but

$$\begin{aligned} &\sum_{k_0}^{k_0+M-1} |S[k-k_0]|^2 \\ &= \sum_{n=0}^{M-1} |S[n]|^2 = const \end{aligned}$$

So we have to minimize

$$\sum_{k=k_0}^{k_0+M-1} -y[k]S^*[k-k_0] - y^*[k]S[k-k_0]$$

or maximize the :

$$\sum_{k=k_0}^{k_0+M-1} +y[k]S^*[k-k_0] + y^*[k]S[k-k_0]$$

If we set $\alpha = y[k]S^*[k-k_0]$ and $\alpha^* = y^*[k]S[k-k_0]$ we have $\sum_{k=k_0}^{k_0+M-1} \alpha+\alpha^* = \sum_{k=k_0}^{k_0+M-1} 2\Re(a) = 2\Re(\sum_{k=k_0}^{k_0+M-1} \alpha)$. We can maximize:

$$2\Re(\sum_{k=k_0}^{k_0+M-1} y[k]S^*(k-k_0)) \tag{3.3}$$

with $|\Re(z)| \leq |z|$, while with $z = \sum_{k=k_0}^{k_0+M-1} y[k]S^*[k-k_0]$ we maximize the following:

$$|\sum_{k_0}^{k_0+M-1} y[k]S^*[k-k_0]| \tag{3.4}$$

$$= |\sum_{k=0}^{M-1} y[k+k_0]S[k]| \tag{3.5}$$

with preamble $S[k-k_0] = S^*[k-k_0]$.
So finally, $k_0 = \text{argmax}_{k_0} |\sum_{k=0}^{M-1} y[k+k_0]S[k]|$

The proof for the Eq. (3.2) is:

$$\begin{aligned}
h &= \underset{h\in C}{\text{argmin}} \sum_{k=\tau^*}^{\tau^*+N_p-1} |y[k] - hS[k-\tau^*]|^2 \\
&= \underset{h\in C}{\text{argmin}} \sum_{k=\tau^*}^{\tau^*+N_p-1} (y[k] - hS[k-\tau^*])(y[k] - hS[k-\tau^*])^* \\
&= \underset{h\in C}{\text{argmin}} \sum_{k=\tau^*}^{\tau^*+N_p-1} (y[k] - hS[k-\tau^*])(y^*[k] - h^*S^*[k-\tau^*]) \\
&= \underset{h\in C}{\text{argmin}} \sum_{k=\tau^*}^{\tau^*+N_p-1} (|y[k]|^2 - h^*y[k]S^*[k-\tau^*] - hS[k-\tau^*]y^*[k] + |h|^2|S[k-\tau^*]|^2) \\
&= \underset{h\in C}{\text{argmin}} \sum_{k=\tau^*}^{\tau^*+N_p-1} (|y[k]|^2 - h^*y[k]S^*[k-\tau^*] - hS[k-\tau^*]y^*[k] + hh^*|S[k-\tau^*]|^2) \\
&= \underset{h\in C}{\text{argmin}} \sum_{k=\tau^*}^{\tau^*+N_p-1} (-h^*y[k]S^*[k-\tau^*] - hS[k-\tau^*]y^*[k] + hh^*|S[k-\tau^*]|^2)
\end{aligned}$$

we set $f(h) = -h^* y[k]S^*[k-\tau^*] - hS[k-\tau^*]y^*[k] + hh^*|S[k-\tau^*]|^2$ and h, h^* as independent $\frac{d}{dh}(hh^*) = h^*, \frac{d}{dh^*}(hh^*) = h$ with $\frac{df(h)}{dh} \neq \frac{df(h)}{dh^*}$.

