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Uncertainty of models in intelligent systems under stochastic loading

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Abstract. *In this paper, we address the problem of vibrations of intelligent structures. Stimuli may come from external perturbations of the system, disturbances or excitation that may cause structural vibrations, such as wind loading or earthquake. First, an accurate model of a piezocomposite intelligent structure with special boundary conditions is derived by using of FEM analysis. Then the robustness of the uncertain closed-loop model performances has been investigated. Obtained results show the higher performance of $H_{infinity}$ design approach in rejection of disturbances.*

1 INTRODUCTION

The study of algorithms for active vibrations control in intelligent structures became an area of enormous interest, mainly due to the countless demands of an optimal performance of mechanical systems. Many researchers are investigated in the field of intelligent structures [1, 2, 3, 4, 5, 6]. A smart structure in one that monitors itself and its environment in order to respond to changes in its conditions. [7,8] Smart structures, formed by a structure base, coupled with piezoelectric actuators and sensor are capable to guarantee the conditions demanded through the application of several types of controllers. This article shows some steps that should be followed in the design of a smart structure. In our paper a cantilever slender beam with rectangular cross-sections is considered. Thirty six pairs of piezoelectric patches are embedded symmetrically at the top and the bottom surfaces of the beam. The beam is from graphite- epoxy T300 − 976 and the piezoelectric patches are PZT G1195N. The top patches act like sensors and the bottom like actuators. The resulting composite beam is modelled by means of the classical laminated technical theory of bending. Let us assume that the mechanical properties of both the piezoelectric material and the host beam are independent in time. The thermal effects are considered to be negligible as well [8, 9].

The beam has length L, width W and thickness h. The sensors and the actuators have width b_S and b_A and thickness h_S and h_A, respectively. The electromechanical parameters of the beam of interest are given in the table 1.

Parameters	**Values**
Beam length, L	0.7m
Beam width, W	0.07m
Beam thickness, h	0.0096m
Beam density, ρ	$1800 kg/m^3$
Young s modulus of the beam, E	1.5×10^{11} N/m^2
Piezoelectric constant, d_{31}	254×10^{-12} m/V
Electric constant, ξ_{33}	11.5×10^{-3} V m/N
Young's modulus of the piezoelectric element	1.5×10^{11} N/m^2
Width of the piezoelectric element	$b_S = b_a = 0.07$m
Thickness of the piezoelectric element	$h_S = h_a = 0.0002$m

Table 1: Parameters of the composite beam.

In order to derive the basic equations for piezoelectric sensors and actuators [1, 2], we assume that:

- The piezoelectric sensors actuators (S/A) are bonded perfectly on the host beam;

- The piezoelectric layers are much thinner then the host beam;
- The piezoelectric material is homogeneous, transversely isotropic and linearly elastic;
- The piezoelectric S/A are transversely polarized [1, 2, 7].

2 SYSTEM MODELLING

This classical finite element procedure leads to the approximate discretized variation problem. For a finite element the discrete differential equations are obtained by substituting the discretized expressions into the first variation of the kinetic energy and strain energy [8, 10]. Integrating over spatial domains and using the Hamiltons principle [8], the equation of motion for a beam element are expressed in terms of nodal variable q as follows,

$$M\ddot{q}(t)+D\dot{q}(t)+Kq(t)=f_m(t)+f_e(t) \tag{1}$$

where M is the generalized mass matrix, D the viscous damping matrix, K the generalized stiffness matrix, f_m the external loading vector and f_e the generalized control force vector produced by electromechanical coupling effects. The independent variable q(t) is composed of transversal deflections ω_1 and rotations ψ_1, i.e., [10, 12]

