3D Visualization of Uncertainty in Archaeological Reconstructions using Bayesian Probabilities

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Abstract

This work aims to explore, describe, quantify, and visualize uncertainty in a cultural informatics context, especially on archaeological reconstructions. For many years, archaeologists have been criticizing the often too-realistic appearance of three-dimensional reconstructions. Most of the times, they highlight one of the unique features of archaeology: the information we have on our heritage, about an archaeological site, will always be incomplete. All attempts to visualize the reconstructions should be based on this archaeological feature, incompleteness.

This view of archaeologists is the driving force behind this work. The thesis is based on research of archaeological theory and inferential process and provides insight into computer visualization. It describes how these two areas, of archaeology and computer graphics, have formed a useful, but many times conflicting, relationship.

By studying the previous work, the existing background and after research, the three main areas proposed through this thesis, for the visualization of multiple versions of a monument based on uncertainty, are: identification, quantification and visualizing.

Firstly, through the design, distribution, and analysis of a questionnaire, the thesis identifies the importance of uncertainty in archaeological interpretation and discovers potential preferences among different evidence types. This interpretation derives from the relative importance of findings that have been unearthed from the excavations, such as artifacts like vases, tools, devotional objects. These findings mainly lead to the interpretation of the use of each archeological structure or the positioning of its structural elements. Wall remains can help archaeologists to understand which and where were the main structural elements of a building in the past.

Secondly, the thesis analyses and evaluates, in relation to archaeological uncertainty, a belief quantification model. By uncertainty we define an archaeological expert's level of confidence in an interpretation of how an archaeological site was in the past. The thesis examines another way to evaluate the belief of archaeologists in a reconstruction, by using the Dirichlet distribution.

Thirdly, multiple uncertainty levels visualized, related to how an archaeological structure may have existed in the past, at the same time, based on dynamically uncertainty calculations, bringing the element of the archaeologist's uncertainty in the visualization. Moreover, in a single 3D archaeological reconstruction, we could also differentiate through diverse visualization the reconstructed parts for which there is strong evidence that they existed as visualized, as opposed to the ones that there is less evidence and knowledge about their past form.

The novelty of this project is based on the fact that we incorporate the element of archaeological uncertainty in a 3D reconstruction of a past archeological structure, now in ruins, as opposed to the monolithic view of current 3D archaeological reconstructions which offer mainly 'pretty' images and a single visualization of the past.

Abstract in Greek

Αυτή η δουλειά στοχεύει στην διερεύνηση, την περιγραφή, τον ποσοτικό προσδιορισμό και την οπτικοποίηση της αβεβαιότητας σε ένα πλαίσιο πολιτιστικής πληροφορικής, ειδικά σε αρχαιολογικές ανακατασκευές. Για πολλά χρόνια, οι αρχαιολόγοι ασκούν κριτική στην υπερβολικά ρεαλιστική εμφάνιση τρισδιάστατων ανακατασκευών. Τις περισσότερες φορές, επισημαίνουν ένα από τα μοναδικά χαρακτηριστικά της αρχαιολογίας: οι πληροφορίες που έχουμε σχετικά με την κληρονομιά μας, σχετικά με ένα αρχαιολογικό χώρο, θα είναι πάντα ελλιπείς. Όλες οι προσπάθειες οπτικοποίησης των ανακατασκευών θα πρέπει να βασίζονται σε αυτό το αρχαιολογικό χαρακτηριστικό, την έλλειψη.

Αυτή η άποψη των αρχαιολόγων είναι η κινητήρια δύναμη πίσω από αυτήν τη διπλωματική εργασία. Η διπλωματική βασίζεται σε έρευνα αρχαιολογικής θεωρίας και συμπερασματικής διαδικασίας και παρέχει πληροφορίες για την οπτικοποίηση υπολογιστή. Περιγράφει πως αυτές οι δύο περιοχές, της αρχαιολογίας και των γραφικών υπολογιστών, έχουν διαμορφώσει μια χρήσιμη, αλλά πολλές φορές συγκρουόμενη, σχέση.

Μελετώντας τα προηγούμενα έργα, το υπάρχον υπόβαθρο και κάνοντας έρευνα, οι τρεις κύριες περιοχές που προτείνονται μέσω αυτής της διπλωματικής για την οπτικοποίηση πολλαπλών εκδόσεων ενός μνημείου που βασίζεται στην αρχαιολογικής αβεβαιότητα, είναι: ταυτοποίηση, ποσοτικοποίηση και οπτικοποίηση.

Αρχικά, μέσω του σχεδιασμού, της διανομής και της ανάλυσης ενός ερωτηματολογίου, η διπλωματική εργασία προσδιορίζει τη σημασία της αβεβαιότητας στην αρχαιολογική ερμηνεία και ανακαλύπτει πιθανές προτιμήσεις μεταξύ διαφορετικών τύπων στοιχείων. Αυτή η ερμηνεία προέρχεται από τη σχετική σημασία των ευρημάτων που έχουν ανακαλυφθεί από τις ανασκαφές αντικειμένων όπως αγγεία, εργαλεία, λατρευτικά αντικείμενα. Αυτά τα ευρήματα οδηγούν κυρίως στην ερμηνεία της χρήσης κάθε αρχαιολογικής δομής ή στην τοποθέτηση των δομικών στοιχείων της. Τα τείχη μπορούν να βοηθήσουν τους αρχαιολόγους να κατανοήσουν ποια και πού ήταν τα κύρια δομικά στοιχεία ενός κτηρίου στο παρελθόν.

Δεύτερον, η διατριβή αναλύει και αξιολογεί, σε σχέση με την αρχαιολογική αβεβαιότητα, ένα μοντέλο ποσοτικοποίησης πεποιθήσεων. Ως αβεβαιότητα ορίζουμε το επίπεδο σιγουριάς ενός αρχαιολογικού εμπειρογνώμονα σε μια ερμηνεία του πώς ήταν ένας αρχαιολογικός χώρος στο παρελθόν. Η εργασία εξετάζει έναν άλλο τρόπο αξιολόγησης της πεποίθησης των αρχαιολόγων σε μια ανοικοδόμηση, χρησιμοποιώντας τη κατανομή Dirichlet.

Τρίτον, οπτικοποιήθηκαν πολλαπλά επίπεδα αβεβαιότητας, που σχετίζονται με το πώς μπορεί να υπήρχε μια αρχαιολογική δομή στο παρελθόν, ταυτόχρονα, με βάση δυναμικούς υπολογισμούς αβεβαιότητας, φέρνοντας το στοιχείο της αβεβαιότητας του αρχαιολόγου στην οπτικοποίηση. Επιπλέον, σε μια μόνο τρισδιάστατη αρχαιολογική ανακατασκευή, θα μπορούσαμε επίσης να διαφοροποιήσουμε μέσω διαφορετικής οπτικοποίησης των ανακατασκευασμένων τμημάτων για τα οποία υπάρχουν ισχυρές ενδείξεις ότι υπήρχαν ως οπτικοποιημένα, σε αντίθεση με αυτά που υπάρχουν λιγότερα στοιχεία και γνώσεις για την προηγούμενη μορφή τους.

Η καινοτομία αυτής της εργασίας βασίζεται στο γεγονός ότι ενσωματώνουμε το στοιχείο της αρχαιολογικής αβεβαιότητας σε μια τρισδιάστατη ανακατασκευή μιας προηγούμενης αρχαιολογικής δομής, τώρα σε ερείπια, σε αντίθεση με τη μονολιθική άποψη των τρεχουσών τρισδιάστατων αρχαιολογικών ανακατασκευών που προσφέρουν κυρίως «όμορφες» εικόνες και μια μοναδική απεικόνιση του παρελθόντος.

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1 Introduction

1.1 Motivation

For a long time, archaeologists have been criticizing the too-realistic appearance of three-dimensional reconstructions of archaeological sites. Most of the times, they highlight one of the unique features of archaeology: the information we have on our heritage, about an archaeological site, will always be incomplete. All attempts to visualize the reconstructions should be based on this archaeological feature, incompleteness. This view of archaeologists leads to this work. This thesis described how the area of archaeology and the area of computer graphics have formatted a useful and tumultuous relationship through the years. An archaeologist's level of confidence in an interpretation deriving from gathered evidence is defined as uncertainty. In general, archaeologists and computer scientists recommend caution in three-dimensional archaeological reconstructions, as some other possible hypotheses may not be acknowledged. This thesis presents a 3D visualization system of archaeological uncertainty, based on Bayesian priors (Chalkiadakis & Boutilier, 2003) which takes as input archaeologists' confidence in relation to evidence uncovered in excavations and calculates the relative probability in relation to how these parts existed in the past, reconstructed and visualized in 3D (Sifniotis et al., 2010). Therefore, it is now possible different parts of a building to be associated with varied probabilities in relation to whether the actual reconstruction is accurate and at which degree. In this sense, the visualization offers multiple reconstructed options in relation to how an architectural structure existed in the past.

Based on the design, distribution, and analysis of a questionnaire to expert archaeologists, this work initially identifies the relative importance among different evidence types for archaeological interpretation. This interpretation is derived from the relative importance of findings that have been unearthed from excavations, such as artifacts, textual evidence, ceramics, etc. A belief quantification model is analyzed and evaluated, in relation to archaeological uncertainty.

Multiple uncertainty levels are visualized, related to how an archaeological structure may have existed in the past, at the same time, based on dynamic, mathematical uncertainty calculations, bringing the element of the archaeological expert's uncertainty in the visualization. In a single 3D archaeological reconstruction, we can now differentiate through diverse visualization the reconstructed parts for which there is strong evidence that they existed as visualized, as opposed to the ones that there is less evidence and knowledge about their past form. The test case we used was the remaining structure of the Palace of Zakros in Crete, Greece.

The archaeologist, who helps us to interpret the evidence unearthed during the excavation and communicate to us her belief in relation to how the Palace of Zakros existed in the past, is Dr. Anna Semantraki-Grimshaw. Dr. Anna Semantraki-Grimshaw is a multilingual archaeologist studying Greek Archeology (Prehistoric, Classical and Byzantine). She specializes in the Bronze Age of the Aegean, especially in Minoan Crete.

There are findings and evidence which can give information about how the Palace of Zakros was in the past. First of all, the remnants of the walls indicate the positioning of the structure of the Palace. This evidence belongs to the category of "Features" evidence, because it is an element of man-made structure. However, excavations haven't yet been completed. Many tools and vases have been found discovered, therefore, the scope and function of certain rooms is evident. Certain frescoes that have been found depict how the palace was in the past in some places. However, archeologists are not certain whether these depictions are true. Devotional objects, which were found in the ruins, give us information in relation to which rooms had a devotional character. These belong to the "Artefacts" evidence type, as they are objects made by a human.

After a thorough discussion with the archeologists, certain elements of the palace are of high uncertainty in relation to their function or exact positioning. These are the wall of Western façade, the central court and the room of the tank. In the first case, there is uncertainty about the existence of windows on the wall. In the second case, in the central court a stone structure could be used as an altar or there could be a tree. In relation to the room of the tank, it is controversial whether this was an outdoor space or not. Certainly, there exist other spots in the Palace of Zakros with uncertainty. Some of the evidence contradict each other. This can lead to alternative scenarios concerning the positioning as well as the function of parts of palace affecting the reconstruction of the Palace of Zakros.

Archaeologists have to give their opinion on how likely each scenario is. The goal of this project is to develop a user interface based on which archaeologists will input their belief about a scenario and this belief will be the input of a probabilistic mathematical model. The archaeologists will give their belief about every different alternative scenario for specific locations of the Palace where there is uncertainty, by answering questions about how confident they are about every hypothesis, which will form the necessary input for the mathematical model. Then they will be able to visualize these different hypotheses based on distinct visualization outputs, each one depending on the results of the mathematical model.

1.2 Contribution

By studying the previous work, the existing background and after research, the three main areas proposed through this thesis, for the visualization of multiple versions of a monument based on uncertainty, are: identification, quantification and visualizing.

First of all, the importance of uncertainty in archaeological interpretations was identified through the design of a questionnaire and the analysis of archaeologists' answers. This interpretation derives from the relative importance of findings that have been unearthed from the excavations, such as artifacts like vases, tools, devotional objects. These findings mainly lead to the interpretation of the use of each archeological structure or the positioning of its structural elements. Wall remains can help archaeologists to understand which and where were the main structural elements of a building in the past.

Then, a belief quantification model was analyzed and evaluated, in relation to archaeological uncertainty. The level of confidence of an archaeologist expert, about how an archaeological site was in the past, is defined as uncertainty. The thesis examines another way to evaluate the belief of archaeologists in a reconstruction, by using the Dirichlet distribution.

Finally, multiple uncertainty levels visualized, related to how an archaeological structure may have existed in the past, at the same time, based on dynamically uncertainty calculations, bringing the element of the element of the archaeological expert's uncertainty in the visualization. Moreover, in a single 3D archaeological reconstruction, we could also differentiate through diverse visualization the reconstructed parts for which there is strong evidence that they existed as visualized, as opposed to the ones that there is less evidence and knowledge about their past form. The novelty of this project is based on the fact that we incorporate the element of archaeological uncertainty in a 3D reconstruction of a past archeological structure, now in ruins, as opposed to the monolithic view of current 3D archaeological reconstructions which offer mainly 'pretty' images and a single visualization of the past.

This thesis presents a complete visualization system for demonstrating archeological uncertainty in 3D reconstructions, by providing multiple reconstructed options of specific spatial locations, driven by an uncertainty mathematical model based on Bayesian probabilities. Expert archaeologists offer their opinion to statements through a questionnaire in order to identify the relative importance among a set of archeological evidence which relates to Minoan archeology. Evidence or the so-called influencing factors which were selected to represent types of archeological evidence of significance for the Minoan civilization are features (pits, walls, ditches, etc), artefacts (tool, work of arts, etc), comparisons (comparison with other structures of the same era or type of building), topography (area elevation, region characteristics, etc), peer review (discussion with experts). Archaeologists also insert their belief about whether the visualized reconstructions are accurate. They select the evidence types which formed their belief in the accuracy of the visualized reconstruction.

Based on this input, the output of the system offers a preview of the Probability Density Function (PDF) of the Dirichlet Distribution, where archaeologists can see the density of the points and the point with higher density is the most likely combination of probabilities. The density of the points but also their spatial location and concentration in space (in a triangle, for which the three vertices represent three differently reconstructed options for the same spatial location) also reveals which reconstructed option is most probable. The system creates random values (random sampling) from this distribution and these values appear as points on a triangle. Every point of the triangle is a different combination of probabilities. When archaeologists hover a point, they can simultaneously view all reconstructed options for the same spatial location, associated to a specific combination of probabilities. The color from red to green employed signifies a low to high probability for the range of reconstructed options offered, for the same spatial location.

It is important to note that when numerous expert archeologists provide input in relation to the significance of evidence types and their belief in the accuracy of the visualized reconstructions, their data is combined and the distribution, especially if the all agree, converges to one of the three reconstructed options. Even if they do not agree, it is spatially evident in the Dirichlet distribution's simplex the convergence of their opinions (whether the concentration of points approaches a vertex of the simplex meaning one of the three reconstructed options is highly probable, while when the distribution of points is more in the middle ground of the simplex, it means that the archaeologists interacting with the system have diverse opinions).

In the end, future extensions are suggested and represented through the design and development of new plugins to a search engine.

1.3 Mathematical Modeling of Archaeological Uncertainty

The project extends earlier work conducted at the University of Sussex, UK - specifically the PhD thesis of Maria Sifniotis (Sifniotis et al., 2010) which introduced methods based on Bayesian probabilities and possibility theory to quantify and visualize the archaeological expert's uncertainty in the reconstructed interpretation. This was conducted based on archeologists' subjective impressions in relation to their confidence in relation to whether varied reconstructed parts of archaeological structures were indeed as reconstructed or whether there was doubt (and at which level) that reconstructed parts were as visualized. The archeologists in the case of Sifniotis et al. 2010, were interviewed and based on linguistics' assessment, the researchers assigned a numerical value signifying their confidence in the archeologist's interpretation of whether a reconstructed part existed as reconstructed and to which degree.

In the work introduced by (Sifniotis et al., 2007), authors identified how important the uncertainty is, in archaeological interpretation and discovered possible preferences among many different evidence types as unearthed during excavations. These evidence factors were features, artefacts, biofacts, textual evidence, absolute comparisons, contextual comparisons, topography, peer review. These factors influence differently the reconstruction, as the weight of the importance of every factor is different according to the opinion of archaeologists. The weight of significance resulted from a questionnaire, in which archaeologists answered questions, prioritizing the importance of these factors. For example, in Sifniotis' thesis archaeologists place more importance on peer feedback and evidence such as features and artefacts. Accordingly, less importance was placed to textual evidence and biofacts. Although past work explored the historical significance of evidence unearthed in excavations, researchers did not incorporate such information in their proposed explorations of how archeological uncertainty should be mathematically represented.

Two different mathematical approaches were analyzed, by (Sifniotis et al., 2010). The first one was based on Bayesian probabilities and the other was Possibility theory. Possibility Theory complements probability theory. The difference between the two is that Possibility Theory uses a pair of dual-set functions, known as Possibility (Pos) and Necessity (Nec), instead of the single function (Pr) used by probability. The Possibility measure reflects how plausibility, belief and Necessity relates to certainty. In most cases, the results showed that they did not deviate much. A Romano British building was used as a test case and its 3D reconstruction was employed so that based on varied transparency and pseudocolour visualization, the goal was for the reconstructed parts with a higher probability that they appeared in that form in the past to be differentiated visually in relation to the ones with a lower probability. Such visualization differentiations were based on the Bayesian and Possibility theory methodologies and were compared.

More recently, (Danielova et al., 2016) proposed a fuzzy logic model for calculating the archaeological uncertainty. For visualization of uncertainty the study case was the Temple of Diana in Demi the Temple of Diana in Demi of Italy. The visualization models were based on transparency and colour visualization. The work presented gives a three-dimensional reconstruction of an ancient Roman temple with the name "Sanctuary of Diana", which is located in Nemi of Italy. The details of this archaeological site are presented in different levels, quantifying uncertainty by a fuzzy logic approach. The resulting reliability value which this work signifies how reliable is the 3D reconstructed part. In order quantify the reliability of a virtual model (Hermon & Niccolucci, 2003), a scale for reliability was established. This scale has values from 0 to 1, where the value 0 means that something is totally unreliable, while the value 1 means that something is absolutely reliable. The main disadvantage of this work is that there is a strong dependence of the final result of reliability on the order in which the building's historical evidence of the reconstruction and objects are added to the fuzzy logic model. This means that depending on the order of input of evidence in the fuzzy logic model, the reliability value changes, therefore, the model did not produce accurate reliability results. For this reason, the objects should not be added randomly but following a specific order so that the resulting reliability value is acceptable by archeologists when the reconstructed parts are visualized based on these values.

The goal of this thesis is to quantify archaeological expert's uncertainty for multiple interpretations and to visualize these interpretations based on a sophisticated Bayesian approach. As a case study, we are focusing on the Palace of Zakros in Crete, Greece. The structure is located at the eastern end of Crete. It is one of the Minoan Palaces which have been excavated.

The Bayesian approach, which was used in Sifniotis et al., 2010, has also resulted in certain issues. First of all, this mathematical model assigns an equiprobability to ignorance. For example, for two cases A and B of 3D reconstructactive options of the same spatial spot in the building in relation to how it could be in the past, with no prior knowledge based on evidence, the expert initially, without the evidence, should assign a prior of {0.5, 0.5} meaning that the visualization of a particular 3D reconstruction has a 50% probability to be accurate. This is not correct as {0.5,0.5} can also reflect knowledge that either A or B are likely to occur. Another issue is that if there are two cases of A, B the expert could assign a confidence of 0.95 to A meaning that there is strong belief that the reconstructed part

A is almost certainly accurate. If this is true, then a confidence of 0.05 should be assigned to B in relation to how the same reconstructed part should appear, even if the archeologists feel that that reconstruction B of the same spatial spot is more probable (represented by a value higher that 0.05). This is too restrictive and it might force the archaeologist to accept assumptions that they would not be comfortable with.

Also, the Bayesian approach requires at least two alternative assumptions for the calculations. For example, in the PhD thesis of Maria Sifniotis, two interpretations were considered about the use of the Roman-British building which was used as a test case. There are two interpretations: its function was of a military nature or of a religious purpose. In the first scenario, more emphasis is placed on the functional use for the building constructed by ordinary materials. The religious significance suggests the use of more expensive construction materials or decorative items. The one interpretation was arbitrarily taken as input to the model so that calculations can be applied. This is a significant issue, because sometimes archaeologists know that an interpretation of how a specific part of a building used to be isn't certain but they don't have an alternative for the same part of the building. An important problem is that if one assigns a likelihood of 0 to a hypothesis of how a reconstructed part was formed in the past or in relation to its use as in this case, this hypothesis may never be resurrected no matter the evidence since in the presented visualization of Sifniotis et al., 2010, the outcome was a 3D model and now a complete visualization system as in this thesis where archeologists can input their beliefs as well as assessment of the evidence significance as unearthed in excavations. In the end, the existence of a 'closed world' restricts the archaeologists in case they would need the option of adding a new hypothesis. Therefore, there is need to apply a mathematical model which will give different probability values to all the possible hypothesis which have emerged from the evidence and the belief of archaeologists. These values must correspond to how much confidence there is in each hypothesis.

The possibilistic approach adopted by Sifniotis et al., 2010 attempted to address the issues above (Chen, 1995). The difference between the Bayesian approach is that Possibility Theory uses a pair of dual-set functions, known as Possibility (Pos) and Necessity (Nec), instead of the single function (Pr) used by probability. The Possibility measure reflects how plausibility, belief and Necessity relates to certainty. As it is described by Sifniotis et al., 2010 Possibility Theory does not contradict probability theory; rather, it complements it. Possibility Theory offers ways to express total ignorance about prior beliefs. This removes a serious constraint of subjective probability. This approach does not inherit the additive axiom of probability. Thus, among two different reconstructed options, A and B, if A is quite possible, B could be quite possible as well. One does not negate the other. In most cases, the results showed that they did not deviate much.

In our work, we tried to face many of these issues. First of all our mathematical model can handle more than two reconstruction options for the same spatial location. As we concentrate on the Palace of Zakros for this thesis, we decided to visualize three different potential reconstructions for every spatial location point of interest, of the Palace of Zakros visualized in 3D. Also, in Sifniotis 2012 archaeologists couldn't update their belief related to new evidence uncovered in excavations, because neither Bayesian Theory nor Possibility Theory as described in her thesis had this ability. In this sense, updated probabilities in Sifniotis 2012 replaced the previous input by experts in relation to the experts' beliefs, among two options rather than add to their collective scientific knowledge as in the presented system. Finally, in our work, distribution does not give 0 probability values in any of the reconstructed options. After archaeologists communicate their belief in relation to whether a reconstructed option is accurate, the distribution changes and it is clear which is the preferable option based on opinions by numerous experts combined. Even when the simplex of the Dirichlet distribution function showcases that one of the three reconstructed options is highly probable, the probability that the remaining two reconstructed options (vertices of the simplex) does not take zero values, therefore, there is lower probability that this was the form of the specific spatial location in the past but the probability is not zero. This mathematical rationale is much closer to how archeologists think in relation to different hypotheses compared to previous work by Sifniotis 2012 and Danielova 2016.

In order to satisfy the fact that numerous archaologists may offer the belief in specific reconstructions in relation to whether they are accurate and these beliefs should be collectively combined and taken into account, we focus on sampling-based posterior updating. Getting inspired from the way archaeologists criticize the 3D reconstructions, we focus on their need to continuously offer their belief and opinions in relation to the significance of evidence and their belief about whether a reconstruction is accurate, based on their new findings or on a new way of thinking. In this thesis, we present a mathematical model which is based on Dirichlet priors. Using Bayes statistics, a prior distribution can be updated as soon as new observations are available.

1.4 Thesis Outline

Our thesis has seven main chapters, which have the following structure:

In the second chapter, we focus on previous research which puts forward 3D reconstructions of archeological sites based on interpreting archeological uncertainty. Previous work including the work which is mentioned in the previous section by Sifniotis et al., 2010 and by Danielova et al., 2016 is analyzed in detail. The novelty of the work presented in this thesis is substantiated by detailing research issues with past work.

In the third chapter, the theoretical background is described. More specifically, in this chapter some essential definitions are given for the completion of this thesis, such as Probability Reasoning, Bayes' Theorem, Hyperparameters.

In the fourth chapter, we present the software which was used for the implementation of the uncertainty visualization system. Firstly, the Unity, our selected game engine, is described. Finally, the tools which were used for the creation of a MySQL Database for saving archaeologists' data as input, the platforms XAMPP for running a web server which takes as input archeologists' input and PhpMyAdmin for the online database utilized, are included in this chapter.

