Optimization of PE Patches in SS using Multi-Objective GA

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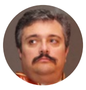
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Optimization of Piezoelectric Patches in Smart Structures using Multi-Objective Genetic Algorithms

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Abstract

In this paper multi-objective genetic algorithms have been used to search for the optimal placement of the piezoelectric sensors and actuators bonded on smart beams. A finite element method based on Timoshenko beam theory is used accounting for the piezoelectric layers. The discrete optimal sensor and actuator location problem is formulated in the framework of a zero-one optimization problem with multi-objective functions as performance measures. A cantilever beam example is considered to demonstrate the performance of the selected multi-objective genetic algorithm which is NSGAII. It is shown that the proposed algorithm is effective in developing optimal Pareto front curves for optimal placement and number of actuators and sensors such that the performance on dynamic responses is also satisfied.

Keywords: multi-objective genetic algorithms, active vibration control, smart structures, actuator placement, NSGAII.

1 Introduction

Piezoelectric actuators are commonly considered for active vibration control of smart structures. In active vibration control of structures using piezoelectric materials, the locations of sensors and actuators have significant influence on the performance of the control system as well as the controlled response. Many studies have been developed on optimal locations of actuators and sensors. Different cost functions and performance measures have been used. Wang et al. [1] proposed a controllability index so as to guide designers in seeking the optimal locations and size of the piezoelectric actuators for structural control. This index was related to the amount of input energy required in the control design.

Many research works in the literature applied genetic algorithms (GAs) to find efficient locations of piezoelectric sensors and actuators of smart structures. Rao et al. [2] developed a GA approach to solve the discrete optimal actuator location problem in actively controlled structures. Han and Lee [3] used a genetic algorithm to seek the optimal locations of piezoelectric sensors and actuators of a smart composite plate from the perspectives of controllability and observability. The optimal placement of collocated piezoelectric sensor-actuator pairs on flexible beams using a model-based linear quadratic regulator (LQR) controller was studied by Kumar et al [4]. The problem was formulated in the framework of a zero-one optimization problem and solved using GAs. Schulz et al [5] studied the optimal discrete piezoelectric patch allocation on composite structures for vibration control based on GA and modal LQR.

Most of research works in the literature considering the optimal location of sensor-actuator pairs for active vibration control of structures are based on a particular performance criteria. But not much work is available on the optimal placement of sensors and actuators based on multi-criteria [6, 7]. Dhuri and Seshu [6] used genetic algorithm to multi-objective optimize piezoelectric actuator placement based on controllability index and minimal change of natural frequencies.

In this work the vibration control of smart beam structures has been formulated as a multi-objective optimization problem to find out the optimal locations and number of collocated piezoelectric actuators - sensors (S/As) pairs that are surface bonded onto beam structures. The objectives taken into consideration are maximization of the controllability index [1], which is the singular values of the control matrix, and minimization of changes in natural frequencies.

A finite element method based on Timoshenko beam theory is used. The contributions of piezoelectric sensor and actuator patches to the mass and stiffness of the beam are considered. The number of actuators- sensors pairs and their location are treated as design variables. The discrete optimal actuator- sensor location problem is formulated in the framework of a zeroone optimization problem which is solved using the multi-objective genetic algorithm NSGAII. Pareto optimal solutions have been found out for cantilever beams with multiple sensors/actuators. Finally, the active vibration control performance with optimal solution of the location and number of the sensoractuator pairs is studied.

2 Mathematical model

Consider a cantilever beam with partially attached distributed piezoelectric actuator and sensor layers as shown in Figure 1.

The displacement field Equations for the beam using first order shear deformation theory at any point through the thickness are presented by

$$u_x(x, y, z, t) = u_0(x, t) - z\theta_y(x, t), \quad u_y(x, y, z, t) = 0, \quad u_z(x, y, z, t) = w_0(x, t) \tag{1}$$

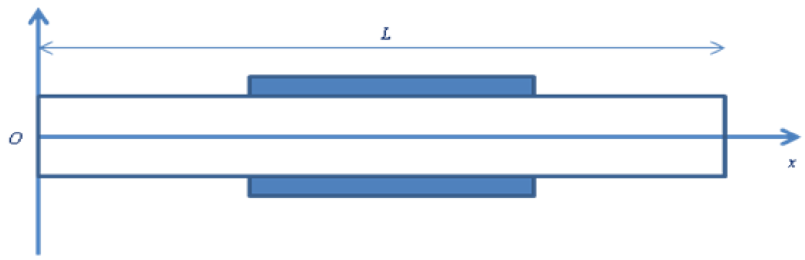


Figure 1: A cantilever beam with partially attached piezoelectric layers

where t denotes time; u_0 and w_0 denote the axial and transverse displacements of the beam's mid-plane, and θ_y is the rotation of normal to the x-axis about the y-axis.

