Traffic flow optimisation in presence of vehicle automation and communication systems - Part II: Optimal control for multi-lane motorways

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Abstract

Integrated motorway traffic flow control considering the use of Vehicle Automation and Communication Systems (VACS) is considered in this paper. VACS may act both as sensors (providing information on traffic conditions) and as actuators, permitting the deployment of Ramp Metering, Variable Speed Limits, and Lane Changing Control. The integrated traffic control problem is addressed through the formulation of a linearly constrained optimal control problem based on the first-order multi-lane model for motorways introduced and validated in a companion paper (Part I). A case study illustrating the potential improvements achievable using this approach is presented.

1 Introduction

In a companion paper (Roncoli et al., 2015), Part I of this work, a first-order multiple-lane traffic flow model for motorways has been developed and validated, mainly for the purpose of supporting the formulation of an integrated optimal control problem for motorways under a mixture of traditional and novel control measures, which is the main subject of this Part II paper. In particular, this work takes into account the potential presence of Vehicle Automation and Communication Systems (VACS) in a portion of vehicles. VACS may provide novel opportunities to improve traffic control performance by use of control actions, some of which would be hardly possible with traditional control actuators. In the last decades, we have witnessed a significant and steadily increasing effort of research and development of various types of VACS, some of which (e.g. Adaptive Cruise Control - ACC) have already been introduced in several car models. In contrast, there has been rather limited research to address the implications of emerging VACS on the traffic flow characteristics and their potential exploitation for improved traffic flow operations on motorways.

The concept of vehicle-platoon based fully-Automated Highway System (AHS) had been widely elaborated by several research groups in the more distant past, and a multi-layer control structure was proposed by Varaiya (1993) to tackle the huge problem complexity. Control decisions addressing the aggregated traffic flow were assigned, within the hierarchical structure, to the link layer. Specifically, the link control layer in these works is decentralised, i.e. it consists of a number of parallel link layer controllers, each of them addressing a corresponding highway link (of about 2 km in length). Each link layer controller decides on vehicle paths (in terms of driving lanes), as well as on targeted vehicle speed and platoon size within the corresponding highway link. One of the first works addressing link-layer control strategies, by Rao and Varaiya (1994), introduced a number of structural simplifications to tackle the problem complexity; specifically, unidirectional lane changings (either inwards or outwards) are allowed within 1-km long 4-lane highway sections, while decisions may differ according to the prevailing (normal or incident) traffic conditions. Lane changing and lane assignment for individual vehicles is the fundamental issue addressed via a set of well-justified and structured heuristic rules.

An alternative approach to local (link layer) lane assignment, based on very similar assumptions, was presented by Lee and Lee (1997). Another interesting, though rather theoretical, work in this context was presented by Li et al. (1997), proposing a space-time continuous decentralised-overlapping control law for the stabilisation of traffic conditions.

Specifically, the authors addressed the problem of driving the traffic conditions back to (pre-fixed) desired speed and density values (per lane) in case of disruptions (e.g. incidents) via appropriate vehicle-speed and lane-changing control decisions. More recently, an MPC (model predictive control) approach was proposed by Baskar et al. (2012) for the integrated control (addressing speed, lane assignment and ramp metering) of platoon-based AHS; the method involves both real-valued and integer variables and leads to a mixed non-convex optimisation problem that may be difficult to be solved in real time. Finally, a number of recent works (e.g., Zhang and Ioannou, 2007, Kesting et al., 2008, Schakel and van Arem, 2014) proposed local reactive rules and feedback laws for mixed traffic flow (comprising both equipped and non-equipped vehicles) under various assumptions and vehicle-to-vehicle and vehicle-to-infrastructure communication architectures.

A number of other works addressed specifically the problem of deciding on vehicle lane-paths for a whole (long) highway under fully automated (AHS) or semi-automated driving. To tackle the problem complexity, a number of assumptions are typically made, such as known and constant prevailing speeds along the highway and absence of traffic congestion, thanks to the assumed (but not addressed) appropriate operation of ramp metering at the highway entrances; also, a number of structural assumptions are made to limit the (otherwise vast) space of potential path assignments. Specifically, Hall and Lotspeich (1996) proposed an AHS model in form of a static trip-based multi-commodity network, with the objective of maximising the total flow served subject to pre-determined O-D (origin-destination) patterns, resulting in a linear programming formulation. The model was extended to the dynamic case by Hall and Caliskan (1999), but it was clearly stated that the method is not intended for real-time usage. Ramaswamy et al. (1997) formulated two static optimisation problems aiming at the minimisation of the total travel time. The difference between them lies in whether the cost for manoeuvring depends on highway congestion (generating a non-linear problem) or not (generating a linear problem); it was shown that the non-linear problem is more appropriate in case of heavily congested networks. Kim et al. (2008) introduced an optimisation problem aiming at identifying the best lane assigning strategies (subject to a partitioning assumption, for simplification) proposing a genetic algorithm for its solution. It should be pointed out that none of these works take into account the traffic dynamics as reproduced by a traffic flow model.

On the other hand, it is worth noting that the integrated control of motorway network traffic via conventional means (road-side variable message signs for route guidance and variable speed limits; traffic signal based ramp metering) was addressed, typically via nonlinear optimal control approaches, in a number of works (see, e.g., Kotsialos et al., 2002, Hegyi et al., 2005, Carlson et al., 2010, Lu et al., 2010). Of course, these works do not include lane changing control, while the assumed granularity of variable speed limits and route guidance actions is limited by the use of road-side VMS (variable message signs) to display the messages, which are visible by all the drivers. In fact, the spatial density of VMS or gantries is limited due to economic and physical reasons; while any provided advice or order is identical for all drivers on all lanes.

This paper proposes an optimal control problem formulation for integrated motorway traffic control under the assumption that a sufficient percentage of vehicles are equipped with VACS, which permits vehicle-to-infrastructure (V2I) communication, to enable variable speed limit control per lane, lane changing control (both at arbitrary spatial resolution) and ramp metering. The approach is based on a novel linear dynamic multi-lane traffic flow model for motorways (developed and validated in Part I) with linear constraints, which allows for consideration of long motorways with moderate computational effort. The employed model is flexible enough to consider a variety of potential cases, objectives, and infrastructures. With appropriate extensions, the proposed method could be used as the kernel for real-time motorway traffic control actions in presence of VACS. In addition, the method may be used in order to study and analyse the complex dynamic interactions among equipped vehicles, traffic conditions and appropriate control actions for various scenarios and settings of VACS penetration and infrastructure type, as well as to guide and provide a reference case for the development and assessment of simpler or complementary control strategies and approaches.

The paper is structured as follows: in Section 2, the adaptation of the traffic flow model for the optimal control problem formulation is described and a quadratic programming problem is developed, aiming at the minimisation of traffic congestion. In Section 3, an application example is worked out for a real motorway stretch, comparing the obtained results with the no-control case. Finally, Section 4 concludes the paper, highlighting the main results and introducing some research challenges for the future.

2 Optimal control problem

2.1 The application framework

We consider motorway traffic flow with a sufficient percentage of VACS-equipped vehicles. "Sufficient percentage" in the present context means that the control variables considered in the optimal control problem can be actually implemented via appropriate actuators. Although the details of this assumption are currently being worked out and will be presented in the near future, it is useful, for better appreciation of the work presented here, to provide some preliminary clarifications. The basic architecture considers a central Decision Maker (DM) that computes the solution of the optimisation problem, disaggregates the results and assigns specific vehicle control tasks that are sent for execution by the corresponding equipped vehicles. It is assumed that the DM has the complete knowledge of the traffic state when necessary; this may be achieved directly if all vehicles are equipped and in communication with the infrastructure (V2I), or via an appropriate state estimation algorithm.

As traffic evolves from the present situation to the future, a number of VACS are expected to appear at diverse and gradually increasing penetration levels. Some of these VACS may be exploited, as novel actuators, for more pertinent traffic control. For example, if some vehicles have V2I capabilities, there is a possibility to impose to them variable speed limits, or even dictate to them their driving speed, according to the real-time decisions of the central DM. In the envisioned scenario, the DM is aware of all the equipped vehicles travelling in each segment-lane considered in the control problem; once the optimal speeds are computed, they can be dispatched through the V2I system, to the on-board system of each vehicle. Then, depending on the specific device and on the level of automation, the speed limit can be visualised on an on-board console, or directly applied in the ACC or CACC control system; in this last cases, the VSL takes part in the local control tasks performed by the automated car, without needing any intervention by the driver. The frequency of updating the specified speed limit must be carefully chosen, particularly in case of non fully-automated vehicles, in order to avoid excessive nuisance to drivers and passengers. This low-nuisance issue is also considered in the presented approach, by penalising the space-time differences of speed limits.