$$
\begin{aligned}
&= \operatorname*{argmin}_{h \in C} f(h) \\
&=> \frac{df(h)}{dh} = \sum_{k=\tau^*}^{\tau^*+N_p-1} (-S[k-\tau^*]y^*[k] + h^*|S[k-\tau^*]|^2) = 0 \\
&=> h^* \sum_{k=\tau^*}^{\tau^*+N_p-1} |S[k-\tau^*]|^2 = \sum_{k=\tau^*}^{\tau^*+N_p-1} S[k-\tau^*]y^*[k] \\
&=> h^* = \frac{\sum_{k=\tau^*}^{\tau^*+N_p-1} S[k-\tau^*]y^*[k]}{\sum_{k=\tau^*}^{\tau^*+N_p-1N} |S[k-\tau^*]|^2} \\
&=> h = \frac{\sum_{k=\tau^*}^{\tau^*+N_p-1} S^*[k-\tau^*]y[k]}{\sum_{k=\tau^*}^{\tau^*+N_p-1} |S[k-\tau^*]|^2} = \frac{\sum_{k=\tau^*}^{\tau^*+N_p-1} S[k-\tau^*]y[k]}{\sum_{k=\tau^*}^{\tau^*+N_p-1} S^2[k-\tau^*]} \\
&= \frac{\sum_{k=\tau^*}^{\tau^*+N_p-1} S[k-\tau^*]y[k]}{|S|^2}
\end{aligned}
$$

because $S[k-\tau^*]$ is real and N_p is the number of the samples of the preamble S and τ^* is from Eq. (3.1).

Chapter 4

Detection

4.1 ML Symbol By Symbol Detection

With parameter h' estimated and known, the ML symbol by symbol detection rule for system of Eq. (2.7) becomes:

$$\underset{\bar{q}' \in \{\bar{q}_H^{(0)}, \bar{q}_L^{(0)}, \bar{q}_H^{(1)}, \bar{q}_L^{(1)}\}}{\operatorname{argmin}} |\bar{r} - h' k \bar{q}'|^2$$

where $k = 2$ for Miller-2, $k = 2\sqrt{2}$ for Miller-4, $k = 4$ for Miller-8. When $\bar{q}' = \bar{q}_H^{(0)}$ or $\bar{q}_L^{(0)}$ decide bit '0' else if $\bar{q}' = \bar{q}_H^{(1)}$ or $\bar{q}_L^{(1)}$ decide bit '1'.

4.2 BER Proof For AWGN

It is assumed that $BER = 2I_1$ and I_1 will be calculated as:

$$\begin{aligned}
I_1 &= \int_{r_2=0}^{\infty} \int_{r_1=-r_2}^{r_1=r_2} f(r_1, r_2|S_1) dr_1 dr_2 \\
&= \frac{1}{2\pi\sigma^2} \int_{r_2=0}^{\infty} \int_{r_1=-r_2}^{r_1=r_2} e^{-\frac{(r_1-k)^2+(r_2)^2}{2\sigma^2}} dr_1 dr_2 \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{r_2=0}^{\infty} e^{-\frac{r_2^2}{2\sigma^2}} \int_{r_1=-r_2}^{r_1=r_2} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r_1-k)^2}{2\sigma^2}} dr_1 dr_2
\end{aligned}$$

set $t = \frac{r_1-k}{\sigma}$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{r_2=0}^{\infty} e^{-\frac{r_2^2}{2\sigma^2}} \int_{t=\frac{-r_2-k}{\sigma}}^{\frac{r_2-k}{\sigma}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2}} \sigma dt dr_2$$

set $dt = \frac{1}{\sigma} dr_1 => dr_1 = \sigma dt$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{r_2=0}^{\infty} e^{-\frac{r_2^2}{2\sigma^2}} [Q(\frac{-r_2 - k}{\sigma}) - Q(\frac{r_2 - k}{\sigma})] dr_2$$

