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MOTORWAY FLOW OPTIMISATION IN PRESENCE OF VEHICLE AUTOMATION AND COMMUNICATION SYSTEMS

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Abstract. *This paper describes a novel approach for defining optimal strategies in motorway traffic flow control, considering that a portion of vehicles are equipped with vehicle automation and communication systems. An optimisation problem, formulated as a Quadratic Programming (QP) problem, is developed with the purpose of minimising traffic congestion. The proposed problem is based on a first-order macroscopic traffic flow model able to capture the lane changing and the capacity drop phenomena. An application example demonstrates the achievable improvements in terms of the Total Time Spent if the vehicles travelling on the motorway are influenced by the control actions computed as a solution of the optimisation problem.*

1 INTRODUCTION

The mitigation of traffic congestion on motorway systems is a complex and useful task that could generate massive economical and environmental advantages for the modern society. As a matter of fact, the main motorways, particularly around metropolitan areas, suffer from congestion for long periods during a day and, ironically, the major congestions appear during the period of maximum traffic demand. Despite the huge improvements achieved in Information Technology during the last decades, a smart and widespread application of these technologies to alleviate traffic congestion is still not fully achieved.

However, there has been an enormous interdisciplinary effort by the automotive industry as well as by numerous research institutions around the world to plan, develop, test, and start deploying a variety of Vehicle Automation and Communication Systems (VACS) that are expected to revolutionise the features and capabilities of individual vehicles within the next decades. Several research works were realised in the past, sometimes foreseeing future scenarios where self-driving vehicles are part of a completely connected road infrastructure. The milestone work [1] introduced the concept of highly automated Intelligent Vehicle Highway System (Smart-IVHS), introducing a possible hierarchical control strategy with the purpose of increasing highway capacity and safety. The authors defined simple policies for prescribing and regulating lane-changing policies and desired speeds in an interconnected control system. Some studies exploited the concept of Automated Highway System (AHS), defining a set of layers and developing control strategies for each one of them. In this context, an interest work was presented in [2], in which the authors analysed the link-layer control problem and proposed a control law for the stabilisation of traffic conditions. It must be highlighted that the concept of platooning (i.e. the organisation of vehicles into closely spaced groups) is often considered as a good approach, capable of increasing the motorway capacity and reducing instability. Another interesting research work is described in [3]; the authors defined a model based on linear programming for assigning traffic to lanes. The AHS was modelled as a static trip-based multi-commodity network, in which the objective was to maximise the total outflow subject to predetermined O/D patterns.

It is a common opinion that an extensive use of VACS will cause an improvement of traffic conditions, however a lot of effort is required in order to define models and strategies that could generate the expected enhancement. In this paper, it is assumed that the use of VACS permits to exploit new control actions, allowing to have higher improvement of traffic conditions.

The paper is written according to the following structure: in Section 2, the proposed traffic flow model is described, whereas Section 3 presents the formulation of the optimisation problem. In Section 4, the proposed problem is applied to a demonstrative network, stating the obtained improvements and highlighting some aspects to be considered for practical purposes. Section 5 concludes the paper and proposes possible future extensions.

2 A TRAFFIC FLOW MODEL FOR MULTIPLE-LANE MOTORWAYS

2.1 Multiple-lane traffic flow models

The motorway traffic flow models that are commonly studied in literature (e.g., the Cell Transmission Model - CTM [4] and METANET [5]) take into account aggregate dynamics for all the lanes of the network. This model simplification is surely reasonable for most of the control purposes since the control actions normally involve all the lanes together. However, having vehicles equipped with intelligent devices creates the possibility of defining more customised control strategies (e.g. assigning to a specific vehicle a task to be performed). For this reason, the proposed model is defined considering the lanes of the motorway network as different entities, characterised by their own state and control variables to define the dynamic equations.