A constant transverse electric field is assumed for the piezoelectric actuator and the remaining in-plane components are supposed to vanish.

The piezoelectric material considered for the surface layers is orthotropic with the directions of orthotropy coincident with the axes of the beam, x, y, z, and is polarized in the transverse direction z. Assuming that the width in the y-direction is stress free and by using the plane stress assumption, the constitutive equations for this beam model can be written as in Equations (2), where σ_x,τ_{xz} is the normal and shear stress, respectively, ε_x, γ_{xz} is the normal and shear strain respectively, D_z is the electric displacement, E_z is the electric field, $\tilde{Q}_{11}$, $\tilde{Q}_{55}$ are the stiffness coefficients, $\tilde{e}_{31}$ is the piezoelectric constant and $\tilde{\xi}_{33}$ is the permittivity constant [8, 9].

$$\sigma_x = \tilde{Q}_{11}\varepsilon_x - \tilde{e}_{31}E_z, \quad \tau_{xz} = \tilde{Q}_{55}\gamma_{xz}, \quad D_z = \tilde{e}_{31}\varepsilon_x + \tilde{\xi}_{33}E_z \tag{2}$$

2.1 Beam finite element formulation

The finite element formulation of the beam with the piezoelectric layers is formulated on the lines of Foutsitzi et al. [8]. The finite element with the piezoelectric layers consists of two nodes with three mechanical degrees of freedom (DoFs) per node (the axial and transverse displacement u_0, w_0 and the rotation θ_y) and two electric DoFs per element (the electric potential difference of each piezoelectric layer). The contributions of the bonded piezoelectric layers to the mass and stiffness of the beam are taken into account while the contribution of the bonding layers themselves is assumed negligible. The applied voltage is assumed to be constant and uniform over the element. Therefore, the elemental mechanical and electrical DoFs vectors, $\{d\}$ and $\{\phi\}$, are given by Equations (3), where it is assumed that piezoelectric actuators and sensors are bonded in the structure, the electric potential vector is subdivided in a sensor component $\{\phi\}_s$ and an actuator component $\{\phi\}_a$.

$$\begin{aligned} \{d\}_e = \left\{u_0^1, w_0^1, \theta_y^1, u_0^2, w_0^2, \theta_y^2\right\}^T \\ \{\phi\}_e = \{\phi_a, \phi_s\}^T \end{aligned} \tag{3}$$

The axial displacement u_0 and the rotation θ_y are interpolated by quadratic polynomial, while the transverse displacement w_0 by cubic polynomials and are expressed in terms of the finite element shape functions as Equation (4) where $[N_u]$, $[N_w]$ and $[N_\theta]$ are appropriate shape functions given in [8].

$$\{u_0, w_0, \theta_y\}^T = [N]\{d\} = [[N_u]\,[N_w]\,[N_\theta]]^T \{d\} \tag{4}$$

Using Hamilton's principle the resultant global FE spatial model, governing the motion and electric charge equilibrium, is given by Equations (5) where $[M]$ is the global mass matrix, $[K_{uu}]$, $[K_{u\phi}] = [K_{u\phi}]^T$ and $[K_{\phi\phi}]$ are the global mechanical stiffness, mechanical-electrical coupling stiffness and dielectric stiffness matrices respectively. $\{F_m\}$ and $\{F_q\}$ are the respective global mechanical and electrical loads vectors. $\{d\}$ and $\{\phi\}$ are the global generalized displacement and potential vectors respectively.

$$\begin{aligned} [M]\left\{\ddot{d}\right\} + [K_{uu}]\{d\} + [K_{u\phi}]\{\phi\} &= \{F_m\} \\ [K_{\phi u}]\{d\} + [K_{\phi\phi}]\{\phi\} &= \{F_q\} \end{aligned} \tag{5}$$

Considering that in the sensor layers the converse piezoelectric effect is negligible, equations (5) are rewritten as equations (6), where $[K_{u\phi}]_a$, $[K_{\phi u}]_s$ and $[K_{\phi\phi}]_s$ are the global discrete matrices for actuator and sensor, respectively.