set $\frac{r_2}{\sigma} = t$

$$\begin{aligned}
&= \frac{1}{\sqrt{2\pi\sigma^2}} \sigma \int_{t=0}^{\infty} e^{-\frac{t^2}{2}} [Q(-t - \frac{k}{\sigma}) - Q(t - \frac{k}{\sigma})] dt \\
&= \frac{1}{\sqrt{2\pi}\sigma} \sigma \int_{t=0}^{\infty} e^{-\frac{t^2}{2}} [(1 - Q(t + \frac{k}{\sigma})) - Q(t - \frac{k}{\sigma})] dt \\
&= \int_{t=0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} [-\frac{1}{\sqrt{2\pi}} Q(t + \frac{k}{\sigma}) - \frac{1}{\sqrt{2\pi}} Q(t - \frac{k}{\sigma})] dt \\
&= \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_{t=0}^{\infty} e^{\frac{-t^2}{2}} [Q(t + \frac{k}{\sigma}) + Q(t - \frac{k}{\sigma})] dt \\
&= \frac{1}{2} + \int_{t=0}^{\infty} Q'(t) [Q(t + \frac{k}{\sigma}) + Q(t - \frac{k}{\sigma})] dt \\
&= \frac{1}{2} + Q(t) [Q(t + \frac{k}{\sigma}) + Q(t - \frac{k}{\sigma})]_0^{\infty} - \int_{t=0}^{\infty} Q(t) [Q(t + \frac{k}{\sigma}) + Q(t - \frac{k}{\sigma})]' dt \\
&= \frac{1}{2} + Q(t) [Q(t + \frac{k}{\sigma}) + Q(t - \frac{k}{\sigma})]_0^{\infty} - \\
&\int_{t=0}^{\infty} Q(t) [-\frac{1}{\sqrt{2\pi}} e^{-\frac{(t + \frac{k}{\sigma})^2}{2}} - \frac{1}{\sqrt{2\pi}} e^{-\frac{(t - \frac{k}{\sigma})^2}{2}}] dt \\
&= \frac{1}{2} - \frac{1}{2} (Q(\frac{k}{\sigma}) + Q(-\frac{k}{\sigma})) + \frac{1}{\sqrt{2\pi}} \int_{t=0}^{\infty} Q(t) [e^{-\frac{(t + \frac{k}{\sigma})^2}{2}} + e^{-\frac{(t - \frac{k}{\sigma})^2}{2}}] dt
\end{aligned}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{t=0}^{\infty} (\frac{1}{2} - \frac{1}{2} erf(\frac{t}{\sqrt{2}})) (e^{-\frac{(t + \frac{k}{\sigma})^2}{2}} + e^{-\frac{(t - \frac{k}{\sigma})^2}{2}}) dt$$

$$\begin{aligned}
&= \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{t=0}^{\infty} e^{-\frac{(t + \frac{k}{\sigma})^2}{2}} dt + \frac{1}{2\sqrt{2\pi}} \int_{t=0}^{\infty} e^{-\frac{(t - \frac{k}{\sigma})^2}{2}} dt \\
&- \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{t=0}^{\infty} erf(\frac{t}{\sqrt{2}}) [e^{-\frac{(t + \frac{k}{\sigma})^2}{2}} + e^{-\frac{(t - \frac{k}{\sigma})^2}{2}}] dt
\end{aligned}$$

According to the first integral set as $t + \frac{k}{\sigma} = x$, the second integral set as $t - \frac{k}{\sigma} = x$ so, the following equation is concluded:

$$= -\frac{1}{2\sqrt{2\pi}}\int_{x=\frac{k}{\sigma}}^{\infty} e^{-\frac{x^2}{2}}dx + \frac{1}{2\sqrt{2\pi}}\int_{x=-\frac{k}{\sigma}}^{\infty} e^{-\frac{x^2}{2}}dx - \frac{1}{2\sqrt{2\pi}}$$
$$\int_{t=0}^{\infty} erf(\frac{t}{\sqrt{2}})[e^{-(\frac{t}{\sqrt{2}}+\frac{k}{\sigma\sqrt{2}})^2} + e^{-(\frac{t}{\sqrt{2}}-\frac{k}{\sigma\sqrt{2}})^2}]dt$$

The third integral is set as $x = \frac{t}{\sqrt{2}}$ so:

$$\begin{aligned}
&= \frac{1}{2}Q(\frac{k}{\sigma}) + \frac{1}{2}Q(-\frac{k}{\sigma}) - \frac{1}{2\sqrt{\pi}}\int_{x=0}^{\infty} erf(x)[e^{-(x+\frac{k}{\sigma\sqrt{2}})^2} + e^{-(x-\frac{k}{\sigma\sqrt{2}})^2}]dx \\
&= \frac{1}{2}Q(\frac{k}{\sigma}) + \frac{1}{2}[1 - Q(\frac{k}{\sigma})] - \frac{1}{2\sqrt{\pi}}\int_{x=0}^{\infty} erf(x)[e^{-(x+\frac{k}{\sigma\sqrt{2}})^2} + e^{-(x-\frac{k}{\sigma\sqrt{2}})^2}]dx \\
&= \frac{1}{2} - \frac{1}{2\sqrt{\pi}}\int_{x=0}^{\infty} erf(x)[e^{-(x+a)^2} + e^{-(x-a)^2}]dx \\
&= \frac{1}{2} - \frac{1}{2\sqrt{\pi}}\frac{\sqrt{\pi}}{2}[1 + erf(\frac{k}{2\sigma})^2] \\
&= \frac{1}{2} - \frac{1}{4}[1 + erf(\frac{k}{2\sigma})^2] \\
&= \frac{1}{4} - \frac{1}{4}(erf(\frac{k}{2\sigma}))^2 \\
&= \frac{1}{4}[1 - (erf(\frac{k}{2\sigma}))^2] \\
&= \frac{1}{4}[1 - erf(\frac{k}{2\sigma}][1 + erf(\frac{k}{2\sigma}] \\
&= \frac{1}{2}[1 - erf(\frac{k}{s\sqrt{2}\sqrt{2}})]\frac{1}{2}[1 + erf(\frac{k}{s\sqrt{2}\sqrt{2}}) \\
&= Q(\frac{k}{\sigma\sqrt{2}})[1 - Q(\frac{k}{\sigma\sqrt{2}})] \\
&= Q(\sqrt{SNR})(1 - Q(\sqrt{SNR})) = I_1
\end{aligned}$$