Furthermore n is the number of nodes used in analysis and vectors ω and f_m are positive upwards. To transform to state-space control representation, let (in the usual manner),

$$\dot{x}(t)=\begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \tag{2}$$

Furthermore to express $f_e(t)$ as $Bu(t)$ we write it as $f_e^{*}u$ where f_e^{*} the piezoelectric force is for a unit applied on the corresponding actuator, and u represents the voltages on the actuators. Furthermore, $d(t)=f_m(t)$ is the disturbance vector [10].
Then,

$$\dot{x}(t)=\begin{bmatrix} O_{2n\times 2n} & I_{2n\times 2n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}x(t)+\begin{bmatrix} O_{2n\times 2n} \\ M^{-1}f_e^{*} \end{bmatrix}u(t)+\begin{bmatrix} O_{2n\times 2n} \\ M^{-1} \end{bmatrix} \tag{3}$$

$$=Ax(t)+Bu(t)+Gd(t)=Ax(t)+\begin{bmatrix} B & G \end{bmatrix}\begin{bmatrix} u(t) \\ d(t) \end{bmatrix}=Ax(t)+\tilde{B}\tilde{u}(t) \tag{4}$$

The previous description of the dynamical system will be augmented with the output equation (displacements only measured) [5,11],

$$y(t)=\begin{bmatrix} x_1(t) & x_3(t) & \cdots & x_{n-1}(t) \end{bmatrix}^T=Cx(t) \tag{5}$$

The units used are compatible for instance m, rad, sec and N. [7,8]

3 DESIGN OBJECTIVES AND SYSTEM SPECIFICATION

The structured singular value of a transfer function matrix is defined as,

$$\mu(M) = \begin{cases} \dfrac{1}{\min\limits_{k_m}\{\det(I - k_m M\Delta) = 0, \bar{\sigma}(\Delta) \le 1\}} \\ 0, \text{if no such structured exists} \end{cases} \tag{6}$$

In words it defines the smallest structured Δ (measured in terms of $\bar{\sigma}(\Delta)$) which makes det(I−MΔ)=0: then $\mu(\mathrm{M}) = 1/\bar{\sigma}(\Delta)$. It follows that values of μ smaller than 1 are desired (the smaller the better: a larger variation is allowed) [13, 14].

4 SYSTEM UNCERTAINTY

The main sources of uncertainty are:

* Nonlinearity and/or dynamic aspects of the system that are ignored at the modeling phase. The error introduced in modal analysis by using only a few significant eigenmodes leads to an uncertainty of the type discussed here. [15, 16]

* Incomplete knowledge of model values and parameters and/or natural fluctuation of those values during system operation.

* Influence of the system's environment, in the form of disturbances.

Assume uncertainty in the M and K matrices of the form,

$$K=K_0(I+k_p I_{2n\times 2n}\delta_K) \tag{7}$$

$$M=M_0(I+m_p I_{2n\times 2n}\delta_M) \tag{8}$$

Also, since, D=0.0005(K+M), an appropriate form for D is,

$$\begin{aligned} D&=0.0005[K_0(I+k_p I_{2n\times 2n}\delta_K)+M_0(I+m_p I_{2n\times 2n}\delta_M)]= \\ &D_0+0.0005[K_0 k_p I_{2n\times 2n}\delta_K+M_0 m_p I_{2n\times 2n}\delta_M] \end{aligned} \tag{9}$$

Alternatively, by adopting the well-known Rayleigh damping assumption,

$$D=\alpha K+\beta M \tag{10}$$

D could be expressed similarly to K, M, as,

$$D=D_0(I+d_p I_{2n\times 2n}\delta_D) \tag{11}$$

In this way we introduce uncertainty in the form of percentage variation in the relevant matrices. Uncertainty is most likely to arise from terms outside the main matrices (since length can be adequately measured).

Here it will be assumed,

$$\|\Delta\|_\infty \overset{\text{def}}{=} \left\| \begin{bmatrix} \mathrm{I}_{n\times n}\delta_{\mathrm{K}} & \vdots & 0_{n\times n} \\ \cdots & & \cdots \\ 0_{n\times n} & \vdots & \mathrm{I}_{n\times n}\delta_{\mathrm{M}} \end{bmatrix} \right\|_\infty < 1 \tag{12}$$

hence m_p, k_p are used to scale the percentage value and the zero subscript denotes nominal values.