Of course, the optimal control decisions are meant for the aggregate traffic flow in specific space-time windows, not for some individual vehicles only; however, if the percentage of equipped vehicles is sufficiently high, the imposed speed (limit) will necessarily also affect the vehicles following an equipped vehicle.

Similarly, V2I-equipped vehicles may receive from the DM lane-changing advices, so as to implement corresponding lateral flow decisions. Since lateral flows are limited, only accordingly few equipped vehicles need to be present in the traffic flow to receive the corresponding advice; in case of higher percentages of equipped vehicles, "keep lane" advices may also be issued to the rest of the equipped vehicles. In the initial phases with low penetration rates of equipped vehicles, traditional actuators, i.e. road-side VMS, may also be employed in parallel, to the extent possible.

More specifically, the following control actions may be taken into account:

- Ramp-metering (RM): these actions consist in regulating the inflow from the on-ramps to the motorway mainstream and they are currently applied on many motorways (see e.g. Papamichail et al., 2010); since they are applied directly at on-ramps, they do not necessarily require any particular in-vehicle equipment to be performed, as the computed inflow may be directly applied using ordinary traffic signals.

- Mainstream Traffic Flow Control (MTFC) via Variable Speed Limits (VSL): the use of VSL to regulate the mainstream flow with the purpose of mitigating traffic congestion was proposed by Hegyi et al. (2005) and Carlson et al. (2010) and has been exploited in an increasing number of research works. In the present work, it is assumed that the exiting flows (and consequently, the speeds) are specified by the DM for each segment-lane; thus all equipped vehicles travelling on a segment-lane will receive and apply the respective speed or speed limit. For a sufficient penetration of equipped vehicles, this will be sufficient to impose the speed limit to non-equipped vehicles as well.

- Lane Changing Control (LCC): the optimal lateral flows are computed for each segment-lane, but the implementation of this control action is more challenging and uncertain than the previous two, unless all vehicles are under full guidance by the control center; in this latter case, it is not difficult to implement the control action by sending lane-changing orders to an appropriate number of vehicles. In all other cases, an intermediate algorithm should decide on the number and IDs of equipped vehicles that should receive a lane-changing advice, taking into account the compliance rate and the spontaneous lane changes; the latter may be reduced by involving additional "keep-lane" advices to other equipped vehicles. Cooperative possibilities of vehicles equipped with V2V-communication capabilities may further facilitate this control action. It is important to highlight that these lateral flows are intended here as macroscopic variables, which does not involve explicitly the characteristics of different drivers, e.g. the origin-destination of drivers is not considered. This aspect should be accounted for in an intermediate algorithm, that assigns appropriate lane-changing advices to individual vehicles according to their specific needs (e.g., vehicles approaching the desired off-ramp should move towards external lanes). These issues are currently in course of detailed investigation and development.

In order to guarantee an adequate flexibility, it is supposed that each control action is updated according to a specific control time step which may be specified according to human-factors and other operational requirements. The control time steps are assumed to be integer multiples of the traffic flow model time step. Specifically, we denote by T the model time step, T^Q the MTFC (longitudinal flow control) time step, T^F the lateral (lane-changes) flow control time step, and T^R the RM time step. The corresponding discrete time indices are $k = 0, 1, 2, \ldots, K$; $k^Q = \left\lceil \frac{kT}{T^Q} \right\rceil$; $k^F = \left\lceil \frac{kT}{T^F} \right\rceil$; $k^R = \left\lceil \frac{kT}{T^R} \right\rceil$[1].

Since the main objective is the reduction of traffic congestion, these control actions are taken as the decision variables in the optimisation problem stated in Section 2.2. In order to make the problem solvable for large networks, it is formulated as a Quadratic Program (QP), characterised by a convex quadratic cost function and linear constraints. This formulation is derived from the traffic flow model described by Roncoli et al. (2015).

2.2 Problem formulation

The following notation is used for the problem formulation in accordance with the Part I paper:

- $\rho_{i,j}(k)$ density [veh/km] in the segment i, lane j at time step k; i.e. the number of vehicles travelling in the segment-lane divided by the segment length L_i;
- $q_{i,j}(k^Q)$ longitudinal flow [veh/h] leaving segment i and entering segment $i+1$, remaining in lane j, during time interval $\left(k^Q, k^Q+1\right]$; this flow is a control input that reflects the MTFC actions;
- $f_{i,j,\bar{j}}(k^F)$ lateral flow [veh/h] moving from lane j to lane $\bar{j}$ during the interval $\left(k^F, k^F+1\right]$; this is a control input that reflects the LCC actions;
- $r_{i,j}(k^R)$ the flow [veh/h] entering from the on-ramp located in segment i, lane j during time interval $\left(k^R, k^R+1\right]$; this is a control input which represents the RM actions;
- $\gamma_{i,j}(k)$ turning rates for assigning off-ramp flows; for the computation of off-ramp flows, all the lanes of segment i are considered (see Roncoli et al., 2015).

Based on these definitions, we may immediately write the conservation equation for each segment-lane:

$$\begin{aligned}\rho_{i,j}(k+1) = \rho_{i,j}(k) + \frac{T}{L_i}\Big[&q_{i-1,j}(k^Q) + r_{i,j}(k^R) - q_{i,j}(k^Q) - \gamma_{i,j}(k)\sum_{j=1}^{J} q_{i,j}(k^Q) \\ &+ f_{i,j+1,j}(k^F) + f_{i,j-1,j}(k^F) - f_{i,j,j-1}(k^F) - f_{i,j,j+1}(k^F)\Big].\end{aligned} \tag{1}$$

The conservation equation (1) is directly taken from the model defined in Part I, with the

[1] The ceiling function $y = \lceil x \rceil$ is used here, where y is the smallest integer not less than x.

minor difference that, because of the possible ramp-metering actions, the entering external flow is actually the ramp outflow $r_{i,j}$.

Each on-ramp (i, j) receives an external (uncontrollable) demand $D_{i,j}$ [veh/h]; since the respective on-ramp outflows $r_{i,j}$ are controllable via corresponding RM actions, this may lead to the creation of ramp queues $w_{i,j}$ [veh]. Since the motorway sections and the on-ramps have finite storage capacities, to be imposed as hard constraints in the optimal control problem formulation, excessive external demand scenarios may lead to an infeasible optimisation problem, whereby no control action can accommodate the external demands without violating the storage capacity constraints. To avoid such situations, where the admissible control region is void, and enable the computation of optimal control for any arbitrary demand scenario, we introduce two extra variables: $W_{i,j}$ [veh] that represents a virtual extra-queue state variable; and $d_{i,j}$ [veh/h] that represents the demand flow that is capable to enter into the real queue $w_{i,j}$ without violating its corresponding upper bound $w_{i,j}^{max}$. The dynamics at on-ramps are thus stated as follows:

$$w_{i,j}(k+1) = w_{i,j}(k) + T\big[d_{i,j}(k) - r_{i,j}(k^R)\big] \tag{2}$$

$$W_{i,j}(k+1) = W_{i,j}(k) + T\big[D_{i,j}(k) - d_{i,j}(k)\big]. \tag{3}$$

Thus, $D_{i,j}$ is the external demand feeding the extra-queue $W_{i,j}$, while the internal demand $d_{i,j}$ connects the extra-queue with the real queue $w_{i,j}$. The idea is to apply an extra-strong penalty factor to the variables $W_{i,j}$ so that the solution of the optimisation problem is forced to keep the extra-queues equal to zero, if at all possible; but if, due to an excessive demand scenario, there is no other admissible solution, the problem will remain feasible by charging the extra-queues.

The longitudinal lane-inflows entering the segment-lanes $(1, j)$ of the first segment are formally treated as on-ramps, however by setting $w_{i,j}^{max} = 0$, they are actually considered uncontrollable. Anyway, also in this case, in order to avoid infeasibility, the same extra-queue approach is applied.