But, $BER = 2I_1$ so it is proved that:

$$BER = 2Q(\sqrt{SNR})(1 - Q(\sqrt{SNR}))$$

Useful Remarks:

1. $\frac{1}{2} = \int_{t=0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$
2. $Q'(x) = -\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
3. $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$
4. $Q(x) = \frac{1}{2} - \frac{1}{2} erf(\frac{x}{\sqrt{2}})$
5. $Q(-x) = 1 - Q(x)$
6. $SNR = \frac{k^2}{2\sigma^2} => \sqrt{SNR} = \frac{k}{\sigma\sqrt{2}}$
7. $\frac{\sqrt{\pi}}{2}(1 + (erf(\frac{a}{\sqrt{2}}))^2) = \int_{x=0}^{\infty} [e^{-(x-a)^2} + e^{-(x+a)^2}] erf(x) dx$, Eq. (38) in [9]

4.3 Viterbi Algorithm

4.3.1 Introduction

Miller-2 line coding has memory, which means that the ML symbol by symbol detector is not optimal as it only sees the current symbol. Instead, the Viterbi algorithm with sequence length of more than one bit, exploits the memory of this code. Specifically, the algorithm finds the path from the beginning of the trellis-diagram of Miller-2 code to the end of it which maximizes a specific criterion. The basic idea is that beginning from the left to the right of the trellis-diagram (Forward-Pass) the transitions to a common state in any step, should maximize the total weight until this state. This means that, if there are two states $\bar{q'}_n^{(1)}$,$\bar{q'}_n^{(2)}$ with total weight until these states $\Gamma(\bar{q'}_n^{(1)})$, $\Gamma(\bar{q'}_n^{(2)})$ then the transition $(\bar{q'}_n^{(1)},\ \bar{q'}_{n+1})$ with weight $w(\bar{q'}_n^{(1)},\ \bar{q'}_{n+1})$ is preferred to the transition$(\bar{q'}_n^{(2)},\ \bar{q'}_{n+1})$ with weight $w(\bar{q'}_n^{(2)},$ $\bar{q'}_{n+1})$ if the maximization criterion is the maximum additive cost, meaning $\Gamma(\bar{q'}_n^{(1)}) + w(\bar{q'}_n^{(1)}, \bar{q'}_{n+1}) > \Gamma(\bar{q'}_n^{(2)}) + w(\bar{q'}_n^{(2)}, \bar{q'}_{n+1})$. The total weight until the step $n+1$ is updated by adding the weight of the transition that was chosen. Consequently, each state in every step has a total weight and a pointer to the state of the previous step that has been chosen.

The Hidden Markov Model for this algorithm is the following where $\bar{q'}_n, \bar{q'}_{n+1}$ are the hidden states and $\bar{r}_n, \bar{r}_{n+1}$ are the observations.