(it is reminded that for matrix $A_{n\times m}$ the norm is calculated through $\|A\|_\infty = \max\limits_{1\le j\le m} \sum\limits_{j=1}^{n} |a_{ij}|$)

With these definitions Eq. 3 becomes,

$$\begin{aligned} &M_0(I+m_pI_{2n\times 2n}\delta_M)\ddot{q}(t) + K_0(I+k_pI_{2n\times 2n}\delta_K)q(t)+ \\ &+[D_0+0.0005[K_0k_pI_{2n\times 2n}\delta_K+M_0m_pI_{2n\times 2n}\delta_M]]\dot{q}(t) = f_m(t)+f_e(t) \end{aligned} \tag{13}$$

$$\begin{aligned} &\Rightarrow M_0\ddot{q}(t)+D_0\dot{q}(t)+K_0q(t)= \\ &-[M_0m_pI_{2n\times 2n}\delta_M\ddot{q}(t)+0.0005[K_0k_pI_{2n\times 2n}\delta_K+M_0m_pI_{2n\times 2n}\delta_M]\dot{q}(t) \\ &+K_0k_pI_{2n\times 2n}\delta_Kq(t)]+f_m(t)+f_e(t) \end{aligned} \tag{14}$$

$$\Rightarrow M_0\ddot{q}(t)+D_0\dot{q}(t)+K_0q(t)=\tilde{D}q_u(t)+f_m(t)+f_e(t) \tag{15}$$

where,

$$q_u(t) = \begin{bmatrix} \ddot{q}(t) \\ \dot{q}(t) \\ q(t) \end{bmatrix} \tag{16}$$

$$\begin{aligned} \tilde{D} &= -\begin{bmatrix} M_0m_p & K_0k_p \end{bmatrix} \begin{bmatrix} I_{2n\times 2n}\delta_M & \vdots & 0_{2n\times 2n} \\ \cdots & & \cdots \\ 0_{2n\times 2n} & \vdots & I_{2n\times 2n}\delta_{\mathrm{K}} \end{bmatrix} \begin{bmatrix} I_{2n\times 2n} & 0.0005I_{2n\times 2n} & 0_{2n\times 2n} \\ 0_{2n\times 2n} & 0.0005I_{2n\times 2n} & I_{2n\times 2n} \end{bmatrix} = \\ &= G_1\cdot\Delta\cdot G_2 \end{aligned} \tag{17}$$

Writing in state space form, gives,

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0_{2n\times 2n} & I_{2n\times 2n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} 0_{2n\times n} \\ M^{-1}f_e^* \end{bmatrix} u(t) + \begin{bmatrix} 0_{2n\times 2n} \\ M^{-1} \end{bmatrix} d(t) + \begin{bmatrix} 0_{2n\times 6n} \\ M^{-1}G_1\cdot\Delta\cdot G_2 \end{bmatrix} q_u(t) \\ &= Ax(t)+Bu(t)+Gd(t)+G_uG_2q_u(t) \end{aligned} \tag{18}$$

In this way we treat uncertainty in the original matrices as an extra uncertainty term.