The computation of lateral flows is fully delegated to the optimiser; only upper bounds are specified to the non-negative lateral flows as follows:

$$\left[f_{i,j,j-1}(k^F) + f_{i,j,j+1}(k^F)\right] \leq \frac{L_i}{T}\rho_{i,j}(k) \tag{4}$$

$$\left[f_{i,j-1,j}(k^F) + f_{i,j+1,j}(k^F)\right] \leq \frac{L_i}{T}\left[\rho_{i,j}^{jam} - \rho_{i,j}(k)\right] \tag{5}$$

$$f_{i,j,j-1}(k^F) \leq f^{max} \tag{6}$$

$$f_{i,j,j+1}(k^F) \leq f^{max}.$$

Equation (4) represents the upper-bound for lateral flows determined by the number of vehicles in the current lane-segment; (5) is an upper-bound considering the available space (ρ^{jam} is the maximum admissible density) in the segment-lane that is receiving the lateral flow; and (6) are hard constraints considered in order to strictly limit lateral movements avoiding unrealistic values that cannot be suggested in a real case. Moreover, a cost function term will be introduced in order to discourage lane changings in dependence of the specific location of each segment-lane.

The modelling approach for longitudinal flows, proposed and tested in Part I, is based on the piecewise-linear fundamental diagram (FD) displayed in Figure 1. More specifically, Figure 1 displays the demand part, which determines the flow based on the upstream density, and the supply part, which determines the flow based on the downstream density. Note that, as detailed in Part I, the employed FD demand part provides for the possibility to reflect the capacity drop phenomenon if the upstream density is over-critical ($\rho(k) > \rho^{cr}$, where ρ^{cr} is the critical density). This is achieved via the introduction of a linear function that decreases (according to the slope w^D) as the density in the current segment increases. The minimum flow achievable due to the capacity drop is q^{jam}, which occurs when $\rho(k) = \rho^{jam}$. In case no control actions are applied, the actual flow equals the minimum between the demand and supply flows. For optimal control, the longitudinal flows are assumed controllable via corresponding lowering of equipped vehicle speeds as mentioned earlier. Hence, the lines of the piecewise-linear FD of Figure 1 may be simply used as upper bounds for the controllable

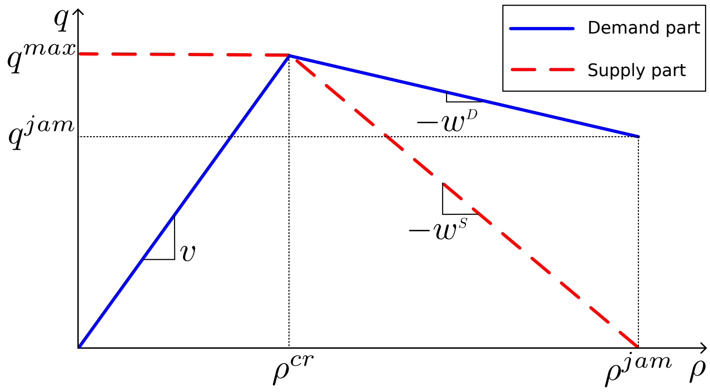

Figure 1: The proposed FD including both the demand and the supply piecewise-linear functions.

longitudinal flows as follows:

$$q_{i,j}(k) \leq v_{i,j}^{free} \rho_{i,j}(k) \tag{7}$$

$$q_{i,j}(k) \leq -\frac{v_{i,j}^{free} \rho_{i,j}^{cr} - q_{i,j}^{jam}}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}} \rho_{i,j}(k) + \frac{\rho_{i,j}^{cr} \left(v_{i,j}^{free} \rho_{i,j}^{jam} - q_{i,j}^{jam} \right)}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}} \tag{8}$$

$$q_{i,j}(k) \leq v_{i+1,j}^{free} \rho_{i+1,j}^{cr} \tag{9}$$

$$q_{i,j}(k) \leq -\frac{v_{i+1,j}^{free} \rho_{i+1,j}^{cr}}{\rho_{i+1,j}^{jam} - \rho_{i+1,j}^{cr}} \rho_{i+1,j}(k) + \frac{v_{i+1,j}^{free} \rho_{i+1,j}^{cr} \rho_{i+1,j}^{jam}}{\rho_{i+1,j}^{jam} - \rho_{i+1,j}^{cr}}. \tag{10}$$

Equations (7) and (8) represent the demand part of the FD, whereas (9) and (10) are the bounds accounting for the supply part of the FD.

Note that, as also mentioned in Roncoli et al. (2015), the left-hand side of the FD may be modelled via any concave piecewise-linear function, rather than one single line as in Figure 1, to better approximate the undercritical speed behaviour of real traffic flow. This would merely increase the number of linear inequalities to be considered, in addition to (7), in the optimal control problem formulation, it should also be noted that the FD is meant to reflect the traffic flow behaviour given the presence and penetration rates of various VACS, irrespective of the traffic control actions that this work aims at optimising. For example, a percentage of vehicles may be equipped with ACC or cooperative ACC systems that alter the inter-vehicle time-gaps, thus influencing flows and capacities. These features would then reflect on the FD, and, of course, would be present with or without the traffic control actions that this work attempt to optimise.

Furthermore, upper-bounds are considered for on-ramp queues ($w_{i,j}(k) \leq w_{i,j}^{max}$), entering flows at on-ramps ($r_{i,j}(k^R) \leq r_{i,j}^{max}$), and exiting flows at off-ramps ($\gamma_{i,j}(k)\sum_{j=1}^{J} q_{i,j}(k^Q) \leq q_{i,j}^{off,max}$). It is worth to highlight that the last constraint provides the possibility to consider a limited off-ramp capacity; this is of particular relevance in the frequent practical case of off-ramp queues that over-spill onto the motorway mainstream and may lead to substantial obstruction and delays. With this constraint, the model is enabled to reflect the potential of over-spilling, and the resulting optimal control may address this phenomenon via appropriate control actions. Finally, non-negativity constraints are also specified for all the variables.

The cost criterion to be minimised is defined by the following equation:

$$
\begin{aligned}
Z = {} & T\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[L_i\rho_{i,j}(k) + w_{i,j}(k)\right] \\
& + M\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J} W_{i,j}(k) \\
& + \sum_{k^F=1}^{K^F}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\beta_{i,j,j-1}\, f_{i,j,j-1}(k^F) + \beta_{i,j,j+1}\, f_{i,j,j+1}(k^F)\right] \\
& + \lambda^r \sum_{k^R=2}^{K^R}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[r_{i,j}(k^R) - r_{i,j}(k^R-1)\right]^2 \\
& + \lambda^f \sum_{k^F=2}^{K^F}\sum_{i=1}^{I}\left\{\sum_{j=2}^{J}\left[f_{i,j,j-1}(k^F) - f_{i,j,j-1}(k^F-1)\right]^2 \right. \\
& \qquad\qquad \left. + \sum_{j=1}^{J-1}\left[f_{i,j,j+1}(k^F) - f_{i,j,j+1}(k^F-1)\right]^2\right\} \\
& + \lambda^{st}\sum_{k=2}^{K-1}\sum_{i=1}^{I}\sum_{j=1}^{J}\frac{\left\{q_{i,j}(k^Q) - q_{i,j}(k^Q-1) + v_{i,j}^{*}\left[\rho_{i,j}(k) - \rho_{i,j}(k-1)\right]\right\}^2}{\left(\rho_{i,j}^{*}\right)^2} \\
& + \lambda^{sl}\sum_{k=1}^{K-1}\sum_{i=2}^{I}\sum_{j=1}^{J}\frac{\left\{q_{i,j}(k^Q) - q_{i-1,j}(k^Q) + v_{i,j}^{*}\left[\rho_{i,j}(k) - \rho_{i-1,j}(k)\right]\right\}^2}{\left(\rho_{i,j}^{*}\right)^2}
\end{aligned}
\tag{11}
$$

The cost function in (11) is composed by the weighted sum of seven different terms, the first three linear and the last four quadratic:

- The first linear term represents the Total Time Spent (TTS) [veh·h]; it considers both the time travelled and the time spent queuing at on-ramps; this is the most important term that is used to evaluate the goodness of the solution in terms of traffic flow efficiency.

- The second linear term (weighted by M) is a penalty value for the extra-queues that may be generated in the optimal solution. The coefficient M must be big enough to obtain that all variables $W_{i,j}$ are as close as possible to 0, if the demand scenario allows.