$$\begin{aligned} [M]\left\{\ddot{d}\right\} + [K_{uu}]\{d\} + [K_{u\phi}]_s\{\phi\}_s &= \{F_m\} - [K_{u\phi}]_a\{\phi\}_a \\ [K_{\phi u}]_s\{d\} + [K_{\phi\phi}]_s\{\phi\}_s &= \{F_q\} \end{aligned} \tag{6}$$

Since the electrical excitation applied to the sensor layer is zero , the voltage from the sensor layer can be statically condensed and the system (6) becomes

$$\begin{aligned} [M]\left\{\ddot{d}\right\} + [K_{uu}]\{d\} &= \{F_m\} - [K_{u\phi}]_a\{\phi\}_a \\ \{\phi\}_s &= -[K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s\{d\} \end{aligned} \tag{7}$$

where $[K^*_{uu}] = [K_{uu}] - [K_{u\phi}]_s [K_{\phi\phi}]_s^{-1} [K_{\phi u}]_s$.

2.2 Modal analysis

In the finite element model, the degrees of freedom of the system are quite large therefore it is required to model the system in modal form. Therefore the governing system dynamics Equation (7) is expressed in modal space by introducing a new variable derived by modal transformation

$$\{d\} = [\Phi]\{\eta\} \tag{8}$$

where $[\Phi]$ is the modal matrix and $\{\eta\}$ is the modal coordinate vector. Substituting Equation (8) into Equations (7) leads to

$$\{\ddot{\eta}\} + \left[\Omega^2\right]\{\eta\} = [\Phi]^T\{F_m\} - [\Phi]^T[K_{u\phi}]_a\{\phi\}_a \tag{9}$$

$$\{\phi\}_s = -[K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s[\Phi]^T\{\eta\} \tag{10}$$

2.3 State space formulation

The state space representation of Equations (9) and (10) is

$$\begin{aligned} \{\dot{x}\} &= [A]\{x\} + [B]\{u_\phi\} + \{f\} \\ \{\phi\}_s &= \{y\} = \{C\}\{x\} \end{aligned} \tag{11}$$

where $\{x\} = \{\eta, \dot{\eta}\}^T$ is the state vector, $[A]$ is the system matrix, $[B]$ is the control matrix, $\{f\}$ is the disturbance input vector and $\{u_\phi\} = \{\phi\}_a$ is the control input to the actuator. These matrices are given by

$$\begin{aligned} [A] &= \begin{bmatrix} [0] & [I] \\ [-\Omega^2] & [0] \end{bmatrix}, [B] = \begin{bmatrix} [0] \\ -[\Phi]^T[K_{u\phi}]_a \end{bmatrix} \\ \{f\} &= \begin{bmatrix} [0] \\ [\Phi]^T\{F_m\} \end{bmatrix}, \ [C] = \begin{bmatrix} -[K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s[\Phi] & [0] \end{bmatrix} \end{aligned} \tag{12}$$

From the expressions of matrices $[A]$ and $[B]$ in the state Equation (11), it is clear that all control designs depend on the placement and size of the piezoelectric actuators as well as the vibration modes used in the modal analysis.

Performing the singular value decomposition of control matrix $[B]$ we get the singular values

$$S = \begin{bmatrix} \sigma_1 & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \sigma_{n_p} \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}, n_p < n \tag{13}$$

where σ_{n_p} is the number of piezoelectric actuators and n is the number of modes used in the modal analysis. The magnitude of σ_i is a function of the location and size of piezoelectric actuators. Based on σ_i, a controllability index (CI) $\hat{\Omega}$ is introduced (14).

$$\hat{\Omega} = \prod_{i=1}^{n_p} \sigma_i \tag{14}$$

The higher the CI, the lower the power consumption required for control i.e. the better the control effectiveness. The index measures the input energy required to achieve a desired structural control by the piezoelectric actuators.

2.4 Active control by sensors and actuators

In order to actively control vibrations, classical controls like direct proportional feedback and constant gain negative velocity feedback are used in this study.

The control law in the case of direct proportional feedback is

$$\{u_\phi\} = [G]\{\phi\}_s \tag{15}$$

where $[G]$ denotes the feedback control gain which can be adjusted depending on the performance requirements of the system.

The control law in the case of constant gain negative velocity feedback is

$$\{u_\phi\} = -[G]_v \left\{\dot{\phi}\right\}_s \tag{16}$$

where $[G]_v$ denotes the feedback control gain.