To express our system consider in the frequency domain Fig. 6

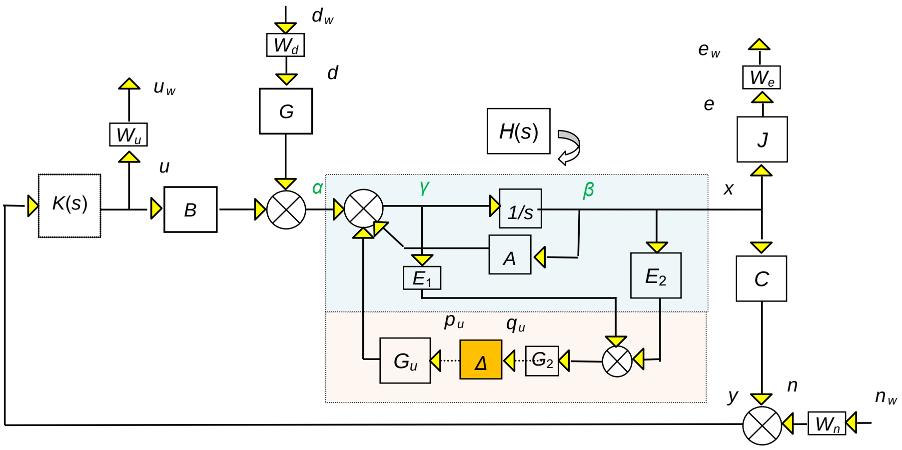

Fig. 6 Uncertainty block diagram

This diagram is the weighted block diagram in the frequency domain. Wd, We, Wu, Wn are the weight of the disturbances, errors, control, noise. *H(s)* is our system, *K(s)*, is the controller and Δ define the uncertainties. [16, 17]

The matrices E_1, E_2 are used to extract,

$$q_u(t) \overset{\text{def}}{=} \begin{bmatrix} \ddot{q}(t) \\ \dot{q}(t) \\ q(t) \end{bmatrix} \tag{19}$$

Since,

$$\gamma = \begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix} \quad \text{and} \quad \beta = \int \begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix} \mathrm{d}t = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \tag{20}$$

appropriate choices for E_1, E_2 are,

$$E_1 = \begin{bmatrix} 0_{2n\times 2n} & I_{2n\times 2n} \\ I_{2n\times 2n} & 0_{2n\times 2n} \\ 0_{2n\times 2n} & 0_{2n\times 2n} \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0_{2n\times 2n} & 0_{2n\times 2n} \\ 0_{2n\times 2n} & 0_{2n\times 2n} \\ I_{2n\times 2n} & 0_{2n\times 2n} \end{bmatrix} \tag{21}$$

The idea is to find an N such that,

$$\begin{bmatrix} q_u \\ e_w \\ u_w \end{bmatrix} = N \begin{bmatrix} p_u \\ d_w \\ n_w \end{bmatrix}, \quad N = \begin{bmatrix} N_{p_u q_u} & N_{d_w q_u} & N_{n_w q_u} \\ N_{p_u e_w} & N_{d_w e_w} & N_{n_w e_w} \\ N_{p_u u_w} & N_{d_w u_w} & N_{n_w u_w} \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \tag{22}$$

or in the notation of Fig. 6,

$$\begin{bmatrix} q_u \\ w \end{bmatrix} = N \begin{bmatrix} p_u \\ z \end{bmatrix} \tag{23}$$

We'll use a methodology known as "pulling out the Δ's". To this end, break the loop at points p_u, q_u (which will be used as additional inputs/outputs respectively) and use the auxiliary signals α, β and γ. [18,19]

To get the transfer function $N_{d_w q_u}$ (from d_w to q_u):

$$q_u = G_2(E_2\beta + E_1\gamma) = G_2(E_2\frac{1}{s} + E_1)\gamma \tag{24}$$

$$\gamma = GW_d d_w + Bu + A\frac{1}{s}\gamma = GW_d d_w + BKC\frac{1}{s}\gamma + A\frac{1}{s}\gamma$$

$$\Rightarrow \gamma = (I - BKC\frac{1}{s} - A\frac{1}{s})^{-1} GW_d d_w \tag{25}$$

Hence,

$$N_{d_w q_u} = G_2(E_2\frac{1}{s} + E_1)(I - BKC\frac{1}{s} - A\frac{1}{s})^{-1} GW_d$$

Now, $N_{p_u q_u}$, $N_{p_u e_w}$, $N_{p_u u_w}$, are similar to $N_{d_w q_u}$, $N_{d_w e_w}$, $N_{d_w u_w}$ with GW_d replaced by G_u, i.e.,

$$\begin{aligned} N_{p_u q_u} &= G_2(E_2\frac{1}{s} + E_1)(I - BKC\frac{1}{s} - A\frac{1}{s})^{-1} G_u \\ N_{p_u e_w} &= W_y JH[I + B[K(I - CHBK)^{-1} CH] G_u \\ M_{p_u u_w} &= W_u K(I - CHBK)^{-1} CHG_u \end{aligned} \tag{26}$$