In the fifth chapter, the influencing factors which signify the archaeologists' belief are described in detail, as well as the way they were selected. In our case study, the influencing factors are features, artefacts, comparisons with similar archaeological sites, topography and peer review. Also, the specific questionnaire which was created to assess how significant these factors are for the archaeological interpretation of historical remains, is presented. We present the User Interface for the input of archaeologists. Then we identify the necessary system's variables. Finally, the sample from Dirichlet distribution is visualized as system's output.

In the sixth chapter, the 3-dimentional visual reconstruction of the palace is presented. User navigation in the Palace of Zakros is analyzed. Information about every reconstructed part of the building is described. 3D reconstructions of objects based on what was found in excavations are included. User interaction with these objects, is described.

In the last chapter, conclusions are presented, as well as certain suggestions about more work in the future that could possibly extend this thesis.

2 Related Work

There are many works which has as object the reconstructions. The work of Calogero et al. 2013 presents the process of reconstructing three facade designs for the east wing of the Louvre using procedural modeling. It is proposed that by formalizing the facade description into a shape grammar with procedural modeling, a systematized approach to a stylistic analysis is possible. It is also asserted that such an analysis is still best understood within the historical context of what is known about the contemporary design intentions of the building creators and commissioners. Another work of Happa et al. 2012 which presents a novel research framework to reverse engineer past sites by extending Predictive Rendering to make use of extant objects and records, experimental data and expert opinion. However, the two fundamental research papers which first put forward the notion of uncertainty visualization in the archeological domain. We will focus on these two papers in this section, providing a detailed analysis of the work presented. These are: the work by Sifniotis, 2012, "Representing archaeological uncertainty in cultural informatics", a PhD thesis of the University of Sussex, UK, Department of Informatics, and the work by Danielova et al., 2016.

2.1 Representing archaeological uncertainty in cultural informatics (Sifniotis et al., 2012)

In Sifniotis, 2012, uncertainty is defined as an archaeological expert's level of confidence related to archaeological evidence. Archaeologists piece together information based on their findings on-site into a speculative version of the past. This version becomes more certain as the evidence increases. The uncertainty associated with an archaeologist's interpretation can be represented using either Bayesian approaches or Possibility Theory and visualized by shader-based information visualization schemes Sifnioti et al., 2012. An increase or decrease in uncertainty is influenced by any related evidence recovered on the archeological site; we define this evidence as "influencing factors".

Every historical period may find an abundance of a specific type of evidence but a complete lack of another. From discussions with archeologists, a list of influencing factors (archeological evidence types) was created (Sifniotis et al., 2007). Briefly this included in Sifniotis et al., 2007: Features as all elements of man-made structures; these can range from ditches to wall remains, to post-holes of wooden structures. Artefacts as any object made, affected, used, or modified in some way by human beings; this usually includes pottery, glass, lithic, etc. Biofacts as Biofacts or ecofacts constitute of human, animal and plant remains which are not changed by human interaction. Textual evidence as ancient texts and documents which may provide information on architecture, decoration, lifestyle etc of a specific culture. Absolute comparisons as a structure is compared to a structure of similar proportions. Contextual comparisons as a structure and its context is compared to similar structures with similar contexts. Topography as natural features of the landscape in which the building is located; elevation of the area, characteristics of the region and landform data in general. Peer Review as the interviews indicated that interpretation is heavily based on discussions and re-discussions with other archaeologists and architecture specialists were considered important.

The questionnaire in Sifniotis et al., 2007 includes 35 related questions (Figure 2.1). It is divided in nine sections. The first section deals with perceptions of uncertainty in the discipline and how alternative hypotheses are handled (e.g. Archaeology is fraught with uncertainty: You can never be too sure about anything.). Sections two to six contain questions on the different identified factors (e.g. While archaeology is uncertain, structural evidence gives me security. , When faced with structural evidence I usually make up my mind very quickly as to how the building looked.). Section seven provides a combination of factors (for example features and artefacts, or artefacts and biofacts) and queries the expert as to how strong he/she considers this combination of evidence to be. In the eighth section, the list of factors is provided and the expert is requested to assign an order of importance for each. Finally, in the last section, the expert enters personal data such as their specialization field, time spent on excavations etc. Most importantly the expert is asked to give an opinion on the completeness of the factor list. Values (Figure 2.2) include never, almost never, sometimes, often, very often, and always. The expert is asked as to how often he/she encounters each statement when making interpretations and reconstructions of a structure. For example:

FEATURES

1. While archaeology is uncertain, structural evidence gives me security.

Never	Almost never	Seldom	Sometimes	Often	Very often	Always

2. When faced with structural evidence I usually make up my mind very quickly as to how the building looked.

Never	Almost never	Seldom	Sometimes	Often	Very often	Always

3. I try to understand the function of the building (i.e. what it was used for) and make my reconstruction around it.

Never	Almost never	Seldom	Sometimes	Often	Very often	Always

4. First I interpret the structural evidence and then I deduce the building's function.

Never	Almost never	Seldom	Sometimes	Often	Very often	Always

5. Most times I am more sure about the ground floors than an existence of a second storey.

Never	Almost never	Seldom	Sometimes	Often	Very often	Always

Figure 2.1: Form of a Questionnaire Page (Sifnioti et al., 2012).

Never	Almost never	Seldom	Sometimes	Often	Very often	Always

Figure 2.2: Scaling of answer values (Sifnioti et al., 2012).

In section 7 the expert is asked to rate a list of factor combinations from 1 (weak) to 9 (strong). Lastly, section 8 follows a ranking system where the expert must rank the eight factors according to how important he/she believes they are in an interpretation. This is done by using numbers from 1 to 8 only once and assigning them to the factors.

The framework for calculating archaeological uncertainty is based on belief of the archaeologist regarding each reconstructed part and whether they are confident that the visualization reflects accurately their perception of how the reconstructed part was formed in the past and formalizes archaeologists' expert knowledge and how this is based on influencing factors in relation to evidence (Sifnioti et al., 2012).

Past work used as a case study a Romano-British building located in the area of Fishbourne, East Sussex, UK. Excavations of this building turned up little evidence besides the stone foundations and hypothesis with what evidence is at hand. There were two interpretations related to its past form: one suggests its function was of a military nature, while the other attaches a religious purpose. The author of this thesis followed two steps for the identification of uncertainty. The first was the discussion with five archaeologists, which was focused solely on the interpretation of structures. From these discussions a list of evidence factors that can influence uncertainty was created. The factors identified are: Features, Artefacts, Biofacts, Textual evidence, Absolute comparisons, Contextual comparisons, Topography, Peer review. The next step involved the creation of a questionnaire with an aim to gather more information on how archaeologists perceive uncertainty as well as define the relative importance of such evidence (named here as influencing factors).

In the work of Sifnioti et al., 2012, the researcher identified the uncertainty factors in the archaeological domain related to unearthed evidence in excavations and other historical evidence through discussions with archaeologists, and evaluated through a questionnaire (Sifniotis et al., 2007). The test case was a Roman-British building which is in ruins at this present time and Sifnioti et al., 2012 aimed to 3D reconstruct by differentiating which parts of the reconstruction are certain that they were formed the way they were reconstructed and which parts are less certain, therefore, avoiding the common pitfall of 3D computer graphics which produce 3D reconstructions of archeological monuments without any reference to whether it is certain that the appeared in the past the way visualized. This work, quantified the archaeological uncertainty using different mathematical models. Three established mathematical models of uncertainty quantification, Probability Theory, Possibility Theory and the Transferable Belief Model, were presented, explained and innovatively applied to archaeological scenarios and then evaluated. Finally, the author of this work visualized the archaeological uncertainty, using a transparency visualization model and a transparency/color visualization model.

This thesis extents earlier work conducted at the University of Sussex, UK e.g., the PhD thesis of Maria Sifniotis (Sifniotis et al., 2010) which introduced methods based on Bayesian probabilities and possibility theory to quantify and visualize the archaeological expert's uncertainty in the reconstructed interpretation. This was conducted based on archeologists' subjective impressions in relation to their confidence in relation to whether varied reconstructed parts of archaeological structures were indeed as reconstructed or whether there was doubt (and at which level) that reconstructed parts were as visualized. The archeologists in the case of Sifniotis et al. 2010, were interviewed and based on linguistics' assessment, the researchers assigned a numerical value signifying their confidence in the archeologist's interpretation of whether a reconstructed part existed as reconstructed and to which degree. However, this questionnaire, which explored the relative importance of historical evidence related to a specific archaeological site, was not incorporated in the visualization of a test-case Roman British building which Sifnioti et al., 2012 visually reconstructed.

In the work introduced by (Sifniotis et al., 2007), authors identified the importance of uncertainty in archaeological interpretation and discovered potential preferences among different evidence types as unearthed during excavations. These evidence factors were features, artefacts, biofacts, textual evidence, absolute comparisons, contextual comparisons, topography, peer review. These factors influence differently the reconstruction, as the weight of the importance of every factor is different according to the opinion of archaeologists. The weight of significance resulted from a questionnaire, in which archaeologists answered questions, prioritizing the importance of these factors. For example, in Sifniotis' thesis archaeologists seem to place more importance on peer feedback and evidence such as features and artefacts. Accordingly, less importance was placed to textual evidence and biofacts. Although past work explored the historical significance of evidence unearthed in excavations, researchers did not incorporate such information in their proposed explorations of how archeological uncertainty should be mathematically represented.

Two different mathematical approaches were analyzed, by (Sifniotis et al., 2010). The first one was based on Bayesian probabilities and the other was Possibility theory. Possibility Theory complements probability theory. The difference between the two is that Possibility Theory uses a pair of dual-set functions, known as Possibility (Pos) and Necessity (Nec), instead of the single function (Pr) used by probability. The Possibility measure reflects how plausibility, belief and Necessity relates to certainty. In most cases, the results showed that they did not deviate much. A Romano British building was used as a test case and its 3D reconstruction was employed so that based on varied transparency and pseudocolour visualization, the goal was for the reconstructed parts with a higher probability that they appeared in that form in the past to be differentiated visually in relation to the ones with a lower probability. Such visualization differentiations were based on the Bayesian and Possibility theory methodologies and were compared.

Possibility Theory offers ways to express total ignorance about prior beliefs. This removes a serious constraint that subjective probability faces. This approach does not inherit the additive axiom of probability. Thus, among two different reconstructed options, A and B, if A is quite possible, B could be quite possible as well. One does not negate the other. Balance is achieved through the dual function of necessity.

2.1.2 Quantifying Uncertainty

Conditional probability is not sufficient for measuring the impact of added evidence in a model, mainly because the alternative hypotheses are not considered. Bayesian conditioning is able to consider all hypotheses. The following example illustrates the difference between conditional probability and Bayesian conditioning.

The roof trusses of the Romano-British building located in the area of Fishbourne, East Sussex, UK are a section of 3D the reconstructed building which is considered here. Two hypotheses currently exist in relation to the past function of the Romano-British building. It was either used as a military building or for religious functions.

The occurrence ranges between 0 and 1 with 0 meaning no occurrence of the roof trusses in buildings of that era and type and 1 meaning that such roof trusses occur in every building of this type. The number Pr Elaborate truss design| Religious)=0.7 reflects the knowledge about the proportion of religious buildings that have such an elaborate truss design at that particular era.

Evidence A	**There were windows**	**There were not windows**

Simple truss design	0.3	0.8
Elaborate truss design	0.7	0.2

Evidence B	**There were windows**	**There were not windows**
Average quality wood	0.4	0.85
High quality wood	0.6	0.15

Table 1: Knowledge about trusses (Sifnioti et al., 2012).

It is easy to calculate the conditional reliability by multiplying between the respective rows. In relation to a military building with material evidence of average quality wood and simply truss, the reliability is 0.68. By adding more evidence, this approach leads to a smaller and smaller reliability (Sifniotis et al., 2010).

Bayes' theorem deals with posterior probability, a conditional probability of a Hypothesis given the Evidence, where the Hypothesis actually occurs first. Bayesian reasoning functions by using a prior condition in order to calculate a posterior. The prior condition is the belief in the hypotheses before the evidence is considered. To calculate this, are required the probability of getting the evidence if the hypothesis is true, required the probability of getting the evidence if the hypothesis is false, and how likely it would be that the hypothesis is true if the particular evidence was not available.

Let $H_1..H_n$ be disjoint events, forming a partition of sample space Ω and $Pr(H_j) >= 0$ for all *j*. Then for every event *E, Pr*(E)>=0 :

$$\Pr(H_j \mid E) = \frac{\Pr(H_j)\Pr(E \mid H_j)}{\Pr(H_1)P(E \mid H_1) + \ldots + \Pr(H_n)P(E \mid H_n)}$$

For this example a prior of Pr(Religious, Military)={0.5,0.5} is assumed, which means that archaeologists expect to encounter as many religious buildings as military ones.

<u>What is the belief that the Romano-British building was of a military nature?</u>

By using the prior {0.5, 0.5} we get:

$$\Pr(Military \mid Simple\ truss\ design) = \frac{0.8\ x\ 0.5}{0.8\ x\ 0.5 + 0.3x0.5} = 0.72$$

The result increases the belief in the Military hypothesis and it is used as an updated prior Pr(Religious, Military)= {0.28, 0.72} for any new evidence (Sifniotis et al., 2010).

The Bayesian approach, which was used in Sifniotis et al., 2010, has also resulted in certain issues. First of all, this mathematical model assigns an equiprobability to ignorance. For example, for two cases A and B of 3D reconstruction options of the same spatial spot in the building in relation to how it could be in the past, with no prior knowledge based on evidence, the expert initially, without the evidence, should assign a prior of {0.5, 0.5} meaning that the visualization of a particular 3D reconstruction has a 50% probability to be accurate. This is not correct as {0.5,0.5} can also reflect knowledge that either A or B are likely to occur. Another issue is that if there are two cases of A, B the expert could assign a confidence of 0.95 to A meaning that there is strong belief that the reconstructed part A is almost certainly accurate. If this is true, then a confidence of 0.05 should be assigned to B in relation to how the same reconstructed part should appear, even if the archeologists feel that that reconstruction B of the same spatial spot is more probable (represented by a value higher that 0.05). This is too restrictive and it might force the archaeologist to accept assumptions that they would not be comfortable with.

Also, the Bayesian approach requires at least two alternative assumptions for the calculations. For example, in the PhD thesis of Maria Sifniotis, two interpretations were considered about the use of the Roman-British building which was used as a test case. There are two interpretations: its function was of a military nature or of a religious purpose. In the first scenario, more emphasis is placed on the functional use for the building constructed by ordinary materials. The religious significance suggests the use of more expensive construction materials or decorative items. The one interpretation was arbitrarily taken as input to the model so that calculations can be applied. This is a significant issue, because sometimes archaeologists know that an interpretation of how a specific part of a building used to be isn't certain but they don't have an alternative for the same part of the building. An important problem is that if one assigns a likelihood of 0 to a hypothesis of how a reconstructed part was formed in the past or in relation to its use as in this case, this hypothesis may never be resurrected no matter the evidence since in the presented visualization of Sifniotis et al., 2010, the outcome was a 3D model and now a complete visualization system as in this thesis where archeologists can input their beliefs as well as assessment of the evidence significance as unearthed in excavations. In the end, the existence of a 'closed world' restricts the archaeologists in case they would need the option of adding a new hypothesis. Therefore, there is need to apply a mathematical model which will give different probability values to all the possible hypothesis which have emerged from the evidence and the belief of archaeologists. These values must correspond to how much confidence there is in each hypothesis.

The possibilistic approach adopted by Sifniotis et al., 2010 attempted to address the issues above (Chen, 1995). The difference between the Bayesian approach is that Possibility Theory uses a pair of dual-set functions, known as Possibility (Pos) and Necessity (Nec), instead of the single function (Pr) used by probability. The Possibility measure reflects how plausibility, belief and Necessity relates to certainty.

Possibility Theory offers ways to express total ignorance about prior beliefs. This removes a serious constraint that subjective probability faces. This approach does not inherit the additive axiom of probability. Thus, among two different reconstructed options, A and B, if A is quite possible, B could be quite possible as well. One does not negate the other. Balance is achieved through the dual function of necessity.

Possibility Theory is specialized in the handling of incomplete information and complements probability theory. Possibility theory is able to support various interpretations: Logical, Feasibility, Plausibility. The possibility measure Pos() on a set Ω for an event A is the degree of possibility that A occurs where Pos(Ø) = 0 and Pos(Ω) = 1 .The disjunction of two events A and B is the maximum of their individual possibilities Pos(A ∪ B) = max{Pos(A), Pos(B)}. The necessity of an event A is the negation of the possibility of complement of A: Nec(A) = 1 −Pos(A¯). Pos(A) = 1 signifies that A is fully possible. The relationship Pos(A) ≥Nec(A) expresses the fact that something must be possible to some extent before it can begin to be certain.

In a possibilistic approach to the Bayes theorem is proposed where the additivity normalization part of Bayes' equation is paralled to the maxitivity axiom of Possibility Theory. The equation, based on subjective possibilities, takes under consideration both possibility and necessity measures, and can be described as:

$$\mathrm{Pos}(H_j \mid E) = \frac{\mathrm{Pos}(H_j)\mathrm{Pr}(E \mid H_j)}{max\{\mathrm{Pos}(H_1)P(E \mid H_1), \mathrm{Pos}(H_n)P(E \mid H_n)\}}$$

There is one restriction; to define the subjective possibilities, one of the possible scenarios must have Pos = 1. This represents the belief that at least one of the hypotheses is fully plausible. This means that the Probability prior was Pr{Religious, Military}={0.5, 0.5}, but the Possibility prior is Pos{Religious, Military}={0.5, 1} and Nec{Religious, Military}={0,0.5}. This represent an undecided prior where a military case is well plausible but not fully certain.

The same knowledge base is used as with subjective probabilities. Using the data from table 1 the possibilities can be calculated. The revised possibility for

$$\mathrm{Pos(Military\ |SimpleTruss\ )} = \frac{1 * 0.8}{\max\{1 * 0.8\,,\ \ 0.5 * 0.3\}} = 1$$

but the revised possibility for

$$\mathrm{Pr}(Religious \mid Simple\ truss\ design) = \frac{0.8\ x\ 0.3}{0.8\ x\ 1 + 0.3x0.5} = 0.19$$

And Nec{Military}=0.81. The outcome of necessity, is the amount of certainty placed on specific interpretation.

	Religious	Military	***Posterior***	Religious	Military
Pos	0.5	1	*Pos*	0.19	1
Nec	0	0.5	*Nec*	0	0.81

Table 2: Subjective possibility prior (Sifnioti et al., 2012).

Discussions with expert archaeologists were held and during these discussions the archaeologists were asked to list the available evidence in relation to each part of the building and report how confident they were concerning their subjective associations between the evidence and the suggested hypothesis. The outcome of these discussions formed a knowledge table (Table 3). The priors Prior A {Religious, Military} {0.5, 0.5} and Prior B {Religious, Military} {0.75, 0.25} were considered.

NSwalls	0.636	0.71	0.37	0.43
EWwalls	0.76	0.84	0.51	0.8
Roofs	0.30	0.21	0.12	0.11
Stairs	0.5	0.5	0.25	0.25
Trusses	0.72	0.81	0.47	0.66
	Prob	**Poss**	**Prob**	**Poss**
Prior	A{0.5,0.5}		B{0.75,0.25}	

Table 3: Subjective probabilities and possibilities for Building 3 Military hypothesis (Sifnioti et al., 2012).

Examination of the results in Table 3 shows that the probabilistic and possibilistic uncertainties do not deviate more than ±0.09 except in three cases: Roofs (0.5 prior), EW walls (0.75 prior) and Trusses (0.75 prior). In the first case, the evidence highly supports the Religious hypothesis, while in the second and third cases, the Military one. This may suggest that in the face of extremely positive evidence, the possibilistic calculations might get overconfident (or under confident in negative) results while the probabilistic ones will not deviate that much. The following section examines the visualization of the results.

Both approaches can't handle more than two hypotheses for reconstruction. Also, the belief of archaeologists is given once orally at the start of the implementation (Table 1) and then archaeologists can't give their new belief in case they find new evidence in excavations. A serious issue that needed to be faced was that when archaeologists give their confidence that they are sure about a reconstruction, this option take the value 1 and the others the value 0.

In our work, we tried to face many of these issues. First of all, our mathematical model can handle more than two reconstruction options and we chose to visualize three of them for every point of the Palace of Zakros because we wanted to show the results in 3D. Also, in Sifniotis' work archaeologists couldn't update their beliefs related to new evidence uncovered in excavations, because neither Bayesian Theory nor Possibility Theory as described in her thesis had this ability. Finally, in our work distribution does not give 0 values in any of the reconstructed options. After archaeologists vote about their belief the distribution change and it is clear which is the preferable option. However, the uncertainty for the other points does not take zero values.

In an attempt to deal with the reality of updating information, we focus on sampling-based posterior updating. Getting inspired from the way archaeologists criticize all the 3D reconstructions, we intend to show attention in the need of archaeologists to give an input based on their new findings. In this thesis we present a mathematical model which is based on Dirichlet priors. Using Bayes' theorem, a prior distribution can be updated as soon as new observations are available.

The Dirichlet distribution is used as a prior distribution in Bayesian statistics because it is conjugate to the multinomial distribution. This means that identifying the posterior will be easy. In Bayesian probability, if the posterior distribution and the prior distribution are from the same probability distribution family, then the prior and posterior are called conjugate distributions, and the prior is the conjugate prior for the likelihood function.

2.1.3 Visualization in Sifniotis et al., 2010

In Sifniotis et al., 2012, two visualization models were employed to differentiate the diverse uncertainty levels associated with specific reconstructed parts of the Roman-British building. The first one was the transparency model. In this model, more transparency indicates more uncertainty. The difference between the probabilistic and possibilistic values for the roofs is apparent, since the possibilistic figure in Figure 2.1 shows much less confidence in the roof reconstruction.

Figure 2.2 shows the specific parts of the reconstructed Building for which uncertainty values calculated using a transparency visualization under Prior B {0.75, 0.25}. While the uncertainty values do not vary considerably, the overconfidence of the possibilistic results for the east-west walls can be clearly observed.

Figure 2.3 shows the specific parts of the reconstructed Roman-British building for which uncertainty values were calculated using a transparency and a Red-Yellow-Green visualization mode under Prior B {0.5,0.5}. The color visualization uses a 10-value scale in equal intervals between Red and Green. This time, the difference between the cautious confidence of the possibilistic one become more apparent, as the east-west walls and trusses appear more yellow, while in the possibilistic version they tend to be greener, which means that the probability is lower than the possibility. As we can see in Table 3 for Trusses the results are Prob = 0.47 and Poss = 0.66 and for EW Walls are Prob = 0.51 and Poss = 0.8 for Military hypothesis. Green signifies more certainty. More specifically, the higher the result turned out to be, the higher the belief resulting in a Military interpretation, derived from the specific part.

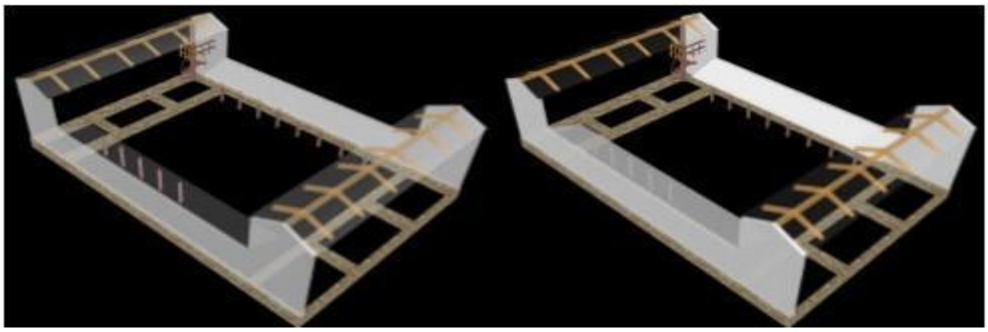

Figure 2.1: Subjective Probabilities and Possibilities with a {0.5,0.5} prior and a transparency visualization (Sifnioti et al., 2012).

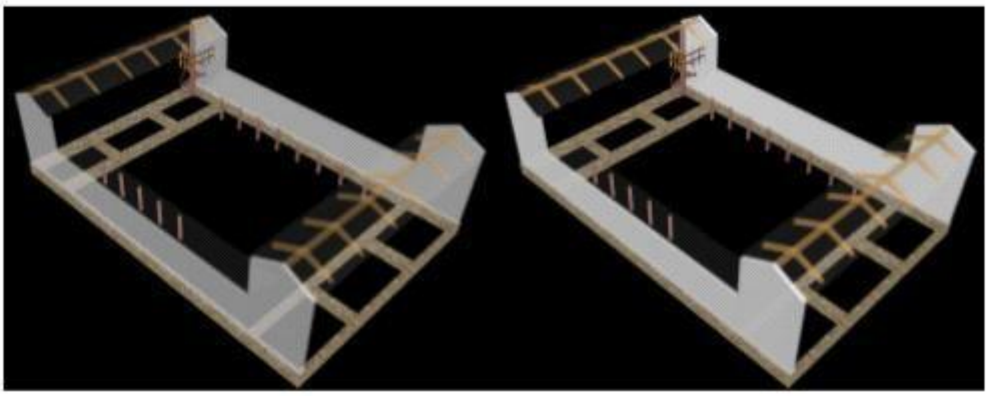

Figure 2.2: Subjective Probabilities and Possibilities with a {0.75,0.25} prior and a transparency visualization. (Sifnioti et al., 2012).