Only a few works on multiple-lane motorways have been carried out in past research. In the first main work, [6], it is assumed that lane densities on a multi-lane highway oscillate around an equilibrium density; the authors developed a methodology to attenuate the disturbances and tried to increase the stability of the system. That work inspired the authors of [7], that proposed three models for capturing the lane changing behaviour. The first model is a continuum model based on the assumption that vehicles change lanes according to the difference of the deviations of their densities from equilibrium values. The second model extends the first one, taking also into account acceleration and inertia effects, obtaining a second-order model. A third extension is also proposed, considering also the street width. However, these models were formulated without applying any discretisation scheme. In the more recent work [8], the authors exploited the kinematic wave (KW) theory, proposing a multi-lane KW-based model as a first module of a more complex model that considers also moving blockages treated as particles characterised by bounded acceleration rates; lane changings are assigned according to the difference of mean speed between two adjacent lanes.

2.2 Model formulation

The multiple-lane motorway is represented introducing the indices $j = 1, \dots, J$ for lanes and $i = 1, \dots, I$ for segments. The simulation time $t = kT$ is defined considering the discrete time step T and the simulation index $k = 1, \dots, K$, where K defines the end of the simulation horizon. The motorway is spatially subdivided introducing the segment-lane entities (see Figure 1), characterised by the following variables:

- the density $\rho_{i,j}(k)$ [veh/km], i.e. the number of vehicles in the segment i, lane j, at time step k, divided by the segment length L_i;
- the longitudinal flow $q_{i,j}(k)$ [veh/h], i.e. the traffic volume leaving segment i and entering segment $i+1$ during time interval $(k, k+1]$, remaining in lane j;
- the lateral flow $f_{i,j,\bar{j}}(k)$ [veh/h] ($\bar{j} = j \pm 1$), i.e. the traffic volume moving from lane j to lane $\bar{j}$ (vehicles changing lane remain in the same segment during the current time interval); and
- the demand flow $D_{i,j}(k)$ [veh/h], i.e. the flow entering from the on-ramp located at segment i, lane j, during the time interval $(k, k+1]$.

The off-ramp flow is determined as a percentage of the total flow passing through all the lanes of the segment, defined by the given turning rates $\gamma_{i,j}(k)$:

$$q_{i,j}^{off}(k) = \gamma_{i,j}(k) \sum_{j=1}^{J} q_{i,j}(k).$$

The following conservation equation is introduced, defining the dynamics of traffic density $\rho_{i,j}(k)$:

$$\begin{aligned} \rho_{i,j}(k+1) = \rho_{i,j}(k) + \frac{T}{L_i}\Big[& q_{i-1,j}(k) + D_{i,j}(k) - q_{i,j}(k) - q_{i,j}^{off}(k) \\ & + f_{i,j+1,j}(k) + f_{i,j-1,j}(k) - f_{i,j,j-1}(k) - f_{i,j,j+1}(k)\Big]. \end{aligned} \tag{1}$$

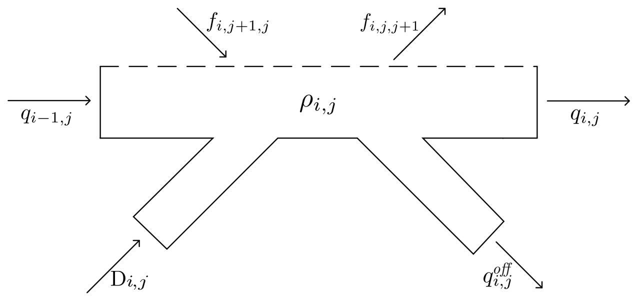

Figure 1: The segment-lane variables used in the model formulation.

In order to ensure numerical stability, the time step T must be defined respecting the Courant-Friedrichs-Lewy (CFL) condition [9]: $T \leq \min_{i,j}\left(L_i/v_{i,j}^{free}\right)$, where $v_{i,j}^{free}$ is the free speed defined for segment i, lane j.

