EVALUATION OF PHY-LAYER MULTICASTING ALGORITHMS USING MEASURED CHANNEL DATA

DIPLOMA THESIS
SUBMITTED TO THE DEPARTMENT OF
ELECTRONIC ENGINEERING AND COMPUTER
ENGINEERING
BY

Georgina Abou-Elkheir

Advisor : Sidiropoulos Nikolaos
Co-advisor : Liavas Athanasios
Co-advisor : Karistinos Georgios

Technical University of Crete
October 2007

Contents

Chapter 1

Introduction

In recent years, multi-input,multi-output (MIMO) systems have received a lot of attention thanks to their ability to achieve improved performance and capacity results for wireless communications by exploiting spatial diversity techniques.

In general, diversity techniques lead to an increase in the number of degrees of freedom in a system . Spatial or in other words antenna diversity refers to the use of multiple antennas at the transmitter (transmit diversity) and/or receiver (receive diversity).

Transmit diversity techniques are of crucial importance especially in scenarios where a base station (multiantenna transmitter) transmits information to several co-channel users, each one equipped with a single antenna element. In this point, note that the above setup is a typical real case scenario, for it is more cost effective to use multiple antennas in a central point. Furthermore, it is well known that today's commercial systems provide mobile devices which are power and space restricted, thus prohibiting the use of multiple antenna elements.

Several techniques and applications have been developed to exploit transmit diversity such as zero force beamforming, power control, minimum squared error beamforming, multicast transmit beamforming etc. Each one of these techniques handles the downlink processing of a MIMO system from a different point of view and by using different criteria and constraints. For instance some algorithms try to suppress multiuser interference while some other algorithms try to guarantee acceptable quality of service to all users.

This thesis is concerned with the design and evaluation of multicast transmit beamforming algorithms from the viewpoint of guaranteeing acceptable quality of service to all users. Multicast services and applications have emerged in recent wireless network technologies, such as the 802.16 standard, and UMTS-LTE.

Specifically, in this thesis we study transmit beamforming to multiple co-channel multicast groups and multicast transmit beamforming with sidelobe constraints.
In the transmit beamforming to multiple co-channel multicast groups algorithm we seek optimal beamforming vectors, one per group, so that all members within each group obtain acceptable quality of service. Note that independent information is transmitted to members of different groups . On the other hand, the multicast transmit beamforming with sidelobe constraints algorithm is an extension of the multicast transmit beamforming problem [5]. In this case, our goal is the design of a beamformer that transmits and guarantees quality of service to specific users, while limiting interference to all others.

As we will see later on, both problems can be approximately solved using semidefinite optimization and highly efficient algorithms such as interior point methods whose complexity grows as a polynomial function of the problem size.

1.1 Quality of Service

As previously mentioned, in this thesis we focus on algorithms whose primary issue is to guarantee quality of service to all users. We assume that different users may require different quality of service. This is a very reasonable assumption. As an example consider the Internet. New applications such as streaming and voice over IP have forced internet to alter its basic approach to provide best effort services to all users. Users may require different types of service such as HTTP requests, video streaming etc. each with different quality of service requirements.
Furthemore, in this thesis we have to describe the quality of service constraint in a quantitative manner. In general, quality of service is a quality measure with many alternative definitions. For example, one could express the quality of service in terms

of delay, bit error rate, mean square error or signal to interference plus noise ratio (SINR). In our study, we use the latter as an effective measure of the quality of service since it determines the maximum achievable data rate and probability of error.

1.2 System Model

Consider a wireless scenario, where a base station is equipped with M antenna elements and simultaneously serves G multicast groups. Assume that each receiver is equipped with a single antenna element.
The transmitted signal can be written as:

$$\mathbf{x} = \sum_{k=1}^{G} b_k \mathbf{w}_k^H \tag{1.1}$$

where $\mathbf{w}_k$ is the M-dimensional beamforming vector and b_k is the transmitted information stream intended for the k-th group.
The received signal at a receiver of the i-th multicast group can be written as:

$$r_i = \sum_{k=1}^{G} b_k \mathbf{w}_k^H \mathbf{h}_i + n_i \tag{1.2}$$

where n_i represents noise, and $\mathbf{h}_i$ denotes the $M \times 1$ channel vector.
Note that equation (1.2) can be also written in the following form:

$$r_i = b_i \mathbf{w}_i^H \mathbf{h}_i + \sum_{j \neq i}^{G} b_j \mathbf{w}_j^H \mathbf{h}_i + n_i \tag{1.3}$$

By observing equation (1.3), we can clearly see that the user of the i-th multicast group obtains not only its desired signal, but also interference from other co-channel multicast groups. Specifically the term $b_i \mathbf{w}_i^H \mathbf{h}_i$ represents the desired signal and the term $\sum_{j \neq i}^{K} b_j \mathbf{w}_j^H \mathbf{h}_i$ represents interference.

In general, transmit beamforming affects not only the performance of a desired user but also the performance of all co-channel users. Consequently, unless designed jointly, transmit beamforming could lead to an overall degradation of the network's performance.

1.3 Channel State Information

Throughout our study, we assume that the transmitter can acquire perfect channel state information (CSI). For instance in time division duplex systems (TDD) where the uplink channel is reciprocal of the downlink channel, we can use training data to obtain instantaneous channel estimates.

In general, we assume that a feedback link sends channel state information from the receiver back to the transmitter. Furthermore, we assume that this link is both delay and error free. However, we should note that this is not a realistic scenario since it is almost certain that delays, estimation and quantization errors as well as channel variations will occur, thus leading to a transmitter that possesses outdated or false channel state information most of the time.

1.4 Introduction to convex optimization theory

In recent years, several breakthroughs and developments in algorithms have fueled new interest in convex optimization theory and convex optimization has become a tool of central importance in engineering

Convex optimization has several desirable properties that makes it tractable and suitable in solving optimization problems. First of all, there exist no local minimum which is not global. Therefore, once we find an optimum we are ensured that it is necessarily a global optimum and the problem is solved. Furthermore, we can benefit from results in duality theory to detect the feasibility or not of a problem. Finally, several numerical solution methods such as the interior point methods can handle very large problems in polynomial time.

Unfortunately, in practice non-convex problems turn out to be more common than convex problems. In such cases, we have to devise an approximation of the original problem by dropping or introducing constraints, changing variables etc. In general, obtaining a good approximation of the original problem is a non trivial task. Below we give a brief introduction to the basic concepts of convex optimization theory.

1.4.1 Convex sets and functions

A set C is said to be convex if for any $x, y \in C$ and $0 \leq \theta \leq 1$

$$\theta x + (1-\theta)y \in C \tag{1.4}$$

In other words, a set C is convex if for any points $x, y\in$ C, the line segment between them also lies in C.

In a similar fashion, a function f is said to be convex over a interval (a,b) if for any $x, y \in$ (a,b) and $0 \leq \theta \leq 1$,

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y) \tag{1.5}$$

If equality holds only if $\theta = 1$ or $\theta = 0$, then f is said to be strictly convex.

If f is convex and continuously differentiable, we can use the first order Taylor series approximation to obtain a global underestimator of the function,

$$f(y) \geq f(x) + \bigtriangledown f(x)^T(y-x) \tag{1.6}$$

A convex function has no local minima which is not global. This is one of the key properties that make convexity so special and attractive in optimization.

Affine, quadratic and positive semidefinite functions as well as the Euclidean norm, are all important elementary convex classes that arise frequently in optimization.

1.4.2 Convex Cones

A set C is a convex cone $\mathcal{K}$, if for each $x \in \mathcal{K}$ and each $a \geq 0,\ ax \in \mathcal{K}$.
A convex cone of special interest is the positive semidefinite cone,

$$\mathcal{K} = \{\mathbf{X}|\mathbf{X} \text{ symmetric and } \mathbf{X} \succeq \mathbf{0}\} \tag{1.7}$$

1.4.3 Convex Optimization

Let us consider the following optimization problem:

$$\min\ f_o(x) \tag{1.8}$$

$$\text{subject to}: f_i(x) \leq 0 \quad i = 1, \cdots, m \tag{1.9}$$

$$h_i(x) = 0 \quad i = 1, \cdots, p \tag{1.10}$$

Our goal is to find an x that minimizes the objective function $f_o(x)$ while satisfying all equality and inequality constraints.

Problem (1.8),(1.9),(1.10) is said to be feasible if there exists at least one x for which all equality and inequality constraints are satisfied. In the opposite case the problem is said to be infeasible.

A convex optimization problem is defined as one in which the objective function is convex, the inequality constraint functions are convex and the equality constraint functions are affine.

Optimization problems in which the unknown variable is not a vector but a symmetric matrix which is required to be positive semidefinite are semidefinite optimization problems. Below we give the standard form of a semidefinite problem (SDP):

$$\min\ \mathbf{CX}$$

$$\text{subject to}:\ \mathbf{A}_i\mathbf{X} = \mathbf{b}_i\ i = 1, \cdots, m \tag{1.11}$$

$$\mathbf{X} \succeq 0$$

where $\mathbf{A}_i \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^m$, $\mathbf{C} \in \mathbb{R}^{n \times n}$, $\mathbf{X} \in \mathbb{R}^{n \times n}$

1.5 Outline of the Thesis

In this thesis, we study physical-layer multicasting algorithms and conduct several experiments using outdoor measured channel data to evaluate their performance.