Finally to find $N_{n_w q_u}$,

$$q_u = G_2(E_2\beta + E_1\gamma) = G_2(E_2\frac{1}{s} + E_1)\gamma \tag{27}$$

$$\gamma = Bu + A\frac{1}{s}\gamma = BK(W_n n_w + y) + A\frac{1}{s}\gamma = BKW_n n_w + BKC\frac{1}{s}\gamma + A\frac{1}{s}\gamma$$

$$\Rightarrow \gamma = (I - BKC\frac{1}{s} - A\frac{1}{s})^{-1} BKW_n n_w \tag{28}$$

Hence,

$$N_{n_w q_u} = G_2(E_2\frac{1}{s} + E_1)(I - BKC\frac{1}{s} - A\frac{1}{s})^{-1} BKW_n \tag{29}$$

Collecting all the above yields N:

$$\begin{bmatrix} G_2\left(E_2\frac{1}{s}+E_1\right)\left(I-BKC\frac{1}{s}-A\frac{1}{s}\right)^{-1}G_u & G_2\left(E_2\frac{1}{s}+E_1\right)\left(I-BKC\frac{1}{s}-A\frac{1}{s}\right)^{-1}GW_d & G_2\left(E_2\frac{1}{s}+E_1\right)\left(I-BKC\frac{1}{s}-A\frac{1}{s}\right)^{-1}BKW_n \\ W_eJH\left[I+BK(I-CHBK)^{-1}CF\right]G_u & W_eJ(I-HBKC)^{-1}HGW_d & W_eJ(I-HBKC)^{-1}HBKW_n \\ W_uK(I-CHBK)^{-1}CFG_u & W_u(I-KCHB)^{-1}KCHGW_d & W_u(I-KCHB)^{-1}KW \end{bmatrix} \quad (30)$$

Having obtained N for the beam problem, all proposed controllers $K(s)$ can be compared using the structured singular value relations. [18, 19]

5 INPUTS

A typical wind load (Fig. 7) acting on the side of the structure. The wind load is a real life wind speed measurements in relevance with time that took place in Estavromenos of Heraklion Crete. We transform the wind speed in wind pressure with,

Loading corresponds to the wind excitation. The function $f_m(t)$ has been obtained from the wind velocity record, through the relation

$$f_m(t)=\frac{1}{2}\rho C_u V^2(t) \quad (31)$$

where V=velocity, ρ=density and C_u=1.2.

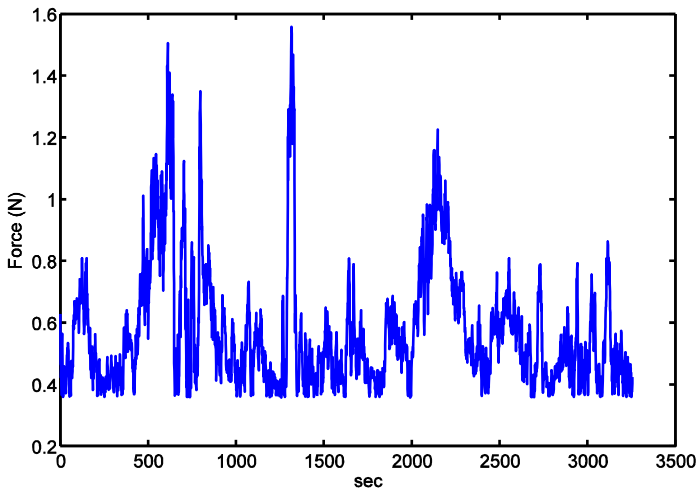

Fig.7 External load

Moreover, in all simulations, random noise has been introduced to measurements at system output locations within a probability interval of ±1%. Due to small displacements of system nodal points, noise amplitude is taken to be small, of the order of 5×10^{-5}. On the other hand, the signal is introduced at each node of the beam by a different percentage, that percentage being lower at the first node due to the fact that the beam end point is clamped. The controller obtained by applying H_∞ control has an order equal to 36. For this controller, $\gamma = 0.074<1$.