The next modelling issue to address is the definition of bounds for the longitudinal flow. The starting basis for this is the well-known CTM [4, 10], that nevertheless does not take into account the capacity drop phenomenon, i.e. the reduction of discharge flow once a congestion is formed. The reasons for this phenomenon are not exactly known, however it seems to be caused by the limited acceleration of vehicles while exiting a congested area. In second-order models, such as METANET [5], the capacity drop is generated by the equations describing the spatiotemporal evolution of speed. This option is not available for first-order LWR models, and in order to overcome this shortcoming, several attempts have been made. The chosen approach is based on [11], where the problem is addressed by imposing an upper bound to the acceleration depending on the traffic phase, distinguishing between LWR and maximum acceleration. The proposed modelling approach, that is represented only by (piecewise) linear equations, is thus represented by a modification of the demand part of the Fundamental Diagram (FD) in the following way: in case of congestion ($\rho_{i,j}(k) > \rho_{i,j}^{cr}$, where $\rho_{i,j}^{cr}$ is the critical density) the demand flow is linearly decreased according to a fixed slope $-w'$. This leads to a flow $q_{i,j}^{jam}$ that is allowed to leave a segment in a completely congested state ($\rho_{i,j}(k) = \rho_{i,j}^{jam}$). A graphical representation of the proposed FD is depicted in Figure 2.

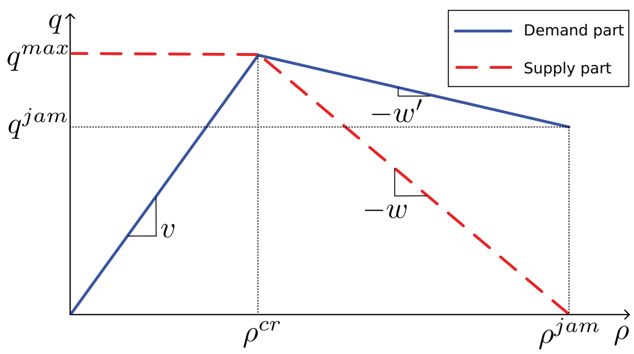

Figure 2: Graphical representation of the proposed Fundamental Diagram.

3 THE OPTIMISATION PROBLEM

The model described in Section 2 is exploited for the definition of an optimisation problem with the aim of improving the motorway conditions by reducing traffic congestion. It is supposed that the following control actions are utilised:

- Ramp Metering (RM) is currently applied on many motorways (see e.g. [12]) and does not necessarily require any additional equipment to be performed.
- Mainstream Traffic Flow Control (MTFC) via Variable Speed Limits (VSL): it is assumed that the exiting flows (and consequently the speeds) are controlled for each segment-lane; thus all equipped vehicles travelling on a segment-lane will receive and apply the respective speed as a speed limit. For a sufficient penetration of equipped vehicles, this will result in the observance of the speed limit by non-equipped vehicles as well.
- Lane-Changing Control: the optimal lateral flows are computed for each segment-lane, but the implementation of this control action is more cumbersome and uncertain than the previous two, unless all vehicles are under full guidance by the control center; in this latter case, it is not difficult to implement the control action by sending lane-changing orders to an appropriate number of vehicles. In all other cases, an intermediate algorithm should decide on the number and ID of equipped vehicles that should receive a lane-changing advice, taking into account the compliance rate and the spontaneous lane-changings; the latter may be reduced by involving additional “keep-lane” advice to other vehicles. These issues are currently in course of investigation and development.

