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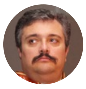
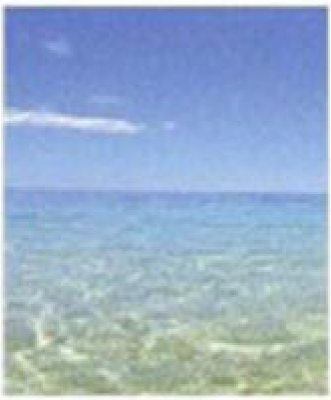
Georgia Foutsitzi
Technological Educational Institute of Epirus



Georgios Eleftherios Stavroulakis
Technical University of Crete

Evangelos P. Hadjigeorgiou
University of Ioannina



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VIBRATION CONTROL ANALYSIS OF SMART PIEZOELECTRIC COMPOSITE PLATES

Georgia A. Foutsitzi [1], Daniela Marinova[2], Georgios E. Stavroulakis[3] and Evagellos Hadjigeorgiou[4]

[1] Technological Educational Institute of Epirus, Dept of Finance and Auditing,
Leof. Ioanninon 210, GR-48100 Preveza, Greece
gfoutsi@cc.uoi.gr,

[2] Technical University-Sofia, Faculty of Applied Mathematics and Informatics, Bulgaria
dmarinova@dir.bg

[3] Technical University of Crete, Dept of Production Engineering and Management, GR-73100 Chania, Greece and
Technical University of Braunschweig, Dept of Civil Engineering, Braunschweig, Germany
gestavr@dpem.tuc.gr

[4] University of Ioannina, Dept of Material Science and Technology,
GR-45110 Ioannina, Greece
ehadjig@cc.uoi.gr

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Abstract. *A finite element analysis for the approximation of the dynamic response of laminate composite plates with piezoelectric layers is presented. The formulation is based on the classical plate theory and Hamilton's principle. The active control of the plate is studied using both the optimal linear quadratic regulator strategy and the robust* H_∞ *control theory. Preliminary numerical simulations are carried out in order to demonstrate the efficiency of the proposed control strategies.*

1 INTRODUCTION

This work outlines the mathematical and computational analysis used for the active vibration control of a composite plate structure with piezoelectric sensors and actuators.

Advanced structures with integrated self-monitoring and control capabilities are very important due to the rapid development of *"smart"* mechanical systems and space structures.

The finite element method has been widely employed in vibration and shape control of piezoelectric smart structures [1], [2]. In most cases, the adhesive layers used to bond the piezoelectric sensors/actuators to the host structures are considered negligible. Nevertheless, it is widely accepted that the adhesive layers are weak points of the structure and, if they are not taken into account in the design, may deteriorate the effectiveness of the structure.

In this study, a finite element model for the active control of a three layered plate system containing piezoelectric sensors and actuator sis studied using the layerwise approach that considers the adhesive layer flexibility [3].

The model is suitable for integration with various optimal control schemes and it has been used in the design of smart structures. Moreover it allows for the investigation of delamination and interlayer damage models, their effects on the structural control as well as the design of robust control schemes and damage identification problems.

The use of the layerwise displacement theory for the finite element modelling of the dynamic response of the system is very important since it provides a more correct representation of the dynamic response of thick laminates.

2 FINITE ELEMENT FORMULATION

Consider a laminated composite plate bonded with piezoelectric sensor and actuator layers and adhesive layers between them, as shown in Fig 1. The adhesive layers between the host plate and the piezoelectric are assumed to be very thin, and their deformation is linear. Only transverse normal stress ($\sigma_z^{(a)}$) and strains ($\varepsilon_z^{(a)}$), and in-plane shear stress ($\tau_{xz}^{(a)}, \tau_{yz}^{(a)}$) and strains ($\gamma_{xz}^{(a)}, \gamma_{yz}^{(a)}$) are taken into account. The in-plane stretching of the adhesive layer is neglected, since its stiffness in that direction is quite small. Also the adhesive layer is treated as an isotropic material. The piezoelectric layers are assumed to be transversely polarized and subjected to transverse electrical fields. The length, width and thickness of the whole plate are denoted by *L*, *b* and *h*, respectively. The mid-plane of the core is set to coincide with the origin of the *z*-axis. The superscripts *p*, *n* and *a* refer to the piezoelectric layer, the non-piezoelectric and the adhesive layer, respectively.