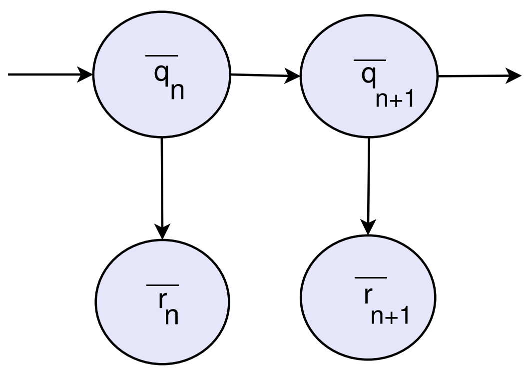

Figure 4.1: Hidden Markov Model

4.3.2 The Algorithm

The trellis-diagram is indicated in the following scheme:

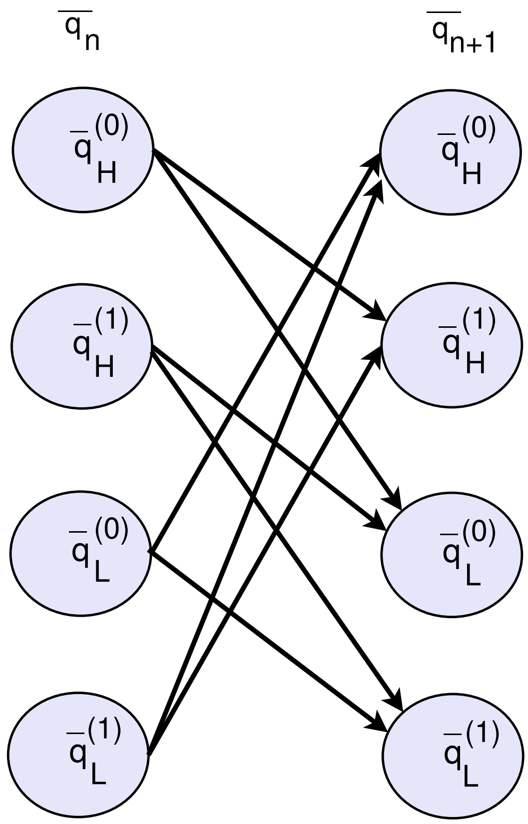

Figure 4.2: Trellis Diagram

The initial distribution of the symbols is $P(\bar{q'}_n) = \frac{1}{4}$. The distribution of receiving $\bar{r}_n = h'2\bar{q'}_n + \bar{w}_n$, if the symbol which is represented by $\bar{q'}_n$ was sent, is the following:

$$f(\bar{r_n}|\bar{q'_n}) \equiv \mathcal{CN}\left(h'2\bar{q'_n}, \frac{L}{2}\sigma_n^2 I_2\right) \propto e^{\frac{||\bar{r_n}-h'2\bar{q'_n}||^2}{\frac{L}{2}\sigma_n^2}}$$

where $\bar{q'_n}$ is in Eq. (2.8) and Eq. (2.9). Also, $\bar{r_n}$ has been described in Eq. (2.7).

The weight for each transition from one state to the other is calculated from this formula:

$$ln f(\bar{r}_n|\bar{q'}_n) \propto -\frac{2}{L\sigma_n^2}||\bar{r}_n - h'2\bar{q'}_n||^2 \equiv w(\bar{r}_n, \bar{q'}_n)$$

Algorithm's steps:

1. The initial total cost to each state is calculated as: $\Gamma(\bar{q'}_n) = ln(\frac{1}{4}) + w(\bar{r_1}, \bar{q'}_n)$

2. In this step the optimal cost must be calculated to reach each state. For example the procedure to reach $\bar{q}_H^{(0)}$ state is as follows:
   From state $\bar{q}_L^{(0)}$ to $\bar{q}_H^{(0)}$ the total cost is:

   $$temp_2 = \Gamma(\bar{q}_L^{(0)}) + ln(P(\bar{q}_H^{(0)}|\bar{q}_L^{(0)})) + w(\bar{r_2}, \bar{q}_H^{(0)}) \tag{4.1}$$

   with $P(\bar{q}_H^{(0)}|\bar{q}_L^{(0)}) = \frac{1}{2}$ as from each state two transitions are possible with equal probability.
   Also, the total cost from $\bar{q}_L^{(1)}$ to $\bar{q}_H^{(0)}$ is:

   $$temp_3 = \Gamma(\bar{q}_L^{(1)}) + ln(P(\bar{q}_H^{(0)}|\bar{q}_L^{(1)})) + w(\bar{r_2}, \bar{q}_H^{(0)}) \tag{4.2}$$

   Finally, the total cost to reach the state $\bar{q}_H^{(0)}$ is updated as:

   $$\Gamma(\bar{q}_H^{(0)}) = \max_{i\in\{2,3\}} temp_i \tag{4.3}$$

   As well, a pointer is kept to show which previous state lead to the current state $\bar{q}_H^{(0)}$.