Since RM actions are applied, the following variables are added considering the creation of queues at on-ramps:

- the queue length $w_{i,j}(k)$ [veh], i.e. the number of vehicles queuing at on-ramp located in segment i, lane j, at time step k; each queue is characterised by a maximum length $w_{i,j}^{max}$;
- the on-ramp flow $r_{i,j}(k)$ [veh/h], as the flow entering the network, leaving the queue generated in segment i, lane j, during the time interval $(k, k+1]$; this variable replaces the demand flow $D_{i,j}(k)$ in Equation 1;
- the extra-queue length $W_{i,j}(k)$ [veh], that represents an additional state variable considering vehicles that cannot enter the queue because it has reached its maximum length; the introduction of this variable permits to avoid the infeasibility of the optimisation problem; and
- the flow $d_{i,j}(k)$ [veh/h], i.e. the demand flow that is capable to enter the real queue; therefore in case the maximum size is not reached, it results $d_{i,j}(k) = D_{i,j}(k)$.

The problem is formalised as a Quadratic Program (QP), characterised by a convex quadratic cost function and uniquely linear constraints, allowing its application also for large networks.

$$
\begin{aligned}
\min_{\substack{\rho, w, W,\\ q, r, f}} \quad Z = {} & T \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[L_i \rho_{i,j}(k) + w_{i,j}(k) \right] \\
& + M \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} W_{i,j}(k) \\
& + \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} \Big[\beta_{i,j,j-1}\, f_{i,j,j-1}(k) + \beta_{i,j,j+1}\, f_{i,j,j+1}(k) \Big] \\
& + \lambda^{r} \sum_{k=2}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[r_{i,j}(k) - r_{i,j}(k-1) \right]^2 \\
& + \lambda^{f} \sum_{k=2}^{K} \sum_{i=1}^{I} \Bigg\{ \sum_{j=2}^{J} \left[f_{i,j,j-1}(k) - f_{i,j,j-1}(k-1) \right]^2 \\
& \qquad\qquad + \sum_{j=1}^{J-1} \Big[f_{i,j,j+1}(k) - f_{i,j,j+1}(k-1) \Big]^2 \Bigg\} \\
& + \lambda^{st} \sum_{k=2}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{\Big\{ q_{i,j}(k) - q_{i,j}(k-1) + v^{free}\big[\rho_{i,j}(k) + \rho_{i,j}(k-1) \big] \Big\}^2}{\left(\rho_{i,j}^{cr}\right)^2} \\
& + \lambda^{sl} \sum_{k=1}^{K} \sum_{i=2}^{I} \sum_{j=1}^{J} \frac{\Big\{ q_{i,j}(k) - q_{i-1,j}(k) + v^{free}\big[\rho_{i,j}(k) + \rho_{i-1,j}(k) \big] \Big\}^2}{\left(\rho_{i,j}^{cr}\right)^2}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
& \rho_{i,j}(k+1) = \rho_{i,j}(k) + \frac{T}{L_i} \Big[q_{i-1,j}(k) + r_{i,j}(k) - q_{i,j}(k) - q_{i,j}^{off}(k) \\
& \qquad\qquad + f_{i,j+1,j}(k) + f_{i,j-1,j}(k) - f_{i,j,j-1}(k) - f_{i,j,j+1}(k) \Big] \\
& w_{i,j}(k+1) = w_{i,j}(k) + T\big[d_{i,j}(k) - r_{i,j}(k) \big] \\
& W_{i,j}(k+1) = W_{i,j}(k) + T\big[D_{i,j}(k) - d_{i,j}(k) \big] \\
& q_{i,j}(k) \le v_{i,j}^{free} \rho_{i,j}(k) \\
& q_{i,j}(k) \le \frac{v_{i,j}^{free} \rho_{i,j}^{cr} - q_{i,j}^{jam}}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}} \rho_{i,j}(k) + \frac{v_{i,j}^{free} \rho_{i,j}^{jam} - \rho_{i,j}^{cr} \rho_{i,j}^{jam}}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}} \\
& q_{i,j}(k) \le v_{i+1,j}^{free} \rho_{i+1,j}^{cr} \\
& q_{i,j}(k) \le \frac{v_{i+1,j}^{free} \rho_{i+1,j}^{cr}}{\rho_{i+1,j}^{jam} - \rho_{i+1,j}^{cr}} \rho_{i+1,j}(k) + \frac{v_{i+1,j}^{free} \rho_{i+1,j}^{cr} \rho_{i+1,j}^{jam}}{\rho_{i+1,j}^{jam} - \rho_{i+1,j}^{cr}} \\
& [f_{i,j,j-1}(k) + f_{i,j,j+1}(k)] \le \frac{L_i}{T} \rho_{i,j}(k) \\
& [f_{i,j-1,j}(k) + f_{i,j+1,j}(k)] \le \frac{L_i}{T} \left[\rho_{i,j}^{jam} - \rho_{i,j}(k) \right] \\
& f_{i,j,j-1}(k) \le f^{max}, \quad f_{i,j,j+1}(k) \le f^{max} \\
& \rho_{i,j}(k) \le \rho_{i,j}^{jam}, \quad w_{i,j}(k) \le w_{i,j}^{max}, \quad r_{i,j}(k) \le r_{i,j}^{max}
\end{aligned}
$$