2.1 The non-adhesive layers

By using the Mindlin plate theory, assuming that the mid-planes are parallel to each other and employing a common coordinate system (x, y, z) given in Fig. 1, the displacements of the non-adhesive layers can be written as

$$
\begin{aligned}
u_1^{(i)}(x,y,z,t) &= u^{(i)}(x,y,t) - \left(z - \tilde{z}^{(i)}\right)\theta_x^{(i)}(x,y,t) \\
u_2^{(i)}(x,y,z,t) &= v^{(i)}(x,y,t) - \left(z - \tilde{z}^{(i)}\right)\theta_y^{(i)}(x,y,t) \\
u_3^{i}(x,y,z,t) &= w^{(i)}(x,y,t)
\end{aligned}
\tag{1}
$$

where $u^{(i)}, v^{(i)}$ and $w^{(i)}$ are the mid-plane deformations of the i-th layer, $\theta_x^{(i)}$ and $\theta_y^{(i)}$ are rotation angles of the normal to the mid-plane about the *y* and *x* axes, respectively and $\tilde{z}^{(i)}$ is the thickness of the mid-plane of the i-th layer.

The bending and shear strains of the i-th layer can be written as

$$
\left\{\varepsilon_b^{(i)}\right\} = \left\{\varepsilon_{0b}^{(i)}\right\} + \left(z - \tilde{z}^{(i)}\right)\left\{\kappa^{(i)}\right\}, \qquad \left\{\varepsilon_s^{(i)}\right\} = \left\{\varepsilon_{0s}^{(i)}\right\}
\tag{2}
$$

where

$$
\begin{aligned}
&\left\{\varepsilon_b^{(i)}\right\} = \left\{\varepsilon_{xx}^{(i)}, \varepsilon_{yy}^{(i)}, \gamma_{xy}^{(i)}\right\}^T, \quad \left\{\varepsilon_s^{(i)}\right\} = \left\{\gamma_{xz}^{(i)}, \gamma_{yz}^{(i)}\right\}^T, \quad \left\{\varepsilon_{0b}^{(i)}\right\} = \left\{\frac{\partial u^{(i)}}{\partial x}, \frac{\partial v^{(i)}}{\partial y}, \frac{\partial u^{(i)}}{\partial y} + \frac{\partial v^{(i)}}{\partial x}\right\}^T, \\
&\left\{\kappa^{(i)}\right\} = \left\{-\frac{\partial \theta_x^{(i)}}{\partial x}, -\frac{\partial \theta_y^{(i)}}{\partial y}, -\left(\frac{\partial \theta_x^{(i)}}{\partial y} + \frac{\partial \theta_y^{(i)}}{\partial x}\right)\right\}^T, \quad \left\{\varepsilon_{0s}^{(i)}\right\} = \left\{\frac{\partial w^{(i)}}{\partial x} - \theta_x^{(i)}, \frac{\partial w^{(i)}}{\partial y} - \theta_y^{(i)}\right\}^T
\end{aligned}
\tag{3}
$$