Repeating the steps (4.1), (4.2), (4.3) the total costs: $\Gamma(\bar{q}_H^{(1)}), \Gamma(\bar{q}_L^{(1)}), \Gamma(\bar{q}_L^{(0)})$ are updated considering the transitions which lead to these states as shown on the trellis-diagram.

3. Step 2 is repeated for the next observations $\bar{r}_3, ..., \bar{r}_N$.

4. In the final step the decision for which $\bar{q'}_n$ was sent is taken by:

$$\underset{\bar{q'} \in \{\bar{q}_H^{(0)}, \bar{q}_L^{(0)}, \bar{q}_H^{(1)}, \bar{q}_L^{(1)}\}}{\operatorname{argmax}} \Gamma(\bar{q'})$$

By following the pointers with backtracking, the exact path is well known.

Chapter 5

Numerical Results

5.1 Simulation Results

At first, a comparison of ML symbol by symbol Miller-2 detector with FM0 coherent 2T is presented below. In the simulation, it is proven that they coincide. The iterations of this simulation were 15000.

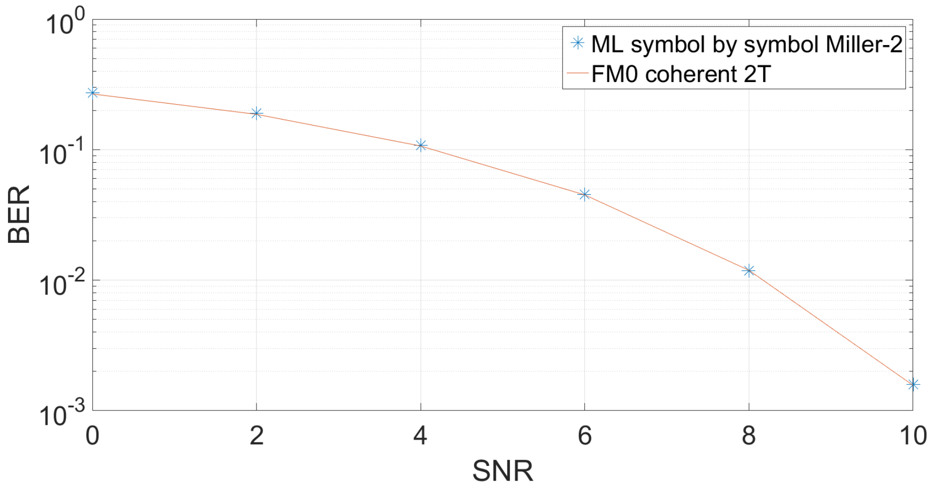

Figure 5.1: ML Symbol By Symbol Miller-2 VS FM0 Coherent 2T

The theoretical BER for the AWGN channel is given by the following formula as shown by Marvin Simon [10]:

$$BER = 2Q(\sqrt{SNR})(1 - Q(\sqrt{SNR}))$$

The plot for this formula is the following and it is observed that it coincides with the previous figure (5.1) as it should.