3 Actuator placement optimization

Performance of active vibration control not only depends upon the control law but also on the placement of piezoelectric sensors and actuators.

In this work the vibration control of smart beam structures has been formulated as a multi-objective optimization problem to find out the optimal locations and number of collocated piezoelectric actuators - sensors (S/As) pairs that are surface bonded onto beam structures.

In order to propose performance criteria for S/A locations, first maximization of CI defined above, has been considered. In the current analysis, CI has been used as a measure of control effectiveness.

On the other hand, when a piezo actuator is mounted on a structure, the mass/stiffness properties of the parent structure are altered. The parent structure is originally designed to have a certain natural frequency spectrum in relation to the disturbance excitation. Hence, in this study, not only the CI is used as a performance criterion but also the minimal change in the natural frequencies of parent structure [6].

3.1 Formulation of the multi-objective problem

In this section, a multi-objective problem for optimal placement of the r piezoelectric pairs bonded onto the beam is formulated. The problem is to determine the optimal placements x_i and the number r of piezoelectric pairs on the beam which minimize the following objective functions [6]:

a) The first objective function to be minimized is the reciprocal of the CI

$$f_1 = \frac{1}{\hat{\Omega}} \tag{17}$$

b) The (i+1)-objective function to be minimized is the change in i^{th} natural frequency

$$f_{i+1} = \frac{\Delta\omega_i}{\omega_i}, i = 1, 2, ..., n \tag{18}$$

The discrete optimal sensor-actuator pairs location problem is cast in the framework of a zero-one optimization problem. A 'one' represents the presence of a sensor-actuator pair and 'zero' indicates the absence of a sensor-actuator pair on the element. Multi-objective genetic algorithms are used to solve this zero-one optimization problem.

3.2 Multi-objective genetic algorithms

Genetic Algorithms (GAs) are population-based methods grounded on the Darwinian idea of "survival of the fittest". The population consists of candidate solutions that evolve by combining existing solutions in an effort to discover high quality ones. This occurs by intermingling beneficial traits of selected individuals. Additional variability is inserted through mutation, which is a mechanism of applying random changes to the solutions. The process continues until a time limit or some other stopping criterion is met.

Furthermore, GAs are an established method for multi-objective optimization. Multi-objective optimization in general tries to detect Pareto optimal solutions (i.e. solutions that cannot be improved in one objective without sacrificing some other objective). These solutions consist an approximation of the Pareto front which is unknown. After the Pareto optimal solutions are generated, a human decision maker selects among alternatives the solution that considers as most preferable.

Several multi-objective genetic algorithms (MOGAs) have been developed through the last decades. A non exhaustive list of them is DENSEA [10], FastPGA [11], NS-GAII [12], PESA2 [13] and SPEA2 [14]. In this work Non-dominated Sorting Genetic Algorithm-II (NSGAII) is used since it is a fast non-dominated sorting approach that results in a set of high fitness solutions that has good spread as documented in several research papers that cite [12].

4 Numerical Results

After validating the present formulation with the existing results in the literature, the optimal locations of piezoelectric sensoractuator pairs for active vibration control of the cantilever beam shown in Figure 1 of dimension 300×25×0.5mm is considered. The host beam is made of aluminium and the piezoceramic is PZT. The material constants for the aluminium and the piezoceramic are: Q_{11}=60.0GPa, Q_{55}=40.0GPa and Q^p_{11}=62.0GPa, Q^p_{55}=30GPa. The piezoelectric constant is d_{13}= 247×10^{-12}. The thickness of each PZT is hp=0.35mm. The length of one piezoelectric patch is assumed to be equal to the length of one finite element. For the finite element analysis the beam is divided into 30 elements and the total number of piezo pairs is assumed to be varied. The stiffness and mass of the piezoelectric patches are taken into account in the model. Structural damping is not included in the formulation so that the effectiveness of only the active control in the vibration response can be assessed

In this study MOEA framework version 2.1 (http://www.moeaframework.org/) was used. In order to search efficiently and determine the optimal placements of the piezoelectric pairs NSGAII operates with the following parameters: population size=100, Simulated Binary Crossover (SBX) rate=0.9, SBX distribution index=15.0 Polynomial Mutation (PM) rate=0.01, PM distribution index=20.0 and maximum number of evaluations=50000.