In chapter 2 we study the problem of transmit beamforming to multicast co-channel multicast groups proposed in [6]. We start with a description of the problem model and next we present the problem formulation. In the third section, we give an analytical description of the proposed relaxation. The fourth section describes the Gaussian randomization technique we use to convert the solution of the relaxed problem to a proper solution for the original problem. Next, we give a detailed description of the algorithm proposed in [6]. In sections 6 and 7 we describe the measurement campaign and the measured channel data we use to conduct our experiments. In section 8 we present the various experiments we have conducted and their results. Finally, in the last section we discuss the results and obtain some conclusions.

In chapter 3, we study the problem of multicast transmit beamforming with side-lobe constraints. At first, we describe the problem and give its formulation. In sections 2 and 3 we give a detailed description of the relaxation and the algorithm we propose to obtain apprimate solutions for the original problem. Next, we present the results we have obtained from various experiments and Monte Carlo simulations. In the last section, we discuss the results and draw some conclusions.

In the final chapter, we give some final conclusions regarding both algorithms.

Chapter 2

Transmit Beamforming to multiple co-channel multicast groups

In this chapter we are concerned with the problem of transmit beamforming to multiple co-channel multicast groups. We give a detailed description of the problem statement and the relaxation proposed in [6]. Finally we test the algorithm under several wireless scenarios using measured channel data

2.1 System Model

Consider a wireless scenario, where a multi-antenna transmitter (base station) simultaneously transmits information to G multicast groups. Note, that the transmitter sends common information to users participating in the same multicast group, and independent information to users of different multicast groups. Assume, that each user is equipped with a single antenna element and participates to a single multicast group. Furthermore, assume that that the transmitter is equipped with N antenna elements.

Let $\mathcal{G}_k$ denote the set of all users allocated in multicast group k and M denote the total number of users in the system.

Clearly:

$$\sum_{k=1}^{G} G_k = M \tag{2.1}$$

where $G_k := |\mathcal{G}_k|$ and $G \in \{1, \cdots, M\}$

Cases of special interest arise when $G = 1$ and $G = M$. Specifically, when $G = 1$, the transmitter sends common information to all users in the system. This is a broadcasting scenario. On the other hand, when $G = M$ the transmitter sends independent information to each user and thus we are dealing with the SINR constrained downlink beamforming problem.

2.2 Problem Statement

Let $\mathbf{h}_i$ denote the $N \times 1$ channel vector for user i and $\mathbf{w}_k^H$ denote the $N \times 1$ beamforming vector generated for multicast group k . Assume that the channel response is frequency flat for all M users. Furthermore, assume that all $N \times 1$ channel vectors are independent of each other.

If all information signals $\{s_k(t)\}_{k=1}^{G}$ are uncorrelated and furthermore if they are zero mean with unit variance, then the total transmitted power can be written as:

$$\sum_{k=1}^{G} ||\mathbf{w}_k||_2^2 \tag{2.2}$$

The SINR at each link can be expressed as:

$$\text{SINR} \ = \ \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{\ell \neq k} |\mathbf{w}_\ell^H \mathbf{h}_i|^2 + \sigma_i^2} \tag{2.3}$$

The beamforming problem can be formulated as one in which the total power radiated by the transmitter is minimized, while the SINR at each user is greater than a target value.

$$
\min_{\{\mathbf{w}_k \in \mathbb{C}^N\}_{k=1}^G} \sum_{k=1}^{G} \|\mathbf{w}_k\|_2^2
$$

$$
\text{s.t} \;:\; \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{\ell \neq k} |\mathbf{w}_\ell^H \mathbf{h}_i|^2 + \sigma_i^2} \geq c_i, \;\; \forall i \in \mathcal{G}_k, \forall k \in \{1, \cdots, G\} \tag{2.4}
$$

where c_i denotes the prescribed minimum SINR at each receiver.

As mentioned earlier, the above setup includes the broadcasting scenario as a special case which as shown in [5] is $NP - hard$. Loosely speaking, this means that we are not likely to devise an algorithm which will solve problem (2.4) both efficiently and optimally, which motivates seeking effective approximate solution.

2.3 Relaxation

Problem (2.4) is a quadratic optimization problem with quadratic non convex constraints. In order to convert problem (2.4) in a convex form, we introduce the following matrices:

$$
\mathbf{Q}_i := \mathbf{h}_i \mathbf{h}_i^H \tag{2.5}
$$

and

$$
\mathbf{X}_k := \mathbf{w}_k \mathbf{w}_k^H \tag{2.6}
$$

Furthermore, by using the rule $trace(AB) = trace(BA)$ we can write:

$$
\begin{aligned}
|\mathbf{w}_k^H \mathbf{h}_i|^2 &= \mathbf{h}_i^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{h}_i \\
&= trace(\mathbf{h}_i^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{h}_i) \\
&= trace(\mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_k \mathbf{w}_k^H) \\
&= trace(\mathbf{Q}_i \mathbf{X}_K)
\end{aligned} \tag{2.7}
$$

Problem (2.4) is reformulated equivalently as follows:

$$\min_{\{\mathbf{X}_k \in \mathbb{C}^{N\times N}\}_{k=1}^{G}} \sum_{k=1}^{G} trace(\mathbf{X}_k)$$

$$\text{s.t} \;:\; trace(\mathbf{Q}_i\mathbf{X}_k) \geq c_i \sum_{\ell\neq k} trace(\mathbf{Q}_i\mathbf{X}_l) + c_i\sigma_i^2, \;\; \forall i \in \mathcal{G}_k, \forall k \in \{1, \cdots, G\} \tag{2.8}$$

$$\mathbf{X}_k \succeq \mathbf{0}$$

$$\text{rank}\,(\mathbf{X}_k) = 1, \forall k \in \{1, \cdots, G\}$$

The notation $\mathbf{X}_k \succeq \mathbf{0}$ means that $\mathbf{X}_k$ is positive semidefinite. Note that the positive semidefinite constraint plus the rank one constraint ensure that $\mathbf{X}_k := \mathbf{w}_k\mathbf{w}_k^H$.

Problem (2.8) has a linear objective function, linear trace inequalities and a convex positive semidefinite constraint. However the rank one constraint is nonconvex. By dropping the rank one constraint, we obtain the following relaxation of the original problem:

$$\min_{\{\mathbf{X}_k \in \mathbb{C}^{N\times N}\}_{k=1}^{G}} \sum_{k=1}^{G} trace(\mathbf{X}_k)$$

$$\text{s.t} \;:\; trace(\mathbf{Q}_i\mathbf{X}_k) - c_i \sum_{\ell\neq k} trace(\mathbf{Q}_i\mathbf{X}_l) \geq c_i\sigma_i^2, \;\; \forall i \in \mathcal{G}_k, \forall k \in \{1, \cdots, G\} \tag{2.9}$$

$$\mathbf{X}_k \succeq \mathbf{0}, \forall k \in \{1, \cdots, G\}$$

In order to put problem (2.10) in the standard semidefinite form (1.11), we have to transform the inequality constraints into equality constraints. Therefore, we introduce M nonnegative slack variables and problem (2.10) can be written as:

$$\min_{\{\mathbf{X}_k \in \mathbb{C}^{N\times N}\}_{k=1}^{G}, \{s_i \in \mathbb{R}\}_{i=1}^{M}} \sum_{k=1}^{G} trace(\mathbf{X}_k)$$

$$\text{s.t} \;:\; trace(\mathbf{Q}_i\mathbf{X}_k) - c_i \sum_{\ell\neq k} trace(\mathbf{Q}_i\mathbf{X}_l) - s_i = c_i\sigma_i^2, \;\; \forall i \in \mathcal{G}_k, \forall k \in \{1, \cdots, G\} \tag{2.10}$$

$$s_i \geq 0, \forall i \in \{1, \cdots, M\}$$

$$\mathbf{X}_k \succeq \mathbf{0}, \forall k \in \{1, \cdots, G\}$$

Problem (2.10) is a semidefinite programming problem (SDP) and we can solve it efficiently by using interior point methods. Interior point methods are implemented in several software packages. In our study, we use the MATLAB Sedumi toolbox to solve problem (2.10) at a complexity cost that is at most $O((GN^2 + M)^{3.5})$

2.4 Randomization

In general, the semidefinite relaxed problem (2.10) is not equivalent to the original problem.

If problem (2.10) yields a feasible solution $\{\mathbf{X}_k\}_{k=1}^G$, then $\{\mathbf{X}_k\}_{k=1}^G$ is the optimal solution to the original problem only if it satisfies the rank one constraint. However, in the general case, if problem (2.10) turns out to be feasible, it is expected to yield a solution $\{\mathbf{X}_k\}_{k=1}^G$ of higher rank. In this case, $\{\mathbf{X}_k\}_{k=1}^G$ will give a lower bound on the cost function of the original problem. The reason is that by dropping the rank one constraint, we have enlarged the feasibility region of the problem.

However, once the solution to the semidefinite relaxed problem is determined, the optimal solution to the original problem can be approximated. The process we use to convert the solution of the relaxed problem into a suitable solution of the original problem is called *randomization*. Several randomization techniques have been proposed in the literature. In our study, we use the Gaussian randomization technique

Gaussian Randomization is an iterative process. At each iteration, we use $\mathbf{X}_k$ to compute a candidate beamforming vector $\mathbf{w}_l$. Below, we give a brief description of its steps.