6 RESULTS

Robust analysis is carried out through the relations:

$$\sup_{\omega \in \mathrm{P}} \mu_{\Delta}\left(N_{11}(\mathrm{j}\omega)\right) < 1$$

for robust stability, and,

$$\sup_{\omega \in \mathrm{P}} \mu_{\Delta_a}\left(N(\mathrm{j}\omega)\right) < 1$$

for robust performance

For the H_∞ found, robust analysis was performed for the following values of *mp*,*kp*.
1. *mp* = 0, *kp* = 0.9. This corresponds to a ±90% variation from the nominal value of the stiffness matrix *K*. In Fig.8 are shown the bounds on the μ values. As seen the system remains stable and exhibits robust performance, since the upper bounds of both values remain below 1 for all frequencies of interest. This result is validated in Fig.9, where the displacement of the free end and the voltage applied are shown at the extreme uncertainty. Comparison with the open loop response for the same plant shows the good performance of the H_∞ controller. Results are very good, and the beam remains in equilibrium even under realistic wind conditions. Reduction of vibrations is observed, while piezoelectric add-ons produce voltage within their tolerance limits (±500volt)

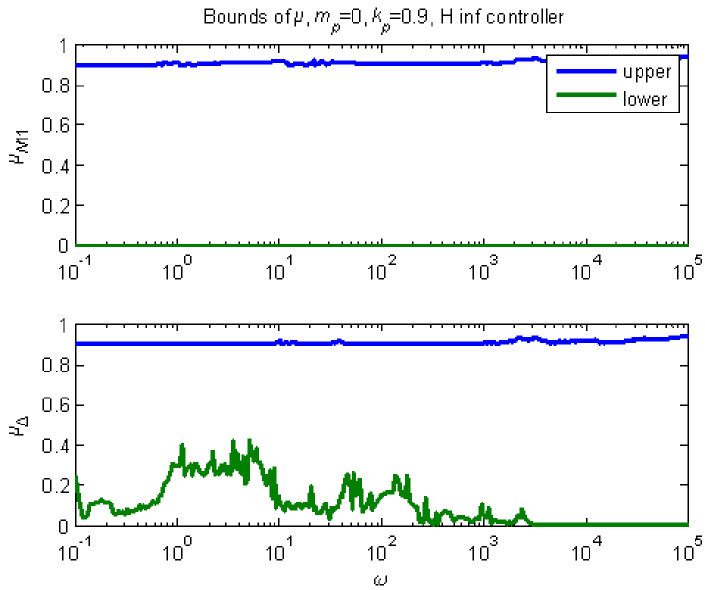

Fig. 8 μ-bounds o H_∞ the controller for m_p=0, k_p=0.9

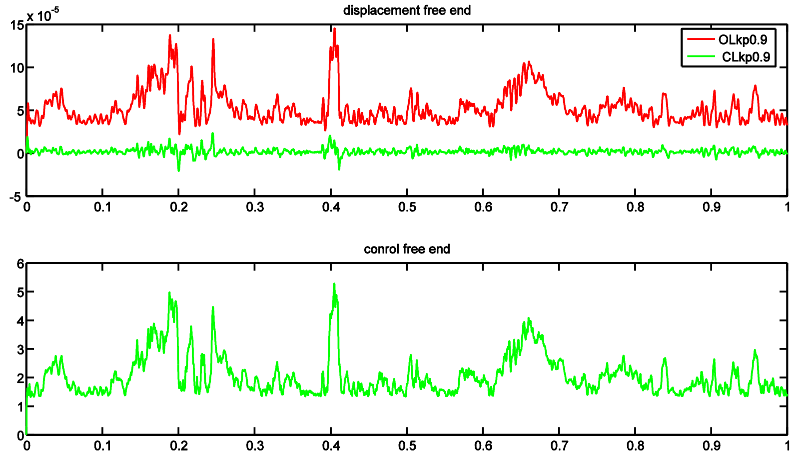

Fig.9 Displacement and control at free end for the H_∞ controller with $m_p = 0$, $k_p = 0.9$

2. $mp = 0.9$, $kp = 0$. This corresponds to a ±90% variation from the nominal value of the mass matrix M.