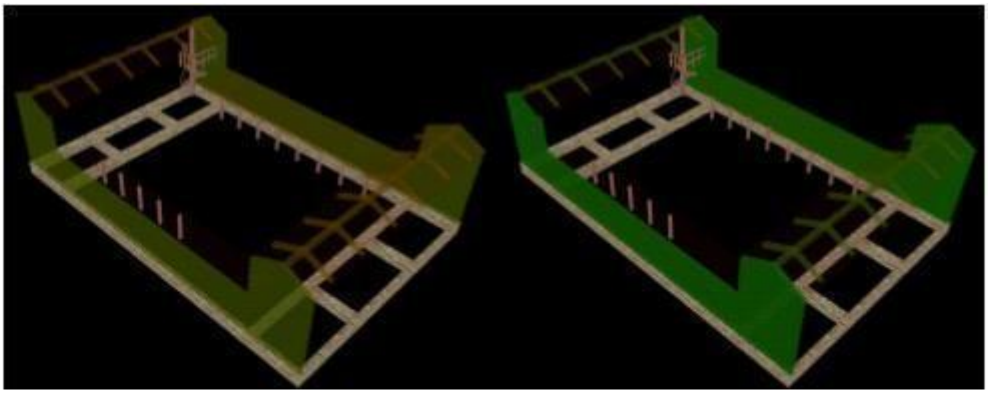

Figure 2.3: Subjective Probabilities and Possibilities with a {0.5,0.5} prior and a transparency/color visualization. (Sifnioti et al., 2012).

2.2 3D Reconstruction and Uncertainty Modelling Using Fuzzy Logic (Danielova et al., 2016)

In this section, we focus on the work employing uncertainty modeling using fuzzy logic for the reconstruction of the Temple of Diana in Nemi, Italy (Danielova et al., 2016). This work is similar to the Sifniotis et al., 2012 research. Both have applied their efforts to mathematically model archeologists' uncertainty in a real archaeological site. They also both used a color visualization model and a transparency visualization model. Danielova et al., 2016 didn't explore the way historical or excavation evidence (influencing factors) affects archeological hypotheses about how a historical structure existed in the past, neither the perception of uncertainty in archaeological sites by archeologists. As explained above, Sifnioti et al., 2012 did. Danielova et al., 2016 placed more emphasis on the visual quality of the 3D reconstruction of the archaeological site, enabling users to interact with this reconstruction.

2.2.1 Case study: The Temple of Diana in Nemi

As case study in the work of Danielova et al., 2016 used the Temple of Diana in Nemi of Italy and in this subsection, we describe the Sanctuary of Diana archaeological site. This archaeological site is located in central Italy. This village is situated 30km southeast of Rome. The name of the temple is related to Diana Nemorensis, also known as Diana of Nemi or Diana of the wood, a goddess of ancient Roman religion. In Figure 2.4 (left) an orthophoto in 2013 is shown, which was captured by a research group from Technische Universität München (Peters et al., 2013), while in Figure 2.4 (right) different parts of the temple ruins are captured.

Figure 2.4: Left, orthophoto; Right, ruins of the Temple of Diana (Danielova et al., 2016).

The work of Danielova et al., 2016 aimed to the three-dimensional reconstruction of the Temple of Diana using the procedural building techniques in CityEngine software. Furthermore, this work had as goal the investigation of uncertainty in archaeological reconstructions, using as case study the Temple of Diana in Nemi and providing multiple solutions expressing this ambiguity that are compatible with the modelling in CityEngine.

2.2.2 3D Temple Reconstruction Modelling

The research of Danielova et al., 2016 based on archaeological interpretations, remains of the archaeological site and photos of the site. There are five (5) successive steps of the three-dimensional (3D) procedural modeling workflow. The first step is the definition of temple parameters. The second step is the import of terminate shapes, which have complicated geometries and they have to be modelled outside the CityEngine software. The third step is the determination of colors and textures. The fourth step is the creation of encoding design rules. The last step is the visualization of three different levels of details. The number of parametered-based rules was 131, which led to the reconstruction of the temple. The parameter modification enabled without the distortion of the entire model. Many shapes, such as statues, podium or structures, were modeled using SketchUp and SketchUp 3D Warehouse because of their complexity and then imported into the CityEngine. Raster templates from many sources were used in order to have more realistic representation of the materials, such as wood or stone.

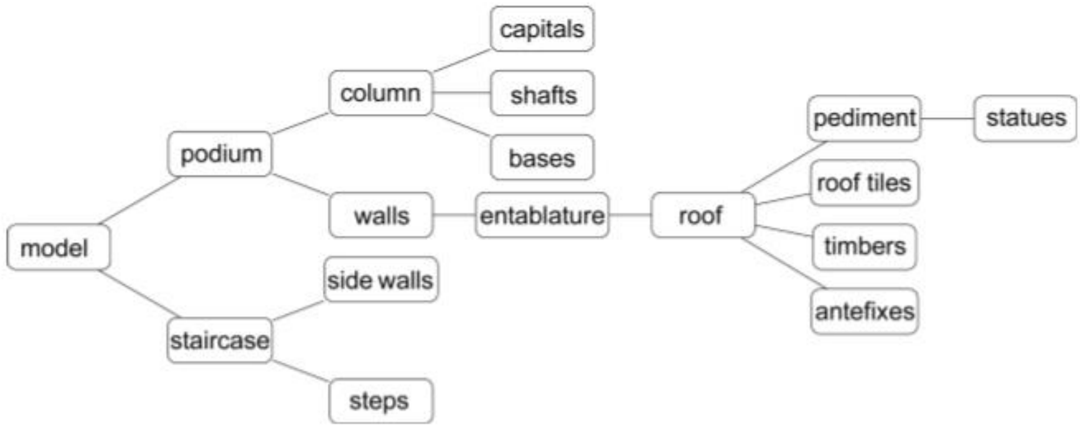

Figure 2.5: Process workflow of the reconstructed temple (Danielova et al., 2016).

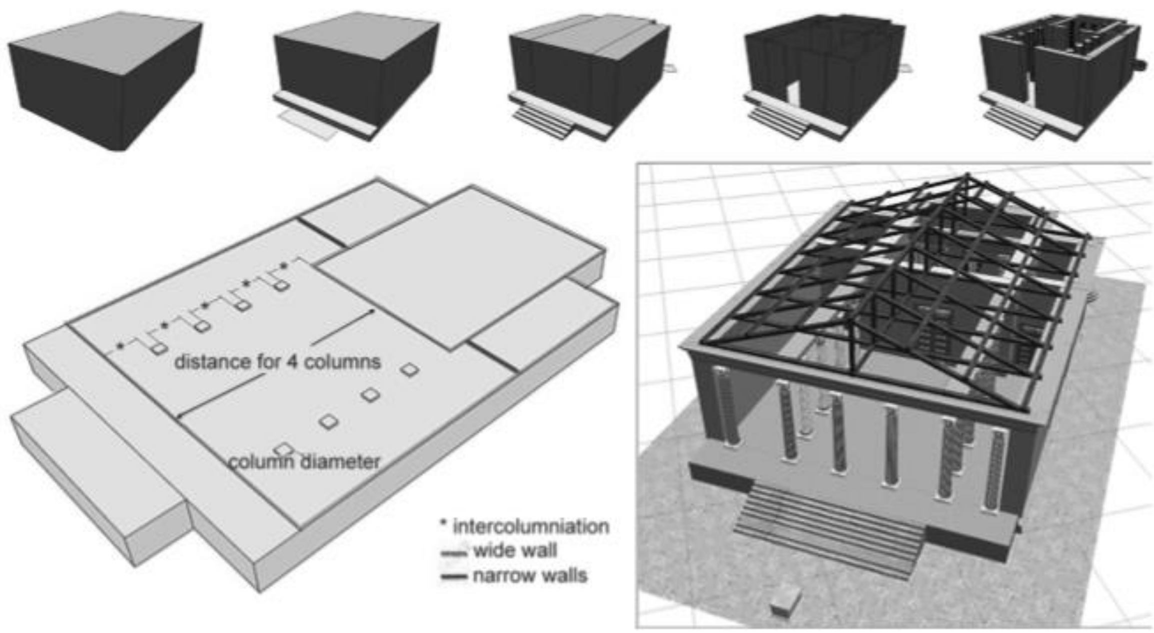

Figure 2.6: Top, evolving temple model; left, temple footprint; right, the solution of reconstruction (Danielova et al., 2016).

Figure 2.7: Left, LoD1; Center, LoD2; Right, LoD3 (Danielova et al., 2016).

The Figure 2.5 illustrates the shape workflow of the Temple of Diana reconstruction. It is clear that the first step of the workflow is the "model" which is the footprint of the temple and is split into two parts: Podium and Staircase. The process continues with the modification, resize and division of these parts leading to other temple parts such as Steps or Walls. Figure 2.6 represents how the temple was modeled. On the top of the figure five steps of the visualization are illustrated. On the left the footprint of the temple is represented, while on the right the result of modelling is shown. Figure 2.7 illustrates three LoDs were adjusted for the implementation of this work. Thus, LoD1 shows only colored visualization without textures. LoD2 includes all details, such as stadues, podium and others, with the same colors as LoD1. Finally, LoD3 represents a more enriched visualization, using many textures.

2.2.3 Quantifying Uncertainty using Fuzzy Logic

A new mathematical approach was developed based on fuzzy logic, in order to calculate the uncertainty in archaeological reconstructions. This approach is based on the methods of Niccolucci & Hermon, 2004, and Tepavcevi, B., & Stojakovic, V., 2013. Danielova et al., 2016 based on work of Niccolucci & Hermon, 2004, the process starts with the initial model M_0. Every step means an addition of a new detail m_n to the model. In this work the details are objects, such as arrows, glyphs, bars, dias, text labels, and buffers. In the initial model there is any detail and the added details for example podium and staircase as it is illustrated in the Figure 2.5. Furthermore, the reliability scale has values between 0 and 1, where the value 0 represents the most uncertainty and 1 the most reliable object, meaning an object for which there is certainty it was formed as visualized in the past, signifying the outmost confidence in the 3D reconstruction. The total object's reliability is derived into three components, which are based on the correction factors of Tepavcevi, B., & Stojakovic, V., 2013 and on the components of reliability of Niccolucci & Hermon, 2004.

The first component is this of reliability r(p), which described the uncertainty of the object's placement. The second component is this of dimension r(d), which is defined as the degree of reliability related to the knowledge of the object's dimension. The third component is this of style r(s), which evaluates the style of added object. Thereafter, the total reliability of an object is defined as the minimum value of these three reliability components.

$$r_n = \ min\ (r_n^{(p)}, r_n^{(d)}, r_n^{(s)})$$

Consequently, the formula of the total reliability for the entire model is the following:

$$r(M_{n+1}) = min\ (r_{n+1}^{(p)}, r_{n+1}^{(d)}, r_{n+1}^{(s)})$$

ID	Object	Variation	r	r(p)	r(d)	r(d)
0	remains	–	1	1	1	1
1	podium	–	1	1	1	1
2	steps	no sides	0.8	1	0.8	0.8
		with sides	0.5	1	0.8	0.5
3	wall	8.5 m (Diastyle)	0.8	1	0.8	0.9
		7.5 m (Eustyle)	0.6	1	0.6	0.9
...						
21	altar	–	0	0.2	0.1	0

Table 4: The values of reliability of the temple object (Danielova et al., 2016)

Table 4 presents 21 steps (ID) of model generation with their numerical evaluations of each reliability component. In every step an object is added, in order to generate the final visualization of the entire temple reconstruction in CityEngine. There is an important disadvantage in the presented uncertainty model in the work of Danielova et al., 2016. The drawback lies in the strong dependence between the resulted reliability and the order in which objects and details are added. To overcome this disadvantage, the added objects should not have random order. For this reason, details and object should be derived into two groups: decoration and construction elements. The decoration elements are added based on their reliability value where the most reliable object should be added sooner. More reliable is an object which was more important for the construction and probably existed, like the podium or roof. The construction objects are added based on their graduating temple reconstructionn, e.g. walls are before the roof is created and after the podium.

2.2.4 Different approaches of uncertainty visualization.

The visualization of uncertainty is an important part of this work, as the viewer should easily understand the uncertainty of every part of this reconstructed archaeological monument (Danielova et al., 2016).

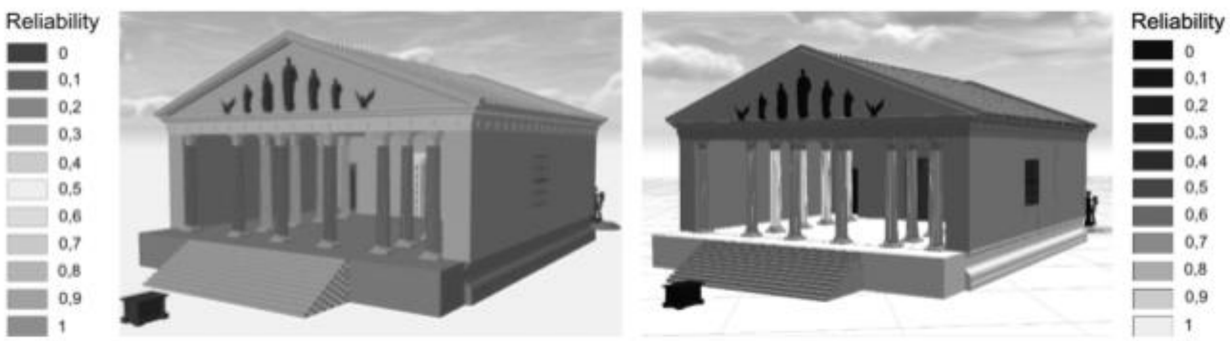

Figure 2.8: Left, red-green color visualization method; Right, one color gradient visualization method (blue) (Danielova et al., 2016).

Figure 2.9: Transparency visualization method (Danielova et al., 2016).

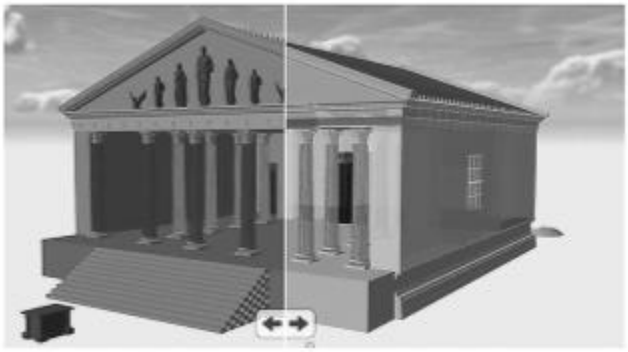

Figure 2.10: Swipe between color and transparency visualization methods (Danielova et al., 2016).

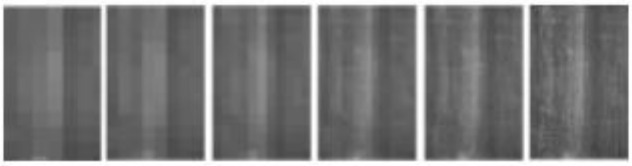

Figure 2.11: Different resolutions of wood textures (Danielova et al., 2016).

Figure 2.12: Uncertainty visualization using different resolution values. Left, pediment reliability is 0.7; Right, pediment reliability is 0 (Danielova et al., 2016).

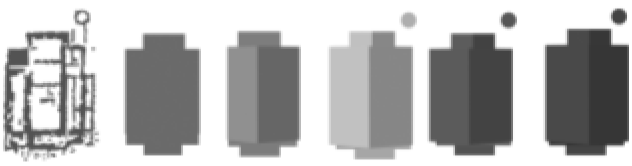

Figure 2.13: Total temple of Diana uncertainty after some steps of construction (Danielova et al., 2016).

In Figure 2.8 (left) is illustrated the visualization of the uncertainty of each reconstructed temple object, using a color gradient from red to green. The most certain is represented by green and the most uncertain reconstruction by red. The drawback of this method is that the color visualization eliminates the real colors and textures of the objects. In Figure 2.8 (right) is presented another method of uncertainty color visualization, using only different values of one color, instead of two.

In Figure 2.9 a transparency method of uncertainty visualization is illustrated. In this method, increased transparency leads to less certainty of a reconstructed object. The advantage of this method is that the object's realistic colors and textures have been preserved and uncertainty information is provided. The main drawback of this method is that objects with high uncertainty can be fully or nearly fully transparent, and thus invisible. Another disadvantage is that semi-transparency can confuse and lead to misinterpretation.

Figure 2.10 shows a swipe tool in order to compare the two visualization methods of color-gradient and transparency. This tool can reduce the disadvantages of these methods which is mentioned above. Furthermore, viewers can detect where objects and details are missing and thus at what point there is the greatest uncertainty, by comparing transparent and realistic layer.

Figure 2.11 illustrates a third visualization method based on the resolution of the object texture, where the higher resolution of an object, the more certain is its reconstruction.

Figure 2.12 shows the textures of the temple based on respective uncertainty values. It is important that the temple appearance data is not lost because the reliability value is encoded straight to the textures. Another way of visualization is the use of different shades of one color, which could lead easily to the comparison of different temple objects. The main disadvantage of using CityEngine software is that the textures lose their real appearance, because stretching. Therefore, the readability is reduced when this software is used or the viewing distance is changed.

Figure 2.13 presents a side-by-side illustrations, which can be a solution for comparing uncertainty of different models using color visualization. In this method, the colors represent the total reliability of the model.

Finally, uncertainty information is provided using interaction by creating clickable objects. The additional information about uncertainty values appearance is displayed when the user clicks on a detail of the reconstructed temple. It is important that a broad exploration of the reconstructed temple details and their uncertainty can be ensured when the interactivity is provided. Interaction can be pan, zoom and rotate, including the option to display or disable certain objects of interest. The proposed future work of Danielova's thesis is a comprehensive user test for the verification of the uncertainty suggestions and assumptions.

2.3 Contribution of this Thesis

In this thesis, it was considered significant, in collaboration with a respected Minoan Archaeology expert (Dr Anna Simandiraki-Grimshaw) to study the type of historical evidence that form archaeologists' opinion about how an archaeological site, today in ruins, used to exist in the past. We adopted how Sifnioti et al., 2012 studied these factors, discussing with archaeologists and creating a questionnaire in order to identify the relative importance among different types of evidence which formed their confidence about a reconstruction option, as we described in section 2.1. However, in our case we created an electronic questionnaire embedded in our system. From discussions with archeologists a number of influencing factors appeared. These influencing factors include: Features; elements made by humans such as ditches or wall remains. Artefacts; objects made or modified by humans such as glass or pottery. Biofacts or ecofacts; humans, plants or animals. Textual evidence; ancient documents or texts. Absolute comparison; comparing structures of similar proportions. Contextual comparison; compering structures with similar contexts. Topography; natural landscape features such as area elevation or general data for the form of land. Peer Review; discussions among archaeologists and architecture specialists.

Archaeologists in our system give answer to these questions giving values among Never, Almost Never, Seldom, Sometimes, Often, Very Often, Always which are values of a dropdown. However, both works didn't use a mathematical model which could meet our needs. A significant need in visualization of archaeological uncertainty is the updating of this uncertainty based on the latest evidence uncovered in excavations. For example, if an archaeologist has new findings, he should be able to introduce them as a new input in our system. Therefore, the value of uncertainty for every reconstruction will change based on new evidence. Moreover, there was need to visualize more than two reconstruction options because in many excavations leads to more than two hypotheses for a place of an archaeological site. In our thesis we wanted a mathematical model which will give us updated information, as updated information we mean the new input of archaeologists which is based in new evidence, which would change the current uncertainty of every point in the Palace, according to the archaeologists' input. The input of archaeologists is the answers to the questionnaire which is described in the previous paragraph. In this thesis, we wanted to create a system where archaeologists could update the uncertainty. Archaeologists can update the uncertainty inserting their belief again to our system, in case they have new evidence and information. In order to achieve the goal of updating data and information, we used the Dirichlet Distribution.

The Dirichlet Distribution meets the needs described above. First of all, this mathematical model can handle more than two options for reconstructions of the same spatial spot. In this thesis, we visualize three reconstructed options for each spatial location of historical interest in the Palace of Zakros. Also, archaeologists can update the distribution giving new input in relation to the significance of evidence types, new unearthed or their updated belief in relation to their perceived accuracy of the presented 3D reconstructions through the system. Finally, in our work, the probability distribution used does not give 0 values in any of the reconstructed options. After archaeologists communicate their belief, the distribution changes and the preferable, more probable reconstructed option is visualized.

In Sifniotis, 2012 work, ther elative significance of evidence types or influencing factors, although identified, they were not taken into account. The relative significance of evidence types for archaeological reasoning did not take part in uncertainty calculations. In this thesis, the belief of archeologists in relation to whether a reconstructed option of a spatial location is probable, is formed differently depending on evidence types' weight which is derived from archeologists' continuous input into the system. In Sifniotis, 2012, this wasn't embedded in the system as in our work. In Sifniotis, 2012 work, multiple uncertainty levels were not visualized related to a specific part of the building. In our work, archeologists can view multiple reconstructed options for the same spatial location of interest and the associated visualization of uncertainty using colour visualization, from green to red, depending on high to low probability that a reconstructed option is probable. This is showcased for three specific locations of the Palace of Zakros in Crete, Greece. If a reconstruction option is visualized with redder color archaeologists believe that is less possible to have existed in this form in the past. However, reconstructions visualized with greener color are more probable, based on archeologists' input.

3 Theoretical Background

In the following chapters we provide the theoretical background required for this thesis. We discuss uncertainty, Probabilistic Reasoning, Bayes' Theorem and Dirichlet Distribution.

3.1 What is uncertainty?

Uncertainty is everywhere, not only in the philosophical sense but also in everyday life. Uncertainty has an important influence on decisions we make and for this reason is a key dimension of our everyday life (Yoshida and Ishii, 2006). It also is connected with psychopathological disorders and emotions, as it is mentioned by Holaway et al., 2006 and Boelen and Reijntjes, 2009. For a long time, uncertainty has huge scientific interest (Bertelson and Boons, 1960) and especially the last years the investigation of uncertainty has been increased.

In the natural sciences a wide range of defined, approved and standardized means for complex calculations and meaningful representation of uncertainties are utilized on a regular basis (e.g. standard deviation, error bars, confidence interval, color coding, or glyphs).

Most likely, every research field is affected or involved with uncertainty. As a consequence, a considerable amount has been published over the past decades, indicating the effort to somehow get a trip on the intangible phenomenon of uncertainty. The correlated terms thereby differ as widely as the related research fields and it seems impossible to find a generally valid definition. Despite this variety and diversity, these approaches share at least one general insight: uncertainty is widely understood as a phenomenon associated with the terms error, accuracy and imperfection.

In general, uncertainty is defined as a realization that our representations of the world cannot predict the future events in our environment. However, in fields like behavioral and cognitive sciences, uncertainty has been defined as the difficulty to predict events that are the consequences of what we do (Volz et al., 2003; Hsu et al., 2005; Huettel et al., 2005; Yu and Dayan, 2005).

3.2 Archaeology

Archaeology is the study of human cultures through the excavation, collection and interpretation of cultural and environmental remains (Renfrew et al., 1991). The width and breadth of archaeological study is wide, encompassing areas such as cultural history, human evolution, cultural and individual behaviour and ecology.

The most common conception of the first archaeologists brings to mind antiquarians of the 18^{th}-19^{th} century. In reality, archaeology has existed for thousands of years. However, until 19^{th} century, archaeology was mostry expressed as antiquarianism and the hoarding of strange and ancient objects for personal collections. One of the most technological developments for archaeology in the 20^{th} century is considered to be radiocarbon dating (Libby, 1949). It brought a revolution in archaeological dating, allowing archaeologists to reasonably date organic material and reassess past discoveries. (Sifniotis et al., 2012)

3.3 Uncertainty and archaeological reconstructions

If one looks at the history, along with advances in the world of IT, the archaeological community has been exploring the benefits of 3D modelling since the 1980s. In 1990, P. Reilly first proposed the concept of virtual archaeology, referring to 3D models of archaeological formation. The word virtual was to represent a replica, a surrogate that would efficiently describe the original formation. Reconstructions were made in effect to describe the original artefact as closely as possible, by putting more and more effort in realistic representations and details (Reilly, 1990).