First, we calculate the eigenvalue decomposition $\mathbf{U\Sigma U^H}$ of $\mathbf{X}_k$. Then, at each iteration we generate a random, zero mean, unit variance, complex circularly symmetric Gaussian vector $\mathbf{v}_l$ and compute the candidate beamforming vector $\mathbf{w}_l$ as follows:

$$\mathbf{w}_l = \mathbf{U}\boldsymbol{\Sigma}^{\frac{1}{2}}\mathbf{v}_l \tag{2.11}$$

Each iteration produces a candidate beamforming vector which may be worse or better than previous checked candidate vectors. In the following section we explain under which criteria we choose the best candidate beamforming vector.

2.5 The Multi-Group Power Control Problem

Gaussian randomization was used in [5], to obtain candidate beamforming vectors for the broadcasting scenario. Furthermore, in order to satisfy the constraints, all candidate beamforming vectors were scaled to the minimum length necessary. Unfortunately, in our study we cannot adopt the above strategy since scaling up one group's beamforming vector would increase interference to other groups.

To this end, let $\alpha_{k,i} := |\mathbf{w}_k^H h_i|^2$ and $\beta_k := ||\mathbf{w}_k||^2$. For each multicast group, we seek the minimum power boost for which no constraint will be violated. Thus, we can formulate the following Multi-Group Power Control problem [6]:

$$\begin{aligned} &\min_{\{p_k \in \mathbb{R}\}_{k=1}^G} \sum_{k=1}^{G} \beta_k p_k \\ \text{subject to : } &\frac{p_k \alpha_{k,i}}{\sum_{\ell \neq k} p_l \alpha_{l,i} + \sigma_i^2} \geq c_i, \ \forall i \in \mathcal{G}_k, \forall k \in \{1, \cdots, G\} \\ &p_k \geq 0, \forall k \in \{1, \cdots, G\} \end{aligned} \tag{2.12}$$

Introducing M slack variables in order to convert the inequalities expressed in (2.12) into equalities, results in the following reformulation:

$$\begin{aligned} &\min_{\{p_k \in \mathbb{R}\}_{k=1}^G, \{s_i \in \mathbb{R}\}_{i=1}^M} \sum_{k=1}^{G} \beta_k p_k \\ \text{subject to :} &p_k \alpha_{k,i} - c_i \sum_{\ell \neq k} p_l \alpha_{l,i} - s_i = c_i \sigma_i^2, \forall i \in \mathcal{G}_k, \forall k \in \{1, \cdots, G\} \\ &p_k \geq 0, \forall k \in \{1, \cdots, G\} \\ &s_i \geq 0, \forall i \in \{1, \cdots, M\} \end{aligned} \tag{2.13}$$

Problem (2.13) has a linear objective function and linear constraints. Note that the best candidate beamforming vector, is the one that yields the minimum objective : $\sum_{k=1}^{G} \beta_k p_k$.

The overall algorithm is summarized below:

1. Solve the semidefinite relaxation problem (2.10). Let $\{\mathbf{X}_k\}_{k=1}^{G}$ denote its solution.

2. If $\text{rank}(\mathbf{X}_k) \neq 1$ for any k, use Gaussian randomization to generate a set of candidate beamforming vectors. Let $\{\mathbf{w}_k\}_{k=1}^{G}$ denote the set of candidate beamforming vectors. If $\text{rank}(\mathbf{X}_k) = 1$ use the principal component of $\mathbf{X}_k$.

3. Use the set $\{\mathbf{w}_k\}_{k=1}^{G}$ to solve the Multi-Group Power Control problem. If, some set $\{\mathbf{w}_k\}_{k=1}^{G}$ yields the minimum objective function compared to all previous checked, store the solution as well as the associated objective function.

Note that for $G = M$, the transmitter sends independent information to each receiver and the relaxed problem is in fact equivalent to the original problem as shown in [1].

2.6 Testbed Description

In order to evaluate the performance of the proposed algorithm, we have conducted several experiments using measured channel data. Measured channel data were downloaded from the iCORE HCDC lab web site university of Alberta in Edmonton
(http://www.ece.ualberta.ca/ mimo/).

2.6.1 Measurement Campaign description

The HCDC Laboratory at the university of Alberta has developed a portable 4×4 MIMO testbed to obtain real time measurements of the 4×4 channel matrix $\mathbf{H}$. All measured channel gains $\mathbf{h}_{ij}$ (from transmit antenna i to receive antenna j) include not only the characteristics of the physical propagation channel but also effects caused by certain antenna configurations.

The portable 4 x 4 MIMO testbed uses a narrowband communication system operating in the unlicensed 902-928 MHz ISM band. The multiantenna structure of both receiver and transmitter consists of a set of four dipole antennas vertically polarized with adjustable antenna spacing.

At the transmitter, the four baseband signals are shaped by a raised cosine pulse with a roll-off factor of 0.31, upconverted to an intrmediate frequency of (IF) of 12.5 MHz and sampled using 50 MHz digital to analog (D/A) conversion cards. Once the outputs of the (D/A) cards are low pass filtered with a cutoff frequency of 15 MHz, the four independent IF waveforms are upconverted to the 902-928 band for transmission.

The receiver has a similar multiantenna structure. The 4 receive waveforms are firstly downconverted to an intermediate frequency (IF) of 12.5 MHz and once they are sampled by the analog to digital (A/D) cards, code synchronization is performed. The receiver performs the measurements non coherently. Finally we should note, that the chip rate of each channel is low enough to assume that the channel is not frequency selective. For a detailed description of the MIMO testbed see

2.7 The 'Quad' Dataset

The iCORE HCDC lab web site contains numerous indoor and outdoor campaign descriptions. In our study, we have used selected outdoor measurements which were collected from the Quad court illustrated in figure 2.1.

Quad is an open area surrounded by trees and buildings. Its width is about 60 meters and its length about 150 meters. The heights of the buildings surrounding the field range from 15 to 30 meters. As we can see the transmitter's location is fixed, whereas the receivers can be placed in 6 different locations. Both transmitter and receiver antennas have a $\lambda/2$ spacing. For each receiver location we have 9 data collections, each one containing about 100 4×4 channel snapshots. The 9 data collections were obtained by shifting the receiver array on a 3x3 square grid with $\lambda/4$ spacing. Finally note that three channel snapshots were recorded per second.

The proposed algorithm has been tested using an average over all grid locations

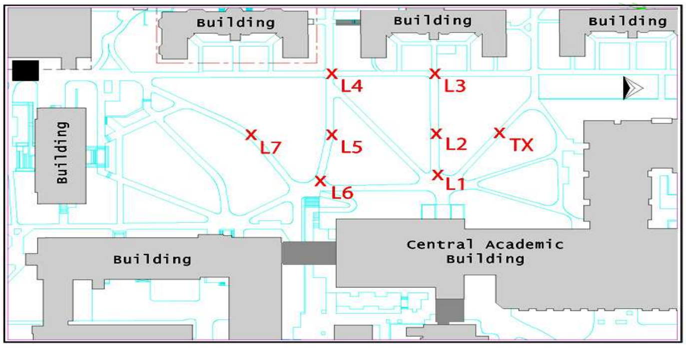


Figure 2.1: Quad Field

and over all channel snapshots. Furthermore, in order to facilitate comparison with the Rayleigh channel simulations conducted in [6], all channel gains have been normalized by the average amplitude over all snapshots.

2.8 Experimental Results

In this section, our goal is to obtain quantitative results regarding the overall performance of the algorithm and furthermore to detect the different factors that may affect it. Throughout our experiments, performance is defined in terms of:

1. The feasibility percentage of the relaxed and the multi group power control problem
2. The percentage for which the relaxed problem is equivalent to the original problem
3. The quality of approximate solutions

The feasibility of the relaxed problem is of paramount importance. Specifically, if problem (2.10) is feasible, then we can use the Gaussian randomization technique and solve the multi group power control problem to obtain an approximate solution to the original problem. On the other hand, if problem (2.10) is infeasible, then so is the original problem, which is intuitive. At this point, note that feasibility of the relaxed problem alone, does not establish feasibility of the original problem. In particular, if the multi group power control problem turns out to be infeasible, the feasibility of the original problem is not guaranteed.

Cases for which the relaxed problem is equivalent to the original problem are of great importance too. Such cases occur when the solution of the semidefinite relaxed problem is rank one. As previously discussed, the principal component of the semidefinite problem solution can be used to provide the optimum solution of the original problem.

Finally, note that the quality of approximate solutions is defined as the ratio of the total transmitted power to the lower bound of transmitted power obtained by solving the relaxed problem.

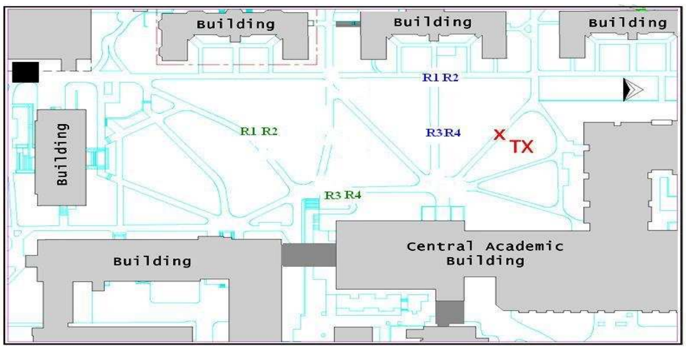
In order to detect the factors that have impact on the performance, we have used several scenarios, which were designed in the basis of:

1. the total number of multicast groups
2. the populations of the multicast groups
3. interference. Specifically, we have tested both directional and non directional scenarios. Note that directional scenarios correspond to cases where the users of each multicast group are spatially close, whereas non directional scenarios correspond to cases where all users are scattered.