In Fig. 10 are shown the bounds on the μ values. As seen the system remains stable and exhibits robust performance, since the upper bounds of both values remain below 1 for all frequencies of interest. This result is validated in Fig. 11, where the displacement of the free end and the voltage applied are shown.

Comparison with the open loop response for the same plant shows the good performance of the controller. By employing the H_∞ control, vibration reduction is achieved, while the voltage applied is significantly lower that 500 V.

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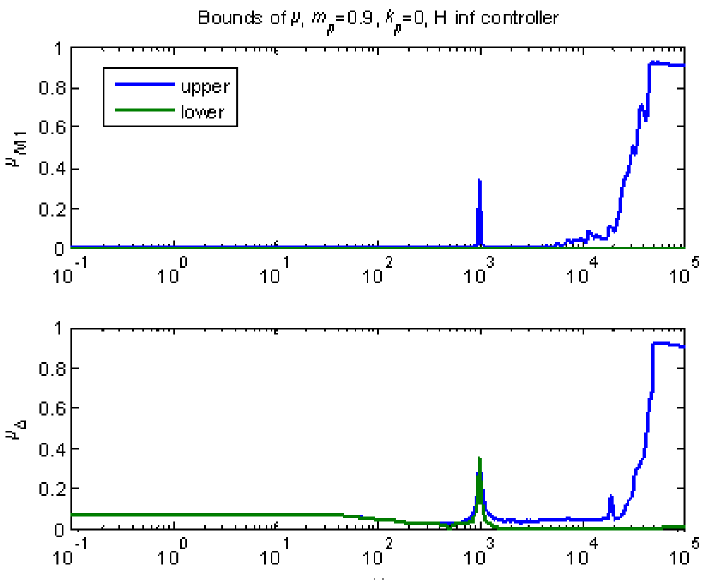

Fig. 10 μ-bounds of the H_∞ controller for $m_p = 0.9$, $k_p = 0$.

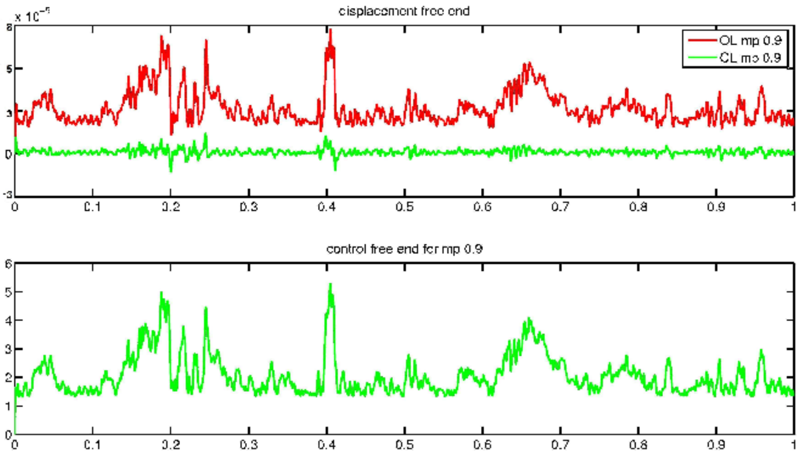

Fig. 11 Displacement and control at free end for the H_∞ controller with m_p =0.9, k_p =0

3. *mp* = 0.9, *kp* = 0.9. This corresponds to a ±90% variation from the nominal values of both the mass and stiffness matrices *M*, *K*.

In Fig. 12 are shown the bounds on the μ values. As seen the system remains stable and exhibits robust performance, since the upper bounds of both values remain below 1 for all frequencies of interest. This result is validated in Fig. 13, where the displacement of the free end and the voltage applied are shown.