For a long time, archaeologists have been criticizing the too-realistic appearance of three-dimensional reconstructions of archaeological sites. Most of the times, they highlight one of the unique features of archaeology: the information we have on our heritage, about an archaeological site, will always be incomplete. All attempts to visualize the reconstructions should be based on this archaeological feature, incompleteness. This view of archaeologists leads to this work. This thesis described how the area of archaeology and the area of computer graphics have formatted a useful and tumultuous relationship through the years. An archaeologist's level of confidence in an interpretation deriving from gathered evidence is defined as uncertainty. In general, archaeologists and computer scientists recommend caution in three-dimensional archaeological reconstructions, as some other possible hypotheses may not be acknowledged. This thesis presents a 3D visualization system of archaeological uncertainty, based on Bayesian priors (Chalkiadakis & Boutilier, 2003).

The last developments in computer graphics provide strong tools for modelling aspects with many dimensions. Computer graphics techniques can be used for the reconstruction and visualization of a site which can be difficult to appreciate. In case we want to avoid misleading impressions of a site, we need to be able to evaluate the realism of the reconstructions that have occurred (Chalmers, A., 2002).

3.4 Quantifying Uncertainty

Until well into the late 19th century, scientific uncertainty was completely undesirable. The development of statistical methods and its acceptance as a valid scientific tool, had two major consequences. Uncertainty was starting to be something quite normal and expected for scientific enquiry, and no longer negative. Moreover, because of the direct relationship between probability and statistics, probability was the only way of dealing with uncertainty (Sifniotis et al., 2012; Thomson, W. 1889).

In this work, Dirichlet distribution was used for the treatment of uncertainty in archaeological reconstruction. As we described above, the field of archaeology is connected with uncertainty, as archaeologist cannot be sure about how every archaeologist site was in the past. In this work, we created a system based on Dirichlet distribution which accepts the knowledge of archaeologists as an input, we implemented GUIs in order to manipulate the α hyperparameter and we had some random variables from Dirichlet distribution as the output of our system. In the next subsections, some important definitions for Dirichlet Distribution and the creation of our algorithm are analyzed.

3.4.1 Probabilistic Reasoning in Cognitive Sciences

In general, in Cognitive sciences like AI and Machine learning, there is always the need of uncertainty reasoning theory as these sciences try to understand human intelligence which is about using uncertain information to make reasonable inferences.

Therefore, knowledge can be represented through probabilistic reasoning, where probability is the applied concept for indicating the uncertainty in knowledge. Generally, probabilistic reasoning is used in Cognitive Sciences when the predicates are uncertain.

Because of the relationship between probabilistic reasoning and probability, the definition of probability is important. Therefore, probability is defined as a chance that an uncertain event can occur. Probability can take values greater or equal to zero and less or equal to 1.

$$0 \leq P(A) \leq 1, where\ P(A)\ the\ probability\ of\ an\ event\ A$$

$$P(A) = 0, total\ uncertainty\ in\ an\ event\ A$$

$$P(A) = 1, total\ certainty\ in\ an\ event\ A$$

The formula of calculating the probability of an uncertain event is:

$$Probability\ of\ occurrence = \frac{Number\ of\ desired\ outcomes}{Total\ number\ of\ outcomes}$$

Some important terms which are related with probabilities are evert, sample space, random variables, prior probability, posterior probability and conditional probability. Each possible outcome of a variable is defined as event. Sample space is the collection of all impossible events. In real world, the events and objects are represented as random variables. An important definition for probabilistic reasoning is that of prior probability which is defined as the computed probability before new observations and information, while posterior is the probability that is calculated after the observed new information. In other words, posterior is the combination of prior probability and new information. Conditional probability is the probability of occurring an event when another event has already happened.

When an event B has occurred and it is necessary to calculate the event A, the formula is the following:

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

$$P(A \wedge B) = joint\ probability\ of\ A\ and\ B,$$

$$P(B) = marginal\ probability\ of\ B$$

If the probability A is known and the requirement is the calculation of probability B, then we need the following formula:

$$P(B|A) = \frac{P(A \wedge B)}{P(A)}$$

Taking a look at literature, one can find a large variety of algorithms in which agents use probabilistic reasoning to decide migration strategies under terrain-related uncertainty (specifically, they estimate their policies via approximating the solution to a Markov Decision Process describing their problem. Firstly, Duff, 2002, developed computational procedures that retain the Bayesian formulation, but sidestep intractability by utilizing techniques from reinforcement theory.

Dearden et al., 1999, presented the investigation of ways to represent and reason about agents' uncertainty in algorithms where the system attempts to learn a model of its environment. More specifically, they gave the definition of reinforcement learning which try to deal with the problem of the way that agent should learn to act in dynamic environments. The framework underlying much of reinforcement learning is that of Markov Decision Processes (MDPs) Therefore, in this paper the authors show how to maintain Bayesian belief states about MDPs and it was discussed how to use Bayesian belief state to choose actions in a way that balances exploration and exploitation.

Chliaoutakis & Chalkiadakis, 2016 presented the development of a generic agent-based model (ABM) for simulating ancient societies. This model includes agents that are autonomous and utility-based. The agent-based modeling has been used in archeology because it has the ability to reflect properties of the real work.

Moreover, Chliaoutakis & Chalkiadakis, 2017, extended a generic-based model for simulating ancient societies, by blending evolutionary game theory with multiagent systems' self-organization. The model, which was presented by the authors, can provide intuitions to archaeological research, and help resolve open questions regarding the socio-economic dynamics at work in past societies.

Chalkiadaki & Boutilier, 2003, proposed a Bayesian model for optimal exploration in MARL problems that allows the exploration costs to be weighed against their expected benefits using the notion of value of information. The authors developed tractable approximations to optimal Bayesian exploration, and report on experiments illustrating the benefits of this approach in identical interest games.

In this thesis, the knowledge of archaeologists can be represented through probabilistic reasoning, where probability is the applied concept for indicating the uncertainty in this knowledge.

3.4.2 Bayes' Theorem

There are many names for Bayes' theorem such as Bayes' Rule, Bayes' Reasoning. Bayes' theorem determines the probability of an event with uncertain knowledge. It is a theory of probability condition that takes into account the probability of a case depending on others (Berger ,2013; Bernardo & Smith, 2009). Basically, the theorem states that future cases can be predicted with the requisite of previous cases which have occurred. Bayesian Analysis digs two sources of information about the parameters of a statistical model – One source comes from a sample, called sample information, while another is derived from expert's opinion (researcher), called prior information. The combination of both information sources forms posterior information (posterior distribution). Sample information is extracted from a likelihood function (Bolstad & Curran; 2016). Bayes' theorem allows two sources of information on the parameter of statistical models to be combined into posterior information (posterior distribution) (Berger ,2013; Bernardo & Smith,2009; Van de Schoot et al., 2014).

Bayes' theorem deals with posterior probability, a conditional probability of H given E, where actually H actually occurs first. In other words, given that E occurred, what is the probability that it happened through H_j? The following formula is the Bayes Theorem:

$$\Pr(H_j|E) = \frac{\Pr(H_j)\Pr(E|H_j)}{\Pr(H_1)\,Pr(E|H_1) + \cdots + \Pr(H_n)\Pr(E|H_n)}$$

The concept behind Bayesian reasoning is the use of a prior condition in order to calculate a posterior-termed as a priori and posteriori. Bayes' Theorem is used to calculate the probability that a hypothesis is true based on the available evidence.

3.4.3 Dirichlet and Sparse-Multinomial Priors

In most cases, Dirichlet distributions (Dearden et al., 1999) are used as distribution in Bayesian statistics. The Dirichlet Distribution is a generalized Beta distribution and each case of this distribution is a vector with elements which can take values greater or equal to 0 and lower or equal to 1. Moreover, Dirichlet distribution is a conjugate prior distribution for multinomial event (Cabamlit et al. 2004).

A well-known definition for Dirichlet distribution is "a measure of measures". A probability measure over the k-simplex is defined by Dirichlet distribution. Each point on surface of k-simplex describes a probability distribution.

A variable can be represented as the difference between the constant and the sum of the other variables. For this reason, a degree of freedom is lost and Dirichlet distribution has k-1 degrees of freedom. This means that when we have k variables, k-1 dimensional simplex is using.

Let's assume that X is a random variable which can take L possible values from a set Σ and $\Sigma = \{1,..,L\}$. We are given a training set *D* that contains the outcomes of *N* independent draws $x^1, \ldots, x^N$ of *X* from an unknown multinomial distribution *P**. The multinomial estimation problem is to find a good approximation for *P**.

This problem can be stated as the problem of predicting the outcome x^{N+1} given $x^1, \ldots, x^N$. Given a prior distribution over the possible multinomial distributions, the Bayesian estimate is:

$$P(x^1, \ldots, x^N, \xi)$$

$$= \int \quad P(\theta, \xi) P(x^1, \ldots, x^N, \xi) d\theta$$

Where θ=$\langle\theta_1, \ldots, \theta_L\rangle$ is a vector that describes the possible values of the (unknown) probabilities *P*(1), …, P*(L),* and ξ is the "context" variable that denote all other assumptions about the domain.

The posterior probability of θ can be rewritten as:

$$P(x^1, \ldots, x^N, \xi) \propto P(\theta, \xi) P(\xi) = P(\xi) \prod_i \quad \theta_\iota^{N_i} \qquad (4)$$

Where N_ι is the number of occurrences of the symbol 'in the training data.

Dirichlet distributions are a parametric family that is conjugate to the multinomial distribution. That is, if the prior distribution is from this family, so is the posterior. A Dirichlet prior for X is specified by hyper-parameters $a_1, \ldots, a_L$, and has the form:

$$P(\xi) \propto \prod_i \quad \theta_i^{a_i - 1} \left(\sum_i \quad \theta_\iota = 1 \; and \; \theta_i \geq 0 \; for \; all \; i \right)$$

where the proportion depends on a normalizing constant that ensures that this is a legal density function (i.e., integral of P(ϑ|ξ) over all parameter values is 1). Given a Dirichlet prior, the initial prediction for each value of *X* is

$$P(\xi) = \int \quad \theta_i P(\xi) d\theta = \frac{\alpha_\iota}{\sum_j \quad a_j}$$

It is easy to see that, if the prior is a Dirichlet prior with hyper-parameters $a_1, \ldots, a_L$, then the posterior is a Dirichlet with hyper-parameters $a_1+N_1, \ldots, a_L+N_L$. Thus, we get that the prediction for X^{N+1} is

$$P(x^1,\dots,x^N,\xi) = \frac{a_j + N_j}{\sum_{j(a_j+N_j)}}$$

In some situations, we would like to sample a vector θ according to the distribution P($\vartheta|\xi$). This can be done using a simple procedure: Sample values $y_1,\dots,y_L$ such that each $y_i \sim Gamma(a_i,1)$and then normalize to get a probability distribution, where $Gamma(\alpha,\beta)$is the Gamma distribution. Procedures for sampling from these distributions can be found in (Ripley 1987).

Friedman and Singer (1999) introduce a structured prior that captures our uncertainty about the set of "feasible" values of X. Define a random variable V that takes values from the set 2^{Σ} of possible subsets of Σ. The intended semantics for this variable, is that if we know the value of V, then $\theta_i > 0\ iff\ i \in V$.

Clearly, the hypothesis $V = \Sigma'(for\ \Sigma' \in \Sigma)$is consistent with training data only if Σ' contains all the indices i for which$N_i > 0$. We denote by $\Sigma°$ the set of observed symbols. That is, $° = \{i: N_i > 0\}$, and we let $k° = |\Sigma°|$. Suppose we know the value of V. Given this assumption, we can define a Dirichlet prior over possible multinomial distributions θ if we use the same hyper-parameter α for each symbol in V. Formally, we define the prior:

$$P(V) \propto \prod_{i\in V} \quad \theta_i^{a-1}\big(\sum_i \quad \theta_i = 1\ and \theta_i = 0 for\ all\ i \notin V\big) \quad (5)$$

Using Eq. (4), we have that:

$$P(X\text{^}(N+1) = i|x\text{^}1,\dots,x\text{^}N,V) = \{(a+N)/(|V|a+N),\ if\ i \in V\ 0,\ otherwise \qquad (6)$$

Now consider the case where we are uncertain about the actual set of feasible outcomes. We construct a two-tiered prior over the values of V. We start with a prior over the size of V, and assume that all sets of the same cardinality have the same prior probability. We let the random variable S denote the cardinality of V. We assume that we are given a distribution $P(S = k)$ for $k = 1, \dots , L$. We define the prior over sets to be $P(S = k) = \binom{L}{k}^{-1}$. This prior is a sparse-multinomial with parameters a and $P(S = k)$.

Friedman and Singer show that how we can efficiently predict using this prior.

Theorem A.1: (Friedman & Singer 1999) Given a sparse-multinomial prior, the probability of the next symbol is

$$P(X\text{^}(N+1) = i|D) = \{(a+N)/(k°a+N)\ C(D,L), \qquad if\ i \in \Sigma°\ \ 1/(n-k°)(1-C(D,L)), \qquad if\ i \notin \Sigma°$$

Where

$$C(D,L) = \frac{\sum_{k=k°}^{L} \quad (k°a+N)}{ka+N} P(D).$$

Moreover,

$$P(D) = \frac{m_k}{\sum_{k'\geq k°} \quad m_k},$$

Where

$$m_k = \frac{P(S=k)k!}{(k-k°)!}\frac{\Gamma(ka)}{\Gamma(ka+N)}$$

And $\Gamma(x) = \int_0^\infty \quad t^{x-1}e^{-t}dt$ is the gamma function. Thus,

$$C(D,L) = \frac{\frac{\sum_{k=k°}^{L} \quad (k°a+N)}{ka+N} m_k}{\sum_{k'\geq k°} \quad m_k}$$

We can think of $C(D,L)$ as scaling factor that we apply to the Dirichlet prediction that assumes that we have seen all of the feasible symbols. The quantity 1- $C(D,L)$ is the probability mass assigned to novel (i.e., unseen) outcomes.

In some of the methods discussed above we need to sample a parameter vector from a sparse-multinomial prior. Probable parameter vectors according to such a prior are sparse, i.e., contain few non-zero entries. The choice of the non-zero entries among the outcomes that were not observed is done with uniform probability. This presents a complication since each sample will depend on some unobserved states. To “smooth” this behavior we sample from the distribution over V° combined with the novel event. We sample a value of *k* from $P(D)$. We then sample from the Dirichlet distribution of dimension *k* where the first k° elements are assigned hyper-parameter *a*. The sampled vector of probabilities describes the probability of outcomes in V° and additional *k*- *k*°. We combine these latter probabilities to be the probability of the novel event.

In this thesis, Dirichlet distribution was used for the treatment of uncertainty. This distribution gave us some positive aspects. Firstly, Dirichlet distribution solves the problem in visualization of archaeological uncertainty, which was the updating of this uncertainty based on the latest evidence uncovered in excavations. Moreover, through the mathematical model based on Dirichlet distribution, the goal of visualizing more than two reconstruction options for every point of interest can be achieved.

3.4.4 Hyperparameters in machine learning and alpha parameter

Machine learning systems abound with hyperparameters. These can be parameters that control model complexity or parameters that specify the learning procedure itself – step sizes, momentum decay parameters and initialization conditions.

In a machine learning model (ML model), hyperparameter is a configuration out of model and its value cannot be estimated from data. For this reason, hyperparameters are specified by practitioner. Choosing the best hyperparameters is both crucial and frustratingly difficult. However, as it is mentioned from Bengio, 2000 the value of a hyperparameter which is the best for a problem, is never known.

Models, built through the machine learning methods, capture elements of interest of based on given data. The determination of a set of hyperparameters is necessary before training commences, when common learning algorithms are used. The resulting model’s performance can be affected for the values of hyperparameters. However, this determination is not easy, it is quite complex (Claesen & De Moor, 2015).

Alpha parameter is the single parameter of Dirichlet distribution. Dirichlet distribution and its concentration are determined by this parameter. When higher alpha is chosen, the result is a denser distribution. However, when a lower value for alpha hyperparameter is chosen, a denser one results. When this hyperparameter is a vector, both distribution and concentration of Dirichlet are determined.

In this thesis, the definition of hyperparameters is significant definition, as it is essential for Dirichlet distribution Dir(α). This parameter α governs the shapes of the distribution. In our case study, dense distribution means that the most of archaeologists have a similar opinion among themselves and therefore gave similar values on the sliders of the User Interface. If the distribution is sparse, means that there were many different opinions about the three possible reconstruction of a point of interest. In this work, we tried to decide the appropriate value of alpha hyperparameter, in order to continue with the implementation of our system.

3.4.5 Probability Density Function (PDF) Definition

The Dirichlet distribution of order $K \geq 2$ with parameters $\alpha_1, \dots, \alpha_k > 0$ has a PDF (Probability Density Function) with respect to Lebesque measure on the Euclidean space R^{k-1}. The formula is the following:

$$f(x_1, \dots, x_K; \alpha_1, \dots, a_K) = \frac{1}{B(\alpha)} \prod_{i=1}^{K} \quad x_i^{a_i - 1}$$

Where $\{x_k\}_{k=1}^{k=K}$ belong to the standard K-1 simplex, or in other words:

$$\sum_{i=1}^{K} \quad x_i = 1 \; and \; x_i \geq 0 \; for \; all \; i \in [1, K]$$

In this thesis, the probability density function visualized, in order to give to archaeologists, the knowledge about how the Dirichlet distribution changes after others archaeologists’ input. They can have the knowledge for which combination of uncertainty other archaeologists are certain.

3.4.6 Samples from Dirichlet Distribution

Firstly, we provide the definition of the simplex, so as to proceed to the explanation of sampling from Dirichlet Distribution. In geometry, simplex can be defined as a generalization of the notion of a polygon. Specifically, a k-simplex is a k -dimensional polytope which is the convex hull of its k+1 vertex. More formally, suppose the k+1 point $u_0, \dots, u_k \in R^k$ are affinely independent, which means $u_1 - u_0, \dots, u_k - u_0$ are linearly independent. The following formula of C, gives the set of points that determine the simplex.

$$C = \left\{ \theta_0 u_0 + \cdots + \theta_\kappa u_k \middle| \sum_{i=0}^{k} \theta_i = 1 \; and \; \theta_i \geq 0 \; for \; all \; i \right\}$$

In Figure 3.1 are illustrated some kinds of simplex, such as 1-simplex which is a line, 2-simplex which is a triangle etc. A regular n-simplex can be constructed by connecting a new vertex to all original vertices.

The standard simplex or probability simplex is the simplex formed from the k+1 standard unit vector, or

$$\{x \in R^{k+1} \colon x_0 + \cdots + x_k = 1, x_i \geq 0, i = 0, \dots, k\}.$$

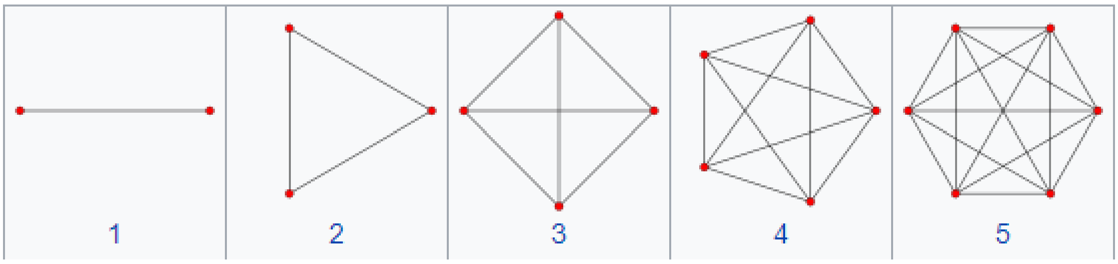

Figure 3.1: (1) 1-simplex, (2) 2-simplex, (3) 3-simplex, (4) 4-simplex, (5) 5-simplex.

The standard n-simplex is the subset of R^{n+1} given by

$$\Delta\text{^}n = \{(t\_0, \dots, t\_n) \in R\text{^}(n+1) \mid \textstyle\sum\_(i=0)\text{^}n \, t\_i = 1 \; and \; t\_i \geq 0 \; for \; all \; i\}.$$

The simplex Δ^n lies in the affine hyperplane obtained by removing the restriction $t\_i \geq 0$ in the above definition.

The n+1 vertices of the standard n-simplex are the points $e_i \in R^{n+1}$, where

$e_0 = (1,0,0, \dots, 0),$

$e_1 = (1,0,0, \dots, 0),$

$\vdots$

$e_0 = (1,0,0 \dots, 0).$

In probability theory, the points of the standard n-simplex in (n+1)-space are the space of possible parameters (probabilities) of the categorical distribution on n+1 possible outcome. The distribution is completely given by the probabilities associated with each number i: $p_i = P(X = i), i = 1, \dots, k$,where $\sum_i \quad p_i = 1$. The possible sets of the probabilities are exactly those in the standard (k-1)-dimensional simplex; for k = 2 this reduces to the possible probabilities of the Bernoulli distribution being the 1-simplex, $p_1 + p_2$ =1, 0 ≤ p_1, p_2≤ 1. In Figure 4.3 are shown the possible categorical distribution with k=3 which are the 2-simplex $p_1 + p_2 + p_3$ =1, embedded in 3-space.

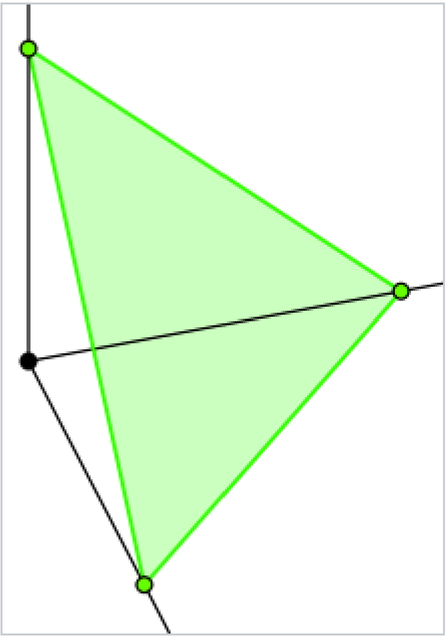

Figure 3.2: The 2-simplex embedded in 3-space.

In our work, there were three different possible reconstructions for every point of interest. For this reason, the 2-simplex was used and each vertex of the triangle corresponds to each possible reconstruction version.

In order to take samples from a Dirichlet Distribution it is necessary to understand the way with which random values from this kind of distribution are generated. Therefore, we describe the process of generating samples, random values, from Dirichlet Distribution. In order to achieve this generation, we followed the next steps.

With a source of Gamma-distributed random variants, one can easily sample a random vector $x = (x_1, \ldots, x_K)$ from the K-dimensional Dirichlet Distribution with parameters $(a_1, \ldots, a_K)$. First, draw K independent random samples $(y_1, \ldots, y_K)$ from Gamma distributions each with density

$$Gamma(a_i, 1) = \frac{y_i^{a_i - 1} e^{-y_i}}{\Gamma(\alpha_i)},$$

And then set

$$x_i = \frac{y_i}{\sum_{j=1}^{K} y_j}.$$

We continue with the description of Gamma Distribution and the generating random values from this. Gamma random variate has a number of applications. One of the most important applications is to generate Dirichlet distributed random vectors. As it is mentioned above, in order to sample from a Dirichlet Distribution we need a source of Gamma-distributed random variates. We used the Marsaglia's simple transformation -rejection algorithm to generate random values from Gamma Distribution.

Marsaglia's simple transformation-rejection method relying on a one normal and one uniform random number:

1. Set up: $d = a - \frac{1}{3}$ and $c = 1/\sqrt{9d}$.
2. Generate: $= (1 + c * x)^3$, with x standard normal.
3. If $v > 0$ and $log\,log\ \ log\,log\,(UNI) \ < 0.5x^2 + d - dv + dlog(v)$ return dv.
4. Else go back to step 2.

With $1 \leq a = k$ generates a gamma distributed random number in time that is approximately constant with *k*. The acceptance rate does depend on *k*, with an acceptance rate of 0.95, 0.98, and 0.99 for k=1, 2, and 4. For *k* < 1, one can use k<1, one can use $\gamma_\alpha = \gamma_{1+\alpha} U^{\frac{1}{a}} k < k <$ to boost *k* to be usable with this method.

For the application of Marsaglia's simple transformation -rejection algorithm it was necessary the generation of values from normal distribution, as it is evident from the second step of this algorithm (Generate: $= (1 + c * x)^3$, with x standard normal). In this subsection are described the algorithms in order to achieve the generation of these values.

It is desirable in Monte-Carlo method used in application, to generate normally distributed values. The following algorithms generate the standard normal deviates, as a $N(\mu, \sigma_z)$ can be generated as $X = \mu + \sigma_z$, where Z is standard normal. These algorithms are related to the random number generator U, which can produce uniform random variables.