Each scenario has been tested for increasing numbers of the target SINR, for 700 iterations of the randomization technique and for users with the same quality of service requirements. Furthermore, both the semidefinite relaxation problem and the linear multi power control problem have been solved using the MATLAB toolbox Sedumi. Last, note that each receive antenna at each location represents a separate user. The results are summarized in the following tables.

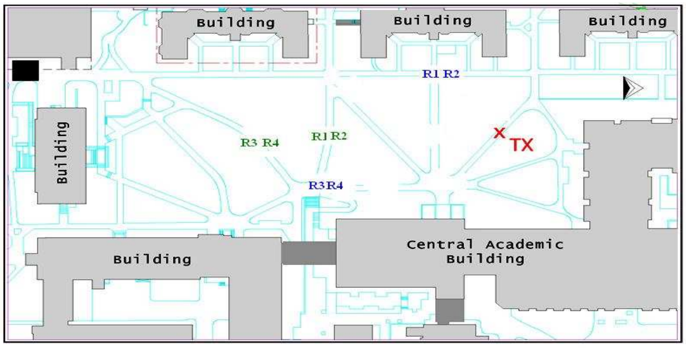
Note that column 2 reports the feasibility percentage of R, column 3 reports the percentage for which R is equivalent to I, column 4 reports the feasibility percentage of MGPC, column 5 reports the average transmitted power over the lower bound obtained from the SDR solution.

1. *Group1* Location 3: user1, user2 / Location 2: user3, user4
   Group2 Location 6: user1, user2 / Location 7: user3, user4



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	98.3274	87.1203	99.5140	1.2642
12db	87.3357	91.3817	99.5896	1.2889

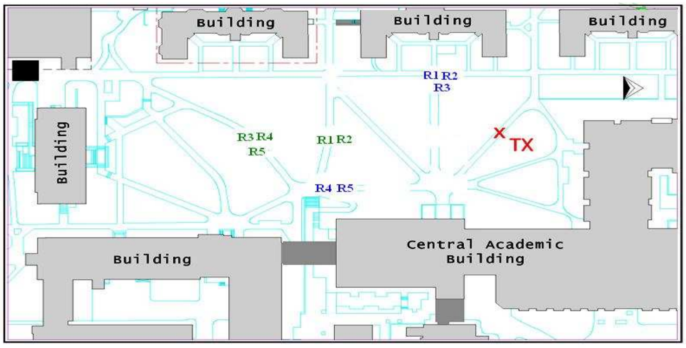
2. *Group1* user1, user2: Location 3 / user3, user4: Location 6
   Group2 user1, user2: Location 5 / user3, user4: Location 7



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	98.3274	85.7837	99.3925	1.2509
12db	88.6499	90.5660	99.4609	1.3336
14db	75.6272	96.6825	99.6840	1.3298
16db	51.3740	97.4419	100	1.7926
18db	27.7180	99.1379	99.5690	1.6830
20db	13.6201	98.2456	99.1228	3.4784
22db	5.4958	100	100	-

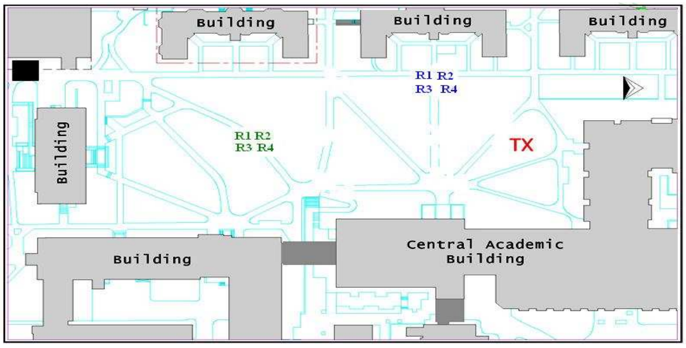
3. *Group1* user1, user2, user3: Location 3 / user4, user5: Location 6

Group2 user1, user2: Location 5 / user3, user4, user5: Location 7



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	51.2545	81.3520	98.6014	2.1182

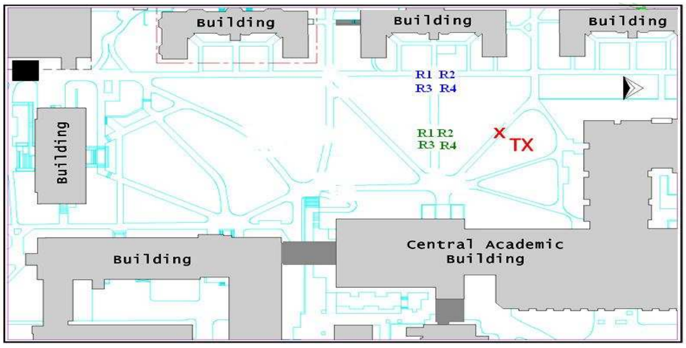
4. *Group1* user1, user2, user3, user4: Location 3
   Group2 user1, user2, user3, user4: Location 7



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	95.8184	94.1397	99.6259	1.2136
12db	91.0394	92.6509	99.6063	1.1923
14db	86.3799	92.8077	99.7234	1.7262
16db	68.3393	95.6294	99.8252	1.4273
18db	43.0108	96.9444	100	1.7063
20db	30.4659	98.8235	99.2157 2	1.2652
22db	17.0848	100	99.3007	1.2404

5. *Group1* user1, user2, user3, user4: Location 3

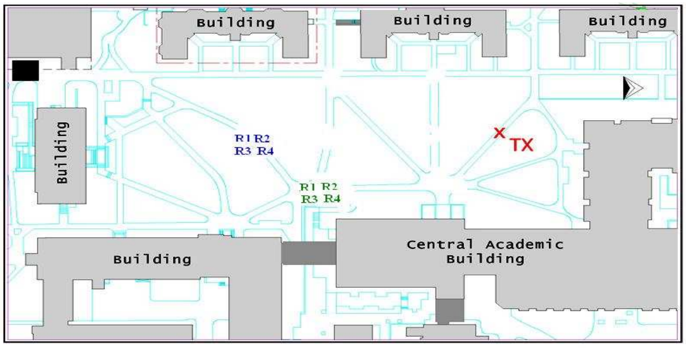
Group2 user1, user2, user3, user4: Location 2



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	100	99.4026	100	1.0508
12db	100	98.0884	99.5221	1.0316
14db	100	97.9689	99.5221	1.1045
16db	100	97.0131	99.7611	1.0631
18db	99.6416	98.0815	99.8861	1.0885
20db	96.7742	99.0123	99.7531	1.1510
22db	86.1410	99.8613	99.0291	1.0102
24db	76.8220	100	99.5334	-
26db	63.0824	99.8106	99.8106	1.1982
28db	43.8471	100	99.4550	-
30db	27.4791	100	97.5632	-
32db	15.0538	100	99.2063	-

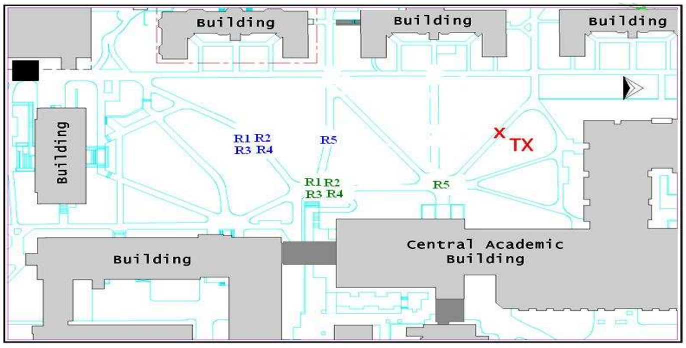
6. *Group1* user1, user2, user3, user4: Location 7

   Group2 user1, user2, user3, user4: Location 6



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	74.7909	80.319	99.3610	1.3096
12db	59.6177	87.3747	99.5992	1.5518
14db	35.8423	91.6667	93.3333	1.2460
16db	14.9343	92.8000	99.2000	1.1634

7. *Group1* user1, user2, user3, user4: Location7 / user5: Location 5
   Group2 user1, user2, user3, user4 : Location6 / user5: Location 1

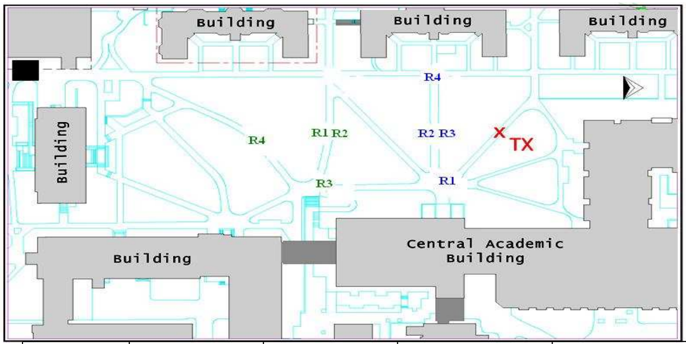


SINR	R%	R=I%	MGPC%	mean_Gauss
10db	26.1649	84.4749	96.8037	1.2825

8. *Group1* user1, user2, user3, user4, user5: Location 7
   Group2 user1, user2, user3, user4, user5: Location 6