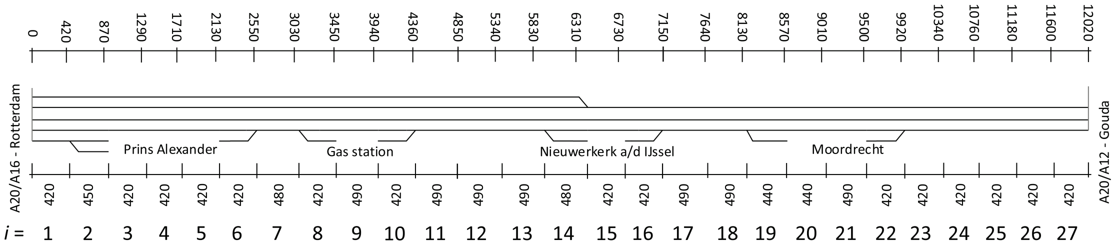

Figure 3: The A20 motorway stretch from Rotterdam to Gouda used to test the proposed approach.

The cost function to be minimised is composed by different terms:

- the first and most important one is the Total Time Spent (TTS), that considers the overall time spent by vehicles both travelling and queuing at the on-ramps;
- the other linear terms are penalty terms defined with the purpose of reducing extra queues and lateral flows; and
- the quadratic terms are introduced in order to penalise time and space oscillations in the control values; among them, the last two terms have the purpose of reducing the oscillations of the speed values (as a matter of fact, they represent a linearisation of the non-linear constraints that consider speed variation).

The first set of constraints represents the dynamics for the densities (that derives from Equation 1, however replacing the external demand with the on-ramp flow), for the queues generated at on-ramps because of the RM actions, and for the extra-queues.

The following set of constraints corresponds to the FD described in Section 2; it is important to highlight that, having the possibility of controlling the flow (and indirectly the speed) of vehicles, the constraints could simply be described by linear inequalities that represent upper-bounds for the segment outflow.

The third set of constraints is related to the lateral flows. In this case, they appear only in the form of upper-bounds, allowing the optimiser to assign the best lateral flow.

The last set of constraints contains the upper-bounds for densities, on-ramp queues, and ramp flows.

4 APPLICATION EXAMPLE

In order to evaluate and illustrate the potential improvements that could be obtained by applying the described methodology, a stretch of the motorway A20 from Rotterdam to Gouda, the Netherlands, taken from [13], is used. The topological characteristics of this network (lane-drops, on-ramps and off-ramps) make it a very interesting test-bed evaluating the results of the proposed optimisation problem.

The stretch, about 12 km in length, is subdivided into 27 segments of 450 m in average, as shown in Fig. 3. The time step is set to $T = 15$ s. The lanes are numbered $j = 1, \ldots, 4$ from the inner lane (close to the roadside) to the outer lane (close to the road median).