- The first method, Box-Muller method, uses the U and V as independent random numbers, which are distributed uniformly on (0,1). The values of these two variables are given from the following formulas

$$X = \sqrt{-2lnU}\cos(2\pi V) \ ,$$
$$Y = \sqrt{-2lnU}\ sin\ (2\pi V)$$

Both are independent and have normal distribution. This formulation arises because a bivariate normal random vector (X,Y) the squared norm $X^2 + Y^2$ will have the chi-squared distribution with two degrees of freedom, which is an easily generated exponential random variable corresponding to the quantity -2ln(U) in these equations; and the angle is distributed uniformly around the circle, chosen by the random variable *V*.

- The other method is named Marsaglia polar method, which is a modification of the Box-Muller method algorithm, which is necessary for functions sin () and cos () to be computed. The uniform (-1,1) distribution draw U and V, and then $S = U^2 + V^2$is computed. In case S is greater or equal to one then the method starts over. In other case two quantities

$$X = U\sqrt{\frac{-2lnS}{S}}\ , Y = V\sqrt{\frac{-2lnS}{S}}.$$

are returned. Same as in the previous method, X and Y are normally distributed and independent.

In this thesis, the samples from Dirichlet distribution are the output of our system. These samples are points on the 2-simplex which has the form of the triangle. These samples can give to archaeologists a clear picture of what the general opinion of archaeologists is and what is the degree of uncertainty of each version of a point. We sampled from a random vector $x = (x_1, x_2, x_3)$ from the 3-Dimensional Dirichlet Distribution with parameters (a_1, a_2, a_3). As mentioned earlier, we are talking about three-dimensional distribution as we have three versions for each point, we want to create its reconstruction. In our system archaeologists see the random values as points in a triangle and hovering them they can see multiple visualization of uncertainty for a place in the Palace.

4 Utilized software

In this chapter, the software and programming API used for the implementation of the uncertainty visualization system, are presented in detail. In the first section, the Unity development platform for interactive 3D graphics is presented. In the second section of this chapter, the modeling software SketchUP employed in this thesis, is described. Finally, the last section includes the description of the database used to store the necessary data.

4.1 Unity Game Engine

Unity development framework for interactive 3D graphics has been selected for the visualization system's software implementation. There are many online tutorials and guides, for creating 2D graphics for user interfaces, using the programming language C# and buying easily objects from Unity's Asset Store, without having to create anything from scratch. Moreover, there is thriving and supportive community, which can answer to your questions and help anytime and last but not least it is a free platform. This environment has the following main components: Hierarchy, Project, Inspector, Console, Scene, Game, Scripting mechanisms

4.1.1 Hierarchy

The Hierarchy window by default is the window for opening a new project in the platform of Unity. This window contains a number of every GameObject in Scene. Some of the GameObjers or objects are custom objects which build most of the game. When an object is added in the Scene, it will appear in the Hierarchy, as well as when an object is removed, it will disappear from the Hierarchy window.

4.1.2 Project

The project window is the part of Unity where user can manage all the assets. This window contains the folders of the project as a hierarchical list, as defined in section 4.1.1. This list is organized in a panel to the right part of this window and the users clicks to expand and collapse the folder.

In the right-hand panel of the project window, the individual assets are shown as icons, which indicate assets' type, such as material or script, and can be resized, or be replaced by a hierarchical list. Moreover, users can keep their favorite – frequently used- items as a project structure list. Finally, a "breadcrumb trail" panel exists that shows the currently viewed folder path.

4.1.3 Inspector

Gameobject consist a project in the Unity platform. These Gameobjects can contain sounds, graphical elements or scripts. When a user selects a Gameobject, he can see detailed information about it, in the Inpector window. This information includes the components, materials and properties of the currently selected GameObject.

4.1.4 Components and GameObjects

As it is mentioned above, GameObjects or "objects" are various objects which represent props, characters or scenery. They act as containers for Components, including the implementation of the real functionality. The Transform Component defines the rotation, position, scale of a GameObject and the Scene View.

The main graphics primitive of Unity is the 3D Meshes and there are many components for rendering regular meshes or 3D lines etc. Unity does not include modelling tool, but it has interaction with 3D modelling packages. A Skinned Mesh Renderer or a Mesh Filter is created automatically by Unity, when mesh assets are imported. In case Mesh renderer is not present, the Mesh will continue to exist in the scene but it will not be drawn. The Mesh Filter gives the geometry of Mesh Renderer and it is rendered at the position defined by the Transform component.

The main component which enables physic behaviors is called RigidBody. It helps objects to respond to gravity. Another component is Collider which defines the shape of an object, something important for the report of physical collisions. This component must be the same shape as the object's mesh. Primitive collider is the most straightforward collider, such as box or sphere collider. Many of them can be added to an object, not only one. Joint component is used in order to attach a rigidbody to another. This helps to allow a freedom of motion.

Character Controller is used for the control and movement of a character. It is a really important controller for first- or third- person games, as collision-based physics are needed. Moreover, character controller gives to the player's collider the shape of capsule. In this case, RigidBody is not needed and there are not realistic effects.

In Figure 4.1 is illustrated a cubemap which is a collection of six squares textures. In general, cubemap can be used for capturing of reflections or the surroundings of an object, such as environment reflections or skyboxes.

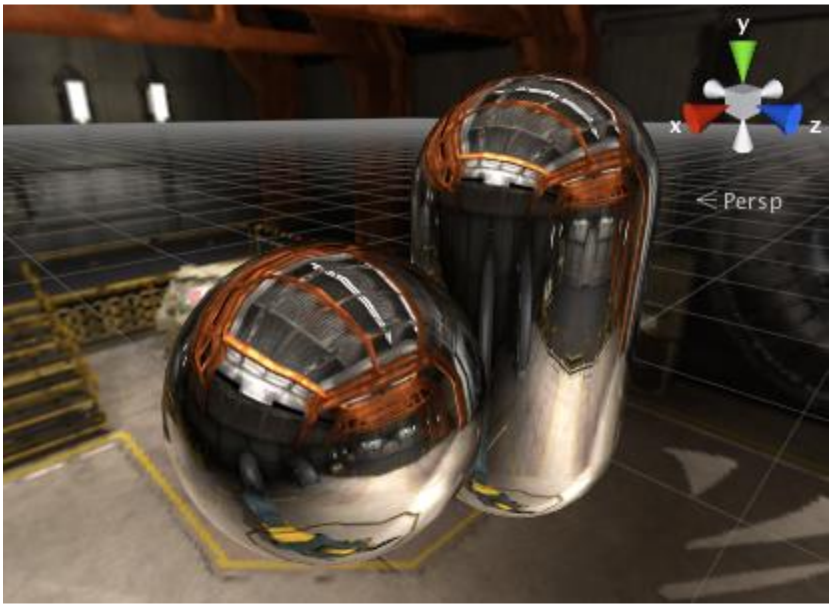

Figure 4.1: Skybox and reflection (https://unity.com).

In Figure 4.2 the Skybox is illustrated. It shows how the world looks like. In general, Skyboxes are rendered around the entire scene, as a complex scene should be presented. The mesh which is used for their rendering is either a box or a sphere.

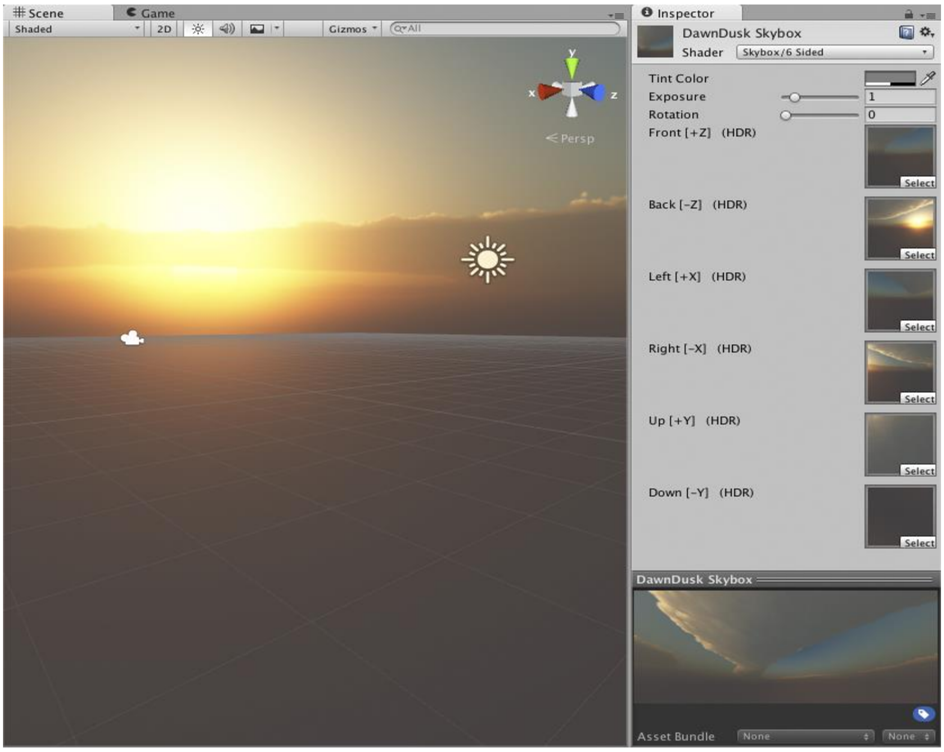

Figure 4.2: SkyBox (https://unity.com).

In Figure 4.3 is presented how the reflections of an object can change according to the environment, a Reflected Probe. This is like a camera which captures a spherical view of the surroundings. The captures image can be used by objects with reflected materials as it is stored as a Cubemap.

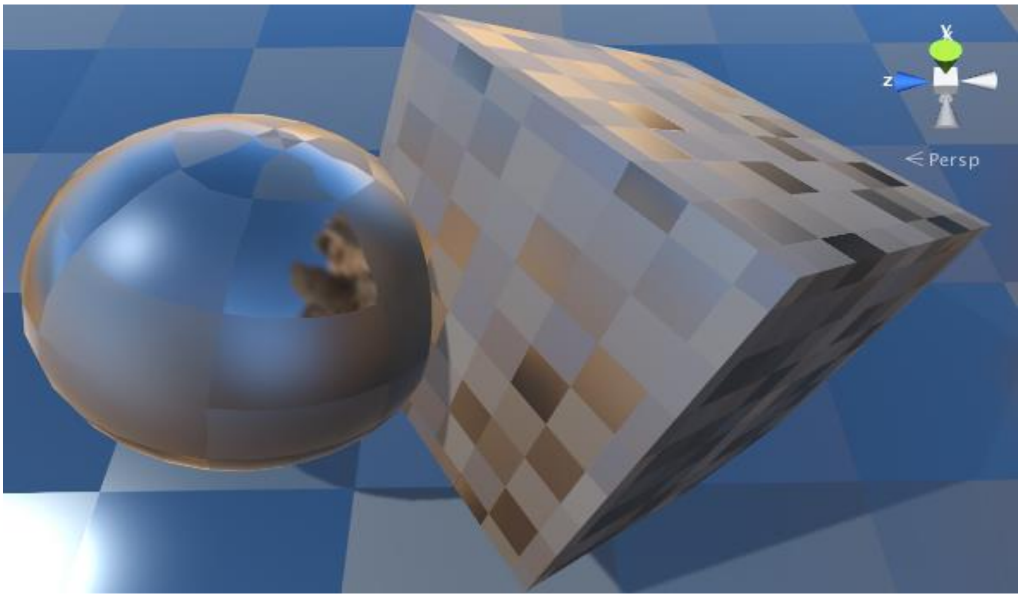

Figure 4.3: Reflection Probe (https://unity.com).

Moreover, Unity includes real-time mixing, full 3D spatial sounds etc. The audio is essential for the completeness of a game and Unity contains a powerful audio system. The audio recording is available in Unity, using any available microphone connected to user's machine.

In our thesis, GameObjects are used for every object of the Zakros Palace, which represent props, characters or scenery. Moreover, the skybox was used for a realistic environment. The Skybox was rendered around the entire scene.

4.1.5 Shaders, Materials and Textures

Shades, Materials and Textures are necessary for rendering in Unity. How a surface should be rendered is defined by materials, by using textures, tiling information etc. The using Shader determines the options which are available for a Material.

The Color of each pixel rendered is calculated by small scripts called Shaders. While, Textures are images applied over the surface. The textures can be used by the Material's Shader, the materials can contain textures. The used Shader is specified by a material and it determines the available options in the Material. Users can assign custom Textures to the Texture variables of a shader. Standard Shader is the best choice for normal rendering.

In this thesis, shaders, materials and textures helped us to create the three-dimensional environment and realistic objects, which there are scattered inside the Zakros Palace.

4.1.6 Scripting

One of the most important ingredients in all games is scripting, which is needed even in the simplest game. Some of the facilities of scripting in Unity is to control objects' physical behavior, to create graphical effects and to implement custom systems based on AI. Effort and time are needed to learn and be familiar with scripting. This chapter explains how the users can start to write script code from scratch in Unity.

Figure 4.4 : C# Script (https://unity.com).

Components control the behavior of GameObjects and they are attached to these kinds of objects. The users can create their own components with the help of scripting in order to accomplish the desired functions. These allow them to modify sone Component properties, trigger many game events and respond to user input. The supported languages are two: C# and UnityScript. C# is similar to C++ and Java, while UnityScript is designed for use with Unity.

MonoBeghaviour is a class which help the connection between a script and the internal workings. When a programmer attaches a script component to a Gameobject, a new instance of the object is created, and the class name is taken from the file name, where these two must be equal in order to enable the script component.

It is necessary the two functions defined inside the class to be noted. The one is the Update function which handle the frame for the GameObject, which can include triggering actions or movement, in other words it includes anything that needs handling during the game. The other is the Start function, which is called before the game begins by Unity itself and it is used for initializations.

These two functions are not the only functions used in a game, as programmers can create custom functions to determine or control the behavior of a GameObject, modify the properties of a component or alter the entire state of the application.

The traditional idea of a program is to have a script which stops running only when its task is completed. However, in Unity this idea of scripting does not apply, as a script has the control by calling functions which are declared in it and when the execution of this function is stopped, Unity has the control again. These are well known as event functions because they are created for the events that occur during the game. There is a number of event functions in Unity and some of the most important events follow below.

In Figure 4.5 the Update function is presented which is the main place for code related to state, behavior and position of objects. This function is called before the frame is rendered and animations are calculated. In Figure 4.6 is shown the FixedUpdate function which is called before every update of physics. More accurate results can be achieved from physics code if they will be included in the FixedUpdate function. In Figure 4.7 is presented the LateUpdate function which can be used for objects in the scene and all animation have been stopped.

```
void Update() {
    float distance = speed * Time.deltaTime * Input.GetAxis("Horizontal");
    transform.Translate(Vector3.right * distance);
}
```

Figure 4.5: Update function (https://unity.com).

```
void FixedUpdate() {
    Vector3 force = transform.forward * driveForce * Input.GetAxis("Vertical");
    rigidbody.AddForce(force);
}
```

Figure 4.6: FixedUpdate function (https://unity.com).**

```
void LateUpdate() {
    Camera.main.transform.LookAt(target.transform);
}
```

Figure 4.7: LateUpdate function (https://unity.com).

An important function id the Awake function which is called when the scene loads. It is called for every object in the scene and in arbitrary order. Another essential function is the Start function which is called for initialization code. It is usual code in Start function uses initializations carried out in the phase of Awake function.

In Figure 4.8 OnGUI function is presented, which is called periodically and contains code related to the system for rendering GUI. Some examples of using this function is for targeting weapons or displaying information while the users hover the necessary objects. Some events such as OnMouseOver or OnMouseDOwn allow user actions to react with the mouse, through a script. For instance, when the mouse button is pressed, an OnMouseDown function is executed.

```
void OnGUI() {
    GUI.Label(labelRect, "Game Over");
}
```

Figure 4.8: OnGUI function (https://unity.com).

In Figure 4.9 the OnCollisionEnter function is presented which is called when collisions against an object will be detected. In case an object's collider is configured as a Trigger, functions such as OnTriggerEnter or onTriggerStay will be called. When more than one collision is reported, these functions will be called more than once. As it is clear in the function of this figure, a parameter is passed (otherObj) which gives details of this collision. These details can be the identity or the position of the incoming object and more.

```
void OnCollisionEnter(otherObj: Collision) {
    if (otherObj.tag == "Arrow") {
        ApplyDamage(10);
    }
}
```

Figure 4.9: OnCollisionEnter function (https://unity.com).

In this thesis, many functions were implemented in order to manipulate the environment and create our three-dimentional system.

4.1.7 Scene View

The interactive view in the world users have created is called Scene View. Users use this View in order to mve characters, light and many other GameObjects. The Scene Gizmo (Figure 4.10) displays the camera's current orientation of the Scene View. Users can easily change the projection and angle mode.

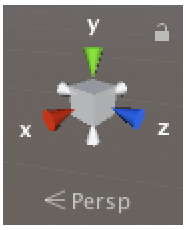

Figure 4.10: The Scene Gizmo (https://unity.com).

Some basic operations in navigation of Scene View is orbiting, moving and zooming. Many ways for maximum accessibility are provided by Unity.

There are arrow keys which helps users to move around the Scene. The Camera can be moved with the up and down arrows, while the left and right arrows pan the view sideways. The user speed changes by holding down the Shift key with an arrow.

The "Move", "Orbit" and "Zoom" of Figure 4.11 are mouse controls available when the Hand tool is selected. Users can navigate by flying, using Flythrough mode. This mode is designed for Perspective Mode.

Figure 4.11: Move, Orbit and Zoom (https://unity.com).

In this thesis, Unity was used for the creation of the 3D reconstruction of the Zakros Palace giving to users the ability to navigate the Palace and to interact with it, viewing information and multiple reconstructions of specific spatial locations.

4.2 Modeling

4.2.1 Sketch up

Google Sketchup is well known as a three-dimensional (3D) modeling computer program for a variety of application with the need of drawing such as civil and mechanical engineering, interior design, film, video game design or architectural. SketchUp Make is a freeware version and SketchUp Pro is a paid version of this program. Using SketchUp users can create three-dimensional (3D) models (buildings, interiors etc.), customize the interface of SketchUp, share or print on a 3D printer three-dimensional (3D) model.

In this thesis, we used SketchUp in order to model specific objects that evidence has showed that they existed in the palace in the past such as a pithos in the court of the Palace.

4.3 MySQL Database

4.3.1 phpMyAdmin

PhpMyAdmin is known as a free software tool, which manage the administration of MySQL. It is written in PHP and supports various operations. The user interface of this tool provide frequently used operations and users can execute Sql statements directly.

Some of the facilities of phpMyAdmin are: (1) browse and drop databases, views, columns etc., (2) display multiple results sets, create databases, views etc., (3)execute or edit any Sql statement, (4) handle multiple servers, (5)create PDF graphics of their database layout, (6) support InnoDB tables and mysqli.

In this thesis, archaeologists have to give values to statements about influencing factors in order to identify the relative importance of this factors through a questionnaire. The influencing factors which were selected to represent types of archeological evidence of significance for the Minoan civilization are features (pits, walls, ditches, etc.), artefacts (tool, work of arts, etc.), comparisons (comparison with other structures of the same era or type of building), topography (area elevation, region characteristics, etc.), peer review (discussion with experts). Furthermore, they have to give their belief about reconstructed options and to select which factors formed their opinion. For this reason, a database was necessary in order to save and to manage their belief in data format.

4.3.2 XAMPP

XAMPP is a free and open source cross-platform web server solution stack package developed by Apache Friends, consisting mainly of the Apache HTTP Server, MariaDB database, and interpreters for scripts written in the PHP and Perl programming languages.

XAMPP stands for Cross-Platform (X), Apache (A), MariaDB (M), PHP (P) and Perl (P). It is a simple, lightweight Apache distribution that makes it extremely easy for developers to create a local web server for testing and deployment purposes. Everything needed to set up a web server – server application (Apache), database (MariaDB), and scripting language (PHP) – is included in an extractable file. XAMPP is also cross-platform, which means it works equally well on

Linux, Mac and Windows. Since most actual web server deployments use the same components as XAMPP, it makes transitioning from a local test server to a live server extremely easy as well.

Officially, XAMPP's designers intended it for use only as a development tool, to allow website designers and programmers to test their work on their own computers without any access to the Internet. To make this as easy as possible, many important security features are disabled by default. XAMPP has the ability to serve web pages on the World Wide Web. A special tool is provided to password-protect the most important parts of the package. XAMPP also provides support for creating and manipulating databases in MariaDB and SQLite among others.

Once XAMPP is installed, it is possible to treat a localhost like a remote host by connecting using an FTP client. Using a program like FileZilla has many advantages when installing a content management system (CMS) like Joomla or WordPress. Itis also possible to connect to localhost via FTP with an HTML editor.

In this thesis, we save and manage data which come from the input of archaeologists in our system. We save these data in a database. In order to create a connection between the UI and the database, we needed a temporary hosting for the implementation of this thesis. For this reason, the XAMPP was used.

4.4 The Programming Language R

4.4.1 What is R?

As it is well known, R is a programming language and is widespread for statistical computing and graphics. It is a free software environment supported by the R foundation for Statistical Computing. The most of statisticians and data miners use this programming language, mostly for data analysis or developing statistical software.

R is a GNU package and the source code for this software environment is written mainly in Fortran, C and R. It is available for free under the GNU General Public License, including precompiled binary versions. Moreover, R includes various graphical front- ends available.

For this thesis the version 3.0.3 was employed to implement GUIs to see the form the Dirichlet Distribution takes with different values for α-hyperparameter and for different values for c-counter. This step was necessary in order to select the appropriate values for α-hyperparameter and c-counter.

4.4.2 The statistical features

A number of graphical and statistical techniques are implemented by this programming language and the libraries of the software of R. These techniques can be nonlinear or linear modeling, statistical tests, classification, time-series analysis, and others. Most of the R’s standard functions are written in R and for this reason the algorithmic choices made, can easily be followed. C, C++ and Fortran code can be called at run time, for computationally intensive tasks. User-submitted packages make R highly extensible and due to its S heritage; it has strong object-oriented programming facilities.

R is a strong software because it includes static graphics. This is important as publication-quality graphs can be produced and interactive graphs can be included using additional packages. R has its own documentation format, called Rd, for supplying comprehensive documentation.

The users have access in R through a command-line interpreter, so it is an interpreted language. Languages like APL and MATLAB which are similar to R support matrix arithmetic. The structure of this programming language includes matrices, arrays, vectors, data frames and lists. Objects for time-series or geo-spatial coordinates are included in this system. A vector with length one represents a scalar.

This programming language supports procedural programming and for functions of it supports object-oriented programming too. The behavior of a generic function depends on the classes of arguments passed to it. This means that the generic function dispatches the function or method specific to that class of object.

In this work, R was used to create the Dirichlet distributions plots parameterized by a vector α of positive reals. We produced plots with three dimensions. We have developed specific scripts based on the function rdirichlet(n, alpha), where n is the number of random generated variables and alpha (α) is the vector of Dirichlet hyperparameter.

5 Beyond point estimates: A Bayesian treatment of uncertainty for archeological reconstructions

Two different approaches for the treatment of uncertainty for archaeological reconstructions were described in Chapter 2. The first work was Danielova's based on fuzzy logic (Danielova et al., 2016) and the second work was Sifniotis' based on subjective (Bayesian) probability (Sifniotis et al., 2012). In subchapter 5.1 we recall these two works and explain why there was a need for a new mathematical model. In section 5.2 we describe the User Interface which was created for archaeologists, so they could give their opinion on any reconstruction of a particular part of an archaeological site based on the findings they have in their hands in real time. In section 5.3 the selection of some necessary variables for our mathematical model and the system we created are described. Finally, in section 5.4 we describe the output of our system.

The Dirichlet distribution is used as a prior distribution in Bayesian statistics because it is conjugate to the multinomial distribution. The distribution used in this thesis has as input the relative importance of archeological evidence as communicated by the archeologists as well as their opinion in relation to whether parts of the reconstruction appear accurate and how that formed their opinion (which type of evidence substantiates the reconstructed part as is). After the analysis of the mathematical model, the Dirichlet distribution with hyperparameters (simplex) was carried out for the selection of the values of the parameters α and c necessary for the presented mathematical model.

5.1 Bayesian Approach in our case study

In Danielova et al., 2016, the author applied a fuzzy model in order to quantify archaeological uncertainty. The application of this mathematical model to the reconstruction of an archaeological site, e.g., the Sanctuary of Diana put forward a problem with the calculations presented. The main disadvantage of this approach lies in the strong dependence of the resulting reliability value on the order in which the building's historical evidence of the reconstruction and objects (added/integrated objects such as arrows, glyphs, bars, charts, text labels, dials, and objects of different shapes such as buffers) are added to the fuzzy logic model.

In an older approach introduced by Maria Sifniotis in 2012, the author proposes that one way to approach archaeological uncertainty is through subjective (Bayesian) probability. This work shows how it is possible, by using Bayesian approach, to evaluate the belief in a reconstruction, or part of it. Through the application of Bayes' Theorem to archaeological examples certain issues were identified. More specifically, the strict totality of distribution is one of these issues, as there are cases where archaeologists are forced to accept assumptions that they would not be comfortable with. For example, that between two cases, A and B if one assigns a confidence of 0.95 to A, they should assign a confidence of 0.05 to B. Another issue is that if one assigns a likelihood of 0 to a hypothesis, this hypothesis may never be resurrected no matter the evidence.