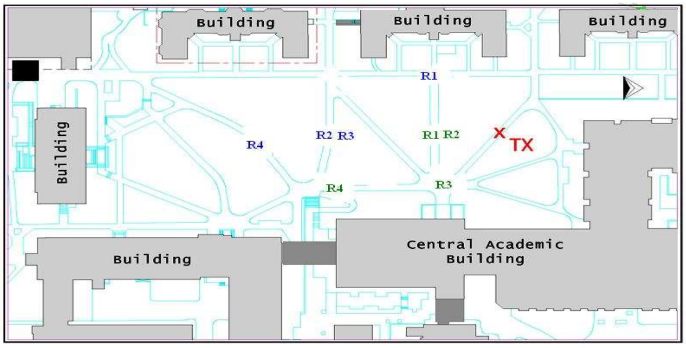
SINR	R%	R=I%	MGPC%	mean_Gauss
10db	74.7909	80.0319	99.3610	1.5620

9. *Group1* user1: Location 1 / user2, user3 : Location 2 / user4: Location 3
   Group2 user1, user2: Location 5 / user3 : Location 6 / user4: Location 7



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	95.5926	86.1935	99.2258	1.6058
12db	71.8041	89.5175	98.8353	6.4524
14db	44.5639	92.7614	97.8552	2.1041
16db	25.3286	96.2264	98.5849	1.3436

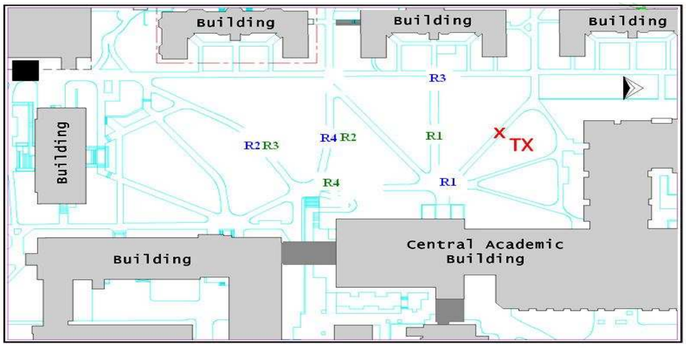
10. *Group1* user1: Location 3 / user2, user3: Location 5 / user4: Location 7
    Group2 user1, user2: Location 2 / user3: Location 1 / user4: Location 6



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	74.3130	91.1576	99.6785	1.7264
12db	49.5818	93.2530	98.7952	1.2868
14db	28.6738	97.0833	99.5833	1.7673
16db	13.1422	100	100	1.0457

11. *Group1* user1: Location 1 / user2: Location 3 / user3: Location 5 / user4: Location 7

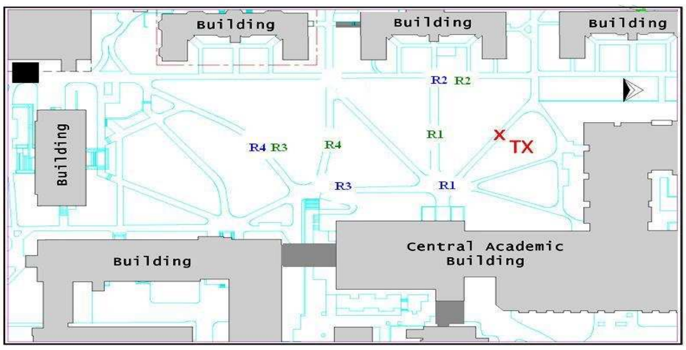
Group2 user1: Location 2 / user2: Location 5 / user3: Location 7 / user4: Location 6



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	60.0956	89.6620	99.4036 2	1.3599
12db	40.1434	91.3690	99.4048	1.4602
14db	29.8686	94	98.8000	1.7876
16db	21.5054	93.8889	99.4444	1.1631
18db	14.6953	99.1870	98.3740	1.2342

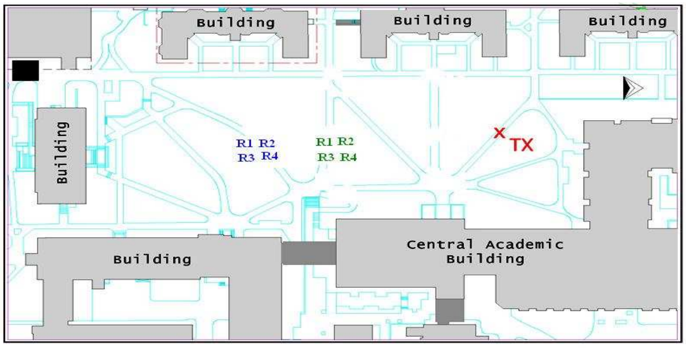
12. *Group1* user1: Location 1 / user2: Location 3 / user3: Location 6 / user4: Location 7

Group2 user1: Location 2 / user2: Location 3 / user3: Location 5 / user4: Location 7



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	53.5245	89.9554	98.6607	1.9458
12db	38.1123	96.2382	99.3730	2.3394
14db	27.5986	97.8355	98.7013	8.3910
16db	18.5185	99.3548	99.3548	6.3769

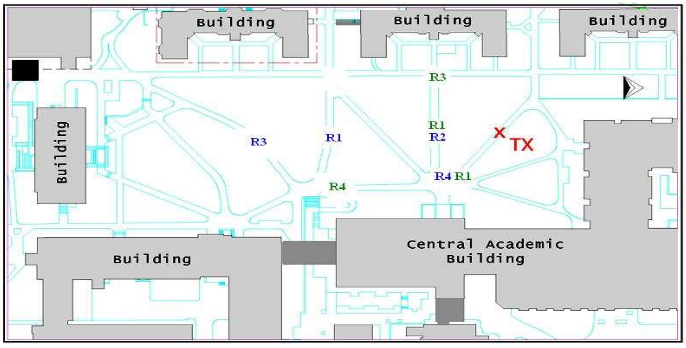
13. *Group1* user1, user,user3,user4 : Location 7
    Group2 user1, user2, user3, user4 : Location 5



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	81.2425	86.6176	99.5588	1.4035
12db	49.8208	90.4077	99.2806	1.7554
14db	18.8769	91.7722	100	1.2302

14. *Group1* user1: Location 5 / user2: Location 2 / user3: Location 7 / user4: Location 1

Group2 user1: Location 3 / user2: Location 2/ user3: Location 1/ user4: Location 6

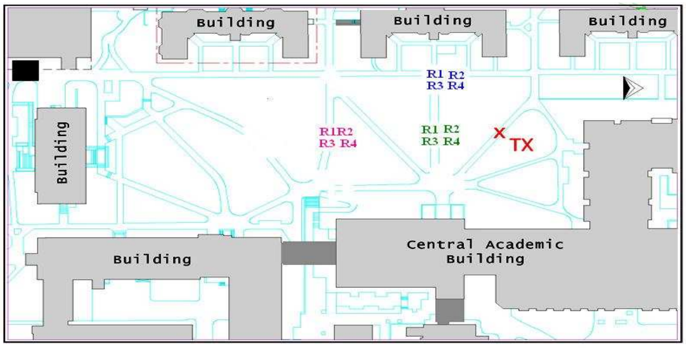


SINR	R%	R=I%	MGPC%	mean_Gauss
10db	69.5341	89.1753	99.1409	1.4055
12db	44.3250	91.1051	99.4609	1.1935
14db	24.6117	96.1165	97.5728	1.5061
16db	11.4695	96.8750	100	1.2151

15. *Group1* user1, user2, user3, user4: Location 3

Group2 user1, user2, user3, user4: Location 2

Group3 user1, user2, user3, user4: Location 5



Infeasible

16. *Group1* user1, user2, user3: Location 3

Group2 user1, user2, user3: Location 2

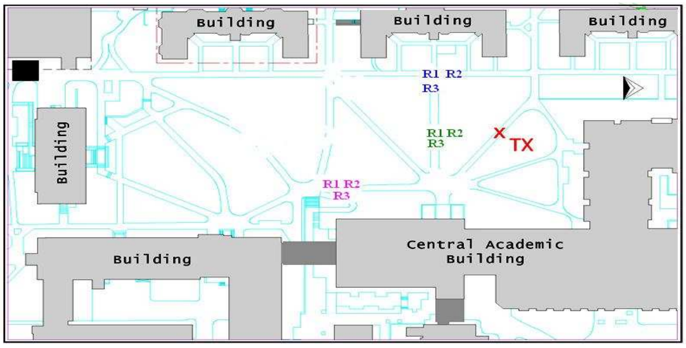
Group3 user1, user2, user3: Location 5

SINR	R%	R=I%	MGPC%	mean_Gauss
10db	5.2569	100	100	-

17. *Group1* user1, user2, user3: Location 3

Group2 user1, user2, user3: Location 2

Group3 user1, user2, user3: Location 6

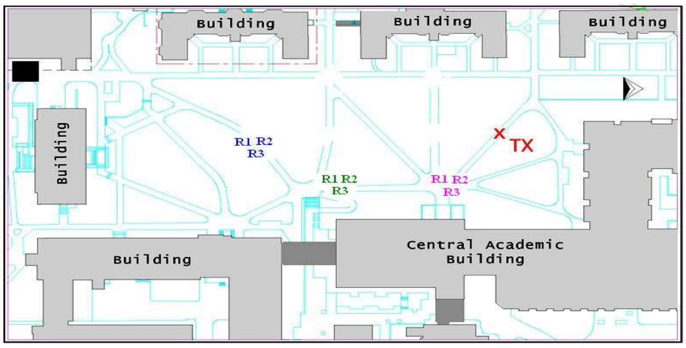


SINR	R%	R=I%	MGPC%	mean_Gauss
10db	32.1386	94.4238	98.5130	2.0651

18. *Group1* user1, user2, user3: Location 7

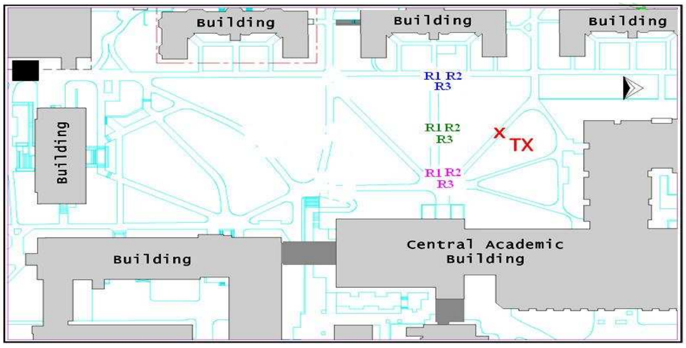
Group2 user1, user2, user3: Location 6

Group3 user1, user2, user3: Location 1



Infeasible

19. *Group1* user1, user2, user3: Location 3
Group2 user1, user2, user3: Location 2
Group3 user1, user2, user3: Location 1



SINR	R%	R=I%	MGPC%	mean_Gauss
10db	12.7838	100	97.1963	-

2.9 Conclusions

In the present section, we discuss the various experimental results.