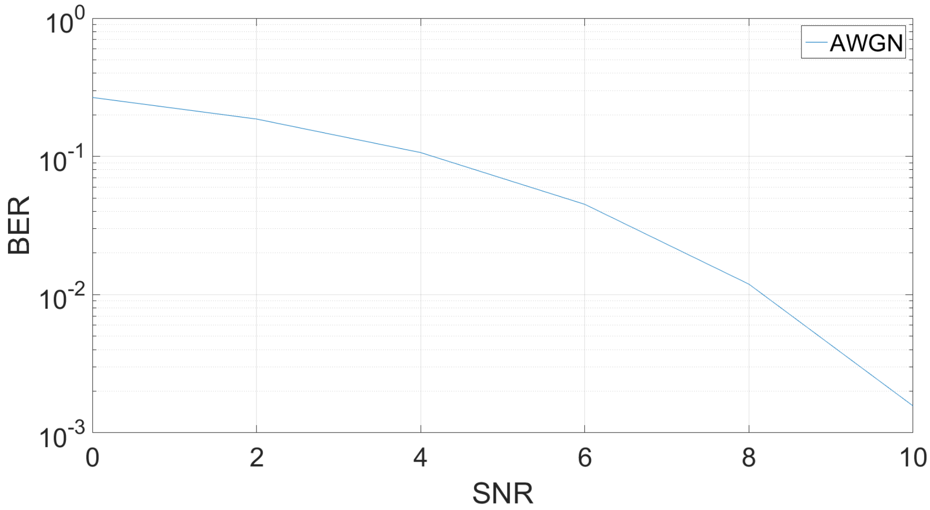

Figure 5.2: AWGN Channel

Furthermore, the Viterbi algorithm is presented for a sequence length of 1 bit ($n = 1$) with perfect channel estimation and is compared with the ML symbol by symbol detector for Miller-2 line coding. These two results coincide because Viterbi detector is the same as the ML. The iterations that were used for this result were 70000 and the plot which represents them is the following:

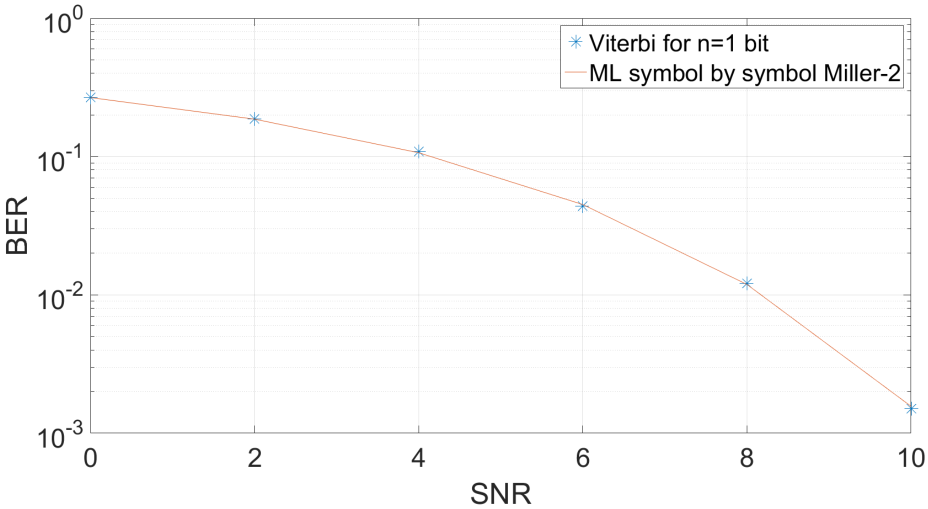

Figure 5.3: Viterbi Detector For A Single Bit VS ML symbol by symbol Detector

Viterbi algorithm for sequence length of $n = 1, 2, 4, 8, 16, 32, 120$ with perfect channel estimation is shown in the following diagram. For $SNR = 0 : 8$, 300000 iterations were used while, for $SNR = 10$, 500000 iterations were used. It is shown that the BER improves as the sequence of bits increases which is a characteristic of an algorithm with memory.

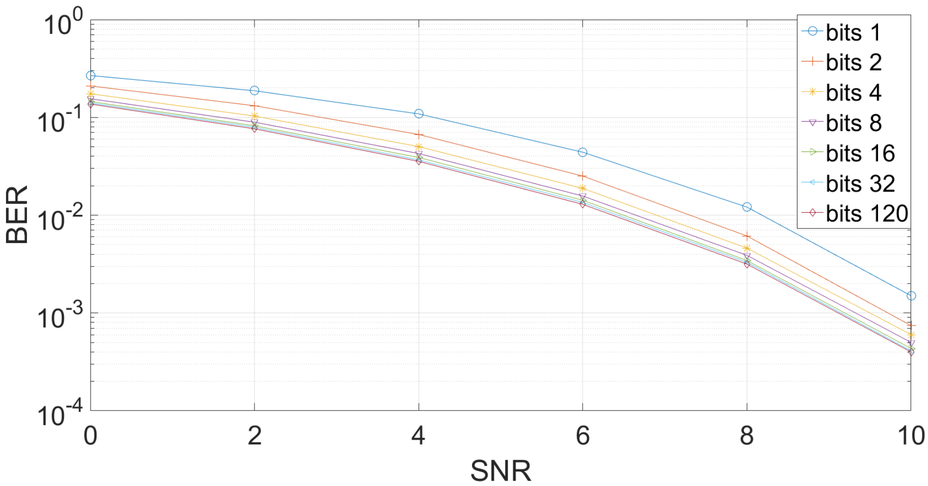

Figure 5.4: Viterbi

For the Viterbi algorithm , $n = 1$ with BER 2 10^{-3} the SNR is equal to 9.724. Also, for $n = 16$ at the same BER, the SNR is equal to 8.523. So, it is observed that 1.2 dB improvement has been achieved.