Comparison with the open loop response for the same plant shows the good performance of the controller. Results are very good, and the beam remains in equilibrium even under realistic wind conditions. Reduction of vibrations is observed, while piezoelectric add-ons produce voltage within their tolerance limits.

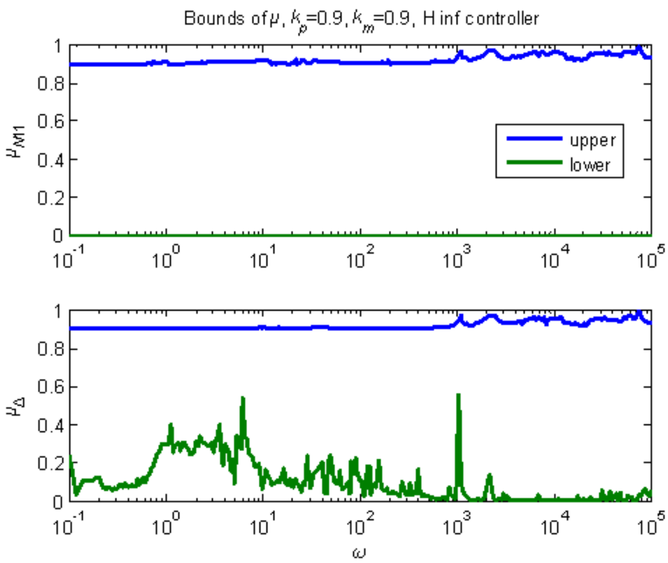

Fig. 12 Displacement and control at free end for the H_∞ controller with m_p =0.9, k_p =0

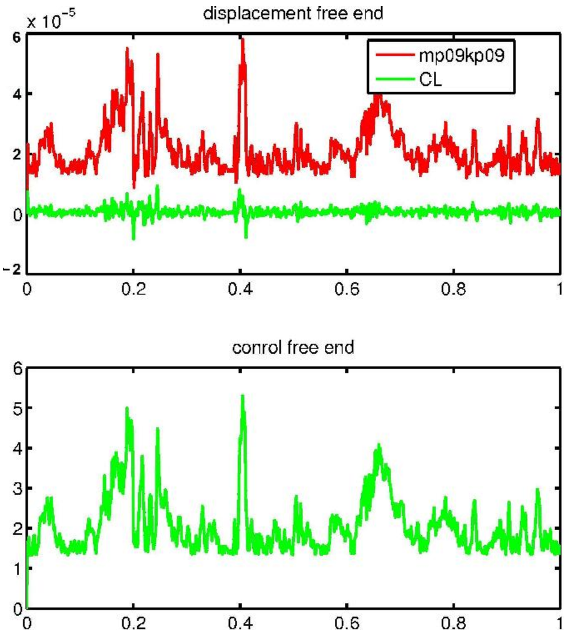

Fig.13 Displacement and control at free end for the H_∞ controller with m_p =0.9, k_p =0

Furthermore, we control the structure with variations of the nominal values of the mass matrix M, stiffness matrix K and matrices A and B. We take into consideration nonlinearities and system dynamics neglected in modeling, incomplete knowledge of disturbances, environment influence in the form of disturbances, and unreliability of system sensor measurements.

7 CONCLUSIONS

This paper describes an integrated approach to design and implement robust controllers for intelligent structures. The mathematical model derived using robust control is compared with models obtained by more conventional and well known methods. Using this model, a $H_{infinity}$ (H_∞) controller is designed for vibration suppression purposes. This robust controller accommodates the limited control effort produced by actuators. These designs are all then realized as digital controllers and their closed-loop performances have been compared. In particular, the robustness properties of the controller have been verified for variations in the mass of the test article and the sampling time of the controller. Complete vibration reduction was achieved even for variations of beam mass and stiffness up to 90%. H_∞ controller results were very satisfactory and prove that H_∞ control can reduce smart structures vibrations and deal with modeling uncertainty, external disturbances, and noise in measurements.

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