In our work, the aim was to fill some of the gaps left the work of Maria Sifniotis in 2012, using Bayesian Probabilities and more specifically, the Dirichlet distribution. First of all, we solve the problem with the 0 likelihood, as if an archaeologist assigns a likelihood 0 in a reconstruction option, this does not mean that the reconstruction cannot be resurrected. Moreover, the mathematical model Sifniotis used, forces her to limit her research to only two reconstructions, which was solved in our thesis. Finally, in Sifniotis work the belief could not be updated, which we were able to handle in our own work by choosing the Dirichlet distribution.

More specifically, in contrast with previous work examined, Dirichlet distribution can handle more than two reconstruction options and we chose to visualize three of them for every point of interest of the Palace of Zakros because we wanted to show the results in 3D. Moreover, in this work archaeologists can update the Dirichlet prior by adding archaeologists' new beliefs related to new evidence uncovered in excavations. Because we examine three reconstruction options, we can represent the beliefs as θ_1, θ_2, θ_3. Their belief is updated given the new evidence from excavations.

In our thesis, we tried to apply a mathematical model, in order to solve the problems of previous work. A problem in visualization of archaeological uncertainty was the updating of this uncertainty based on the latest evidence uncovered in excavations, which couldn't be supported from previous work. Moreover, there was need to visualize more than two reconstruction options for every point of interest, because many excavations lead to more than two hypotheses for a place of an archaeological site. Therefore, we tried to find the appropriate mathematical model for these purposes. We came up that Dirichlet Distribution was the most suitable for the implementation of our project.

First of all, this mathematical model can handle more than two reconstruction options and we chose to visualize three of them for every point of the Palace of Zakros because we wanted to show the results in 3D. Furthermore, archaeologists can update in real time the distribution giving new input through the system. Finally, in our work distribution does not give 0 values in any of the reconstructed options. After archaeologists vote about their belief the distribution change and it is clear which is the preferable option.

Firstly, archaeologists have to offer their knowledge about influencing factors through a User Interface (Figures 5.4-5.8), which are the types of evidence that form archaeologists' opinion. Then, archaeologists have to offer their belief about three reconstructed options for every interest point of the Palace. The three points of interest we examine in this thesis are in Central Court, in Tank Room and at the Entrance of the Palace. For each of these points, there are three different reconstruction options. To introduce the opinion of archaeologists, a User Interface was created.

5.2 User's Input

In order to apply the mathematical model described below, it was necessary to create a User Interface so that archaeologists could offer their opinion on any influencing factor, identified through discussions with Dr. Anna Semantraki-Grimshaw, and any reconstruction of a particular part of the archaeological site, in our work Zakros Palace, based on the findings they have in their hands at any given time.

In our project, there was need to treat archaeological uncertainty in relation to which reconstructed area of a specific point of interest in the Palace of Zakros was more or less probable. For this reason, expert archaeologists have to insert their knowledge and their belief about how likely it was for a specific area to be formed in a certain way out of three reconstructed options. The first step is firstly to answer a questionnaire with which archaeologists give their opinion in relation to identifying the relative importance among a set of archaeological evidence related to Minoan archaeology and then to give a ranking number to every influencing factor, which number can be an integer from 1 for the least significant influencing factor to 5 for the most significant. During this step, for each type of evidence which is significant to Minoan archaeology, a separate questionnaire investigates the relative importance of this evidence for this historical era. Evidence or the so-called influencing factors which were selected to represent types of archaeological evidence of significance for the Minoan civilization are features (pits, walls, ditches, etc.), artefacts (tool, work of arts, etc.), comparisons (comparison with other structures of the same era or type of building), topography (area elevation, region characteristics, etc.), peer review (discussion with experts). This step is explained in section 5.2.1 and the implementation of the questionnaire is illustrated in Figures 5.1- 5.9. The second step is for expert archaeologists to utilize a separate user interface giving their belief through sliders about whether the visualized reconstructions are accurate. They do that for each reconstructed option of every specific point of interest in Zakros Palace separately. Moreover, they select the influencing factors (evidence types) which formed their belief in the accuracy of the visualized reconstruction. Every influencing factor (evidence type) correspond to a checkbox in the panel which archaeologists select in order to give how their belief formed. This step is explained in section 5.2.2 and the implementation of the second stage of the User Interface is illustrated in Figures 5.10- 5.19.

Archaeologists have to offer their confidence for each reconstructed option for every interest point of the Palace which is studied in this thesis. The three points of interest which are examined in this work are in Central Court, in Tank Room and at the Entrance of the Palace. As in this work we have focused on these three points of interest of the palace, there is this type of User Interface for each point.

5.2.1. Influencing factors ranking

As it is mention in previous sections, after discussions with Dr. Anna Semantraki-Grimshaw, firstly we identified the evidence types form the view of archaeologists. The list of the influencing factors (archeological evidence types) is: Features as all elements of man-made structures; these can range from ditches to wall remains, to post-holes of wooden structures. Artefacts as any object made, affected, used, or modified in some way by human beings; this usually includes pottery, glass, lithic, etc. Biofacts as Biofacts or ecofacts constitute of human, animal and plant remains which are not changed by human interaction. Comparisons as a structure is compared to a structure of similar proportions. Topography as natural features of the landscape in which the building is located; elevation of the area, characteristics of the region and landform data in general. Peer Review as the interviews indicated that interpretation is heavily based on discussions and re-discussions with other archaeologists and architecture specialists were considered important.

After identifying these influencing factors and after discussions with Dr. Anna Semantraki-Grimshaw, there was the need to identify how important each of these factors was, as it was clear through these discussions that the types of evidence which form the opinion of archaeologists have different significance. This means that some kind of evidence are more important and reliable than others. Based on the design, distribution, and analysis of a questionnaire to expert archaeologists, we tried to identify the relative importance among different evidence types for archaeological interpretation. This interpretation is derived from the relative importance of findings that have been unearthed from excavations, such as artifacts, textual evidence, ceramics, etc. The questions that make up the questionnaire were decided with the help of the archaeologist Dr. Anna Semantraki-Grimshaw.

This questionnaire is based on the questionnaire which was created by Sifniotis et al., 2012, adapted to the Minoan archaeological monument of study, e.g., the Palace of Zakros in Crete, Greece, with the help of Dr. Anna Semantraki-Grimshaw. This questionnaire comprises of five (5) parts including questions associated to each influencing factor. Also, there is another part with five (5) variables, each accounting for a specific identified factor.

As we mentioned above, in the first part of the questionnaire, archaeologists offer their judgment about each influencing factor, so as to quantify how important every factor is. There are eight (8) questions about features, four (4) questions about artefacts, seven (7) questions about comparative data and five (5) questions about peer review. The archeologists are offering a judgment on a 7-point scale, ranging from Never to Always for each statement.

More specifically, the specific statements about features were: While archaeology is uncertain, structural evidence gives me security. When faced with structural evidence I always make up my mind very quickly as to how the building looked. I try to understand the function of the building and make my reconstruction around it. First, I interpret the structural evidence and then I deduce the building's function. Most times someone can be surer about the ground floors than an existing of a second floor. Interpreting basic structural shape (e.g. walls) is easy even when he/she has just the foundations. I am comfortable with placing exits in a building when the structural evidence suggests it. I am comfortable with placing exits in a building in the absence of structural evidence. The archeologists are offering a judgment on a 7-point scale, ranging from Never to Always for each statement.

The statements about artefacts were: Scarcity of artefacts in a site greatly hinders my interpretation of the site. Interpreting the function of a room mostly depends on what artefacts I have found. Artefacts mostly help in the details of a structure. I can interpret the look of a room without any artefact. The archeologists are offering a judgment on a 7-point scale, ranging from Never to Always for each statement.

The statements about comparative data were: I often base my interpretation on the surrounding topography of the area. I rely on similar structures to establish a layout of the site. Documented structures contemporary to the one I work on greatly influence my interpretation. I often change my interpretation of the structural features when faced with contradictory contextual comparisons. The surrounding environment greatly influences the structure and layout of a building. I often place exits on a building according to the surrounding landscape. I will change my interpretation of the structural features if textual evidence suggests it. The archeologists are offering a judgment on a 7-point scale, ranging from Never to Always for each statement.

Finally, the statements about peer review were: I find it useful to discuss my interpretations with my peers. I rely upon peer interpretations and comments. I sometimes find that my peers' opinions change my interpretation. Peer discussion results in better interpretations and results. How often do you discuss interpretations with your peers?

The archeologists are offering a judgment on a 7-point scale, ranging from Never to Always for each statement: never, almost never, seldom, sometimes, often, very often, always. These answers offer a large range of choices. After completing the first part of the questionnaire, the average for each factor is calculated. In the next step, archaeologists are asked to rank the evidence in order of importance. They give values to each evidence type from one (1) to five (5), one (1) being the least important and five (5) the most important. Every archaeologist can use a number only once. The number completed in the second part is added to the resulting average for each type of evidence of the first part.

The goal of this part was to identify the order of importance of the influencing factors, according to archaeologists' opinion. More specifically, we wanted to know who is the most important of these factors, who is the next important and so on. For example, if the "features" are more important than "artefacts" or if "artefacts" are more important than "comparative data". This identification is important for our system, as the opinion of archaeologists will have other significance depending on how important are the findings that shaped his opinion. For example, the assumption of an archaeologist about a reconstruction based on peer review has not the same importance as this of an archaeologist which is based in the remains of the Palace. This information changes dynamically and in real time as archaeologists continue to fill this questionnaire. Therefore, the information of our system is always updated.

The numerical results obtained from the above description are not used in our system, they only determine the order of importance of the influencing factors as it is described in the previous paragraph. The values corresponding to each of this factor are given by us and resulted from implementing GUIs as described in the next chapter. These values are the weight of every evidence type. Every factor will take a number as its weight depends of its importance. For example, if in the next steps we decide that the most important type will take the value 2 as a weight, if the results of the questionnaire show that the most important factor is the “features”, then this factor will take the value 2 as its weight. However, if the results of the questionnaire show that the most important factor is the “artefacts”, then this factor will take the value 2 as its weight. The decision of which will be the weight of every type of evidence is described in details in chapter 5.3.3.

This visualization system presented will be used by mainly archaeologists, so the development of a simple and understandable User Interface (UI) was necessary.

A welcome window appears when the application starts (Figure 6.1). In this window users are required to answer whether they are archeologists. If they are not archaeologists, they immediately enter the reconstruction of the monument without giving any information on uncertainty or answering a questionnaire. If they are indeed archaeologists, they press the “YES” button and they enter the main menu (Figure 6.2), where they can select their next action. They can select to view the complete 3D reconstruction of the Palace of Zakros, or to provide their view on statements included in the questionnaire regarding the various types of archaeological evidence (influencing factors). They can then press the ‘Uncertainty’ button and view varied reconstructions of specific locations, based on their input previously, in relation to the significance of the influencing factors. They are now required to provide feedback in relation to the resulted 3D reconstructions as explained in Section 6.1.2. If they are not archaeologists, they should press the “NO” button and they will enter a complete 3D reconstruction of the Palace of Zakros, which they can navigate.

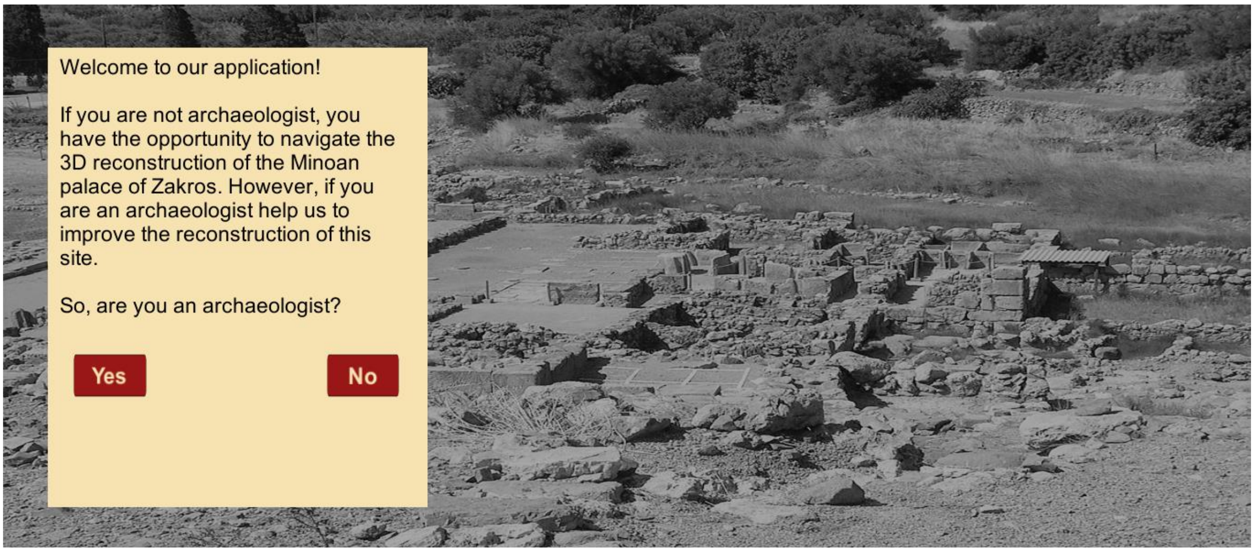

Figure 5.1: The welcome window of our application.

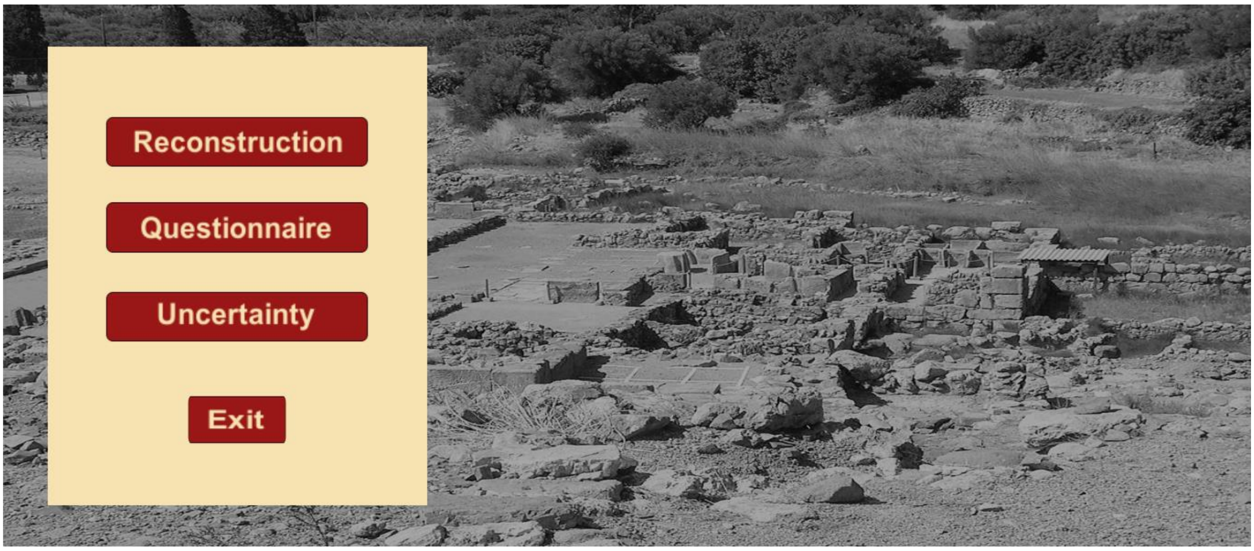

Figure 5.2: The main menu.

If users press the button “Questionnaire”, they are able to provide views about the evidence types (influencing factors) in order to assess the importance of each factor. The questionnaire is directly linked to the uncertainty estimation analyzed below. More specifically, the archaeologist provides judgments to statements which relate to each type of evidence on a 7-point scale, ranging from Never to Always. For each factor, an average is formed, depending on the answers of the archaeologists. These averages determine how significant each factor is for modulating archaeologists’ belief for each alternative reconstruction of a point. Also, the archeologists communicate their belief in relation to the accuracy of the three reconstructed options of the same spatial location as well as provide which evidence types have led them to form their opinion about whether each specific reconstruction is probable. Such input signifies, in the model, how the uncertainty visualization would be formed for each reconstructed option, adopting a visualization scheme from red to green depending on archeologists’ input. For example, if the archaeologists are confident enough about a reconstruction, but the evidence they have in their hands does not justify this certainty, model driving the uncertainty visualization will drive the mathematical distribution to be less certain because the evidence types entered by the archaeologist does not just justify their communicated level of high confidence in the reconstruction.

In the first part of the questionnaire in relation to statements related to the significance of evidence values, the user can select among the options: never, almost never, seldom, sometimes, often, very often, always. These options are displayed to the user in the form of a dropdown menu, as shown in the Figure 6.3.

Since archaeologists have given the uncertainty about the points they are asked for, this uncertainty is graded differently depending on which factors have led him to these conclusions. For example, if the archaeologists are confident enough about a reconstruction, but the evidence they have in their hands does not justify this certainty, the prices that represent their certainty will fall.

In the first part of the questionnaire, the user can select among the options: never, almost never, seldom, sometimes, often, very often, always. These options are displayed to the user in the form of a dropdown, as shown in the Figure 5.3.

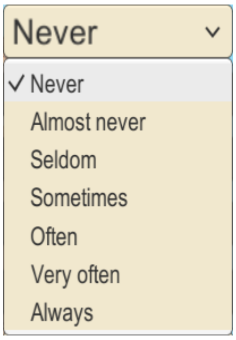

Figure 5.3: User’s options for the Part 1

The first eight (8) questions are about Features (Figure 5.4). The next four (4) questions are for artefacts (Figure 5.5). Then there are seven (7) questions about comparative data (Figure 5.6). The last five (5) questions are about peer review (Figure 5.7).

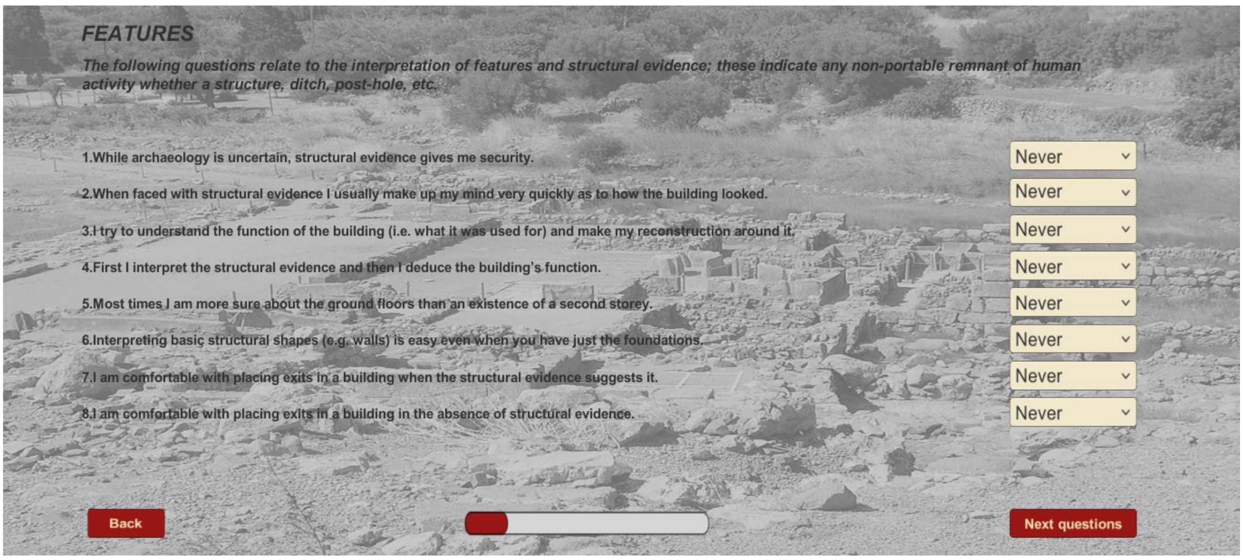

Figure 5.4: Questions about features.

Figure 5.5: Questions about artefacts.

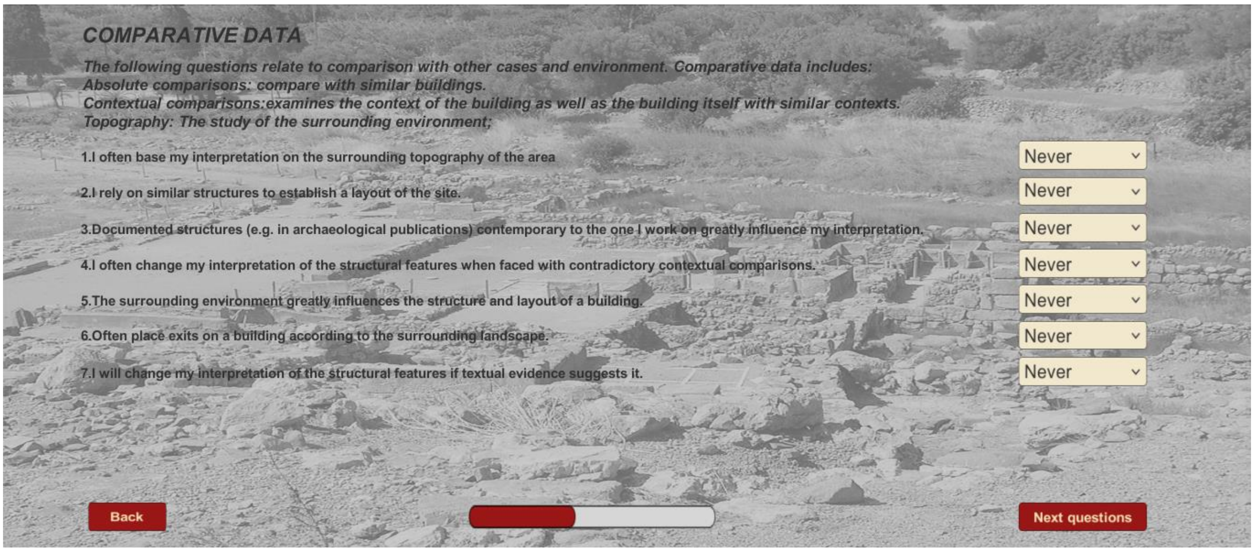

Figure 5.6: Questions about comparative data.

Figure 5.7: Questions about peer review.

In the second part, there is the list of evidence types and the user can enter a value to every type (Figure 5.8). This value can be an integer number, which number can be an integer from 1 for the least significant influencing factor to

5 for the most significant. If the user enters a number smaller than 1 or greater than 5, or he uses a number more than once, a message is displayed (Figure 5.9).

Figure 5.8: Evidence type ranks.

Figure 5.9: Error page.

5.2.2 Archaeologists' belief

In relation to archeologists communicating their belief or confidence for a particular reconstruction, the presented uncertainty visualization system provides three parts: The parameter setting panel, the view of the Dirichlet distribution and the 3D preview.

First of all, archaeologists are required to select which spatial location (point of interest) of the Palace out of three which are examined in this work, they offer their belief in relation to their perceived confidence in relation to whether the reconstruction is accurate. They have to select among three points of interest, as we can see in Figure 5.10, which are located in the Palace of Zakros. If they hover the Buttons "Point 1", "Point 2", "Point 3", they can see which point each button refers to (Figure 5.10 – 5.13). After they select the point of interest signifying a specific spatial location in the Palace of Zakros, they are presented with information about the excavation findings, leading to the multiple reconstructions of each point. Figure 5.14, Figure 5.15 and Figure 5.16 show how information concerning each point of interest is communicated to the archeologists.

Then, through this stage of the User Interface, archaeologists are able to offer their confidence, through a panel, for every reconstruction for each point of interest, necessary information for the treatment of uncertainty through the Dirichlet distribution. First of all, in the upper side of the panel there are sliders, one for each reconstruction option, and archaeologists can move them depending on their confidence for every hypothesis. For example, if archaeologists believe that a reconstruction is quite probable, then, they move the slider to the right. On the other hand, if they believe that a reconstruction is not so likely, they move the slider to the left. At the bottom of the panel, there is a list of evidence types, all the same for the three reconstruction options of one specific point of interest. As it is described in details in the section 5.2.1, after a short discussion with the archaeologist Dr Maria Simantiraki, we concluded that the factors which affect the opinion of Minoan archaeologists concerning how a historical site used to be formed in the past, are features, artefacts, absolute comparisons, peer review, topography. The archeologists select the factors that lead them to their confidence or belief in relation to the hypotheses concerning the past form of the archeological site. The panels for the three points, representing three distinct spatial locations, are shown in Figure 5.17, 5.18 and 5.19.