We begin our discussion with the feasibility percentage of the semidefinite relaxation problem. As expected, the feasibility of the relaxed problem depends on the number of multicast groups and their populations. As the number of multicast groups or their populations increases, the feasibility of the relaxed problem drops. For example, consider scenarios (15-19) and (7,8) respectively. Note, that even for scenarios dealing with relatively few users, an increase in the number of multicast groups yields a substantial decrease of the feasibility.

Another factor of great importance is the target SINR. For increasing values of the target SINR, the feasibility percentage of the relaxed problem decreases (scenario 4). We observe that directional scenarios tend to be more feasible than non directional scenarios since they can be solved for higher values of the target SINR. For example, compare scenarios 5 and 9. Note that typical values of the highest SINR that can be achieved for directional scenarios range between 18-22 dB, whereas for non directional scenarios range between 14-16 dB. The feasibility percentage of the multi power control problem is for most cases higher than 95 %. Therefore, for almost all cases, if the semidefinite relaxation is feasible, an approximate solution to the original problem can be obtained.

A surprising result in our study, is that the percentage of rank one solutions $\{\mathbf{X}_k\}_{k=1}^{G}$ is at most cases much higher than 90%. In such cases, the relaxed problem is equivalent to the original problem and thus by solving the semidefinite relaxed problem we obtain an exact solution of the original problem. Furthermore, we observe that some relationship seems to exist between the rank one and feasibility percentages of the relaxed problem. In particular, as the feasibility percentage of the semidefinite relaxation problem drops, the percentage of rank one solutions is constantly increasing. For example consider scenario (2).

Last but not least, the quality of approximate solutions defined as the ratio of the total transmitted power corresponding to the approximate solution we obtain, to the lower bound of transmitted power obtained by solving the semidefinite relaxation is in most cases at most 3dB away of the lower bound.

Unfortunately, the transmitter is equipped with only 4 antennas and thus we have not been able to fully analyze the impact of the number of transmit elements to the performance of the algorithm. However it is intuitive that transmitters equipped with more antenna elements will be able to serve more multicast groups and more users. In fact, Monte Carlo simulation in [6], indicates that the feasibility of the relaxed problem depends on the ratio $\frac{N}{G}$ and the populations of the multicast groups.

Chapter 3

Multicast Trasnmit Beamforming with Sidelobe constraints

In this chapter we are concerned with the problem of multicast transmit beamforming with sidelobe constraints. Our goal is to find an optimal beamforming scheme which transmits to specific users, while limiting interference to certain directions. This problem is an extension of the multicast transmit beamforming problem posed in [5]. In the following sections, we give a detailed description of the problem model and the proposed relaxation. Finally, we conduct several experiments to evaluate and verify the adequacy of the proposed algorithm.

3.1 Problem Statement

Consider a wireless scenario, where a multiantenna transmitter (base station) simultaneously sends common information to K users. Assume that the transmitter is equipped with N antenna elements, and that each receiver is equipped with a single antenna element. Let $\mathbf{w}^H$ denote the $N \times 1$ beamforming vector and $\mathbf{h}_i$ denote the $N \times 1$ channel vector from each transmit antenna to the receive antenna of user i. The channel response is frequency flat for all K users. Note that since the transmitter sends common information to all users we have no interference. In this case, quality of service can be described in terms of signal to noise ratio (SNR).

If the transmitted information signal is zero mean with unit variance, then the total transmitted power is equal to $||\mathbf{w}_k||_2^2$. It follows that the SNR at each link can be written as:

$$\text{SNR} = \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \tag{3.1}$$

where σ_i^2 denotes the variance of the white, zero mean noise.

Let the set $\{1, \cdots, M\}$ denote the set of users we are interested to satisfy the prescribed minimum SNR, while on the other hand, let the set $\{1, \cdots, S\}$ denote the set of users for which we wish to limit interference. Furthermore, let c_i denote the minimum target SNR for the i-th user in $\{1, \cdots, M\}$ and d_j denote the maximum SNR for the j-th user in $\{1, \cdots, S\}$. The design of a beamformer, that minimizes the total power and transmits to specific users, while limiting interference to certain directions can be posed as:

$$\begin{aligned} &\min_{\{\mathbf{w} \in \mathbb{C}^N\}} \|\mathbf{w}\|_2^2 \\ &\text{subject to}: \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \geq c_i, \forall i \in \{1, \cdots, M\} \\ &\frac{|\mathbf{w}^H \mathbf{b}_j|^2}{\sigma_i^2} \leq d_j, \forall j \in \{1, \cdots, S\} \end{aligned} \tag{3.2}$$

where $\mathbf{h}_i$ and $\mathbf{b_j}$ correspond to the channel vectors of users in $\{1, \cdots, M\}$ and $\{1, \cdots, S\}$ respectively.

At this point, we define the normalized channel vectors $\tilde{\mathbf{h}}_i := \frac{\mathbf{h}_i}{\sqrt{c_i \sigma_i^2}}$ and $\tilde{\mathbf{b}}_j := \frac{\mathbf{b}_j}{\sqrt{d_j \sigma_i^2}}$, and problem (3.2) can be written as:

$$\begin{aligned} &\min_{\{\mathbf{w} \in \mathbb{C}^N\}} \|\mathbf{w}\|_2^2 \\ &\text{subject to}: |\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 \geq 1, \forall i \in \{1, \cdots, M\} \\ &|\mathbf{w}^H \tilde{\mathbf{b}}_j|^2 \leq 1, \forall j \in \{1, \cdots, S\} \end{aligned} \tag{3.3}$$

As mentioned, the above problem is an extension to the multicast beamforming problem which is NP-hard. Thus problem (3.3) is NP hard too. As discussed in chapter 2, NP hard problems cannot be exactly solved in reasonable time.

3.2 Relaxation

Problem (3.3) has a quadratic cost function and quadratic constraints. In order to cast problem (3.3) in a convex form we introduce the following matrices :

$$\mathbf{Q}_i = \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H \tag{3.4}$$

and

$$\mathbf{B}_j = \tilde{\mathbf{b}}_j \tilde{\mathbf{b}}_j^H \tag{3.5}$$

The problem can be written as :

$$\begin{gathered} \min_{\{\mathbf{w} \in \mathbb{C}^N\}} trace(\mathbf{w}\mathbf{w}^H) \\ \text{subject to} : trace(\mathbf{w}\mathbf{w}^H Q_i) \geq 1, \forall i \in \{1, \cdots, M\} \\ trace(\mathbf{w}\mathbf{w}^H \mathbf{B}_j) \leq 1, \forall i \in \{1, \cdots, S\} \end{gathered} \tag{3.6}$$

By defining $\mathbf{X} = \mathbf{w}\mathbf{w}^H$ we obtain:

$$\begin{gathered} \min_{\{\mathbf{X} \in \mathbb{C}^N \times N\}} trace(\mathbf{X}) \\ \text{subject to} : trace(\mathbf{X}\mathbf{Q}_i) \geq 1, \forall i \in \{1, \cdots, M\} \\ trace(\mathbf{X}\mathbf{B}_j) \leq 1, \forall j \in \{1, \cdots, S\} \\ \mathbf{X} \succeq \mathbf{0} \\ \text{rank}\,(\mathbf{X}) = 1 \end{gathered} \tag{3.7}$$

Problem (3.7) has a linear cost function, linear trace inequalities and a convex semidefinite constraint. However, the rank one constraint is non convex. By dropping the non convex constraint, problem (3.7) can be written as :

$$\min_{\{\mathbf{X}\in\mathbb{C}^{N}\times N\}} trace(\mathbf{X})$$

$$\text{subject to : } trace(\mathbf{X}\mathbf{Q}_i) \geq 1, \forall i \in \{1, \cdots, M\} \tag{3.8}$$

$$trace(\mathbf{X}\mathbf{B}_j) \leq 1, \forall j \in \{1, \cdots, S\}$$

$$\mathbf{X} \succeq \mathbf{0}$$

In order to cast problem (3.8) in the standard semidefinite form, we have to transform the inequality constraints into equality constraints. Therefore, we introduce M+S non negative slack variables and problem (3.8) can be written as :

$$\min_{\{\mathbf{X}\in\mathbb{C}^{N}\times N\}, s_i\in\mathbb{R}, r_j\in\mathbb{R}} vec(\mathbf{I}_N)^T vec(\mathbf{X})$$

$$\text{subject to : } vec(\mathbf{Q}_i^T)^T vec(\mathbf{X}) - s_i = 1, \forall i \in \{1, \cdots, M\} \tag{3.9}$$

$$vec(\mathbf{B}_j^T)^T vec(\mathbf{X}) + r_j = 1, \forall j \in \{1, \cdots, S\}$$

$$s_i \geq 0, \forall i \in \{1, \cdots, M\}$$

$$r_j \geq 0, \forall i \in \{1, \cdots, S\}$$

$$\mathbf{X} \succeq \mathbf{0} \tag{3.10}$$

Problem (3.9) is a semidefinite programming problem (SDP) and it can be efficiently solved by using interior point methods.

