RETAINING WALL-SOIL-STRUCTURE INTERACTION EFFECTS DUE TO SEISMIC EXCITATION

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ABSTRACT

Retaining walls are very frequently used in engineering practice to support, apart from soil layers, structures founded on the retained soil layers. During a seismic event it is evident that the dynamic response of each component of this complex system (i.e., wall, soil and structure) may affect substantially the response of the rest and vice versa. This phenomenon, which could be adequately described as "dynamic wall–soil–structure interaction" (DWSSI), is a rather complicated issue that combines: (a) the dynamic interaction between the wall and the retained soil layers, and (b) the "standard" dynamic interaction of a structure with its underlying soil layers. In the present study, using two-dimensional numerical simulations, the impact of the wall flexibility and structure stiffness on the dynamic response of the overall system is investigated. Emphasis is given on the relation between the dynamic behavior of the overall system and the dynamic properties of each component (one-dimensional soil layer-structure and wall-soil-structure). Primarily, the response of a simple structure lying on a single infinite soil layer is investigated. Subsequently, a retaining wall is included in the numerical models. A parametric study is performed in order to examine at what extend the presence of the wall may affect the amplification factors of the various components, and consequently the distress imposed on the structure (with respect to its position and its fundamental eigenperiod). In addition, the eigenperiod of the soil-structure system is calculated via analytical methods. Despite the fact that there exist many open issues to be resolved, the results of the current study provide a clear indication of the direct dynamic interaction between a retaining wall and an adjacent structure.
Keywords: dynamic response, soil-structure interaction, amplification factor, soil spring.

INTRODUCTION

It is generally accepted that during a seismic event structures founded on soft soil exhibit much different dynamic behaviour than those founded on rigid rock. Therefore, structures founded on soft soil cannot be modelled as if they were founded on rigid rock, by assuming that they are totally fixed at their base. Their dynamic characteristics change substantially due to soil compliance, which has to be taken into account during their seismic design. This problem is widely known as *dynamic soil-structure interaction problem* (DSSI). Soil compliance is usually taken into account by using a number of springs and dashpots, which are located at the level of the structure's foundation, and are supposed to substitute totally the underlying soil. The calculation of these srings and dashpots is a sophisticated procedure, which has been coped with (semi-) analytical solutions (Veletsos and Meek, 1974, Gazetas, 1983). Using spring constants, various seismic norms (e.g., NEHRP-97) provide simple formulas to calculate the modified (due to DSSI) structural eigenperiod. Nevertheless, dynamic interaction between the soil and the structure remains a complex and unresolved issue, given that the nonlinearities (material and/or geometrical) present in the overall system behavior are usually not taken into account. However, the issue of DSSI (taken

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into consideration in a simplistic way by most seismic norms) is considered a-priori to be beneficial for a structure, which seems not to be always the case (Mylonakis and Gazetas, 2000).

In contrast, retaining walls are extensively used worldwide for serving various purposes in structures and infrastructures. Deep excavations, bridge abutments, or harbor quay-walls are some of the cases where a rigid or a flexible retaining wall is constructed. Despite their structural simplicity, the seismic response of retaining walls (that retain even a single soil layer) is a rather complicated problem. The major complexity of the problem is the dynamic interaction between the wall and the retained soil, especially when material and/or geometry nonlinearities are present (Kramer, 1996, Wu and Finn, 1999). This interaction is known as *dynamic wall-soil interaction* (DWSI). Consequently, the performance of retaining walls during earthquakes is a subject being still examined by many researchers, experimentally, analytically, or numerically (Veletsos and Younan, 1997, Psarropoulos et al., 2005). However, regarding the seismic design of retaining systems, the DWSI is generally ignored by the seismic norms (EC8, 2004, EAK, 2000).

In both the aforementioned cases (DSSI and DWSI), the phenomenon of “simple” dynamic interaction with soil material occurs. Nevetheless, in many real cases retaining walls are used to support structures founded on the retained soil. It is evident that during a seismic event the dynamic response of each component of this complex system (wall, soil layer, and structure) may affect substantially the response of the others. The presence of the retaining wall will affect not only the ground surface shaking of the retained soil, but the dynamic response of any type of retained structure as well. In addition, the existence of a structure behind the wall is expected to alter the dynamic earth pressures developed on the wall. Therefore, the phenomenon of *dynamic wall–soil–structure interaction* (DWSSI) is a complicated problem that includes: (a) the dynamic interaction between a wall and the retained soil layers, and (b) the “standard” one-dimensional dynamic soil–structure interaction of a structure with its underlying soil.