4.1 Multi-objective problem with two objectives

In order to have a picture of Pareto-optimal (PO) solutions, the problem with only the first two objectives, namely the CI and the change in first natural frequency (NF), is considered first. In this case, the total number of piezo pairs is assumed to be varied from 1 to 10. The Pareto front obtained using NSGAII is shown in Figure 2, for 50000 evaluations. Figure 2 clearly shows that the maximum controllability and minimum NF change are contradictory criteria. It should be noted that although our algorithm was run for a number of piezo patches between 1 to 10, all Pareto optimal solutions obtained have ten number of piezos in different locations. The maximum CI is reached with maximum NF change of about 70 percent and the piezo patches in the locations 2, 5, 9, 12, 15 , 19, 22, 25, 28, 30. The minimum NF change is 47 percent with minimum CI of 0.19 percent and the optimal distribution located at the free end. The Pareto optimal solutions discovered by the algorithm seem to be well distributed in the feasible area.

4.2 Multi-objective problem with five objectives

Next, the first five objectives, that are the CI and the change in the first four frequencies, are considered as objective functions in order to determine the optimal number and locations of actuators. In this case, the total number of sensor-actuator pairs is assumed to be varied from 1 to 15. From the results, it is observed that the optimal number of piezos varies between 10 to 15. Figure 3 shows two PO solutions, that exhibit maximum and minimum controllability respectively. It is to be noted that the number of the piezo patches for the first case is 15 while for the second case is 10. Thus, placement and number of piezo patches i.e. disposition of additional stiffness and mass plays important role in order to obtain maximum controllability and minimum NF change.

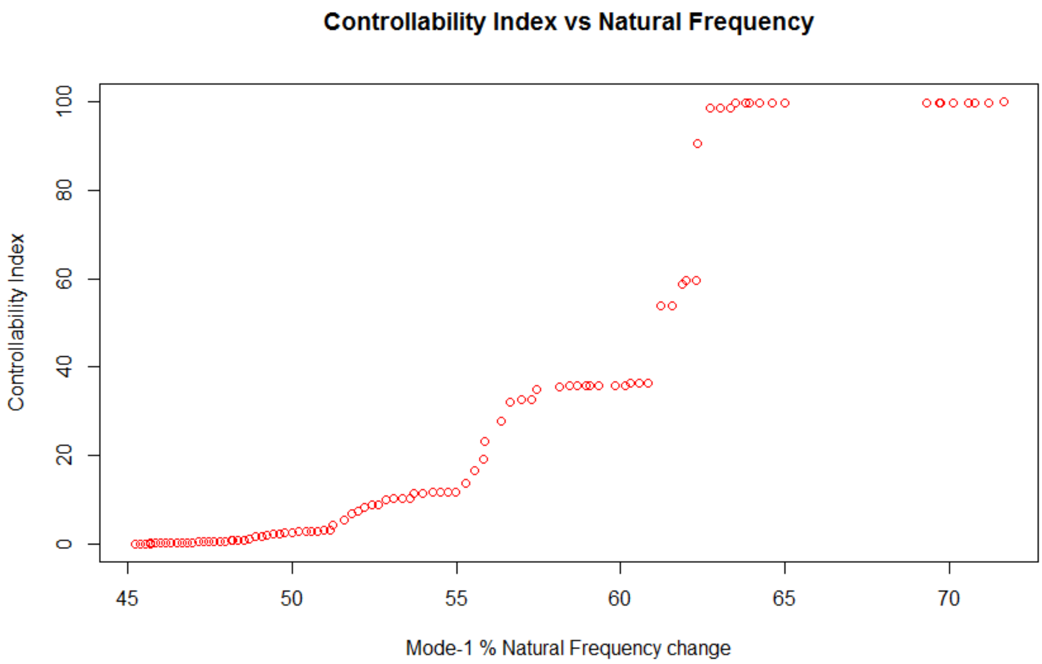


Figure 2: Pareto optimal objective set for the cantilever beam with 1-10 numbers of collocated piezo pairs