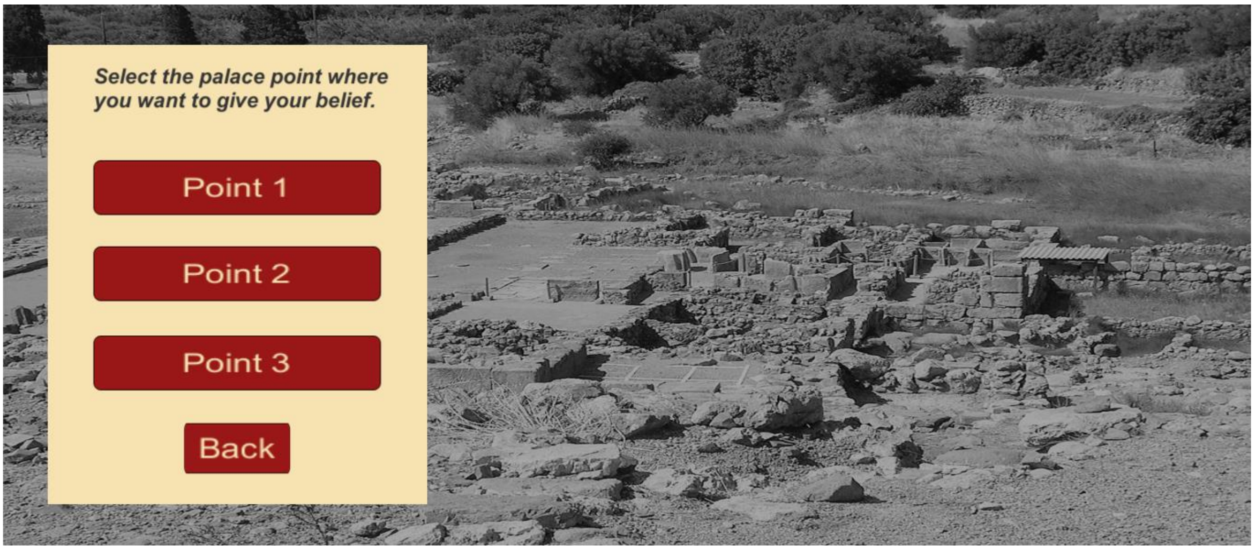

Figure 5.10: Menu of Uncertainty Points.

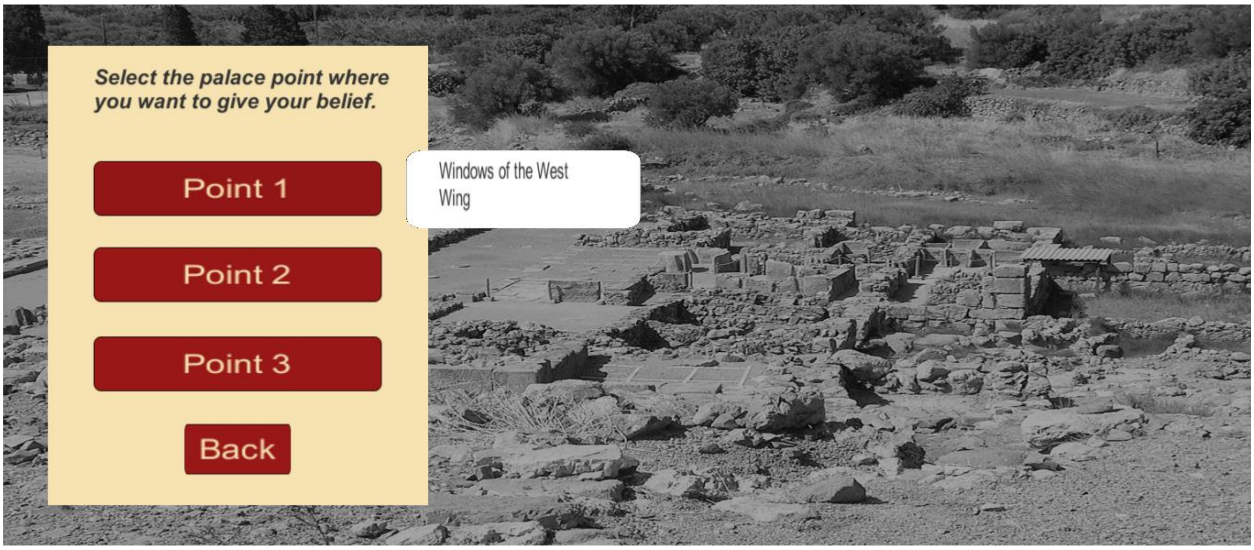

Figure 5.11: Menu of Uncertainty Points, hovering Point 1.

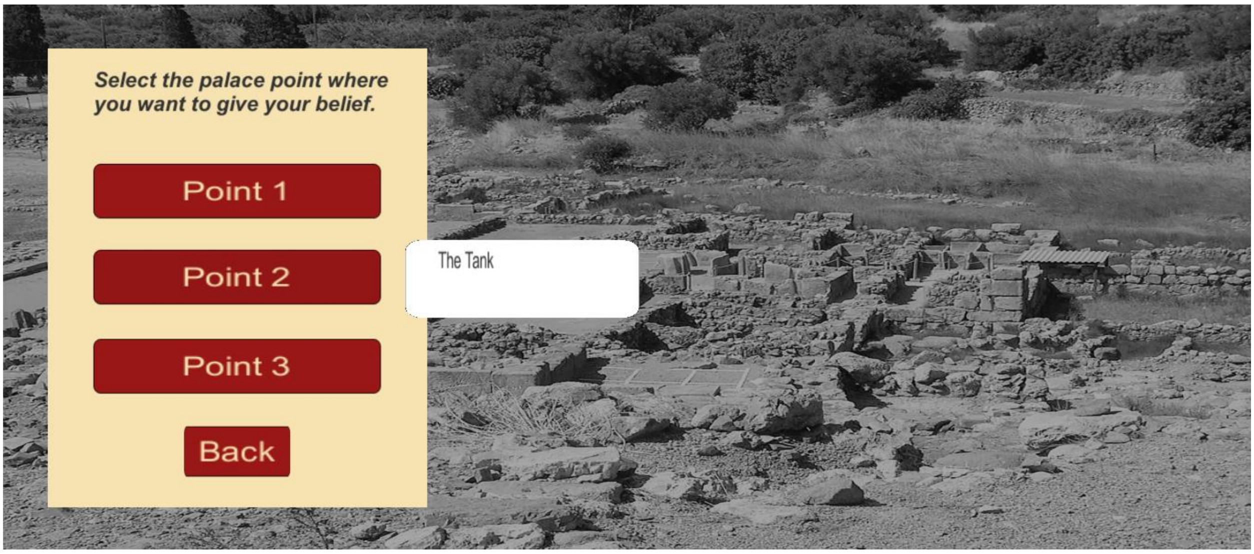

Figure 5.12: Menu of Uncertainty Points, hovering Point 2.

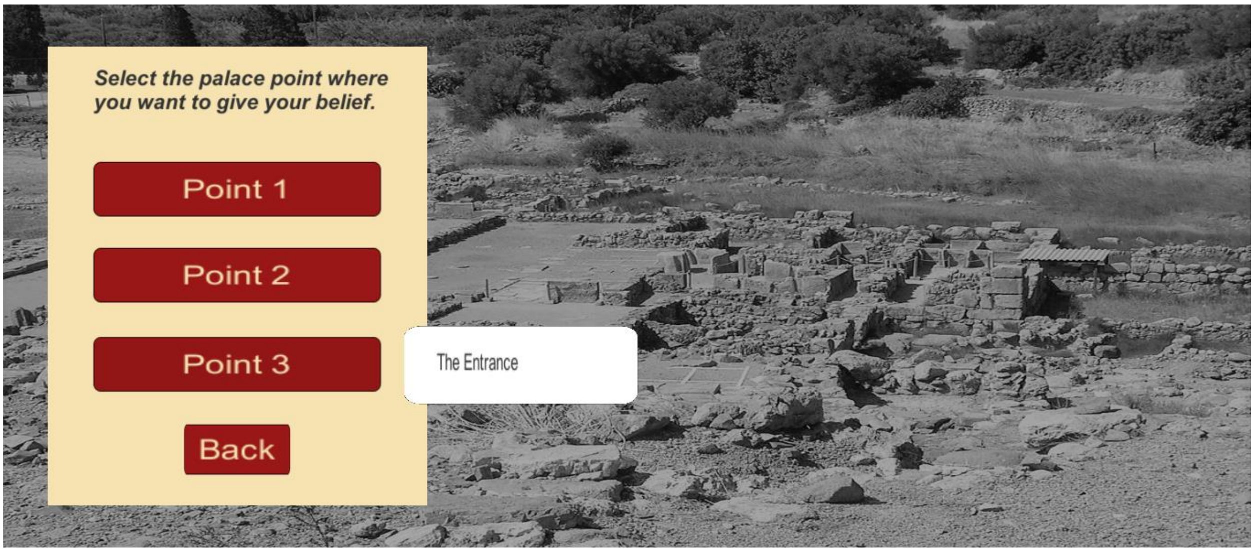

Figure 5.13: Menu of Uncertainty Points, hovering Point 3.

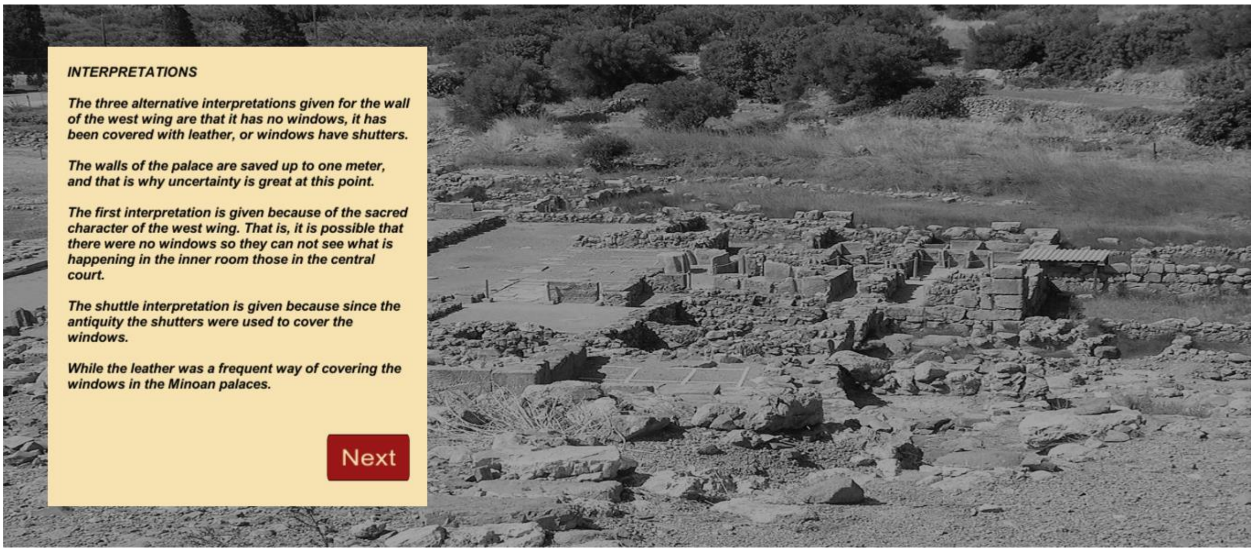

Figure 5.14: Information about the interpretations of Point 1.

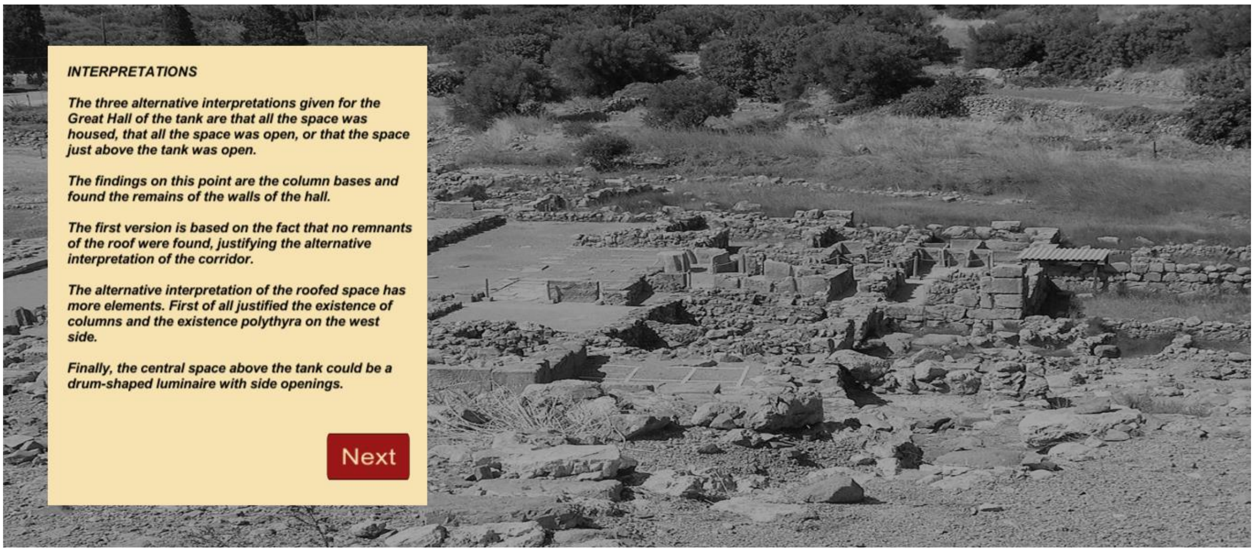

Figure 5.15: Information about the interpretations of Point 2.

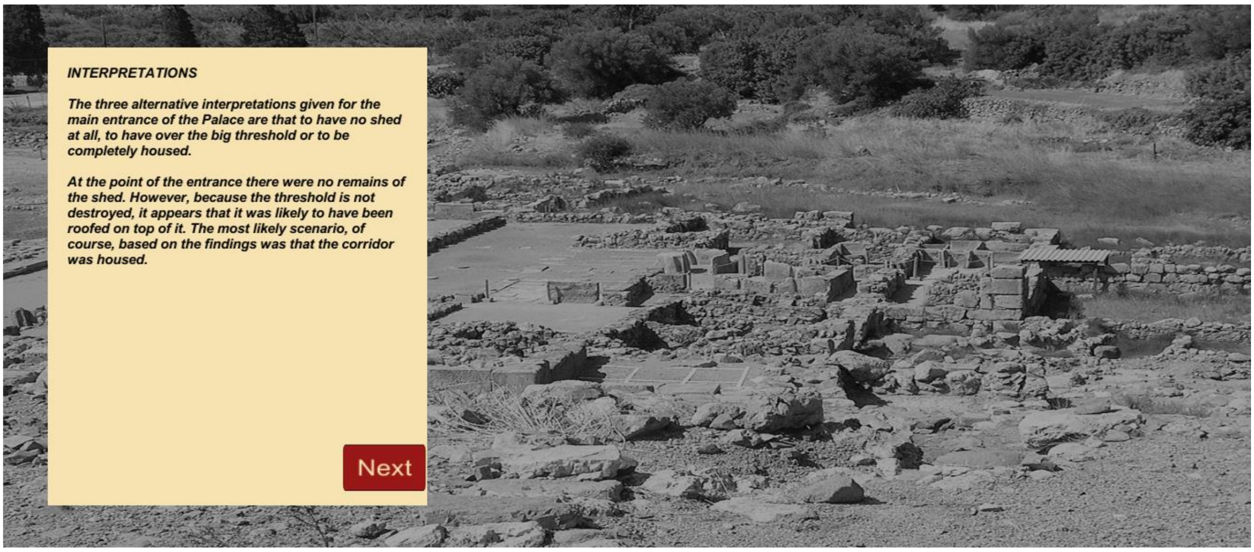

Figure 5.16: Information about the interpretations of Point 3.

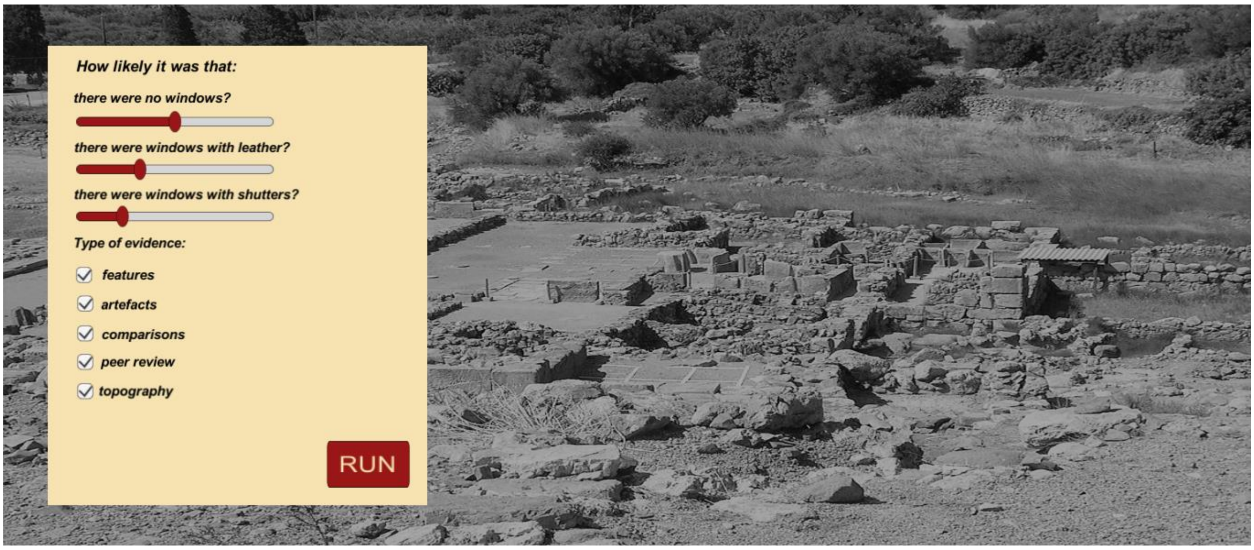

Figure 5.17: Parameter Setting Panel about Point 1.

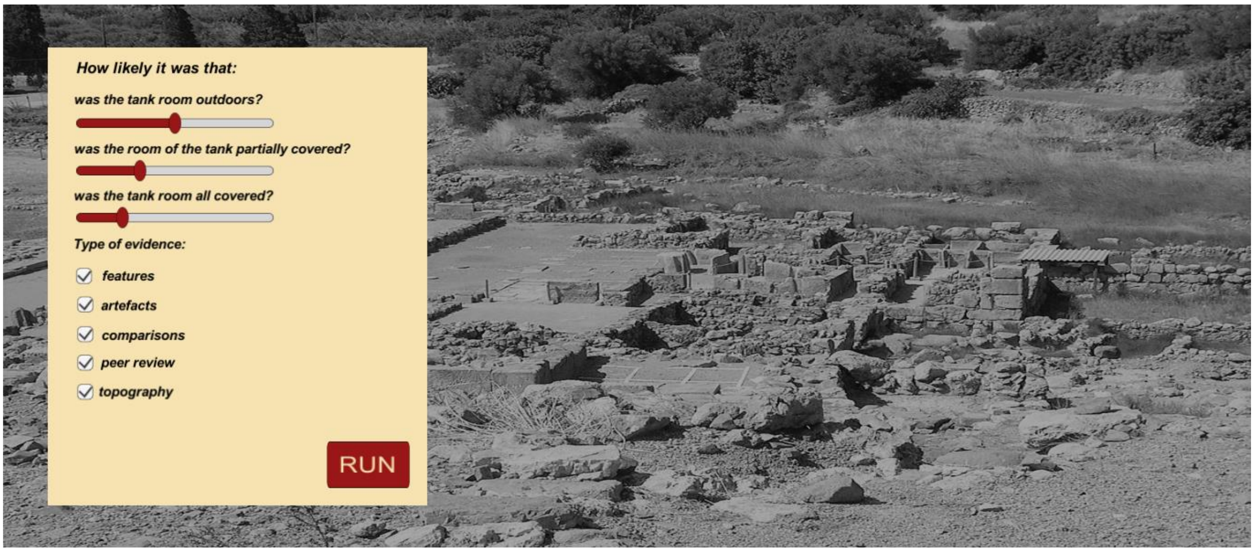

Figure 5.18: Parameter Setting Panel about Point 2.

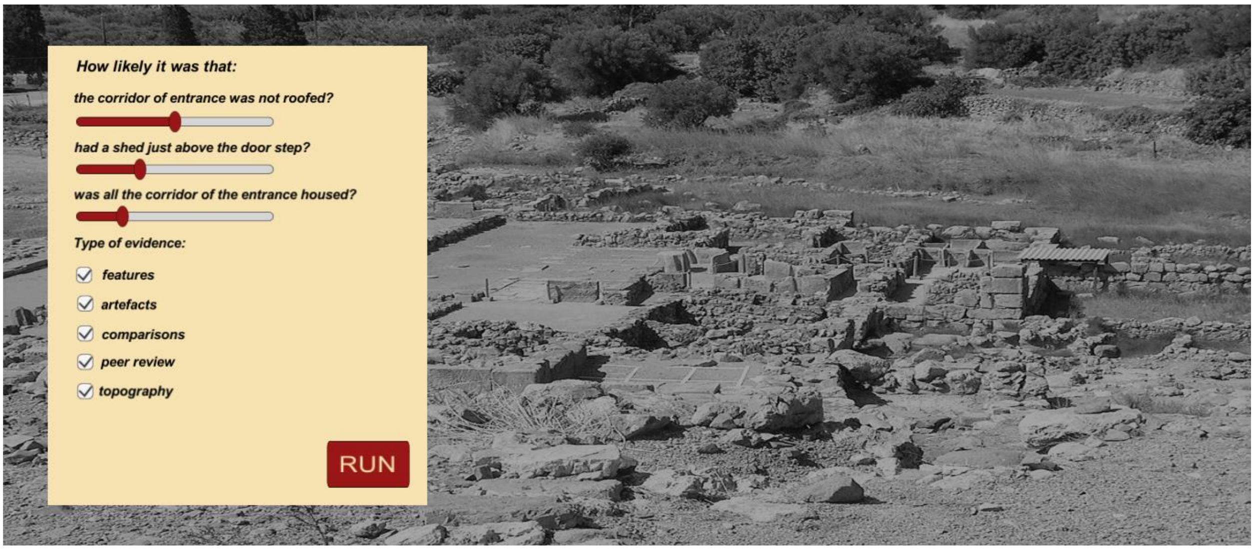

Figure 5.19: Parameter Setting Panel about Point 3.

The goal of this part was to create a User Interface which offers to our system the belief of archaeologists. More specifically, the confidence of archaeologists in the form of numbers was essential for the treatment of uncertainty and as it is impossible for an archaeologist to offer a number which correspond to his opinion, the use of the sliders was the most user-friendly option for the User Interface. Moreover, as we described above, the opinion of archaeologists will have other significance depending on how important are the findings that shaped his opinion. For example, the assumption of an archaeologist about a reconstruction based on peer review has not the same importance as this of an archaeologist which is based in the remains of the Palace. For this reason, we the checkboxes can give us the information about which type of evidence led every archaeologist to his belief about how a point of interest could be in the past. Again, checkboxes were the most user-friendly option, in order to offer to our system this information.

5.3 System's Variables

In the previous section, chapter 5.2, the archaeologists' input was described. This input is the opinion of every archaeologist for each influencing factor and their knowledge and confidence for the possible reconstructions for every point of interest in Zakros Palace. The first step was to answer a questionnaire and give a ranking number for each influencing factor, in order to identify the relative importance among a set of archaeological evidence related to Minoan archaeology. During this step, for each type of evidence which is significant to Minoan archaeology, a separate questionnaire investigates the relative importance of this evidence for this historical era. Evidence or the so-called influencing factors which were selected to represent types of archaeological evidence of significance for the Minoan civilization are features (pits, walls, ditches, etc.), artefacts (tool, work of arts, etc.), comparisons (comparison with other structures of the same era or type of building), topography (area elevation, region characteristics, etc.), peer review (discussion with experts). This step is explained in section 5.2.1 and the implementation of the questionnaire is illustrated in figures 5.1- 5.9. The second step is for expert archaeologists to utilize a separate user interface giving their belief through sliders about whether the visualized reconstructions are accurate. They do that for each reconstructed option of a specific point of interest separately. Moreover, they select the evidence types which formed their belief in the accuracy of the visualized reconstruction. Every evidence type has a checkbox in the panel which archaeologists select in order to give how their belief formed. This step is explained in section 5.2.2 and the implementation of the questionnaire is illustrated in figures 5.1- 5.9.

In our work the system which was created for the treatment of uncertainty for the three points of Zakros palace (Central Court, Tank Room and the Entrance of the Palace) is based on the Dirichlet Distribution $Dir(\alpha)$. As it is mentioned in the theoretical background of chapter 3, a Dirichlet distribution is defined as a measure on measures. Specifically, a Dirichlet distribution defines a probability measure over the k-simplex. The k-simplex is the convex hull constructed so that each point on the surface of the simplex describes a probability distribution over k outcomes. Therefore, our first decision was the kind of simplex for our case study. This is described on section 5.3.1.