The aforementioned issues are not considered with the proper realism in the current seismic norms used in modern engineering practice, such as the Eurocode 8 (EC8 2004) or the Greek Seismic Code (EAK 2000). However, when structures (even with a single degree of freedom) have foundation that do not lie on a homogeneous halfspace, but nearby a rigid or a flexible retaining wall, their seismic response becomes much more complicated, mainly due to the existence of the aforementioned double interaction, rather than the simple well-known interaction between the structure and the underlying soil. In essence, the existence of a wall imposes a vertical boundary on the overall soil-structure system which makes the whole model stiffer, and consequently reduces its eigenperiod. Thus, an eigenperiod lower than the one described by the NEHRP-97 provisions is expected.

The objective of the present study is to examine more thoroughly the phenomenon of *dynamic wall–soil–structure interaction*. For this purpose, two-dimensional plane-strain numerical simulations are performed, utilizing the finite-element method, in order to investigate some of the most important aspects of this complex phenomenon. Firstly, the influence of the soil compliance on the structural response is investigated, with the absence of the retaining wall (as shown in Figure 1), while emphasis is given on the amplification of the base acceleration, a fact generally underestimated by the seismic norms. The structure is essentially a single-degree-of-freedom (SDOF) system, comprised of a concentrated mass, a column, and a rigid footing. The resonant frequencies of the structure can be estimated, and the interpretation and verification of the results, with the well-established NEHRP-97 provisions, can be easily performed. Subsequently, a retaining cantilever wall adjacent to the structure is included in the numerical models. A parametric study has been conducted in order to examine how the location of the structure may affect the amplification factors in characteristic locations of the model. In addition, the parametric study investigates at what extent the presence of the wall may affect the response of the structure with respect to its position. In all cases, the wall is characterized by its height H and its relative flexibility d_w, while the soil is considered as uniform viscoelastic material with constant shear-wave velocity V_S, density ρ, and critical damping ratio ξ. In general, dynamic response of any system depends on the seismic excitation characteristics (both in the time domain and in the frequency domain). Without any loss of generality, harmonic excitations have been considered in this study, since any arbitrary seismic excitation can be analysed into a series of harmonic functions with selected amplitudes and frequencies (Fourier series). Results provide a clear indication of the direct dynamic interaction between a retaining wall and its retained structures. That fact justifies the necessity for a more elaborate consideration of this interrelated phenomenon on the seismic design of the retaining walls and the nearby structures.

EFFECTS OF DYNAMIC SOIL-STRUCTURE INTERACTION (DSSI)

Initially, the retaining wall was not considered during the analysis of the dynamic system. Thus, the soil layer behaves as if it was extended to infinity for both horizontal directions. Such cases, in which structures have to be founded on compliant soil, are not rare in civil engineering practice. In order to examine the effects of DSSI on structures, one-dimensional (1-D) numerical simulations of the system depicted in Figure 1 were conducted. The simulations were performed utilizing the finite-element code ABAQUS (2003), which is capable of performing dynamic linear analyses using Rayleigh type of material damping (resulting to a critical damping ratio of ξ for the frequencies of interest). The soil was discretized using four-node quadrilateral plain-strain elements. Kinematic constraints were used at the two vertical boundaries of the soil layer in order to simulate the 1-D soil layer response. For more accurate simulation, the vertical kinematic constraints were placed far away from the structure. Regarding the ground motion, harmonic excitation has been selected as the dynamic loading. Although soil nonlinearity is expected to have a significant impact, it was not examined in this preliminary investigation of this complex interaction phenomenon. To avoid inertial effects on the response of the structure (which would make the interpretation of the results of the investigated dynamic interaction issues much more complicated) the structure was modelled as a lumped mass m on top of a weightless column of flexural stiffness K_{STR}, discretized with beam elements. The weightless column was considered to be fixed at a rigid, weightless foundation, which is located on the surface of the soil layer (i.e., without any embedment). The interface between the foundation and the underlying soil is considered to be fully bonded, thus no sliding of the footing is allowed and its dynamic response is identical to the response of the surrounding soil. This assumption is generally valid for cohesive soils and small displacements.