- The third linear term (weighted by coefficients $\beta_{i,j,\bar{j}}$) aims at penalising lateral flows; it has the purpose of reducing the lane changings (thus giving priority to the other control actions); however, the use of lower values for the weights $\beta_{i,j,\bar{j}}$ at specific locations (e.g. upstream of lane drops) may facilitate reasonable and beneficial lane changing actions.

- The quadratic terms are penalty terms aiming at penalising the variation of control variables from a time step to the next one or between neighbouring segments. These terms are introduced in order to reduce, or even suppress, space-time fluctuations of the control variables, that have a minor contribution to the resulting traffic flow efficiency. The first penalty term (weighted by λ^r) is related to time-variations of the on-ramp flows; the second one (weighted by λ^f) penalises the time-variation of lateral flows; and the last two terms have the purposes of penalising, respectively, time and space variations of speed values. Since the speed is not a problem variable and could only be included via the non-linear relation $v = q/\rho$, a linearisation approximation of the speed is utilised in the proposed approach. The term weighted by λ^{st} represents the penalisation of speed from one step to the next one, whereas the one weighted by λ^{sl} is related to the penalisation from a segment to the next one. The derivation of the last two terms is detailed here below.

The penalisation of space-time variations of the mean speed is deemed necessary in order to accordingly limit, or even suppress, driving speed changes for the individual vehicles, whenever such changes would lead to only marginal TTS improvements. Since the mean speed is not an explicit model variable, the non-linear relation $v = q/\rho$ is considered; this, however, would introduce an undesired non-linearity; therefore, the following approximating linearisation is used instead, which was found in several test examples to lead to excellent results. In the case of time-variations, the target is to penalise the difference between the speeds $v(k)$ and $v(k+1)$; the abbreviation $\Delta v = v(k+1) - v(k)$ is introduced, with analogous definitions Δq for the corresponding flow and $\Delta\rho$ for the density. $v(k+1)$ may then be expressed as:

$$v(k+1) = v(k) + \Delta v = \frac{q(k) + \Delta q}{\rho(k) + \Delta\rho}, \tag{12}$$

which implies

$$\Delta v = \frac{\Delta q - v(k)\Delta\rho}{\rho(k) + \Delta\rho} = \frac{\Delta q - v(k)\Delta\rho}{\rho(k+1)}. \tag{13}$$

This relation is again non-linear; however fixed values for $v(k)$ and $\rho(k+1)$ may be considered in order to treat this penalty term as a linear approximation around nominal values:

$$v(k+1) - v(k) \approx \frac{\Delta q - v^*\Delta\rho}{\rho^*}. \tag{14}$$

Possible nominal values may be the free speed and the critical density; nevertheless a different linearisation point can also be chosen if considered more appropriate. Equation (14) is thus utilised as quadratic penalty term for time variations of the mean speed, and the corresponding penalty term for space variation of the mean speed may be derived in the same way.

The formulated discrete-time optimal control problem can be written in matrix form as:

$$\min_{\mathbf{x}} z = \mathbf{c}^T\mathbf{x} + \frac{1}{2}\mathbf{x}^T H\mathbf{x} \tag{15}$$

subject to

$$A_i\mathbf{x} \le \mathbf{b}_i \tag{16}$$

$$A_e\mathbf{x} = \mathbf{b}_e \tag{17}$$

$$\mathbf{x} \le \mathbf{d} \tag{18}$$

$$\mathbf{x} \ge 0 \tag{19}$$

where the vector $\mathbf{x}$ contains all the state and control variables previously introduced. In (15), the vector $\mathbf{c}^T$ contains coefficients for all the linear terms of the cost function (11), whereas matrix H gathers the specifics of the quadratic terms; since H derives from the weighted sum of quadratic terms (each one of them characterised by a positive-semidefinite matrix), H is positive-semidefinite, which implies that the cost function is convex. The linear inequality (16) derives from (4), (5), (7), (8), and (10); while the linear equality (17) represents the conservation equations (1) - (3). The formulation is completed by upper-bounds (18), including (6) and (9), and non-negativity constraints (19). This formulation corresponds to a convex QP with very sparse matrices (due to the time dimension). Such a problem can be solved very efficiently (even for large-scale infrastructures) using available numerical solution codes.

2.3 Extensions

A number of extensions of the formulated optimal control problem may be envisaged, when needed, under the requirement of preserving the quadratic structure of the cost criterion and the linearity of the constraints.

To start with, mainstream congestion may sometimes reduce the level of flow that can merge from on-ramps; in other words, mainstream congestion may, in some cases, also spill back into merging on-ramps. This situation may be modelled via a corresponding limitation of the on-ramp flow $r_{i,j}$ in dependence of the density $\rho_{i,j}$ in the merging segment-lane. The following on-ramp flow constraint, which is linearly decreasing with the mainstream density, was proposed and validated by Papageorgiou et al. (1990) (see also Bar-Gera and Ahn, 2010 for similar modelling):

$$r_{i,j}(k^R) \leq r_{i,j}^{max} \frac{\left[\rho_{i,j}^{jam} - \rho_{i,j}(k)\right]}{\left[\rho_{i,j}^{jam} - \rho_{i,j}^{cr}\right]}. \tag{20}$$

The free flow speeds $v_{i,j}^{free}$, introduced in Part I and in Section 2.2, may differ for different lanes. This differentiation is essential for European motorways, since overtaking is only allowed on one side, and trucks (or other slow vehicles) usually drive on the shoulder lane; for North-American freeways, although often present, this phenomenon is less accentuated since vehicle overtaking is allowed on any lane. It should be noted that, if different free speeds for different lanes are actually foreseen in the model, the attempted TTS minimisation may imply, at low densities, that fast lanes are favoured in the optimal control results, which would contradict the current practice. This undesirable implication may be largely circumvented via additional limitation of the density differences among lanes (e.g. $\rho_{i,j} \geq \rho_{i,\bar{j}}$), forcing a more homogeneous flow distribution across (slow and fast) lanes, mainly via appropriate limitation of the corresponding lateral flows. However, in case of ordinary traffic, some studies addressed the problem of modelling lane distribution, e.g. Wu (2006), Lee and Park (2010), Knoop et al. (2010), Duret et al. (2012), Samoili et al. (2015), showing that the lane distribution is affected, among others, by some characteristics of the network layout (e.g., the total number of lanes); however this choice is also behavioural, since every single driver may autonomously decide to stay in a slower lane accepting the lower speed, stay in the slower lane and overtaking when necessary (for lower densities), or choosing to travel constantly in a faster lane (in higher densities). To account for these characteristics, without affecting the linear structure of the proposed problem, an additional parameter $P_{i,j,\bar{j}}$ can be included (similarly to the one included in Part I to account for special infrastructure conditions), that can be properly tuned in order to alter the equilibrium of lane distribution, according to $\rho_{i,j} \geq P_{i,j,\bar{j}}\rho_{i,\bar{j}}$. Moreover, as mentioned earlier, a piecewise-linear FD for undercritical densities may also be used as a tool to influence the optimal results towards more realistic lane distribution at low densities in presence of different free speeds for different lanes.

Lateral flow into a segment-lane may affect its capacity. To reflect this possibility within our approach, we may readily render the flow capacity of a segment-lane linearly dependent on the entering lateral flows (and analogously on the flow entering from on-ramps). The following linear relation, that replaces (8), can be considered, with the purpose of limiting the capacity proportionally to the flow entering a segment-lane:

$$q_{i,j}(k) \leq -\frac{v_{i,j}^{free}\rho_{i,j}^{cr} - q_{i,j}^{jam}}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}}\rho_{i,j}(k) + \frac{\rho_{i,j}^{cr}\left(v_{i,j}^{free}\rho_{i,j}^{jam} - q_{i,j}^{jam}\right)}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}} - \alpha_{ef}\left[f_{i,j-1,j}(k) + f_{i,j+1,j}(k)\right] - \alpha_r r_{i,j}(k) \tag{21}$$

where α_{ef} and α_r are weigh parameters that reflect the impact of entering flows from adjacent lanes or on-ramps respectively.

3 Case study

3.1 Network description

The motorway used with the purpose of evaluating and illustrating the concepts described in this paper is the same one utilised in Part I for the calibration of the proposed model. The Part I results are therefore useful to provide the real demand data, the calibrated values related to the FD, and also to act as a reference case to evaluate the control actions suggested and the improvements obtained by solving the optimisation problem.