It is supposed that both the densities and ramp queues are initialised to 0 at the beginning of the simulation. All links have the same characteristic values: the critical density is set to $\rho_{i,j}^{cr} = 22$ veh/km, the jam density is set to $\rho_{i,j}^{jam} = 180$ veh/km, the maximum speed is $v_{i,j}^{max} =$

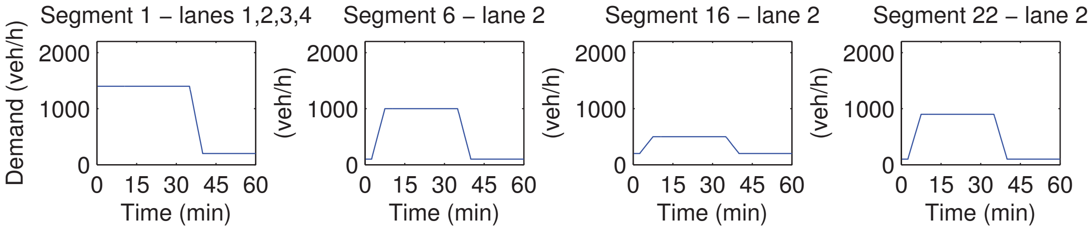

Figure 4: Demand profiles at on-ramps; "Gas station" is omitted due to the very low entering flow (a maximum value of 30 veh/h).

100 km/h, and the maximum flow at jam density is $q_{i,j}^{jam} = 1467.4$ veh/h (obtained by setting a slope $w' = w/3$). The exit rates at off-ramps are set as follows: $\gamma_{2,1} = 0.2$, $\gamma_{8,2} = 0.0085$, $\gamma_{14,2} = 0.3$, $\gamma_{19,2} = 0.2$.

Another significant aspect is represented by the tuning of the cost function weights: once a proper value of M is determined in order to avoid extra-queues (in this case, $M = 10$), the tuning procedure is focused in keeping virtually the same TTS and, at the same time, trying to obtain reduced lateral flows and smooth control actions. For the linear penalty term related to lateral flow, an important aspect is also represented by the locations of these control actions. In locations where strong lateral flow actions are expected (e.g. at lane-drops and on-ramps), vehicles are encouraged to change lane in the segment immediately upstream by setting the weight $\beta_{i,j,\bar{j}} = 0$; in all other segments, the values are set to $\beta_{i,j,\bar{j}} = 0.01$. As a last step, the weigh parameters of quadratic terms were tuned, obtaining the following values: $\lambda_f = 10^{-5}$, $\lambda_r = 10^{-7}$, $\lambda_{st} = 10^{-5}$, and $\lambda_{sl} = 10^{-6}$.

For the resolution of the optimisation problem the solver Gurobi [14] has been utilised, choosing a barrier method algorithm for QP solving. Despite the considerable size of the optimisation problem that is obtained also for small networks, the solution could be achieved in a reasonable time. In fact, as it is shown in Table 1, setting an optimisation horizon of 45 minutes or less, the solution is achieved in a computation time that is smaller than the simulation step, making this approach very attractive also for real-time control, e.g. using this problem as a module in a Model Predictive Control (MPC) framework.

Optimisation horizon (min)	Variables	Equalities	Inequalities	Computation time (s)
60	206820	77760	308880	~ 20
45	154980	58320	96660	~ 13
30	103340	38880	64260	~ 7

Table 1: Comparison of computation time with respect to the reduction of the optimisation horizon (and consequently of the size of the optimisation problem).

In order to highlight the computed control actions, the following example is examined considering an optimisation horizon of 60 minutes. The utilised demand profile is shown in Fig. 4, where the reduced entering flows during the last 20 minutes represent a cool down period that will ensure the disappearance of any congestion at the end of the simulation.