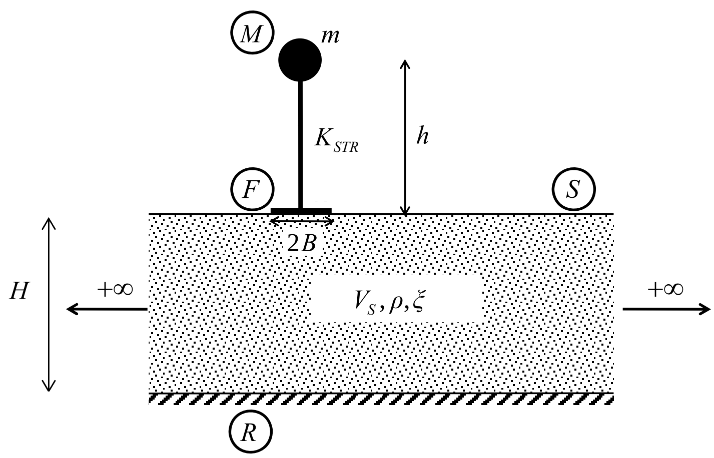

Figure 1. The examined 1-D model: a SDOF system founded on the surface of an infinite soil layer

The response of the structure and its foundation were studied via properly defined transfromation functions that define the amplification in various locations shown in Figure 1: the bedrock (R), the free-field of the soil (S), the foundation (F) of the SDOF and the top (M) of the SDOF where its mass is located. Consequently, the Amplification Factor between two arbitrary points X and Y of the model, denoted as *AF(XY),* is defined by:

$$AF(XY) = \frac{FFT[Y(t)]}{FFT[X(t)]} \quad (1)$$

where *FFT[Y(t)]* and *FFT[X(t)]* are the Fast Fourier Transforms of the corresponding time histories of points X and Y, respectively. Displacement-, velocity-, or acceleration- time histories can be used in the above equation, provided that both Y(t) and X(t) are expressed in terms of the same quantity. Two characteristic amplification factors *AF* are considered here: (a) the *AF(RM)* which is the total amplification between the rigid rock layer and the lumped mass of the SDOF, and (b) the *AF(RF)* which is the amplification between the rigid rock and the foundation of the SDOF.

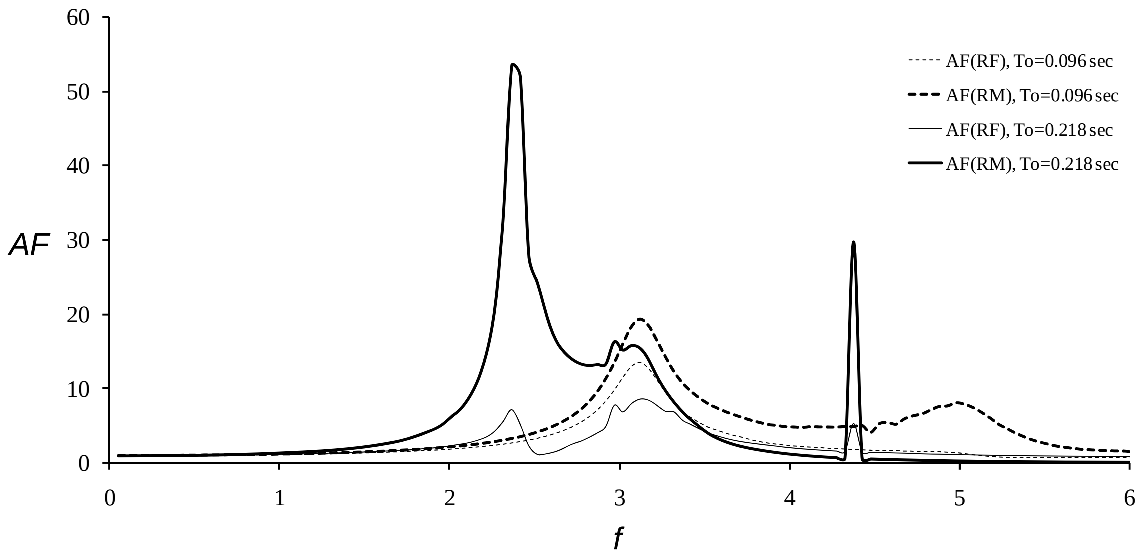

Figure 2. The variation of the steady state *AF*s, versus the frequency of the imposed harmonic excitation

For any structure founded on rigid rock its eigenperiod *To* is given by:

$$To = 2\pi\sqrt{\frac{m}{K_{STR}}} \quad (2)$$

If two structures with arbitrary characteristics and eigenperiods equal to To_1=0.218 sec and To_2=0.096 sec are assumed on the top of the model presented in Figure 1, then the amplification factors depicted in Figure 2 will result. The bold lines correspond to the amplification of the structures' lumped mass, and the thinner ones refer to the amplification of the structures' foundation level. By inspecting Figure 2 the following trends can be noticed:

(a) All curves have a local peak around the frequency equal to $f=3.15$ *Hz*. These peaks are due to the resonance of the underlying soil. The resonant frequency of the soil layer is given by:

$$f_0 = \frac{V_S}{4H} \quad (3)$$

and for $V_s=100m/s$ and $H=8m$ considered in this simulation, Eq. (3) results to $f_o=3.125$ *Hz*, which is very close to the aforementioned value.