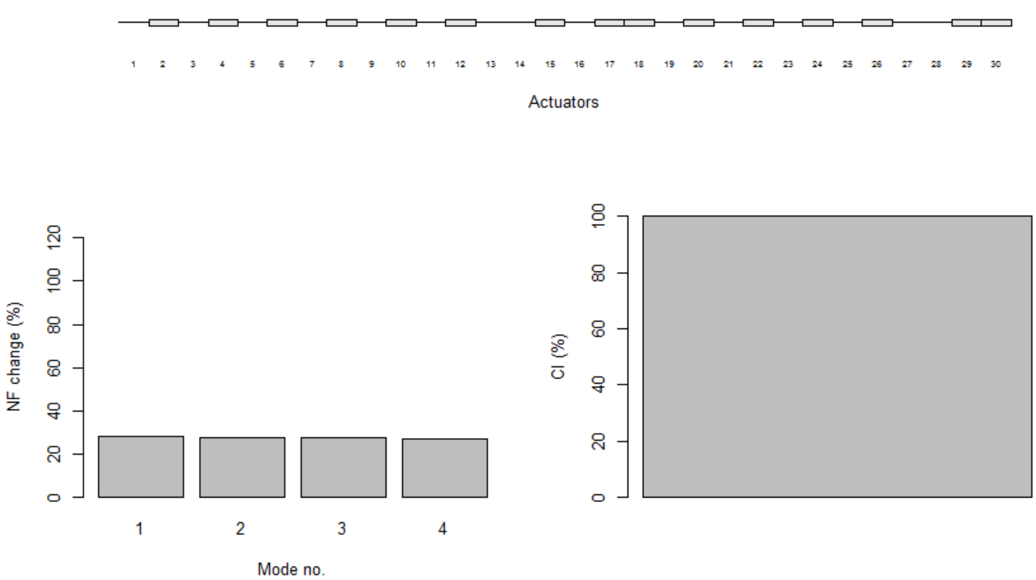
4.3 Active vibration control of cantilever beam

The performance of the present scheme for optimal location and number of piezos is investigated by comparing the active vibration control effectiveness of optimal piezo location with those for randomly selected piezo sensors/actuators. The solution with maximum CI is considered for the study.

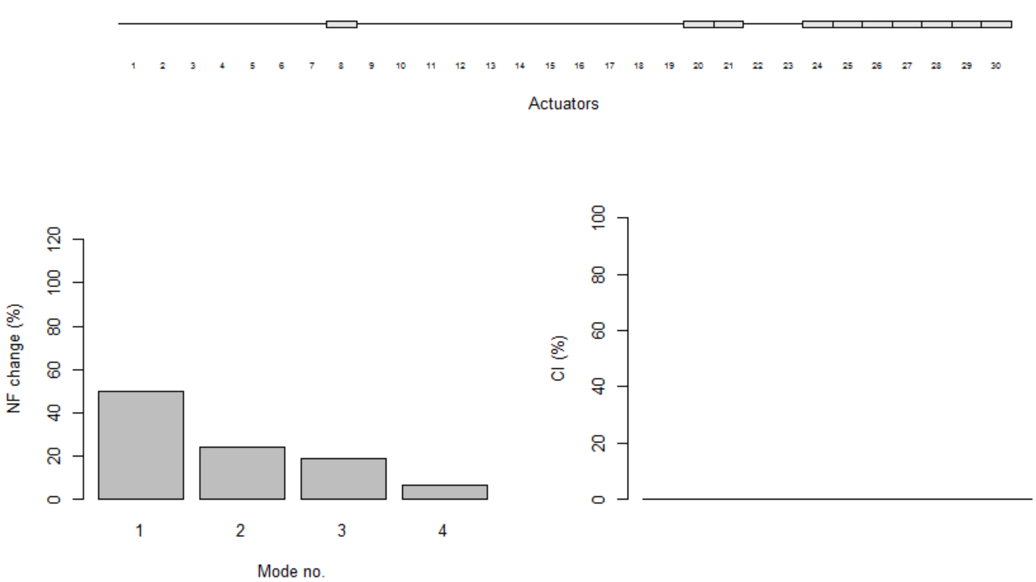
Consider an initial displacement field applied to the beam which is obtained by a mechanical force applied at the free tip that induces a tip displacement equal to 1mm. Structural damping is not included in the formulation so that the effectiveness of only the active control in the vibration response can be assessed. The tip displacement time history and control voltage using velocity feedback gain $G_v = 1$ for the optimal and randomly selected location of 15 sensors/actuators are presented in Figure 4. It can be observed that the control performance is better for the design based on the present scheme than randomly selection of piezo sensors/actuators.