The network is a stretch of 5.26 km in length of the Monash Freeway (M1) located in the area of Melbourne, Victoria, Australia. It is composed by four lanes and it presents three on-ramps and three off-ramps; a graphical representation of its structure may be seen in Figure 2. The lanes are numbered $1, \dots, 4$ from the inner lane (close to the roadside), to the outer lane (close to the road median). The addressed time-horizon is from 5 AM to 9 AM and it includes the morning peak period, when a high flow of vehicles is travelling towards the centre of Melbourne. Figure 3 provides the validated (Part I) simulation data produced with real demands from 14 August 2013. As it may be seen in the contour plots of Figure 3, a first congestion is appearing in proximity of on-ramp R1 (segment 2), due to the increased demand. Then, the main congestion starts in segments 17-20 at about 6:15 AM due to the high number of trucks reducing their velocity because of the slope of the motorway. The increased number of trucks in lane 1 causes most car drivers to decide to move into the adjacent lanes, causing an increase of density that, quite rapidly, triggers a capacity drop due to the starting of a strong congestion. This congestion spills back, covering the whole stretch at about 6:35 AM. This causes further speed reductions in the merging area of all the on-ramps, contributing to the creation of a generalised congestion. Around 8 AM, since the demand decreases, the congestion reduces and finally disappears, restoring the free-flow conditions. A more detailed description of the uncontrolled congestion pattern is presented in Part I, that also includes a detailed description of the model calibration and the model capability to reproduce the different traffic phenomena.

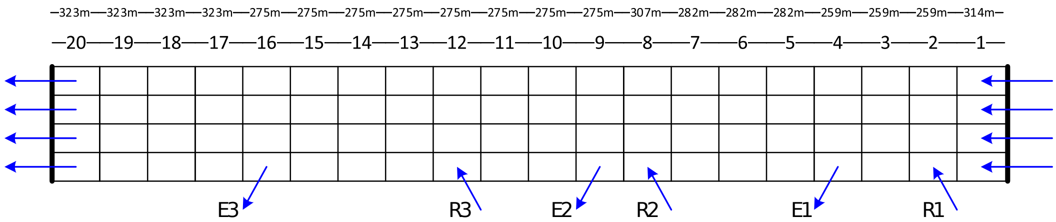

Figure 2: A schematic representation of the motorway stretch used in the case study.

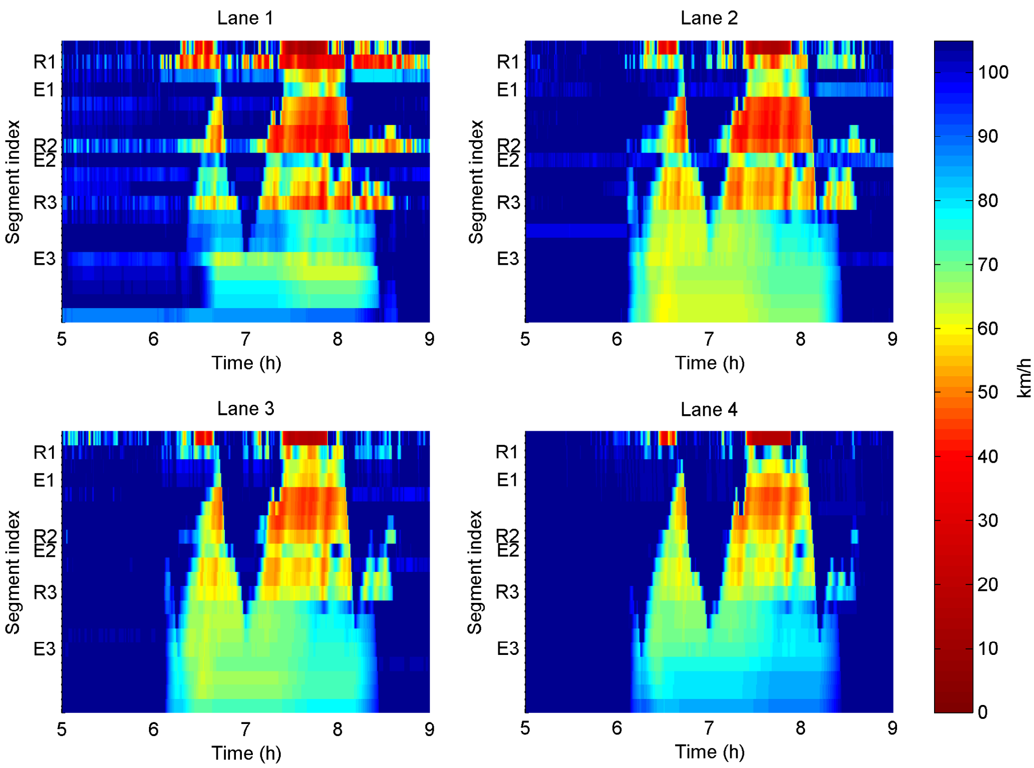

Figure 3: Contour plots of the speed per lane in the no-control case of Scenario 1.

The choice of the simulation step is a crucial aspect that must be carefully taken into account. In fact, a too long simulation step could allow vehicles to travel in more than one cell during its duration, causing numerical instability of the mathematical model; on the other hand, the size of the optimisation problem is affected by this choice. In this case, a value $T = 7.5$ s is set, that represents the maximum value that satisfies the CFL condition (Courant et al., 1928) for model stability.

Once the simulation step is chosen, the control steps are defined as a multiple of the simulation step. A first issue here is to specify the range of control update frequencies that appear suitable for the physical system addressed. Typical control time steps for traditional ramp metering are in the range 20 s - 1 min, while traditional VSL systems typically employ a time step of 1 min. In the case of VACS presence, RM may be implemented either with ordinary traffic signals or via direct communication to the vehicles waiting at the on-ramp (if the penetration rate and the compliance level are 100%). VSLs (or driving speeds) will be communicated to equipped vehicles, preferably for automated application. A time step of 30 s to 1 min appears reasonable for these control applications. As for lane changing, the corresponding advices will be addressing very few vehicles, hence the frequency of updating is not a major concern.

Beyond the physical relevance, the choice of a control time step may be based on a trade-off between the computation time and the resulting cost function value. As a matter of fact, the minimum cost is achieved by setting the control step equal to the simulation step; in this case, the control variables are updated synchronously with the state variables and have the highest degree of freedom to react immediately to any variations. Increasing the control step means that the control variables are kept constant for a period of some simulation steps, therefore the control actions are less able to handle the changes in the state variables, which results in an accordingly degraded cost value. An illustrative comparison is shown in Table 1 where, for the sake of simplicity, all the control steps are set to the same value. It may be seen that, as expected, the cost function deteriorates while the control step grows. Certainly, these results are dependent on all the other parameters, however the cost function value seems

Control step [s]	7.5	15	30	60	120
Computation time [s]	951	1097	703	678	675
TTS [veh · h]	1514	1522	1532	1549	1576
Improvement	23.3%	22.9%	22.4%	21.5%	20.1%

Table 1: Comparison in terms of computation time and TTS value in case different control steps are defined (the same step value is used for all the control actions). The improvement is related to the no-control case. The computation time has not a strictly monotonic trend possibly because of a slower convergence for some specific instances of the problem.

to be quite stable until the control steps are set to $T^Q = T^R = T^F = 4T = 30s$; in which case the cost function is only 1.1% worse than in the best case, but the solution is obtained in a shorter time; whereas for bigger values, the cost function value starts to increase more substantially. Thus, a unique time step of 30 s for all control applications is utilised in the rest of the paper.

Another significant aspect is the tuning of the cost function weights. First priority is given to the parameter M, that is chosen in order to avoid the creation of extra-queues; a value $M = 10$ was found to be appropriate in the related experiments. Then, the tuning procedure is mainly focused in keeping essentially the same optimal TTS value while trying to obtain reduced lateral flows and smoothed control actions. For the linear penalty terms related to LCC, an important aspect is also related to the locations of these control actions. In locations where strong lateral flow actions are expected (e.g. upstream of lane-drops or on-ramps), it is not reasonable to discourage vehicles to change lane; hence, at segments immediately upstream of such locations, the LCC weight is set to $\beta_{i,j,\bar{j}} = 0$; in all other segments, the values are set to $\beta_{i,j,\bar{j}} = 10^{-4}$. In addition, the weigh parameters of the quadratic terms have been tuned and the following values are used in all the simulations: $\lambda_f = 10^{-6}$, $\lambda_r = 10^{-7}$, $\lambda_{st} = 10^{-4}$, and $\lambda_{sl} = 10^{-3}$. Moreover, according to some experimental observations, the linearisation point for suppressing the time-space speed oscillations has been chosen as $v^*_{i,j} = v^{free}_{i,j}$ and $\rho^*_{i,j} = \frac{\rho^{cr}_{i,j}}{4}$.