(b) The curves showing the total amplification factor of the SDOF, *AF(RM)*, have another peak at different frequencies, while at the same frequencies the value of the amplification factor of the foundation, *AF(RF)*, is small. These frequencies are the resonant frequencies of the structures and they appear to be independent of the behavior of the underlying soil. These two frequencies are equal to $\overline{f}o_1 = 2.4$ Hz (corresponding to period $\overline{T}o_1 = 0.417$ sec) for the structure with To_1=0.218 sec, and $\overline{f}o_2 = 5$ Hz (corresponding to period $\overline{T}o_2 = 0.2$ sec) for the structure with To_2=0.096 sec. Note that $\overline{T}o_1 > To_1$ and $\overline{T}o_2 > To_2$, more specifically $\overline{T}o_1 / To_1 = 1.91$ and $\overline{T}o_2 / To_2 = 2.08$, due to the impact of DSSI.

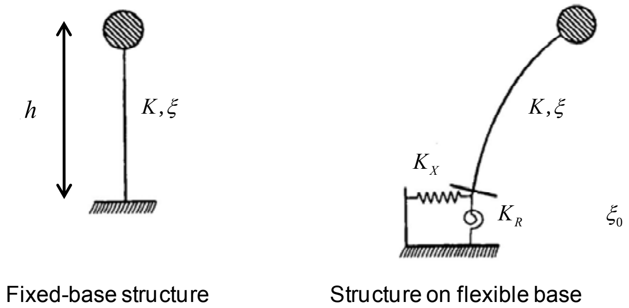

Figure 3. A SDOF system lying on rigid rock (left), and a SDOF system lying on deformable soil (right)

Figure 3 presents two possible modelling types for the foundation of SDOF systems. At the left, the original fixed-base soil-structure system is depicted, while at the right the equivalent SDOF system of the sift soil layer, substituted by two representative springs, is shown. According to the NEHRP-97 provisions (Gazetas, 1991), the eigenperiod of a structure founded on compliant soil is calculated by:

$$\overline{T} = T\sqrt{1+\frac{K_{STR}}{K_X}\left(1+\frac{K_X h^2}{K_R}\right)} \tag{4}$$

where $\overline{T}$ is the eigenperiod of the structure lying on deformable soil and T is the eigenperiod of the same structure being totally fixed at its base, while K_{STR} is the flexural stiffness of the column:

$$K_{STR} = \frac{3EI}{h^3} \tag{5}$$

E, I, h denote the Young modulus, the moment of inertia, and the height of the column of the SDOF. Parameters K_X and K_R are the horizontal and rotational equivalent stiffeness of the springs simulating the underlying soil respectively, which for the strip foundation lying on the surface of a homogeneous soil (see Figure 4) are given by:

$$K_X \simeq \frac{2.1G}{2-\nu}\left(1+\frac{2B}{H}\right) \tag{6}$$

$$K_R \simeq \frac{\pi G B^2}{2(1-\nu)}\left(1+\frac{B}{5H}\right) \tag{7}$$

The parameters in Equations (6) and (7) are shown in Figure 4. For the examined structure they are equal to: $h=6$ m, $E=30GPa$, and $I=0.018$ m^4 (for dimensions *1.0m x 0.6m*) and its foundation has width $2B=3.2$ m. In addition, the underlying soil layer has $G=18MPa$ and $\nu=0.3$. According to Eqs. (5), (6) and (7) the stiffnesses parameters can be easily calculated:

K_{STR}*=7.5MN/m,* K_X*=31.129MN/m,* and K_R*=107.539MNm.* Therefore, Equation (4) results to $\overline{T}/T \simeq 1.94 \simeq \overline{T}o_1/To_1 \simeq \overline{T}o_2/To_2$, thus, the NEHRP-97 provisions are verified.

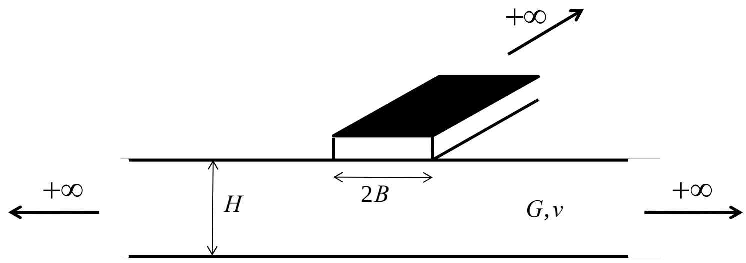

Figure 4. A rigid strip footing lying on an infinite soil layer for plane strain conditions