3.3 Algorithm

The matrix **X** obtained by solving the above relaxed problem will not be rank one in general.

A way of obtaining an approximate solution to the original problem is to apply the Gaussian Randomization technique, discussed in chapter 2. Unfortunately, our problem includes opposing inequality constraints and therefore, we cannot simply

scale up the candidate beamforming vectors as proposed in [5]
In order to resolve this situation let p denote the power scaling factor we seek.
An easy way to compute p comes from the key observation that p should satisfy the following constraints:

$$p \min |\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 \geq 1 \Leftrightarrow p \geq (min(|\mathbf{w}^H \tilde{\mathbf{h}}_i|^2)^{-1} \tag{3.11}$$

$$p \max |\mathbf{w}^H \tilde{\mathbf{b}}_j|^2 \leq 1 \Leftrightarrow p \leq (max(|\mathbf{w}^H \tilde{\mathbf{b}}_j|^2)^{-1} \tag{3.12}$$

Thus, if we can find a power scaling factor p, for which both inequalities (3.11) and (3.12) are satisfied, then the minimum power boost is equal to $min(|\mathbf{w}^H \mathbf{h}_i|^2)^{-1}$. Note that $\mathbf{w}^H$ denotes a candidate beamforming vector.
The overall algorithm is summarized below :

1. Solve the semidefinite relaxation problem (3.9). Denote the solution $\mathbf{X}$.
2. Apply the Gaussian randomization technique to obtain the set of all candidate beamforming vectors. If rank($\mathbf{X}$) = 1 use the principal component of $\mathbf{X}$ instead.
3. For each candidate beamforming vector, check if inequalities (3.11) and (3.12) are simultaneously feasible. If they are, then the minimum power boost is equal to $min(|\mathbf{w}^H \mathbf{h}_i|^2)^{-1}$. Select the candidate beamforming vector that minimizes the total transmitted power.

3.4 Simulation and Experimental Results

In this section, we test the algorithm for numerous Monte Carlo simulations and experiments using measured channel data. The measured channel data were collected at the Quad Field described in chapter 2. Our goal is to evaluate the performance of the proposed algorithm. Furthermore, as in chapter 2, we wish to determine the factors that affect performance.
In our study, performance is defined in terms of :

1. the feasibility percentage of the semidefinite relaxation
2. the percentage for which the solution $\mathbf{X}$ of the semidefinite problem has rank one
3. the percentage for which the power boost problem is feasible
4. the quality of approximate solutions

If the semidefinite relaxation problem is feasible, we can apply the Gaussian randomization technique and solve the power boost problem to obtain an approximate solution to the original problem. If the relaxed problem is infeasible, then so is the original. On the other hand, feasibility of the original problem can be established only if both the relaxation and the power boost problems are feasible. Thus, the feasibility of both the relaxed and the power boost problems is of great importance. In fact, if the power boost problem turns out to be infeasible, then the feasibility of the original problem is not guaranteed.

In cases, where the the solution of the semidefinite problem has rank one, the relaxed problem is equivalent to the original problem. Thus, by solving the semidefinite problem, we obtain the optimum solution of the original problem.

Finally, the quality of the approximate solutions is defined as in chapter 2, as the ratio of the transmitted power to the lower bound on the transmitted power we obtain by solving the semidefinite relaxation.

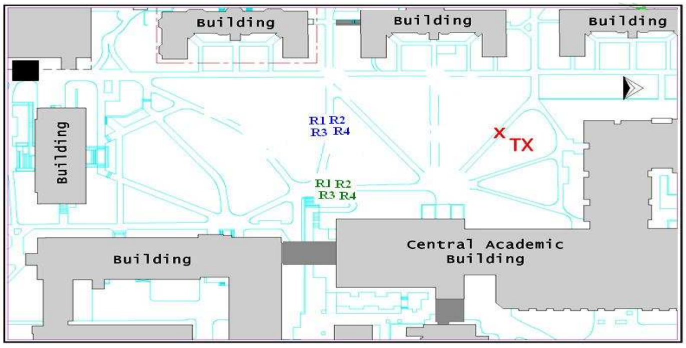
First, we test the algorithm for 1000 Monte Carlo runs and 700 Gaussian Randomization iterations using a variety of choices for N,M,S. We assume that the $\mathbf{h}_i$, $\mathbf{b}_j$,channel vectors are i.i.d. cicularly symmetric complex Gaussian random variables of variance 1.
The results are summarized in the following table.
Note that, column 2 reports the feasibility percentage of the relaxed problem, column 3 reports the rank-one percentage of $\mathbf{X}$, column 4 reports the average transmitted power over the lower bound obtained from the SDR solution and column 5 reports the feasibility percentage of the power boost problem.

users	R%	rank=1%	mean	p%
N=4/M=4/S=4	98.5	78.1726	1.0840	58.6047
N=4/M=4/S=6	89.9	70.7453	1.0619	32.6996
N=4/M=6/S=6	85.4	38.0562	1.13	21.9282
N=4/M=6/S=8	71.9	32.4061	1.0894	15.2263
N=4/M=8/S=8	61.4	12.8664	1.15553	9.5327
N=4/M=10/S=10	35.1	6.5527	1.1196	4.2683
N=6/M=6/S=6	100	46.4	1.1884	37.5
N=6/M=8/S=6	100	19	1.3240	28.7654
N=6/M=8/S=8	99.7	15.6469	1.2353	14.3876
N=6/M=8/S=10	98.9	12.5379	1.1665	9.3642
N=6/M=10/S=10	98.3	4.5778	1.2305	5.1173
N=6/M=12/S=12	92.5	1.1892	1.2652	1.5317
N=8/M=8/S=8	100	19.6	1.3352	28.2338
N=8/M=10/S=10	99.9	4.9049	1.3504	9.4737
N=8/M=12/S=12	100	1	1.3827	3.2323

Furthemore we test the algorithm using selected outdoor measurements.

1. *Group M*: user1, user2, user3, user: Location 5

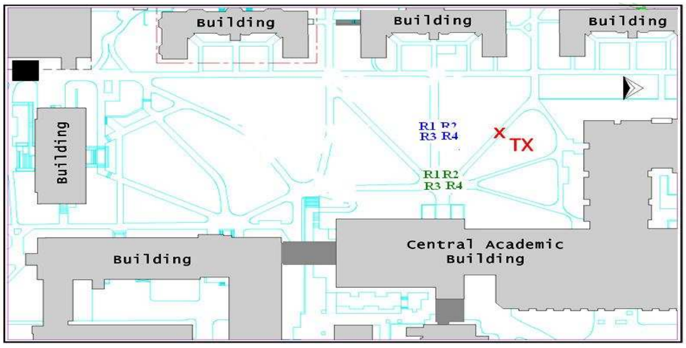
   Group S: user1 ,user2, user3, user4: Location 6



R%	rank=1%	mean	p%
97.0131	85.5911	1.0358	3.4188

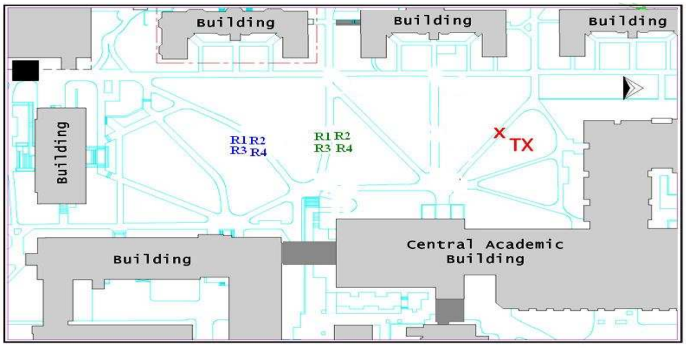
2. *Group M*: user1, user2, user3, user4: Location 2

   Group S: user1, user2, user3, user4: Location 1



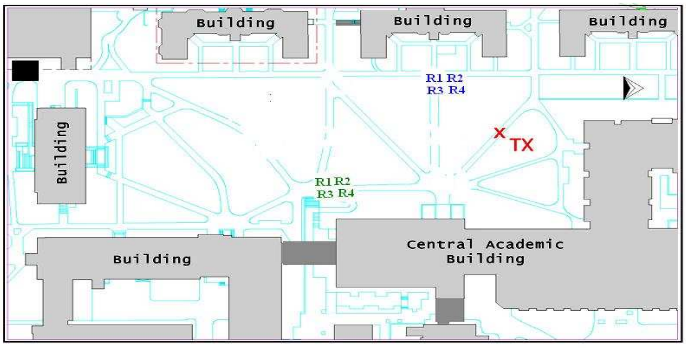
R%	rank=1%	mean	p%
100	100	1	-

3. *Group M*: user1, user2, user3, user4: Location 7
   Group S: user1, user2, user3, user4: Location 5