In addition, upper-bounds are provided for the on-ramps queues, obtained from an estimation based on the length of the real ramp ($w_{2,1}^{max} = 40$ veh, $w_{8,1}^{max} = 20$ veh, and $w_{12,1}^{max} = 20$ veh); for the maximum flow allowed to enter from on-ramps ($r_{2,1}^{max} = r_{8,1}^{max} = r_{12,1}^{max} = 1600$ veh/h); and for lateral movements ($f^{max} = 400$ veh/h).

3.2 Optimisation results

3.2.1 Scenario 1

As previously mentioned, the reference case for the Scenario 1 is taken from the results of Part I, in which it is assumed that no control actions are applied. Computing the value of the cost function according to (11), a TTS value $Z_{TTS}^{*} = 1973$ veh·h is obtained.

The optimisation problem is thus applied to the same network, using the same demand profiles, shown in Figure 4, and the parameters described in the previous section. The demand is fed as the external input $D_{i,j}$ at the mainstream entrance and at the on-ramps. Again, similarly to the reference case, no boundary conditions are specified for the network exits, allowing a fair comparison of the results. The obtained TTS part of the cost function is $Z_{TTS}^{*} = 1532$ veh·h, that represents a 22.4% improvement with respect to the no-control case. The traffic improvement may be seen by examination of the contour plot of Figure 5, where it is clearly visible that speed reductions are almost completely avoided. Hereafter, the intelligent control actions taken at critical locations are analysed and discussed.

- The first congestion appearing in the network is due to the increased flow at on-ramp R1 around 6 AM. It is tackled by the optimiser via a combined use of RM and LCC. As it is shown in Figure 6, some space for the entering flow is created by moving vehicles from lane 1 to lane 2 in the segment 1, upstream of the merging area (this is facilitated by the weight $\beta_{i,j,\bar{j}} = 0$ set for that location); in addition, RM actions are performed in the period of maximum demand. Of course, the complete situation is more complex, since some other actions are also performed while trying to avoid the creation of congestion at downstream locations. The result is a local maximisation of the outflow from the on-ramp location due to the avoidance of congestion.

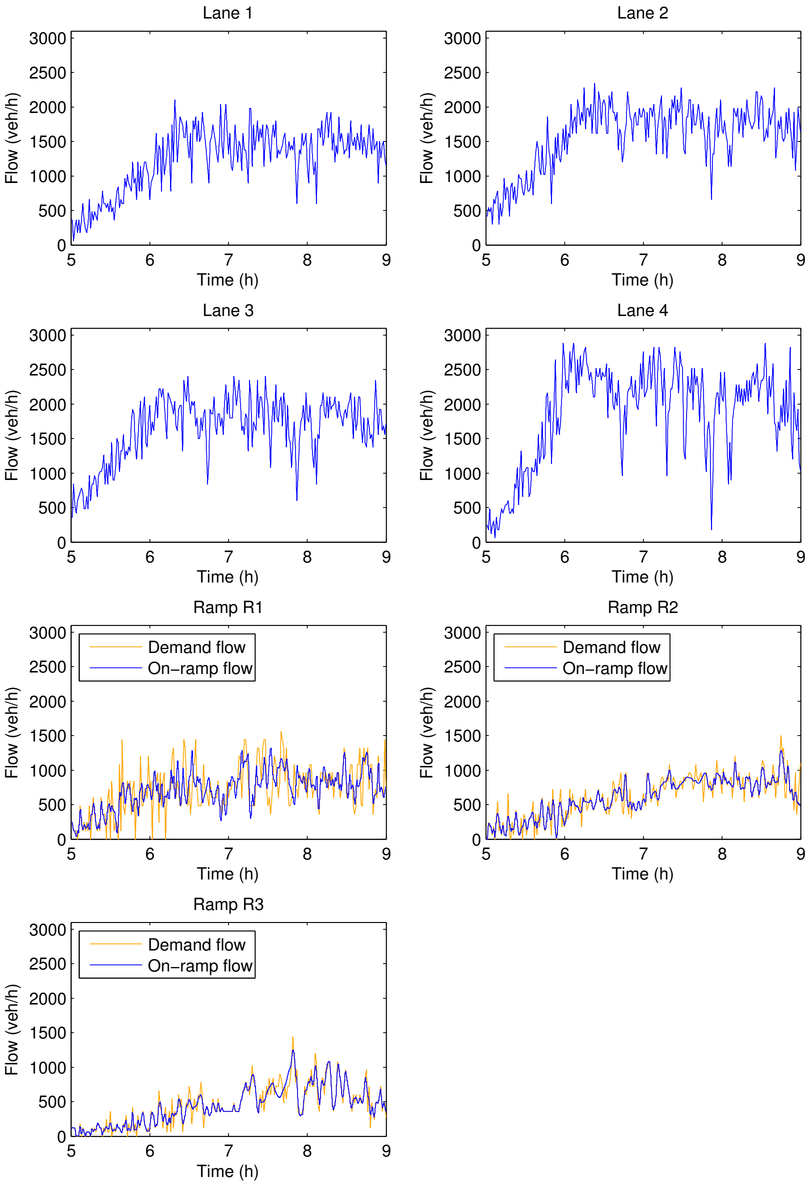

Figure 4: The network demand and the flow computed for the metered on-ramps.

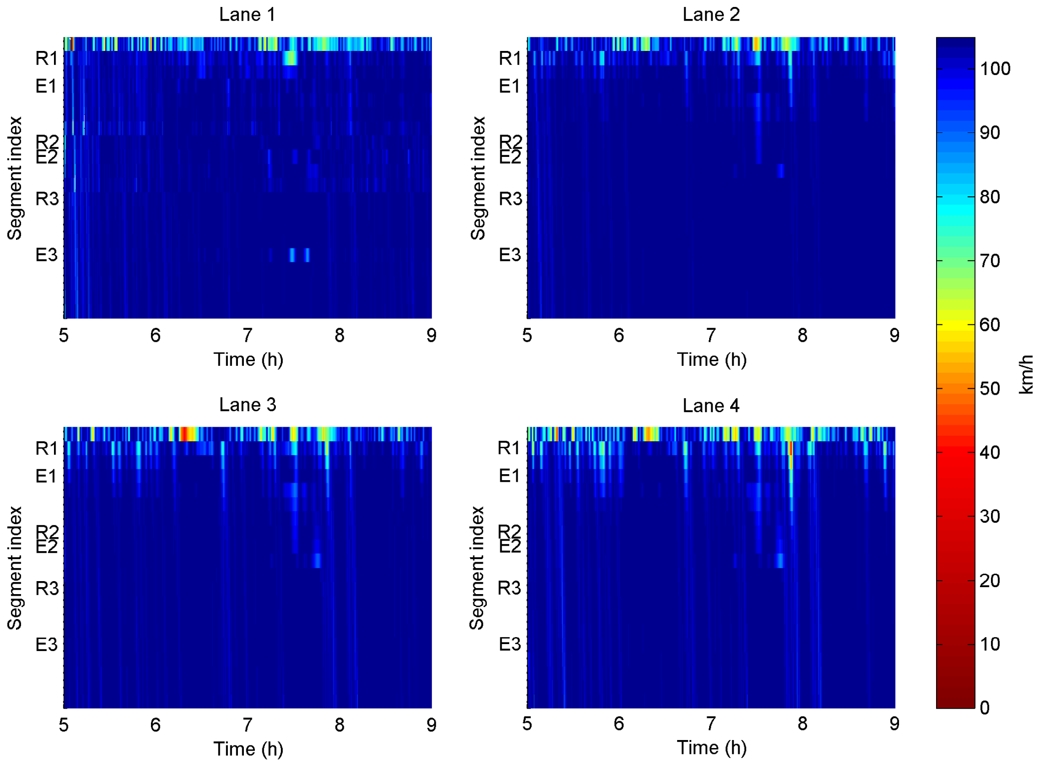

Figure 5: Contour plots of the computed speed in the optimal control case for Scenario 1.