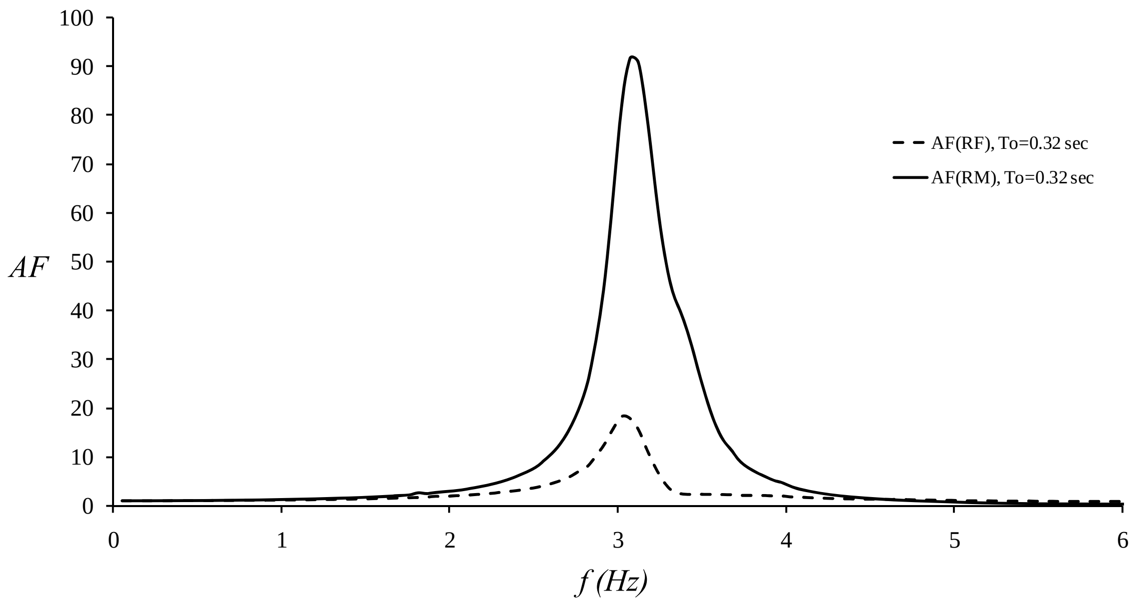

Figure 5. The variation of the steady state AFs versus the frequency of the imposed harmonic excitation for the case of double soil-structure resonance

Considering the case when the structure’s eigenperiod is equal to the eigenperiod of the underlying soil layer. Then, if the amplification factors are plotted against the frequency *f* of the harmonic excitation then the curves shown in Figure 5 are produced. In these curves, both amplification factors (*AF(RM)* and *AF(RF)*) present a peak at the resonant frequency of the soil layer. No other peaks exist, and it can be observed that the amplification factors are both much higher than the ones observed in Figure 2, e.g., *AF(RM)* is almost equal to 100. This phenomenon, in which the amplification factor of the SDOF is very high, compared to the amplification factor of its foundation, is called “double resonance”. If this occurs, then the structure alone, and the soil layer also, are in conditions of resonance and the overall vibrating system is also in resonance. Consequently, the dynamic response of the structure increases dramatically (as one can see by comparing Figures 2 and 5), and failure will ensue. Such a case has occured in the Mexico city, during the 1985 earthquake, the destructive impact of which is widely known. This system, which is characterized by double soil-structure resonance, is selected in order to study the influence of a retaining wall (rigid or flexible) adjacent to the structure.

EFFECTS OF DYNAMİC WALL-SOİL-STRUCTURE INTERACTİON (DWSSI)

As it was previously mentioned, prescriptive seismic norms are not capable of taking realistically into consideration the main components of the dynamic wall-soil-structure interaction: (a) the dynamic interaction between a retaining wall and the retained soil layer, and (b) the "standard" 1-D dynamic soil-structure interaction, i.e., the foundation of a structure on a soil layer and the related kinematic or inertial interaction with it. Therefore, in a case of a complex wall-soil-structure system, elaborate numerical modelling of the whole problem is unavoidable, as it is not realistic to study the wall-soil system and the soil-structure system independently. In order to examine more efficiently the DWSSI phenomenon, numerical analyses were conducted and the results are obtained in terms of amplification factors. The analytical methodology of Veletsos and Younan (1997) has permitted the assessment of the effects and the relative importance of the factors involved. The 1-D soil layer is now considered to be retained by a vertical, rigid or flexible wall; it is free at its upper surface and fixed on a rigid base (thus no radiation damping is expected to occur). The properties of the wall are described by its thickness t_w, mass per unit of surface area μ_w, modulus of elasticity E_w, Poisson's ratio ν_w, and critical damping ratio ξ_w. The base of both the wall and the soil stratum were considered to be excited by a space-invariant horizontal harmonic motion, assuming an equivalent force-excited system.