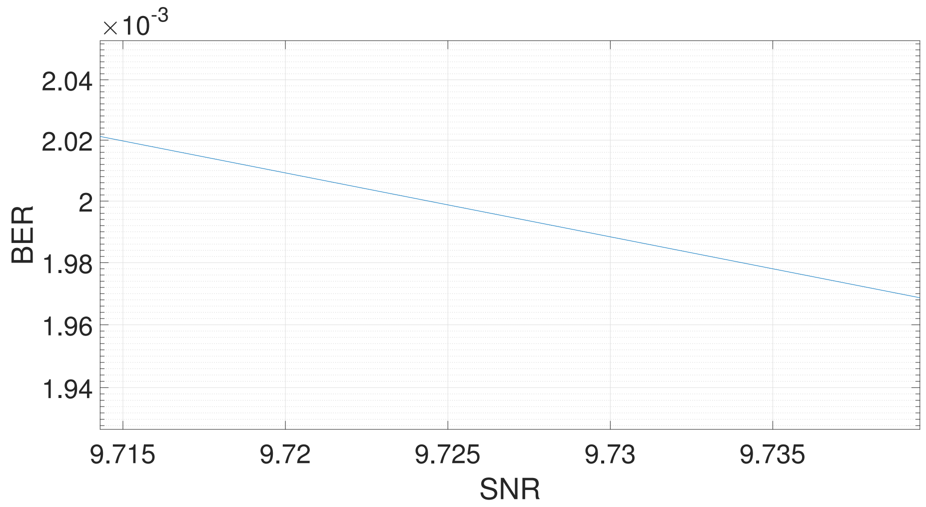

Figure 5.5: Viterbi n=1 with BER 2 10^{-3}

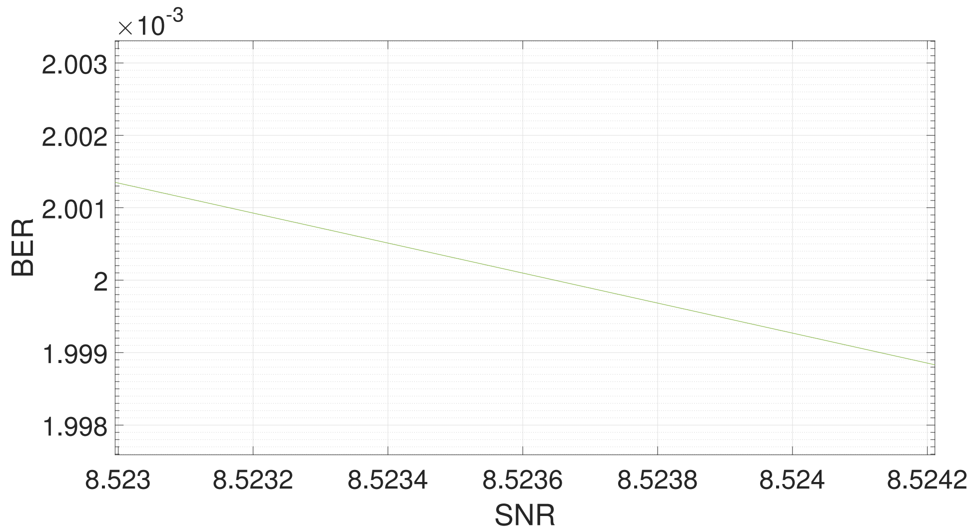

Figure 5.6: Viterbi n=16 with BER 2 10^{-3}

Chapter 6

Conclusions

6.1 Conclusions

This work proves that the simulation results for BER of Miller-2 line coding versus SNR coincide with the theoretical results. It is proved that Miller-2 line coding with ML symbol by symbol detector has the same BER as the FM0 coherent 2T in [1]. Furthermore, the BER for a Viterbi detector with perfect channel estimation is improved as the sequence length of bits increases. Finally, a 1.2 dB improvement is achieved compared to the ML symbol by symbol detector.

6.2 Future Work

Rayleigh Fading channel is not the most appropriate for RFID systems because there is no line of sight signal. Instead, Rice channel should be used as it provides a strong line of sight signal.

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