- In the proximity of the on-ramp R2, a congestion, though not really extensive, appears in the no-control case around 7 AM due to merging. However, since the overall flow is not exceeding the capacity of the whole section, the optimiser simply designates some vehicles to move from lane 1 to lane 2 within segment 7.

- The congestion that is created in the last part of the network (which, in the uncontrolled case, is spilling back triggering stronger congestion at the upstream on-ramp locations), is here managed through a proper assignment of flows among the lanes, performing lateral movements well in advance, so as to avoid the sudden increase of density in segment 17. As it may be seen in Figure 7, some lane-changes from lane 1 to 2 are computed already at segment 12, with the purpose of not exceeding the critical density in the bottleneck area of segments 17-20, thus avoiding the capacity drop. The last part of the network is in fact, as mentioned in Section 3.1, characterised by a significant upward slope, that generates a reduction of mainstream capacity. It should be emphasised that

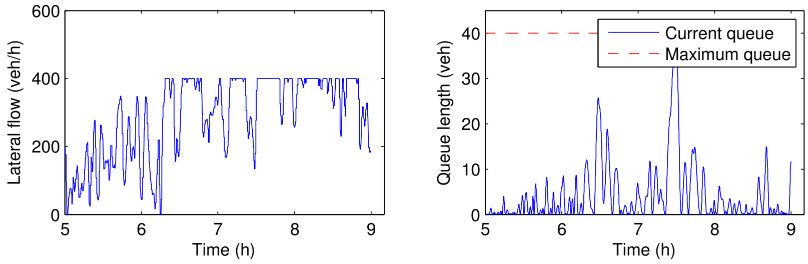

Figure 6: A graphical representation of the optimal results computed for the on-ramp R1. More specifically, in the left diagram the lateral flow from lane 1 to lane 2 at segment 1 is shown; whereas the right diagram displays the on-ramp queue at R1, where the dashed line is the maximum queue length.

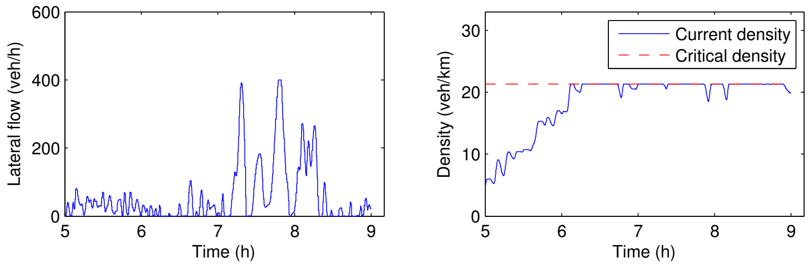

Figure 7: The left picture shows the lateral flow assigned from lane 1 to 2 in segment 11; this allows to keep the density in segment 17, shown in the right picture, below its critical value, thus avoiding the triggering of a capacity drop.

the optimiser has a complete view of the motorway scenario, thus also all other (even minor) actions performed in upstream locations may have a meaningful contribution in improving the overall performance.

3.2.2 Scenario 2

As it can be noticed by the description of the control actions taken in Scenario 1, the coordinated use of RM and LCC is sufficient to completely avoid any congestion, causing a significant reduction of TTS. In this second demonstration example, the demand is increased in order to obtain a scenario where the previously taken control actions are not sufficient to

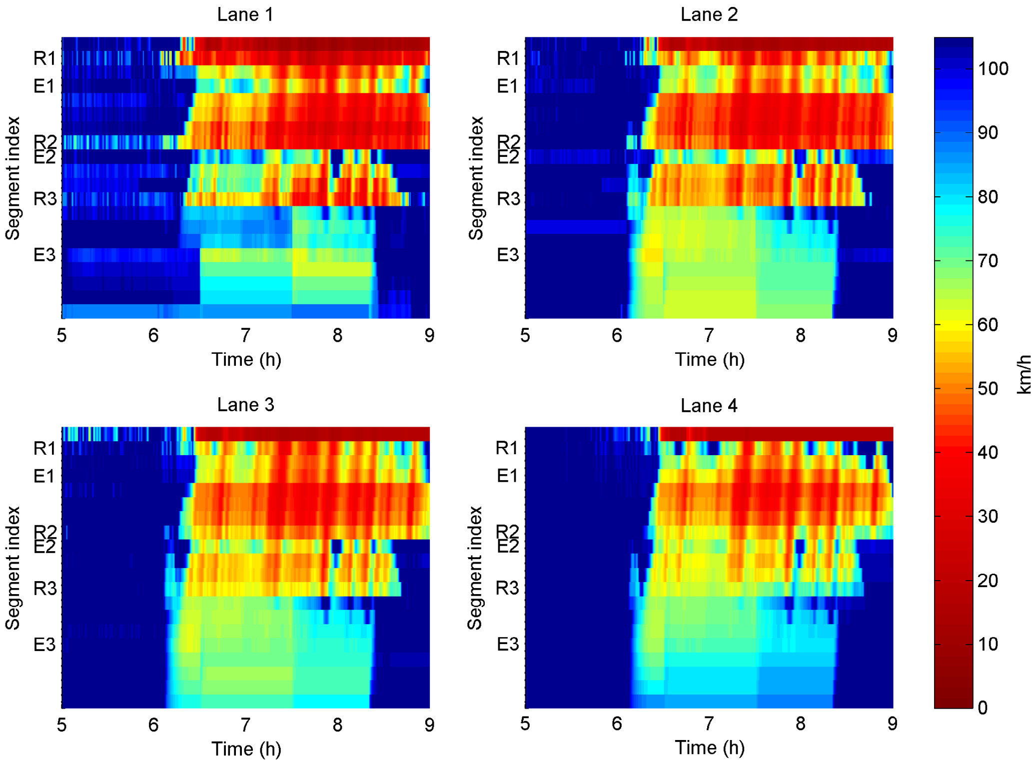

Figure 8: Contour plots of the speed in the no-control case for Scenario 2.

avoid a congestion, expecting therefore that also MTFC actions will be performed. More specifically, the demand at on-ramps R2 and R3 (Figure 4) is increased by 30%.

For this scenario, the no-control case results are shown in the contour plots of Figure 8, and the TTS part of the cost function is $Z^*_{TTS} = 3561$ veh·h. The optimal control problem solution yields a value $Z^*_{TTS} = 1700$ veh·h, which is an improvement of 52.2% with respect to the no-control case.

As expected, the improvement is obtained also because some MTFC actions are performed in order to cope with the increased flow entering from the on-ramps R2 and R3. The overall improvement is again clearly visible through a comparison of the respective contour plots in Figures 8 and 9; in fact, it can be observed that the local speed actions taken upstream of the two on-ramps R2 and R3 manage to avoid the creation of a larger congestion, contributing in the amelioration of traffic conditions. These local control actions are here briefly analysed and discussed:

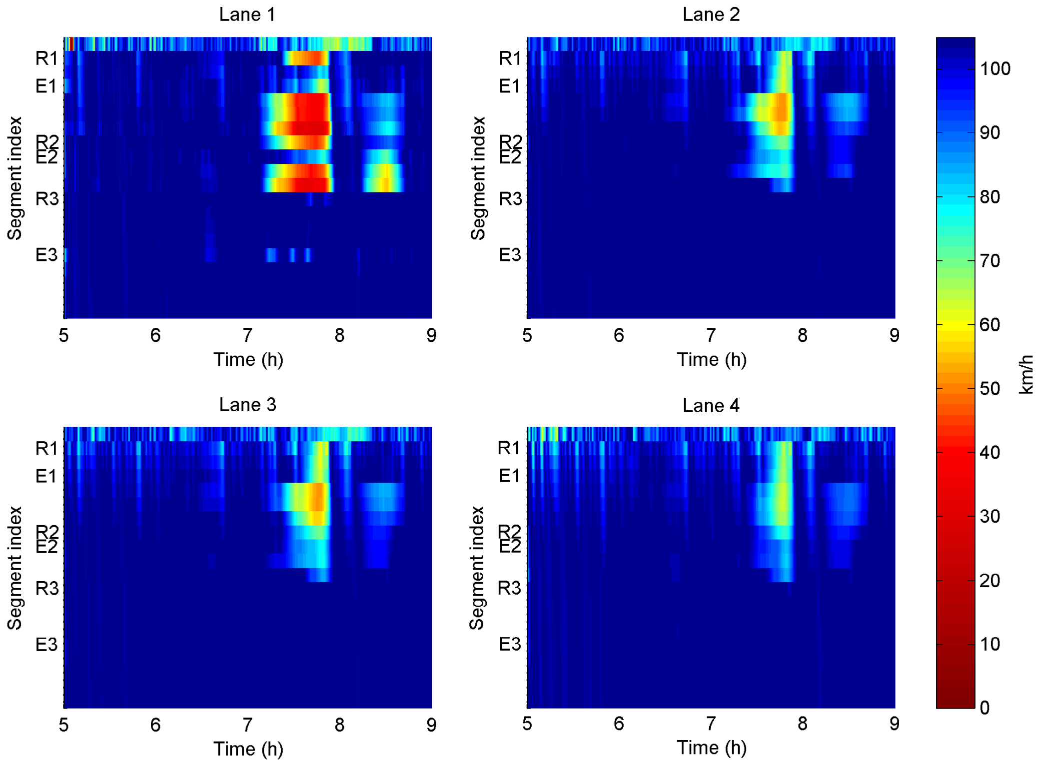

Figure 9: Contour plots of the computed speed in the optimal control case for Scenario 2.