In the present study, in order to examine the effects of DWSSI on the retained structures, two-dimensional (2-D) numerical simulations of the retaining system depicted in Figure 6 were conducted. The simulations were performed utilizing also the finite-element code ABAQUS (2003), which is capable of performing dynamic linear analyses using Rayleigh type of material damping (resulting to a critical damping ratio of ξ for the frequencies of interest). The wall was discretized using beam elements of unit longitudinal dimension and thickness equal to t_w = 0.20m.

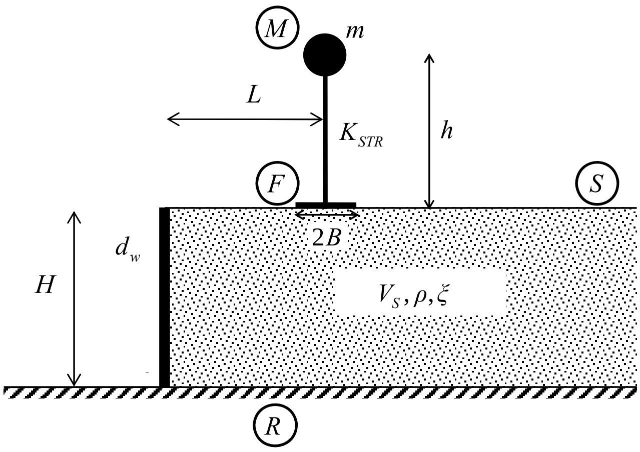

Figure 6. The retaining wall-soil-structure system examined in this study: a wall retaining a soil layer of height H on which a SDOF system is founded at distance L

The main parameters that affect the response of the system are the relative flexibility of the fixed-base wall derived by:

$$d_w = \frac{GH^3}{D_w} \tag{8}$$

and the dimensionless distance of the structure from the wall, defined by the ratio L/H. Parameter D_w in Eq. (8) denotes the flexural rigidity per unit of length of the wall, and is given by:

$$D_w = \frac{E_w t_w^3}{12(1-\nu_w^2)} \tag{9}$$

Two rather extreme cases were examined in this study: (a) a rigid fixed-base wall (d_w = 0), and (b) a very flexible fixed-base wall (d_w = 40). Given the value of d_w, the modulus of elasticity of the wall E_w is evaluated using equations (8) and (9), while the Poisson's ratio v_w is taken as 0.2. The wall is presumed to be massless, in order to avoid inertial effects on the structural response, which would further increase the complexity of the problem. The simplifying assumptions that no de-bonding or relative slip is allowed to occur at the wall-soil and the structure-soil interfaces were used.

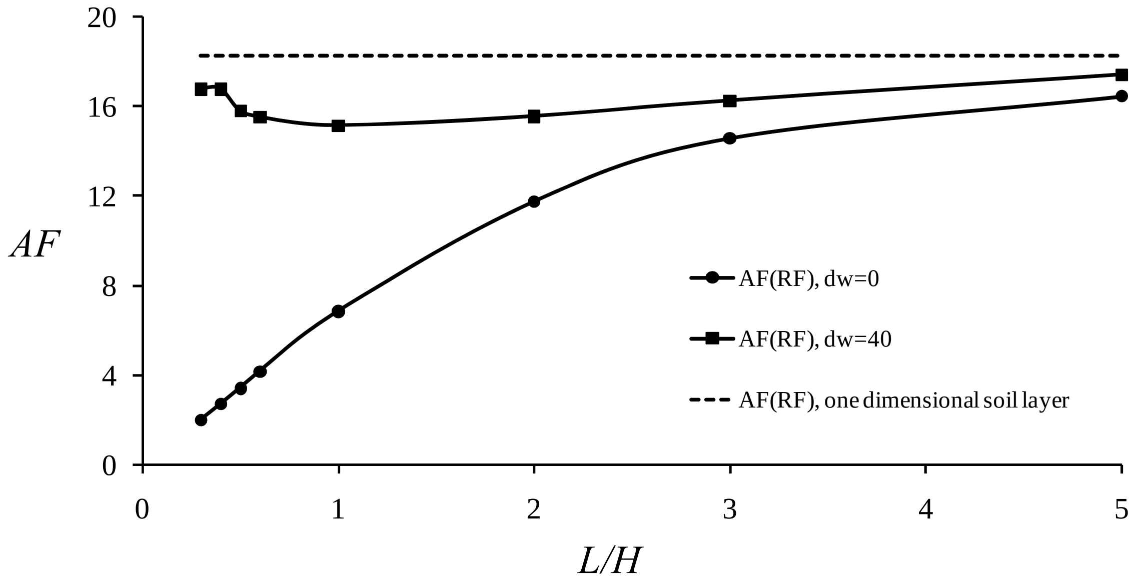

Figure 7. Variation of AF between the rigid rock (R) and the foundation of the SDOF (F) with the dimensionless distance from the wall *L/H*; *AF* for the case of 1-D soil layer is also shown