The following different actions could be identified in the optimal control solution:

- In case the overall segment capacity is sufficient to accommodate the flow entering at the on-ramp, the space is created by assigning lateral flow actions (e.g., this happens at

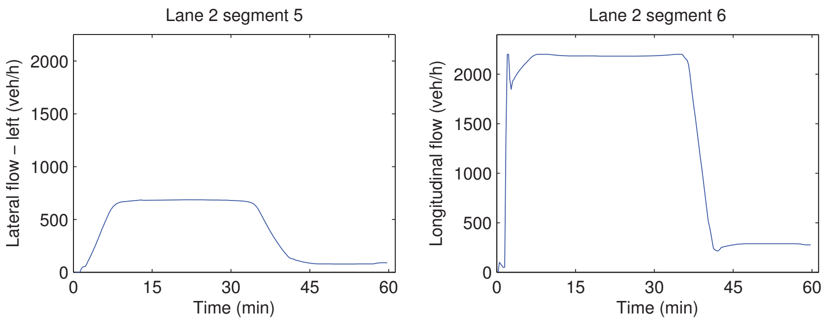

Figure 5: The Lane-Changing control actions taken during the high-demand period in segment 5, lane 2 generate a maximum outflow in segment 6, lane 2 after the vehicles have entered from the on-ramp “Prins Alexander”.

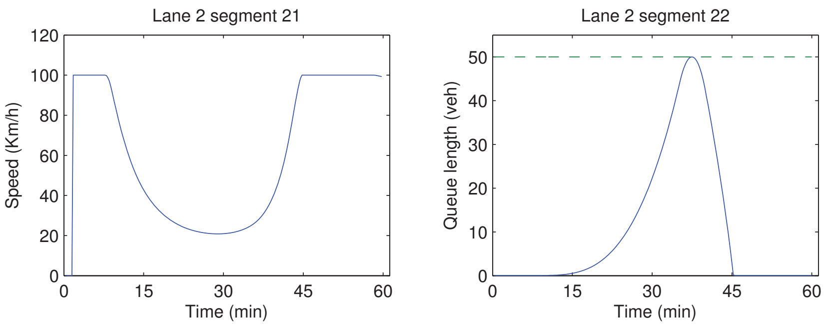

Figure 6: Combined VSL and RM actions are taken in order to maximise the throughput in the area of on-ramp “Moordrecht”. The maximum queue length is represented by the dashed line.

on-ramp “Prins Alexander”). These actions are shown in Figure 5: it is interesting to highlight that they lead to achieve the maximum flow at segment 6 (that includes also the flow entering at the on-ramp). Since the lateral movements are performed in the upstream segment, it is also worthwhile to point out that the phenomenon of having vehicles entering the motorway and changing lane in the same segment is avoided.

- In the lane-drop area of segment 14, the space for vehicles that have to change lane is created through some MTFC actions in the upstream segments, thus avoiding an excessive increase of density and excessive vehicle movements at the segment of the lane-drop.
- In case the demand flow is higher than the segment capacity, RM and VSL actions are jointly applied, allowing to not exceed the capacity flow and to avoid any speed breakdown. These actions appear both at the on-ramp “Nieuwerkerk a/d IJssel” and “Moordrecht” (for the latter one, see Figure 6).

Moreover, it is interesting to point out that MTFC actions are performed only on the inner lane, whereas in the other lanes the flow (and consequently the speed) remains constantly at the maximum value. In addition, some minor VSL actions are taken in order to help vehicles that are changing lanes in proximity of the on- and off-ramps, as well as because of the lane drop.

5 CONCLUSIONS

The paper describes a novel multiple-lane traffic flow model on which an optimisation problem is based. The model includes some simplifications that have allowed to obtain a QP with only linear constraints. The low required computation time makes this methodology suitable for real-time applications, as well as usable in a hierarchical control approach. The exploitation of this work as the optimisation module in a Model Predictive Control (MPC) is the subject of ongoing research activities. However, in order to make possible to implement this strategy also in case of mixed traffic (that contains vehicles equipped with VACS and traditional ones), the definition of a hierarchical control structure seems to be necessary .

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