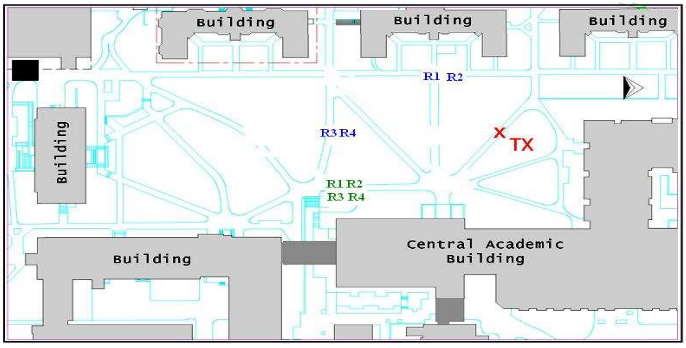
R%	rank=1%	mean	p%
100	87.2162	1.6345	57.0093

4. *Group M*: user1, user2, user3, user4: Location 3
   Group S: user1, user2, user3, user4: Location 6



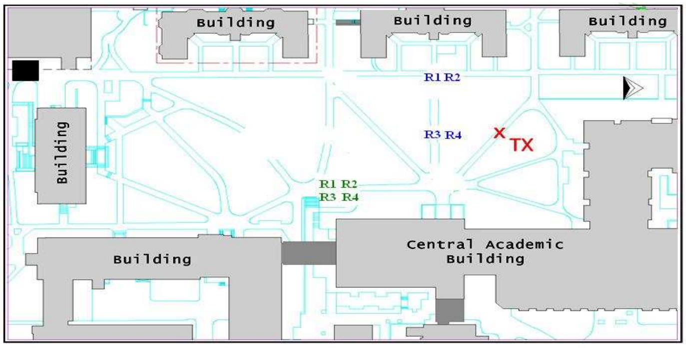
R%	rank=1%	mean	p%
100	96.29	1.1915	54.8387

5. *Group M*: user1, user2:Location 3 / user3, user4: Location 5
   Group S: user1, user2, user3,user4: Location 6



R%	rank=1%	mean	p%
97.1326	96.802	1.2478	23.0769

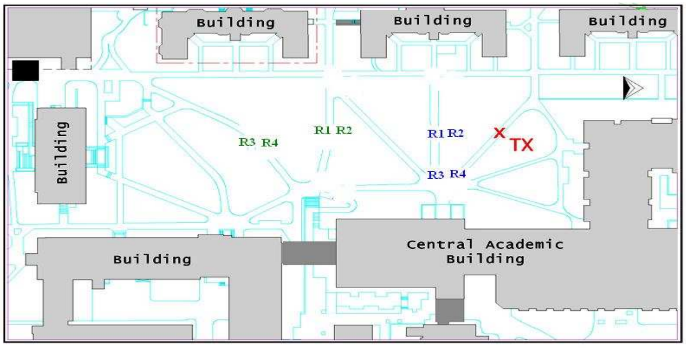
6. *Group M*: user1, user : Location 3 / user3, user4: Location 2
   Group S: user1 ,user2, user3,user4: Location 6



R%	rank=1%	mean	p%
100	100	1	-

7. *Group M*: user1, user2: Location 2 / user3, user4: Location 1

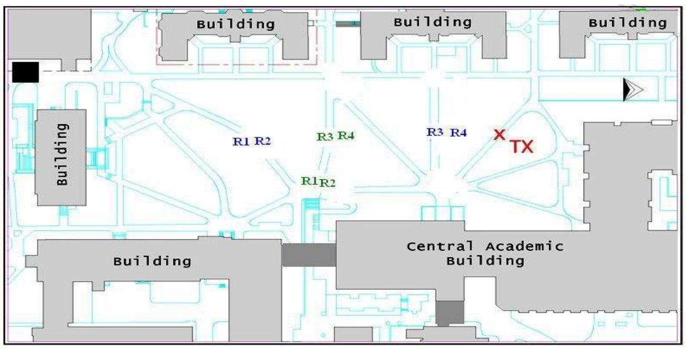
   Group S: user1, user2: Location 5 / user3, user4: Location 7



R%	rank=1%	mean	p%
93.6679	72.3214	1.2134	30.4142

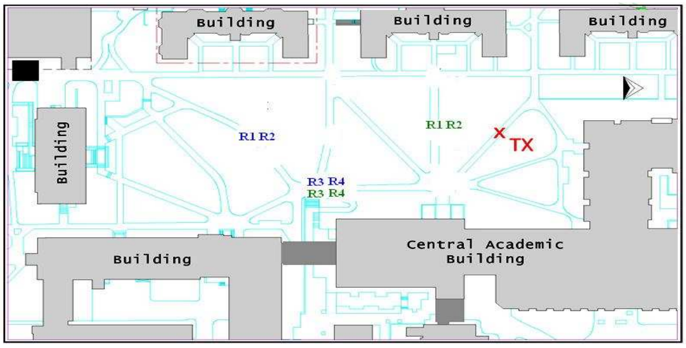
8. *Group M*: user1, user2: Location 7 / user3, user4: Location 2

   Group S: user1, user2: Location 5 / user3, user4: Location 6



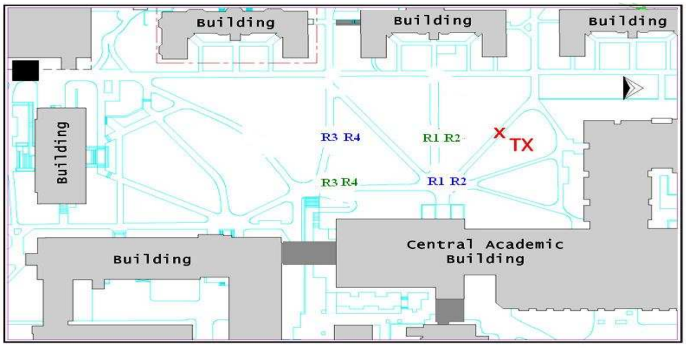
R%	rank=1%	mean	p%
99.0442	73.462	1.4601	18.6364

9. *Group M*: user1, user2: Location 7 / user3, user4: Location 6
   Group S: user1, user2: Location 6 / user3, user4: Location 2



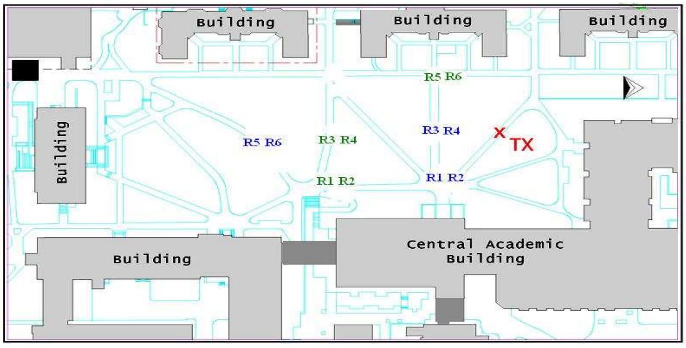
R%	rank=1%	mean	p%
100	79.5699	1.4584	30.9942

10. *Group M*: user1, user2: Location 5 / user3, user4: Location 1
    Group S: user1, user2: Location 2 / user3, user4: Location 6



R%	rank=1%	mean	p%
99.8	88.5167	1.4414	28.1250

11. *Group M*: user1, user2: Location 2 / user3, user4: Location 1 / user5, user6: Location 7
Group S: user1, user2: Location 3 / user3 ,user4: Location 5 / user5, user6: Location 6



R%	rank=1%	mean	p%
56.63	39.0295	2.3716	2.7682

3.5 Conclusions

In this section, we discuss the results presented in the previous section.
First, we observe that it is the ratio $\frac{N}{K}$ that determines the feasibility of the relaxed problem. Specifically, the feasibility of the relaxed problem decreases, for decreasing values of the ratio $\frac{N}{K}$. Furthermore, the feasibility percentage of the relaxed problem is for almost all experiments conducted much higher than 90%. As we can see, the percentages of the power boost problem and the rank one solutions $\mathbf{X}$ are also decreasing for decreasing values of the ratio $\frac{N}{K}$. Note finally that the rank one percentages tend to be higher for directional scenarios than for non directional.
For both experiments conducted with measured channel data and Monte Carlo simulations the quality of service is for most cases much less than 1.5dB away from the lower bound on transmitted power obtained by solving the semidefinite relaxed problem.
Finally, we have tested the algorithm using Vandermonde type channel vectors $\mathbf{h}_i$ and $\mathbf{b_j}$. As shown in [8], if we apply channel vectors of Vandermonde type to the transmit

beamforming to multicast co-channel users problem, then the proposed relaxation is equivalent to the original problem. The same seems to hold for the multicast beamforming with sidelobe problem we study here. We have used Vandermonde channel vectors in several scenarios involving different number of transmit antenna elements and different number of users and in all cases considered, we have obtained rank one solutions.

Chapter 4

Conclusions

In this thesis we have studied the problem of transmit beamforming to multiple co-channel multicast groups and the problem of multicast beamforming with sidelobe constraints. We have approximated both problems using a convex form and used semidefinite programming to obtain solutions of good quality.

At first, we studied the problem of transmit beamforming to multiple co-channel multicast groups and conducted several experiments using measured channel data to evaluate its performance. For the majority of scenarios considered, the relaxed problem yields rank one solutions. Thus, in most cases we are able to obtain the exact solution of the original problem.

Next, we studied the problem of multicast transmit beamforming with sidelobe constraints and implemented an algorithm to obtain an approximate solution to the original problem. We have tested the algorithm under several scenarios. The feasibility percentage of the relaxed problem is in most cases higher than 90%. Furthermore for most cases we are able to obtain approximate solutions of good quality.

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