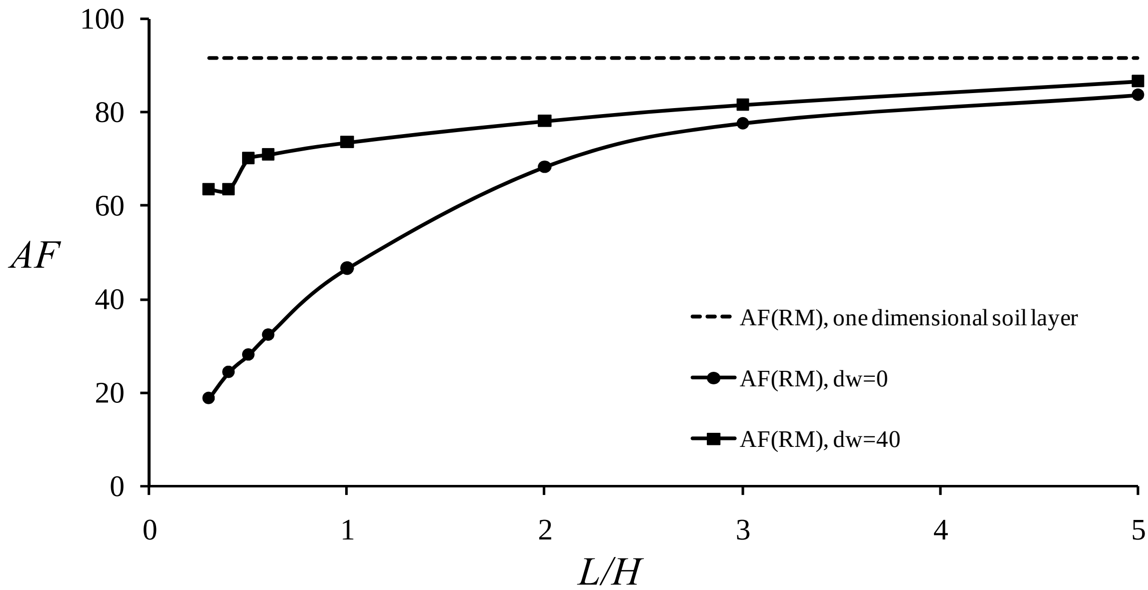

Figure 8: Variation of the Amplification Factor between the rigid rock (R) and the top of the SDOF (M) with the dimensionless distance from the wall *L/H*; *AF* for the case of 1-D soil layer is also shown

It has to be stressed that the eigenperiod of the structure is affected by two primary factors: (a) the underlying soil compliance has an increasing effect on the structure eigenperiod, and (b) the presence of the retaining wall imposes a vertical (rigid or flexible) boundary to the soil layer, thus it makes the soil-structure system stiffer and it has a decreasing effect on the structure eigenperiod.

Therefore, the value of the structure's eigenperiod will be lower than the one given by Equation (4).

In Figure 7, the maximum amplification factor *AF(RF)* is plotted versus the dimensionless distance of the structure from the wall, *L/H*, for the two extreme cases of wall flexibility: $d_w=0$ and $d_w=40$. It is obvious that the presence of a rigid wall has a decreasing effect on the maximum amplification for values of *L/H* lower than 5It can be observed that as the distance ratio *L/H* decreases the system becomes stiffer, so the observation of lower amplification values is rational. For the case of flexible retaining wall, this reduction is not so intense. As the structure moves away from the wall, the wall has gradually lower influence on the structural response and the structural response approaches its behavior when founded on the 1-D soil layer. The same trends are detected in Figure 8 which presents the variation of the maximum amplification factor *AF(RM),* with increasing dimensionless distance of the structure from the wall, for the two extreme values of wall flexibility.

Note that for all values of *L/H*, in both figures (7 and 8) the system is in resonance, thus, the maximum possible amplification factor can be obtained. It is also noteworthy that for *L/H=1*, the amplification factor *AF(RF)* reaches a local minimum of 15. Actually, AF does not get lower than this value, for any value of *L/H>0.3*. The corresponding curve for the *AF(RM)* in Figure 8 does not follow this tendency, as the amplification of the structure's foundation increases and the amplification of the lumped mass decreases. This is attributed to the fact that the inertial effects of the lumped mass of the structure are gradually increasing, provided that the structure is relatively close to the wall. The flexibility of the retaining wall plays also an important role on the dynamic response of the structure. It can be seen in both figures that for all cases of *L/H*, flexible wall produces higher amplifications, both for the structure's foundation and the structure's lumped mass. The lowest amplification corresponds for the perfectly rigid retaining wall ($d_w=0$).