5 Conclusions

The multi-objective simultaneous optimization design of the vibration control system, including the placement and number of the piezoelectric patches has been formulated. Algorithm NSGAII has been used to solve this problem. The multi-objective function including minimization of the controllability index and minimization of the frequency

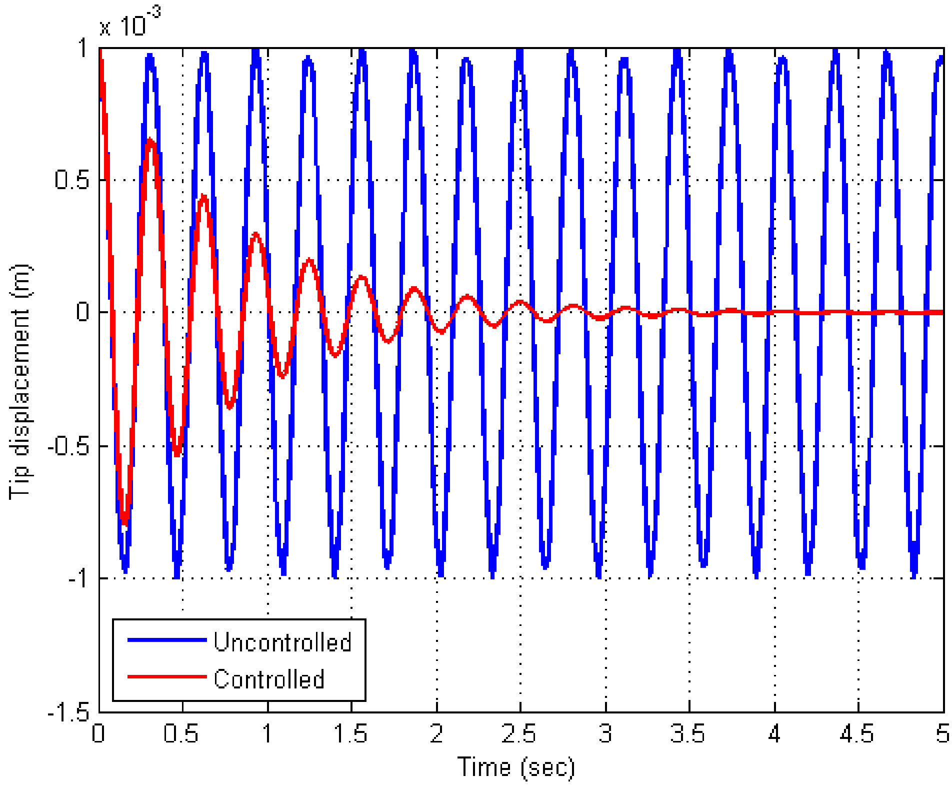


(a)

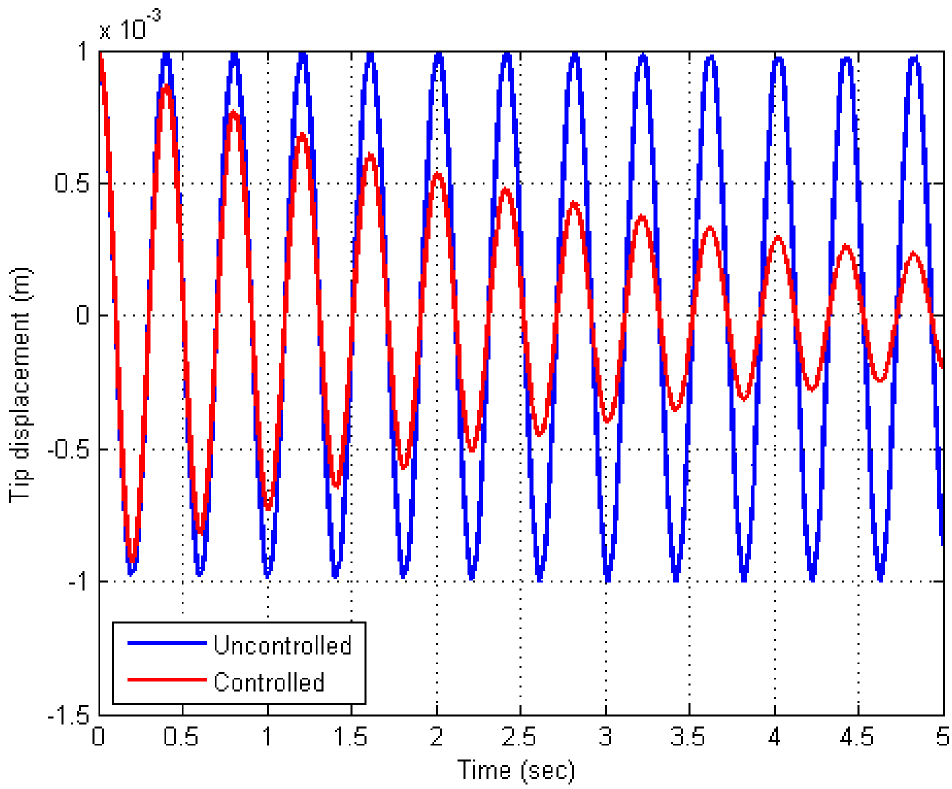


(b)