Equation (3) can be expressed in the following form

$$
\left\{\varepsilon_b^{(i)}\right\} = \left[L_b^{(i)}(z)\right]\left\{\overline{u}^{(i)}\right\}, \qquad \left\{\varepsilon_s^{(i)}\right\} = \left[L_s^{(i)}(z)\right]\left\{\overline{u}^{(i)}\right\}
\tag{4}
$$

where $\left\{\overline{u}^{(i)}\right\} = \left\{u^{(i)}, v^{(i)}, w^{(i)}, \theta_x^{(i)}, \theta_y^{(i)}\right\}^T$ is the generalized displacement vector and

$$\left[L_b^{(i)}(z)\right]=\begin{bmatrix} \partial/\partial x & 0 & -\left(z-\tilde{z}^{(i)}\right)\partial/\partial x & 0 \\ 0 & \partial/\partial y & 0 & -\left(z-\tilde{z}^{(i)}\right)\partial/\partial y \\ \partial/\partial y & \partial/\partial x & -\left(z-\tilde{z}^{(i)}\right)\partial/\partial y & -\left(z-\tilde{z}^{(i)}\right)\partial/\partial x \end{bmatrix},\quad \left[L_s^{(i)}(z)\right]=\begin{bmatrix} 0 & 0 & \partial/\partial x & -1 & 0 \\ 0 & 0 & \partial/\partial y & 0 & -1 \end{bmatrix} \tag{5}$$

A constant transverse electrical field is assumed for the piezoelectric layers and the remaining in-plane components are supposed to vanish. Consequently the electric field intensity can be expressed as

$$\left\{E^{(i)}\right\}=\left\{0,0,-\frac{V^{(i)}}{h^{(i)}}\right\}^T \tag{6}$$

where $h^{(i)}$ is the thickness of the i-th piezoelectric layer and $V^{(i)}$ is the applied voltage across the i-th piezoelectric layer.

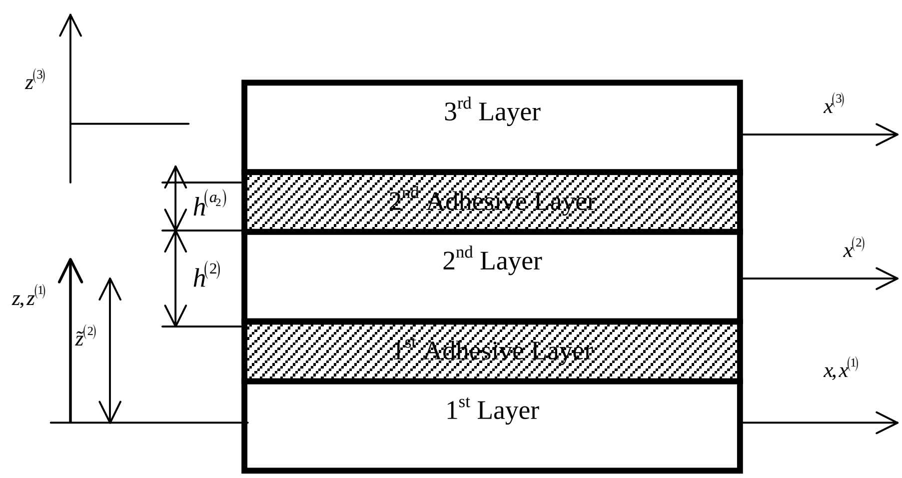


Figure 1. A three-layered composite plate

For simplicity of the notation, all the non-adhesive lavers will be considered piezoelectric. The linear constitutive equations of each layer can be written as

$$\begin{aligned} \left\{\sigma^{(i)}\right\}&=\left[Q^{(i)}\right]\left\{\varepsilon^{(i)}\right\}-\left[e^{(i)}\right]^T\left\{E^{(i)}\right\}, \\ \left\{D^{(i)}\right\}&=\left[e^{(i)}\right]\left\{\varepsilon^{(i)}\right\}+\left[\xi^{(i)}\right]\left\{E^{(i)}\right\}, \end{aligned} \tag{7}$$

where $\{\sigma\}$ is the stress tensor, $\{\varepsilon\}$ is the strain tensor, $\{D\}$ is the electric displacement, $\{E\}$ is the electric field, $[Q]$ is the elastic stiffness matrix, $[e]$ is the piezoelectric matrix and $[\xi]$ is the permittivity matrix.