- The congestion forming in the area of the on-ramp R1 is addressed in a similar way as in Scenario 1, i.e. via synergistic combined RM and LCC actions.

- Because of the increased demand at on-ramps R2 and R3, a coordinated exploitation of the three available control measures is performed in a similar way for both the bottleneck locations. In fact, strong RM actions are needed in the periods of high demand, causing the queues to approach their maximum values (Figure 10a); in addition, lateral movements are performed in the segments immediately upstream the merging areas (the ones characterised by a penalty cost $\beta_{i,j,\bar{j}} = 0$), sending vehicles from lane 1 to 2 (Figure 10b). While the ramp queues are approaching their maximum values, also some strong MTFC actions appear in the upstream segments (Figure 10c); because of the specific penalty term in the cost function, these actions do not feature strong oscillation among consecutive segments. The MTFC actions reduce the flow entering the merging area, avoiding the capacity drop and causing again to obtain the corresponding

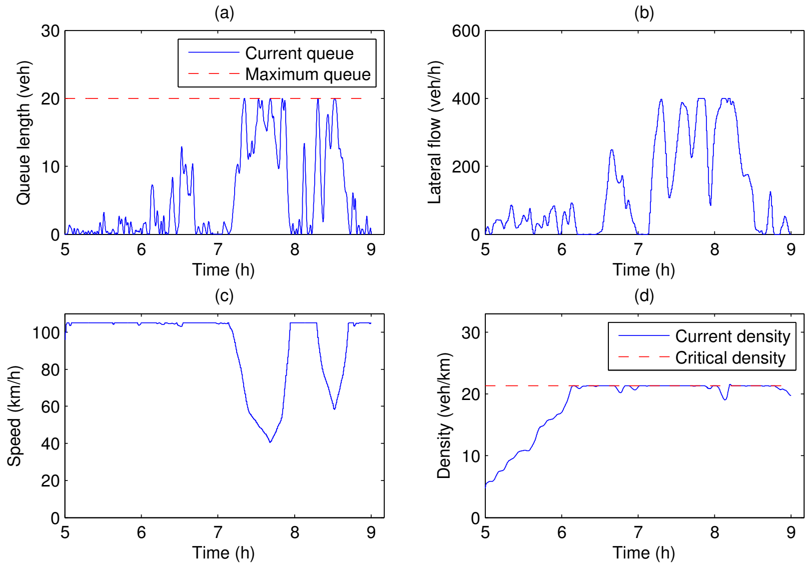

Figure 10: The queue length generated at on-ramp R3 is shown in (a); the lateral movement assigned from lane 1 to 2 of segment 11 is displayed in (b); the speed action performed in segment 11 lane 1 is shown in (c); the density achieved at the final bottleneck location (segment 17) is shown in (d).

critical density (hence maximum flows) at the final bottleneck area of segment 17-20 (Figure 10d).

3.3 Implementation and computation remarks

The optimisation problem has been implemented in MATLAB, using the optimisation solver Gurobi (Gurobi Optimization, 2013), exploiting an algorithm based on the barrier method for QP problems. All the experiments were performed on a personal computer equipped with a processor Intel® Core i5 3.20 GHz and 4 GB of RAM; it should be clear that, obviously, better results in terms of computation time could be achieved using a more powerful workstation. Considering the large size of this problem and the structure of the variables and constraints, very sparse matrices occur, and the employed solver accounts efficiently for this fact. More specifically, the size of the optimisation problem may be computed as following:

- the number of state variables is $N^S = 3\, IJK$;

Optimisation horizon [min]	240	60	30	15
Computation time [s]	592	84	19	10

Table 2: Comparison in terms of computation time in case different different optimisation horizons are considered.

- the number of control variables is $N^C = IJK^Q + IJK^R + 2\,IJK^F + IJK$;
- the number of equality constraints is $N^E = 3\,IJK$;
- the number of inequality constraints is $N^I = 3\,IJK + 2\,IJK$.

For the described example, characterised by $I = 20$, $J = 4$, $K = 1920$, and $K^Q = K^F = K^R = 480$, the following values are obtained: $N^S = 460,800$, $N^C = 307,200$, $N^E = 460,800$, and $N^I = 768,000$. Despite the large size of this optimisation problem (mainly due to the very large optimisation horizon considered), the solver was able to find the optimal solution in a reasonable time (in all the tested scenarios, some minutes; see also Table 1).

As mentioned earlier, the presented methodology could provide a basis for real-time control; this does not appear to be feasible in view of the computation times reported in Table 1. However, the use of optimal control in real-time is usually materialised via a MPC scheme, as envisaged by Burger et al. (2013), whereby the required (rolling) optimisation horizon is in the order of the time needed to drive the considered motorway stretch (rather than the 4-h horizon used here to demonstrate the method); while the update period (for re-computation of the solution) is surely not less than the control time step. Thus, to investigate this issue, the possibility of decreasing the optimisation horizon is considered. The experiments made in this direction led to the results shown in Table 2. A reasonable target is to be able to obtain the optimal solution in a time that is smaller than the control time step, and this is indeed readily achievable as Table 2 indicates.

Interested readers may consult the paper by Roncoli et al. (2014) for optimal control results obtained with the same methodology, but for a different (real) motorway infrastructure that features partly different phenomena and control actions.

4 Conclusions

An optimal control problem has been developed for flow maximisation in multi-lane motorways in presence of VACS. The traffic flow model presented by Roncoli et al. (2015) has been utilised to define the constraints of a QP problem for the coordinated application of ramp metering, variable speed limits, and lane changing control. The choice of introducing some simplifications in the traffic flow model has permitted the formulation of an optimisation problem with only linear constraints that is, as a consequence, solvable in a reasonable computation time.

The optimal control has generated important and useful results, showing that the use of VACS could enable strong benefits for the traffic conditions, alleviating congestion and consequently improving safety. The results of this paper could be therefore utilised as a starting point to better understand the strategies that must be exploited when defining control actions in presence of VACS.

While some other approaches considering "intelligent vehicles", e.g. the AHS (Varaiya, 1993), envision the creation of a novel highway infrastructure, dedicated exclusively to autonomously driven vehicles, the assumption made in this model is the coexistence of fully or partly automated vehicles with manually driven cars. In fact, the proposed control strategy leads to throughput maximisation by reducing (or even avoiding) congestion phenomena, irrespectively of the characteristics of the vehicles on the motorway. Further improvement may be achieved by the adoption of proper control strategies at the vehicle level (e.g., considering vehicles equipped with ACC or CACC systems); the adoption of these systems may also lead to a substantial modification of the traffic behaviour (e.g., obtaining different capacities at different critical densities), that can nevertheless be accounted for via a proper definition of the FD structure and parameters, without compromising the effectiveness of the approach.

Since the direct application of these results is far from reality, because several years may pass before a sufficiently large amount of vehicles are equipped with the necessary devices, the only opportunity to test this control strategy is by use of appropriate simulations. Ongoing work implements the optimisation problem in a MPC framework and applies the results to a microscopic simulator for various cases of penetration rate of VACS. In order to apply the computed optimal strategies and react faster to the perturbations of traffic conditions, a hierarchical control strategy could be applied, solving the QP problem in a high layer and introducing a lower layer that includes a set of local feedback controllers, to obtain a faster reaction to traffic disturbances (see e.g. Papamichail et al., 2010).

Acknowledgement

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