Figure 3: Location and number of piezo patches and first four natural frequency changes: (a) maximum controllability and (b) minimum controllability



(a)



(b)

Figure 4: Tip response using velocity feedback gain G_v = 1 for (a) the maximum CI and (b) randomly selected location of the sensors/actuators

change has been adopted for the vibration suppression of the structure. The results of a cantilever beam show that using this simultaneous optimization of locations and number of piezoelectric patches, the vibration of the structure can be effectively suppressed.

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References

[1] Q. Wang and C. Wang, "Optimal placement and size of piezoelectric patches on beams from the controllability perspective," *Smart Materials and Structures*, vol. 9, no. 4, p. 558, 2000.

[2] S. S. Rao, T.-S. Pan, and V. B. Venkayya, "Optimal placement of actuators in actively controlled structures using genetic algorithms," *AIAA journal*, vol. 29, no. 6, pp. 942–943, 1991.

[3] J.-H. Han and I. Lee, "Optimal placement of piezoelectric sensors and actuators for vibration control of a composite plate using genetic algorithms," *Smart Materials and Structures*, vol. 8, no. 2, p. 257, 1999.

[4] K. R. Kumar and S. Narayanan, "Active vibration control of beams with optimal placement of piezoelectric sensor/actuator pairs," *Smart Materials and Structures*, vol. 17, no. 5, p. 055008, 2008.

[5] S. L. Schulz, H. M. Gomes, and A. M. Awruch, "Optimal discrete piezoelectric patch allocation on composite structures for vibration control based on ga and modal lqr," *Computers & Structures*, vol. 128, pp. 101–115, 2013.

[6] K. Dhuri and P. Seshu, "Multi-objective optimization of piezo actuator placement and sizing using genetic algorithm," *Journal of Sound and Vibration*, vol. 323, no. 3, pp. 495–514, 2009.

[7] M. Marinaki, Y. Marinakis, and G. E. Stavroulakis, "Fuzzy control optimized by a multi-objective particle swarm optimization algorithm for vibration suppression of smart structures," *Structural and Multidisciplinary Optimization*, vol. 43, no. 1, pp. 29–42, 2011.

[8] G. Foutsitzi, E. Hadjigeorgiou, C. Gogos, and G. Stavroulakis, "Modal shape control of smart composite beams using piezoelectric actuators," in *10th HSTAM (Hellenic Society for Theoretical and Applied Mechanics) 2013 International Congress on Mechanics*, Technical University of Crete Publishing House, 2013.

[9] G. A. Foutsitzi, C. G. Gogos, E. P. Hadjigeorgiou, and G. E. Stavroulakis, "Actuator location and voltages optimization for shape control of smart beams using

genetic algorithms," in *Actuators*, vol. 2, pp. 111–128, Multidisciplinary Digital Publishing Institute, 2013.

[10] D. Greiner, J. Emperador, and G. Winter, "Enhancing the multiobjective optimum design of structural trusses with evolutionary algorithms using densea," in *44th AIAA (American Institute of Aeronautics and Astronautics) Aerospace Sciences Meeting and Exhibit, paper AIAA-2006-1474*, 2006.

[11] H. Eskandari, C. D. Geiger, and G. B. Lamont, "Fastpga: A dynamic population sizing approach for solving expensive multiobjective optimization problems," in *Evolutionary Multi-Criterion Optimization*, pp. 141–155, Springer, 2007.

[12] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan, "A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: Nsga-ii," *Lecture notes in computer science*, vol. 1917, pp. 849–858, 2000.

[13] D. W. Corne, N. R. Jerram, J. D. Knowles, M. J. Oates, *et al.*, "Pesa-ii: Region-based selection in evolutionary multiobjective optimization," in *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO2001*, Citeseer, 2001.

[14] E. Zitzler, M. Laumanns, L. Thiele, E. Zitzler, E. Zitzler, L. Thiele, and L. Thiele, "Spea2: Improving the strength pareto evolutionary algorithm," 2001.