After separating the bending and shear related variables, Eq. (7) becomes

$$\begin{aligned} \left\{\sigma_b^{(i)}\right\}&=\left[Q_b^{(i)}\right]\left\{\varepsilon_b^{(i)}\right\}-\left[e_b^{(i)}\right]^T\left\{E^{(i)}\right\}, \\ \left\{\sigma_s^{(i)}\right\}&=\left[Q_s^{(i)}\right]\left\{\varepsilon_s^{(i)}\right\}-\left[e_s^{(i)}\right]^T\left\{E^{(i)}\right\}, \\ \left\{D^{(i)}\right\}&=\left[e_b^{(i)}\right]\left\{\varepsilon_b^{(i)}\right\}+\left[e_s^{(i)}\right]\left\{\varepsilon_s^{(i)}\right\}+\left[\xi^{(i)}\right]\left\{E^{(i)}\right\}, \end{aligned} \tag{8}$$

where $\left\{\sigma_b^{(i)}\right\} = \left\{\sigma_{xx}^{(i)}, \sigma_{yy}^{(i)}, \tau_{xy}^{(i)}\right\}^T$, $\left\{\sigma_s^{(i)}\right\} = \left\{\tau_{xz}^{(i)}, \tau_{yz}^{(i)}\right\}^T$.

The piezoelectric layers are considered to exhibit transverse isotropic properties. In Eqs. (8), a layer can be either a piezoelectric layer or an elastic one. In the latter case, material constants $[e]$ and $[\xi]$ should be zero.

2.2 The adhesive layer

The shear and peel strains of the adhesive layers can be written as [3]

$$
\begin{aligned}
\varepsilon_{zz}^{(a_i)} &= \frac{w^{(i+1)} - w^{(i)}}{h^{(a_i)}} \\
\gamma_{yz}^{(a_i)} &= \frac{1}{h^{(a_i)}}\left[v^{(i+1)} - v^{(i)} + \frac{h^{(i)}}{2}\theta_y^{(i)} + \frac{h^{(i+1)}}{2}\theta_y^{(i+1)}\right] \\
\gamma_{xz}^{(a_i)} &= \frac{1}{h^{(a_i)}}\left[u^{(i+1)} - u^{(i)} + \frac{h^{(i)}}{2}\theta_x^{(i)} + \frac{h^{(i+1)}}{2}\theta_x^{(i+1)}\right]
\end{aligned}
\tag{9}
$$

Equation (9) can be written into the following compact form

$$
\left\{\varepsilon^{(a_i)}\right\} = \left[L_a^{(top)}(z)\right]\left\{\bar{u}^{(i+1)}\right\} + \left[L_a^{(botton)}(z)\right]\left\{\bar{u}^{(i)}\right\}
\tag{10}
$$

where

$$
\left\{\varepsilon^{(a_i)}\right\} = \left\{\varepsilon_{zz}^{(a_i)}, \gamma_{xz}^{(a_i)}, \gamma_{yz}^{(a_i)}\right\}^T
\tag{11}
$$

$$
\left[L_{a_i}^{(top)}(z)\right] = \frac{1}{h^{(a_i)}}\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{h^{(i+1)}}{2} \\ 1 & 0 & 0 & \frac{h^{(i+1)}}{2} & 0 \end{bmatrix}, \qquad \left[L_{a_i}^{(botton)}(z)\right] = \frac{1}{h^{(a_i)}}\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & \frac{h^{(i)}}{2} \\ -1 & 0 & 0 & \frac{h^{(i)}}{2} & 0 \end{bmatrix}
$$

The bending and shear stress in the adhesive layer can be written as

$$
\left\{\sigma^{(a_i)}\right\} = \left[Q^{(a_i)}\right]\left\{\varepsilon^{(a_i)}\right\}
\tag{12}
$$

where $\left\{\sigma^{(a_i)}\right\} = \left\{\sigma_{zz}^{(a_i)}, \tau_{yz}^{(a_i)}, \tau_{xz}^{(a_i)}\right\}^{\mathrm{T}}$ and $\left[Q^{(a_i)}\right]$ is the elastic matrix of the adhesive layer.