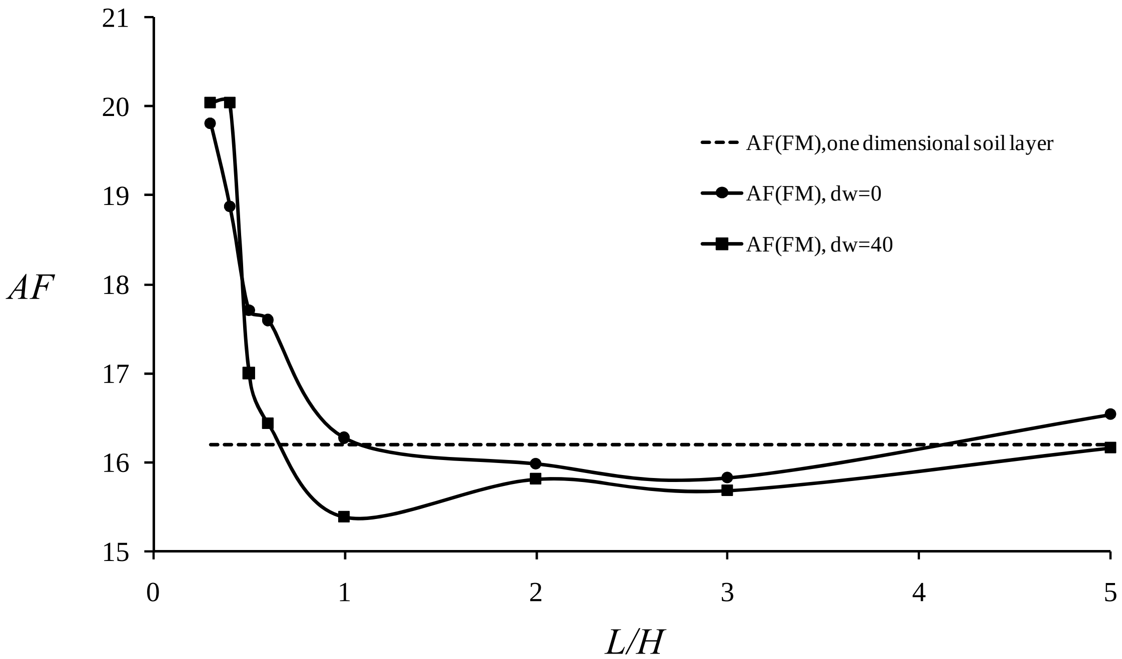

Figure 9: Variation of the Amplification Factor between the foundation (F) and the top of the SDOF (M) with the dimensionless distance from the wall *L/H*; *AF* for the case of 1-D soil layer is also shown

In Figure 9 the amplification factor *AF(FM)* is shown as a function of the dimensionless distance of the structure from the wall *L/H*. In this diagram, a remarkable effect of the presence of the retaining wall is shown: while the other amplification factors generally reduce as the structure approaches the wall, this amplification factor becomes higher. Therefore, it is obvious that if the *AF(RF)* or *AF(RM)* are low, this does not mean that the amplification of the structure alone is also low. Structural engineers must be aware of the DWSSI phenomenon, as most seimic norms regard only the *AF(RF)* (which is not always realistic) only for the DSSI phenomenon, and may lead to unsafe design. Thus, the necessity of a unified retaining wall-soil-structure system and the understanding of the response of its components is imperative, in order to deal realistically with DWSSI phenomena.

CONCLUSIONS

The scope of the present study was to investigate preliminarily the dynamic interaction between retaining walls, retained soil, and retained structures. It has been presented that the existence of a retaining wall may alter considerably the dynamic response of a structure founded on the retained soil. In the examined cases it was proven that the characteristics of the wall affect substantially the dynamic behaviour of the whole system. The presence of a rigid wall imposes a boundary that clearly alters the 1-D conditions of the backfill, while a flexible wall has less impact on the structural response. Furthermore, it has been shown that the amplification of the structure alone can be higher than that of 1-D conditions, as it comes closer to the retaining wall. In general, the results of this investigation provide a clear indication of the direct dynamic interaction between the wall, the retained soil, and the retained structures. That fact justifies the necessity for a more elaborate consideration of this interrelated phenomenon during the seismic design (both in seismic norms and in engineering practice), taking into account not only the DSSI conditions, but the DWSSI imposed by a nearby retaining wall as well.

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