2.3 Hamilton's Principle

To derive the equations of the structure, Hamilton's principle is employed and it can be written as

$$
\int_0^T (\delta T - \delta U + \delta W)\, dt = 0
\tag{13}
$$

where T is the kinetic energy, U is the total strain energy and W is the work done by the loads.

The total kinetic energy of the system is the sum of the corresponding energies of individual layers and can be given by

$$T=\sum_{i=1}^{3}T^{(i)}+\sum_{i=1}^{2}T^{(a_i)} \tag{14}$$

where the kinetic energies of the *i*-th layer and a_i adhesive layer can be expressed as

$$T^{(i)}=\frac{1}{2}\int_{V^{(i)}}\left\{\dot{\bar{u}}^{(i)}\right\}^{T}\left[I^{(i)}(z)\right]\left\{\dot{\bar{u}}^{(i)}\right\}dV\,,\qquad T^{(a_i)}=\frac{1}{2}\int_{V^{(a_i)}}\left\{\dot{u}^{(a_i)}\right\}^{T}\left[I^{(a_i)}(z)\right]\left\{\dot{u}^{(a_i)}\right\}dV\,, \tag{15}$$

with $u^{(a_i)}=\left\{u_1^{(a_i)},u_2^{(a_i)},u_3^{(a_i)}\right\}^T$ and $V^{(i)}$, $V^{(a_i)}$ are the volumes of the *i*-th layer and a_i adhesive layer, respectively.

The total strain energy of the system, is represented as

$$U=\sum_{i=1}^{3}U^{(i)}+\sum_{i=1}^{2}U^{(a_i)} \tag{16}$$

where the strain energy for the *i*-th layer can be written in the form

$$U^{(i)}=\frac{1}{2}\int_{V^{(i)}}\left\{\left\{\bar{\varepsilon}^{(i)}\right\}^{T}\left[D^{(i)}(z)\right]\left\{\bar{\varepsilon}^{(i)}\right\}-\left\{\bar{\varepsilon}^{(i)}\right\}^{T}\left[\mathcal{E}^{(i)}(z)\right]\left\{\bar{\varepsilon}^{(i)}\right\}\right\}dV\,, \tag{17}$$

with $\left\{\bar{\varepsilon}^{(i)}\right\}=\left\{\left\{\varepsilon_{0b}^{(i)}\right\}^T,\left\{\kappa^{(i)}\right\}^T,\left\{\varepsilon_{0s}^{(i)}\right\}^T\right\}^T$. The strain energy for the a_i -th adhesive layer is given by

$$U^{(a_i)}=\frac{1}{2}\int_{V^{(a_i)}}\left\{\left\{\varepsilon^{(a_i)}\right\}^{T}\left[Q^{(a_i)}(z)\right]\left\{\varepsilon^{(a_i)}\right\}\right\}dV \tag{18}$$

The total work is the sum of the work done by the electrical forces $W_E^{(i)}$ and the work done by the mechanical forces $W^{(i)}$,

$$W=\sum_{i=1}^{3}\left(W_E^{(i)}+W^{(i)}\right), \tag{19}$$

where

$$W_E^{(i)}=\frac{1}{2}\int_{V^{(i)}}\left\{E^{(i)}\right\}^{T}\left[D^{(i)}\right]dV=\frac{1}{2}\int_{V^{(i)}}\left\{\left\{E^{(i)}\right\}^{T}\left[\xi^{(i)}\right]\left\{E^{(i)}\right\}+\left\{\bar{\varepsilon}^{(i)}\right\}^{T}\left[\mathcal{E}^{(i)}\right]\left\{E^{(i)}\right\}\right\}dV \tag{20}$$

$$W^{(i)}=\left\{\bar{u}^{(i)}\right\}^{T}\left\{F_c^{(i)}\right\}+\int_{S^{(i)}}\left\{\bar{u}^{(i)}\right\}^{T}\left\{f_s^{(i)}\right\}dS+\int_{V^{(i)}}\left\{\bar{u}^{(i)}\right\}^{T}\left\{f_v^{(i)}\right\}dV-\int_{S^{(i)}}\left\{E^{(i)}\right\}^{T}\left\{q^{(i)}\right\}dS \tag{21}$$

In Eq. (21), $\left\{F_c^{(i)}\right\}$ denotes the concentrated force vector, $\left\{f_s^{(i)}\right\}$ and $\left\{f_v^{(i)}\right\}$ denote the surface and volume vector respectively and $\left\{q^{(i)}\right\}$ denotes the surface charge vector.

2.4 Finite Element Model

From the Hamilton's principle (13), a finite element model was developed for the three layered adhesively bonded plate. The simplest elements, which are used in this study, are rectangular and have four nodes in each layer. Thus a finite element for a three-layered plate has 4×3=12 nodes with five degrees of freedom at each node. The generalized displacement vector $\left\{\bar{u}^{(i)}\right\}$ is interpolated as

$$\left\{\bar{u}^{(i)}(x,y,t)\right\}=[H]\left\{d_e^{(i)}\right\} \tag{22}$$

where $[H]$ is the interpolation matrix and $\left\{d_e^{(i)}\right\}$ is the nodal variable vector given by

$$[H]=\left[[H_1][H_2][H_3][H_4]\right] \qquad \left[H_j\right]=H_j I_5,\ j=1,2,3,4$$
$$\left\{d_e^{(i)}\right\}=\left\{\left\{d_1^{(i)}\right\}^T,\left\{d_2^{(i)}\right\}^T,\left\{d_3^{(i)}\right\}^T,\left\{d_4^{(i)}\right\}^T\right\}^T \qquad \left\{d_j^{(i)}\right\}=\left\{u_j^{(i)},v_j^{(i)},w_j^{(i)},\theta_{xj}^{(i)},\theta_{yj}^{(i)}\right\}^T,\ j=1,2,3,4 \tag{23}$$

where $H_j,\ j=1,2,3,4$ are bilinear isoparametric shape functions and I_5 is the unit matrix.
Substituting Eq. (22) into (13), the following equations for each element can be obtained

$$\left[M_e\right]\left\{\ddot{d}_e\right\}+\left[K_e\right]\left\{d_e\right\}=\left\{F_{e(m)}\right\}+\left[F_{e(el)}\right]^T\left\{V_e\right\} \tag{24}$$

where

$$[M_e]=\begin{bmatrix} \left[M_e^{(1)}\right]+\left[M_{e(bb)}^{(a_1)}\right] & \left[M_{e(bt)}^{(a_1)}\right] & 0 \\ \left[M_{e(tb)}^{(a_1)}\right] & \left[M_e^{(2)}\right]+\left[M_{e(tt)}^{(a_1)}\right]+\left[M_{e(bb)}^{(a_2)}\right] & \left[M_{e(bt)}^{(a_2)}\right] \\ 0 & \left[M_{e(tb)}^{(a_2)}\right] & \left[M_e^{(3)}\right]+\left[M_{e(tt)}^{(a_2)}\right] \end{bmatrix}$$

$$[K_e]=\begin{bmatrix} \left[K_e^{(1)}\right]+\left[K_{e(bb)}^{(a_1)}\right] & \left[K_{e(bt)}^{(a_1)}\right] & 0 \\ \left[K_{e(tb)}^{(a_1)}\right] & \left[K_e^{(2)}\right]+\left[K_{e(tt)}^{(a_1)}\right]+\left[K_{e(bb)}^{(a_2)}\right] & \left[K_{e(bt)}^{(a_2)}\right] \\ 0 & \left[K_{e(tb)}^{(a_2)}\right] & \left[K_e^{(3)}\right]+\left[K_{e(tt)}^{(a_2)}\right] \end{bmatrix}$$

$$\left\{F_{e(m)}\right\}=\left\{\left\{F_{e(m)}^{(1)}\right\}^T,\left\{F_{e(m)}^{(2)}\right\}^T,\left\{F_{e(m)}^{(3)}\right\}^T\right\}^T, \qquad \left\{V_e\right\}=\left\{V^{(1)},V^{(2)},V^{(3)}\right\}^T$$

3 CONTROLLED STRUCTURE AND EXAMPLE

3.1 Active control problem

The finite element model of the dynamical system is rewritten in the state space form

$$\dot{x}=Ax+B_1w+B_2u$$

where x is the state vector, A is the system matrix, and B_1, B_2 are allocation matrices for the disturbances w and control u. Let us denote the measured outputs by $y(t)$. The simples possible linear control (feedback) reads

$$u(t) = Ky$$

The problem is solved for both linear quadratic regulator (LQR) and H_2 optimal performance criteria. Let us take the weighted sum of the energy of the state and of the control as performance criterion. The following quadratic cost function is minimized

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \qquad K_{LQR} = R^{-1} B_2^T P$$

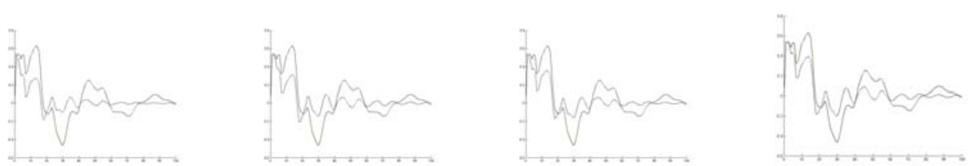
The weights $Q \geq 0$ and $R > 0$ are the main design parameters. K_{LQR} is the control gain and the constant matrix P is the positive solution of the Algebraic Riccati Equation.

Let us assume the measured output vector $y = C_2 + D_{12}w$ and a regulated output in the form $z = C_1 x + D_{12}u$. We need to minimize the influence of the disturbances over the regulated states. An appropriate performance criterion is the H_2 norm of the transfer function T_{zw} between disturbances w and regulated outputs z that must be minimized over all controllers internally stabilizing the plant

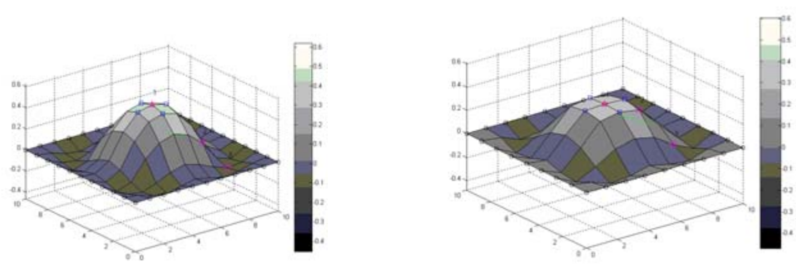
$$\|T_{zw}\|_2 = \left(\frac{1}{2}\int_{-\infty}^{+\infty} trace[T_{zw}^{\ *}(j\omega) \ \ T_{zw}(j\omega)]d\omega \right)^{1/2} \quad \to \quad \min$$

3.2 Representative numerical example

An elastic square plate discretized by a ten by ten mesh is considered. The four boundaries of the plate are fixed. The ambient vibrations are excited by a concentrated force applied in vertical direction to the centre on the plate. In order to check the robustness of the algorithm, the one element of the plate is considered to be damaged. Four possible schemes with four numbers of actuators and two numbers of actuators placed symmetrically have been used here for the numerical investigation. All states or their part are available for measuring. The two control strategies are applied for vibration suppression. The control forces act in vertical direction reducing the effect of the adverse vibrations. The response of the closed loop system is compared with the response of the open loop system with respect to the reduction of the maximum magnitude of the vertical displacement. The figure below displaces the results with LQR control strategy for all schemes.



The figure below depicts the response of the uncontrolled and controlled plate with the H_2 control strategy.



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