Moreover, it was reported that the parameter α governs the shapes of the distribution. This parameter is an hyperparameter and in a machine learning model, hyperparameter is a configuration that is external to model, whose value cannot be estimated from data. For this reason, hyperparameters are specified by practitioner, as it is defined by the laws of Machine Learning. The partitioner in our work is the creator of the system. The machine learning model, in our case study, is based on Dirichlet distribution and this kind of distribution has a single hyperparameter, alpha hyperparameter and we had to consider the appropriate value.

As it is defined in machine learning there is not a perfect value for an hyperparameter. This hyperparameter has the same number of elements as the number of the possible reconstructions for every point we study. Therefore, α has three dimensions, $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. As it is clear from the theoretical background the value of alpha hyperparameter can influence the distribution. For example, if the values of $\alpha_1, \alpha_2, \alpha_3$ are all equal ($\alpha_1 = \alpha_2 = \alpha_3$), the density of the becomes uniformly distributed over the triangle. In our case study, this mean that the three possible reconstructions for an interest point are equally likely. However, the values of $\alpha_1, \alpha_2, \alpha_3$ may not be identical and in this case the distribution is not symmetric. When one of $\alpha_1, \alpha_2, \alpha_3$ has a larger value, the density is higher close to the vertex with the higher value. This means, that the reconstruction which corresponds to this vertex it is more likely to represent how it a specific point of interest in the past. Furthermore, a higher alpha gives a denser distribution whereas a lower alpha gives a sparser distribution. In our case, dense distribution means that the most of archaeologists have a similar opinion among themselves and therefore gave similar values on the sliders of the User Interface. If the distribution is sparse, means that there were many different opinions about the three possible reconstruction of a point of interest. This is described in details in subsection 3.4.4.

In this work we had to wonder what was the right choice for the value of this variable. To achieve the goal of specifying this hyperparameter, we implement GUIs for some different values of α priori and in the section 5.3.2 this GUIs is described in details. In our case, we run implement GUIs in order to choose the appropriate value for our case study. The goal of these GUIs was to select a value for the alpha hyperparameter, which lead to a distribution depended to both alpha hyperparameter and observed data. More specifically, when α is small, then the posterior distribution is more dependent on the observed data, while when it is large the posterior distribution is more dependent on α. This can help archaeologists to know which reconstruction is more likely to represents a point of interest according to the opinion of other archaeologists, who gave an input that explained above in the system.

As it is described in chapter 5.2 and in the previous paragraphs of this section, archaeologists offer their opinion about how significant every influencing factor is through answering questions implemented for each of these influencing factors, and ranking the influencing factors from the most important to the least important of them. The questions can take answers from Never to Always which are represented from integer values between the intervals 1 and 7 and as ranking number can be assigned an integer value from 1 to 5, where 1 is the value for the least important influencing factor and 5 is the value for the most important of these factors. In order to quantify the significance of influencing factors, we calculate the average of the answers which were given for each one of them and then we add to these results the ranking number that correspond to them. The result of this part of the User Interface is a number and it is referred in our work as weight (w_i) of each influencing factor.

This part was very important for our work, as after discussion with the archaeologist, Dr Anna Simantiraki, we came together to the conclusion that all the opinion of archaeologists for the reconstructions cannot be taken into account equally. When archaeologists base their view on important findings, such as the remains of the Palace, walls or frescoes, which are classified as more than one influencing factors (remains and walls belong to the "features" and frescoes to the "artefacts"), it is more important than when they do not base their opinion in findings which belong to any of the influencing factors which have been identified in this work and only they give their opinion to the User Interface, without being able to support this view. This difference was important to be shown in the system created to this work for the treatment of uncertainty. To integrate and make this difference obvious, the input of our system should be a combination of the opinion of archaeologists which is given through the sliders as is it is described above, and the significance of each influencing factor which is represented by the weight (w_i) as it is defined in the previous paragraph. This influenced opinion of archaeologists, which is a combination of sliders' values (θ_ι) and the significance of the influencing factors (w_i) is referred to in our work as counter c. The counter c can be ed from the following equation:

$$c_i = (w_f + w_a + w_c + w_t + w_p) \text{ x } \theta_i \, x \, 10$$

$$\text{where } i = 1,..,3$$

In this equation w_f is the weight of "features" factor, w_a is the weight of "artefacts" factor, w_c is the weight of "comparisons" factor, w_t is the weight of "topography" factor and w_p the weight of "peer review" factor. After the insertion of counter c in our work, the α hyperparameter changes as follow:

$$a_i = a_i + c_i, \; where \; i = 1,2,3$$

Since we have incorporated the definition of counter c in our system, it is clear that the values which represent the weight of influencing factors should match with previous assumptions for alpha hyperparameter, as now the counter c is the variable which influence the alpha hyperparameter as it is shown in the previous equation and not only the opinion of archaeologists which is represented from values θ_i . More specifically, the changes of the distribution should be depended to both alpha hyperparameter and observed data. After the insertion of the definition of counter c, this variable represents the input of archaeologists which is the observed data. As it is mentioned, when α is small, then the posterior distribution is more dependent on the observed data, while when it is large the posterior distribution is more dependent on α. This can help archaeologists to know which reconstruction is more likely to represent a point of interest according to the opinion of other archaeologists, who gave an input that explained above in the system. Moreover, as it is shown from the previous equations, if the counter c (observed data in our case) is large, the posterior distribution is dependent a lot to the counter c. Now, we had to consider that the values of w_f, w_a, w_c, w_t, w_p cannot be large because in that case the distribution will be dependent only on counter c.

Based on this assumption, the numerical results obtained from the User Interface related to the importance of influencing factors cannot be used in our system, they only determine the order of importance of the influencing factors. This is because the results obtained from the questions and the ranking about the influencing factors can be large. As in questions the answers can be from Never to Always which are represented from the values 1 to 7, respectively and the ranking number from 1 to 5, the final number can reach up to number 12, which is a large number for our assumption.

In this work we had to wonder what was the right choice for the value of weights of influencing factors. The values corresponding to each of these influencing factors are given to the system after implementing GUIs as described in the next chapter. Every factor will take a number as its weight depends of its importance. For example, if in the next steps we decide that the most important type will take the value 2 as a weight, if the results of the questionnaire show that the most important factor is the "features", then this factor will take the value 2 as its weight, $w_f = 2$. However, if the results of the questionnaire show that the most important factor is the "artefacts", then this factor will take the value 2 as its weight, $w_a = 2$. The decision of which will be the weight of every type of evidence is described in details in chapter 5.3.3. After this decision, the sum of weights of the selected influencing factors is required in order to identify how accurate is every archaeologist's input, so we calculate the total confidence of an archaeologist for a reconstruction as $w_f + w_a + w_c + w_t + w_p$, where w_f the weight of "features" factor, w_a the weight of "artefacts" factor, w_c the weight of "comparisons" factor, w_t the weight of "topography" factor and w_p the weight of "peer review" factor. As counter c is defined the sum of the weights of archaeologists input multiplied with the values of sliders. More specifically, $c_i = (w_f + w_a + w_c + w_t + w_p)$ x θ_i , where $i = 1,..,3$ and θ_ι is the confidence of archaeologists with every reconstruction for a specific point of interest, represented by the sliders. This counter c, gives to our system the knowledge of archaeologists depending on the evidence which form this knowledge.

Finally, after making the appropriate decisions on the above issues we are ready to implement our system. The first part of our system is the visualization of Probability Density Function (PDF) which allows archaeologists to see the configuration of the distribution based on the data inserted in User Interface, which means how the knowledge of archaeologist, which is the input of our system, changes which is the prevalent reconstruction of a specific point of interest. After this step the samples of Dirichlet Distribution are drawn, where every point of the simplex is a possible combination of uncertainty for every reconstruction. This output is described in details in chapter 5.4.

5.3.1 2-Simplex

The first step was to decide which is the simplex in our case study. In chapter 3 reported that for n probabilities our probability space is on a n-1 simplex. We lose a degree of freedom simply because one probability can be determined by deducting all the other from 1.

In our work, every point of the Palace, we want to create the reconstruction, is connected with three possible reconstructions. The three points which are studied are in Central Court, in Tank Room and at the Entrance of the Palace. Therefore, we have three (3) probabilities for every point and our probability space is on a 2-simplex. Every vertex of the 2-Simplex is one of the reconstruction options and each point on the surface of the simplex describes a probability distribution over 3 outcomes. Each point q in the simplex can be thought of as a probability mass function in its own right. This is because each component of q is non-negative and the components sum to 1 (Frigyik et al., 2010). The simplex is important to be defined as the Dirichlet Distribution can be thought of as a probability distribution over the 2-dimentional probability simplex. The Dirichlet Distribution is sampling over this probability simplex.

5.3.2 α hyperparameter

In the introduction of this chapter it was reported that Dirichlet Distribution has only a hyperparameter, alpha hyperparameter. The Dirichlet distribution parameterized by this vector α of positive reals. As it is mentioned again, hyperparameters are important for a machine learning model, and their value cannot be estimated from data, they are specified by us. Therefore, we have to give a value to the alpha hyperparameter. For this reason, the practitioner has to specify these values. In our case study the machine learning model is based on Dirichlet distribution and this kind of distribution has a single hyperparameter, alpha hyperparameter, and we had to consider the appropriate value. Higher alpha gives a denser distribution whereas a lower alpha gives a sparser distribution.

As we described above, in section 5.3.1, dense distribution means that the most of archaeologists have a similar opinion among themselves and therefore gave similar values on the sliders of the User Interface. If the distribution is sparse, means that there were many different opinions about the three possible reconstruction of a point of interest.

The shape of Dirichlet distribution depends on a parameter referred as α. This parameter is an hyperparameter and in a machine learning model, hyperparameter is a configuration that is external to model, whose value cannot be estimated from data. For this reason, hyperparameters are specified by practitioner, as it is defined by the laws of Machine Learning. The partitioner in our work is the creator of the system. The machine learning model, in our case study, is based on Dirichlet distribution and this kind of distribution has a single hyperparameter, alpha hyperparameter and we had to consider the appropriate value.

As it is defined in machine learning there is not a perfect value for an hyperparameter. This hyperparameter has the same number of elements as the number of the possible reconstructions for every point we study. Therefore, α has three dimensions, $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. As it is clear from the theoretical background the value of alpha hyperparameter can influence the distribution. For example, if the values of $\alpha_1, \alpha_2, \alpha_3$ are all equal ($\alpha_1 = \alpha_2 = \alpha_3$), the density of the becomes uniformly distributed over the triangle and Dirichlet distribution is called symmetric. In our case study, this mean that the three possible reconstructions for an interest point are equally likely. However, the values of $\alpha_1, \alpha_2, \alpha_3$ may not be identical and in this case the distribution is not symmetric. When one of $\alpha_1, \alpha_2, \alpha_3$ has a larger value, the density is higher close to the vertex with the higher value. This means, that the reconstruction which corresponds to this vertex it is more likely to represent how it a specific point of interest in the past. Furthermore, a higher alpha gives a denser distribution whereas a lower alpha gives a sparser distribution. In our case, dense distribution means that the most of archaeologists have a similar opinion among themselves and therefore gave similar values on the sliders of the User Interface. If the distribution is sparse, means that there were many different opinions about the three possible reconstruction of a point of interest.

In this chapter, we want to examine which is the right choice for the value of this variable. To achieve the goal of specifying this hyperparameter, we implement GUIs for some different values of α priori. In our case, we implement GUIs in order to choose the appropriate value for our case study. The goal of these GUIs was to select a value for the alpha hyperparameter, which lead to a distribution depended to both alpha hyperparameter and observed data. In this work, observed data is the input of archaeologists. We want both, alpha hyperparameter and the knowledge of archaeologists to influence the distribution, because if only alpha hyperparameter influence distribution, archaeologists could not know the general opinion of archaeologists and how their opinion has been formed. On the other hand, if the value of α hyperparameter is small, only the archaeologists' input influences the distribution and this can affect the next archaeologists before they give their own knowledge through our system. Therefore, it was necessary for our GUIs to have a combination with small values for this hyperparameter, a combination with large values and a combination which will be in the middle, these combinations we wanted not to be too close.

Moreover, because of no evidence initially, the values of $\alpha_1, \alpha_2, \alpha_3$ should have equal values $\alpha_1 = \alpha_2 = \alpha_3$. After the previous description and this assumption, for these GUIs, we chose three different combinations of $\alpha = (\alpha_1, \alpha_2, \alpha_3)$, which are $\alpha = (1,1,1)$, $\alpha = (10,10,10)$, $\alpha = (100,100,100)$.

The case of $\alpha = (1,1,1)$ shows a sparse distribution, which means that there is not any opinion for the reconstructions and there is no evidence. The case of $\alpha = (10,10,10)$ shows a denser distribution, which means that all three versions of an interest point are equally possible. Finally, the case of $\alpha = (100,100,100)$ shows a very dense distribution, which means that it is more certain reconstruction option to be equal possible, than the case of $\alpha = (10,10,10)$.

Every graph, in this subchapter, shows the change of distribution according to the value of α-hyperparameter. As we can see from the figures and as we mentioned above, the higher the value of the parameter, the denser the points near the top with the highest value. This means that the reconstruction represented by this top is the most possible. Moreover, we can observe that as well as to magnify the values of the parameter, the rest are not excluded as versions and are likely to be valid. In other cases, the α-hyperparameter would have a symmetric Dirichlet Distribution because we start from scratch and we would not have previous knowledge and the values of α would be equal, more specifically $\alpha_1 = \alpha_2 = \alpha_3$. As $\alpha_1, \alpha_2, \alpha_3$ are the three different reconstructions of a point of interest and these values are equals, means that these reconstructions are equal possible. In our work we wanted to start from values of α-hyperparameters which can give values which indicate what the opinion of the archaeologists is up to now, as we know from discussions. However, we implemented these GUIs because we wanted values which are changing the distribution quickly so we can get faster conclusions related to archaeologists' input.

As we described in the previous subchapter 5.2, archaeologists give their opinion about how sure they are for the three alternatives reconstructions of every point through sliders. These sliders have zero as a minimum value and 10 as a maximum value.

All GUIs were conducted for n=1000. We choose n=1000 in order to have a clearer view of distribution in the graphs. More specific, there are 1000 points in every triangle for the needs of the GUIs. For these GUIs we used the software environment R and the library "DirichletReg".

Archaeologists	**Slider 1 (θ_1)**	**Slider 2 (θ_2)**	**Slider 3 (θ_3)**
Archaeologist 1	0.5	0.3	0.2
Archaeologist 2	0.7	0.2	0.1
Archaeologist 3	0.4	0.3	0.3
Archaeologist 4	0.2	0.5	0.3
Archaeologist 5	0.9	0.1	0
Archaeologist 6	0.4	0.4	0.2
Archaeologist 7	0.2	0.2	0.6
Archaeologist 8	0.3	0.3	0.4

Archaeologist 9	0.8	0.1	0.1
Archaeologist 10	0.8	0	0.1

Table 5.1: The sliders' input of ten archaeologists

In table 5.1, we assumed that ten archaeologists gave their belief about the reconstruction of a point of interest through sliders. If they thought that a reconstruction version of a point was very likely then they moved the slider to the right. Suppose we have three alternative reconstructions of a wall. These interpretations are θ_1, θ_2, θ_3 which are the values of slider in the User Interface, after moving them to the right. By θ, we mean the expert archeologist's opinion about a reconstruction option. After the input of every archaeologist the α hyperparameter changes as follow:

$$a_i = a_i + \theta_i x10,\ where\ i = 1,2,3\ \ (1)$$

The ten archaeologists gave values to the sliders ranging from 0 to 1 (Table 5.1) multiplied with the number 10 and the following are derived respectively for each archaeologist: [5,3,2] , [7,2,1] , [4,3,3] , [2,5,3] , [9,1,0] , [4,4,2] , [2,2,6] , [3,3,4] , [8,1,1] , [8,0,1]. The input of archaeologists

In the first case, the hyperparameters for the distribution are α = (1,1,1). The input of the first archaeologist (Archaeologist 1 of Table 5.1) was θ_1 = 0.5, θ_2 = 0.3, θ_3 = 0.2 and these values are multiplied with the value 10 and the numbers [5,3,2] are derived. After this input and according to the previous formula (1), the α hyperparameter takes the values $a_1 = 1 + 0.5x10 = 6$, $a_2 = 1 + 0.3x10 = 3$, $a_3 = 1 + 0.2x10 = 2$, when adding the initial alpha (1,1,1). The second archaeologist (Archaeologist 2 of Table 5.1) gives to the sliders the valuesθ_1 = 0.7, θ_2 = 0.2, θ_3 = 0.1 and we convert these values to 7,2 and 1. The alpha hyperparameter after the input of the first archaeologist has the values α = (6,3,2) and now takes the values $a_1 = 6 + 0.7x10 = 13$, $a_2 = 3 + 0.2x10 = 6$, $a_3 = 2 + 0.1x10 = 4$, $a_3 =$ (13,6,3). Similarly, the values of hyperparameter change after the input of every archaeologist as follow: (6,4,3), (13,6,4), (17,9,7), (19,14,10), (28,15,10), (32,19,12), (34,21,18), (37,24,22), (45,25,23), (53,025,24).

The script of Figure 5.20 is an example of our code for the simulation of the updated Dirichlet Distribution for the archaeologists' input values (6,4,3), (13,6,4) and (17,9,7), starting with α = (1,1,1).

```
library("DirichletReg")

project.to <- function(p1,p2,p3) {

  x = 1.0 / 2
  y = 1.0 / (2 * sqrt(3))
  # Vector 1 - bisect out of lower left vertex
  x = x - (1.0 / sqrt(3)) * p1 * cos(pi / 6)
  y = y - (1.0 / sqrt(3)) * p1 * sin(pi / 6)
  # Vector 2 - bisect out of lower right vertex
  x = x + (1.0 / sqrt(3)) * p2 * cos(pi / 6)
  y = y - (1.0 / sqrt(3)) * p2 * sin(pi / 6)
  # Vector 3 - bisect out of top vertex
  y = y + (1.0 / sqrt(3) * p3)

  c(x,y)
}
attach(mtcars)
par(mfrow=c(2,2))

n <- 100
pts1 <- rdirichlet(n, alpha=c(1,1,1))
pts2 <- rdirichlet(n, alpha=c(6,4,3))
pts3 <- rdirichlet(n, alpha=c(14,6,4))
pts4 <- rdirichlet(n, alpha=c(18,9,7))


plot(pts1,type="n",xlim=c(0,1),ylim=c(0,1),main="a =(1,1,1)")
lines(c(0,1,.5,0),c(0,0,0.866,0))
text(0,.07,bquote(paste(alpha[1])))
text(1,.07,bquote(paste(alpha[2])))
text(.5,.9,bquote(paste(alpha[3])))

for(i in 1:n) {
  projection <- project.to(pts1[i,1],pts1[i,2],pts1[i,3])
  points(projection[1], projection[2], pch=1)
}
```

```
plot(pts2,type="n",xlim=c(0,1),ylim=c(0,1),main="a =(6,4,3)")
lines(c(0,1,.5,0),c(0,0,0.866,0))
text(0,.07,bquote(paste(alpha[1])))
text(1,.07,bquote(paste(alpha[2])))
text(.5,.9,bquote(paste(alpha[3])))

for(i in 1:n) {
  projection <- project.to(pts2[i,1],pts2[i,2],pts2[i,3])
  points(projection[1], projection[2], pch=1)
}

plot(pts3,type="n",xlim=c(0,1),ylim=c(0,1),main="a =(13,6,4)")
lines(c(0,1,.5,0),c(0,0,0.866,0))
text(0,.07,bquote(paste(alpha[1])))
text(1,.07,bquote(paste(alpha[2])))
text(.5,.9,bquote(paste(alpha[3])))

for(i in 1:n) {
  projection <- project.to(pts3[i,1],pts3[i,2],pts3[i,3])
  points(projection[1], projection[2], pch=1)
}

plot(pts4,type="n",xlim=c(0,1),ylim=c(0,1),main="a =(17,9,7)")
lines(c(0,1,.5,0),c(0,0,0.866,0))
text(0,.07,bquote(paste(alpha[1])))
text(1,.07,bquote(paste(alpha[2])))
text(.5,.9,bquote(paste(alpha[3])))

for(i in 1:n) {
  projection <- project.to(pts4[i,1],pts4[i,2],pts4[i,3])
  points(projection[1], projection[2], pch=1)
}
```

Figure 5.20: Script for the simulation of Dirichlet Distribution using R.

The Figure 5.21 Figure 5.22, Figure 5.23 represent the updating of the Dirichlet Distribution after every archaeologist's input. In Figure 5.21 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.2 (b) archeologists have given in the sliders the values [5,3,2] and the α hyperparameter is adjusted as follows $\alpha_1 = 6$, $\alpha_2 = 4$, $\alpha_3 = 3$. In the figure 5.21 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist,α hyperparameter is adjusted as follows $\alpha_1 = 13$, $\alpha_2 = 6$, $\alpha_3 = 4$.In the figure 5.21 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 17$, $\alpha_2 = 9$, $\alpha_3 = 7$. In the figures 5.21 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

3

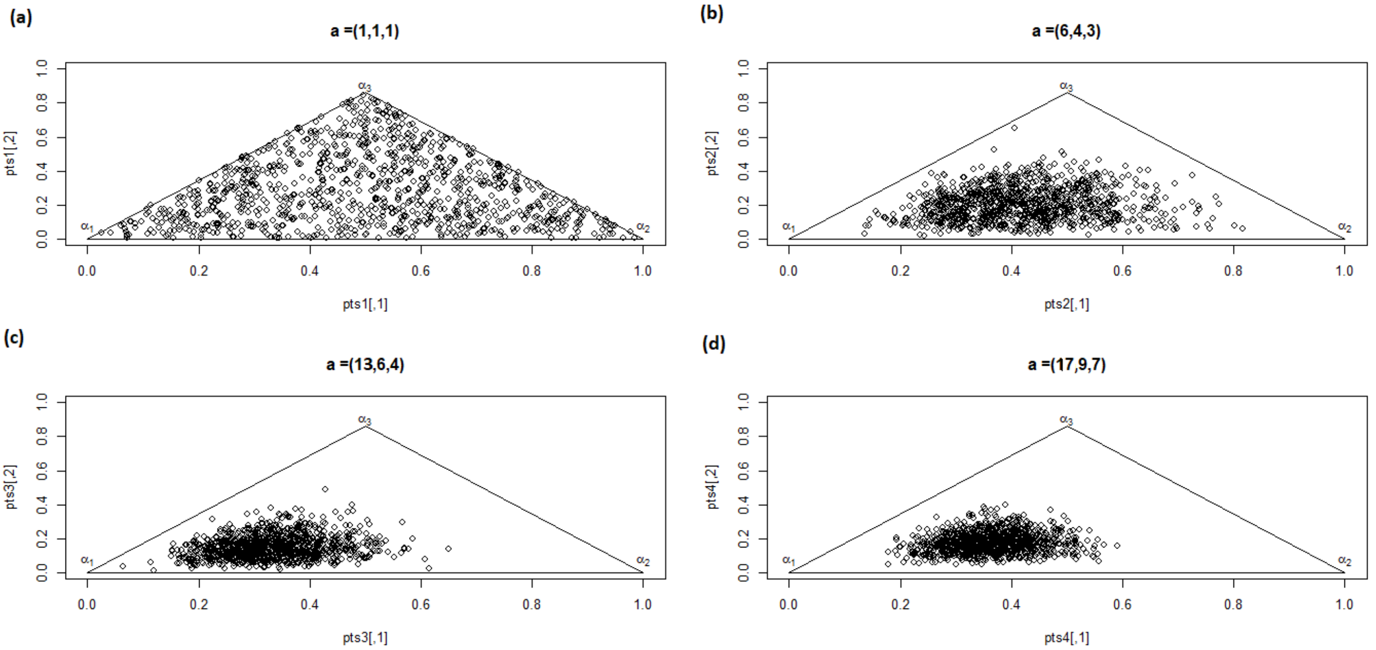

Figure 5.21: Dirichlet Distribution with hyperparameters: (a) α = (1,1,1), (b) α = (6,4,3), (c) α = (13,6,4), (d) α = (17,9,7)

In Figure 5.21 (a) archeologists have given in the sliders the values [2,5,3] and the α hyperparameter is adjusted as follows $\alpha_1 = 19$, $\alpha_2 = 14$, $\alpha_3 = 10$. As we can see the points have been moved to the center of the triangle after the values have increased on all sides. In the figures 5.3 (b), (c) and (d) archaeologists have given the values [9,1,0], [4,4,2] and [2,2,6] respectively. It is clear from these figures that the values of a_1 are much higher from a_2 and a_3, with a significant difference from the other two. For this reason, the points of the triangle are closer to the vertex of a_1.

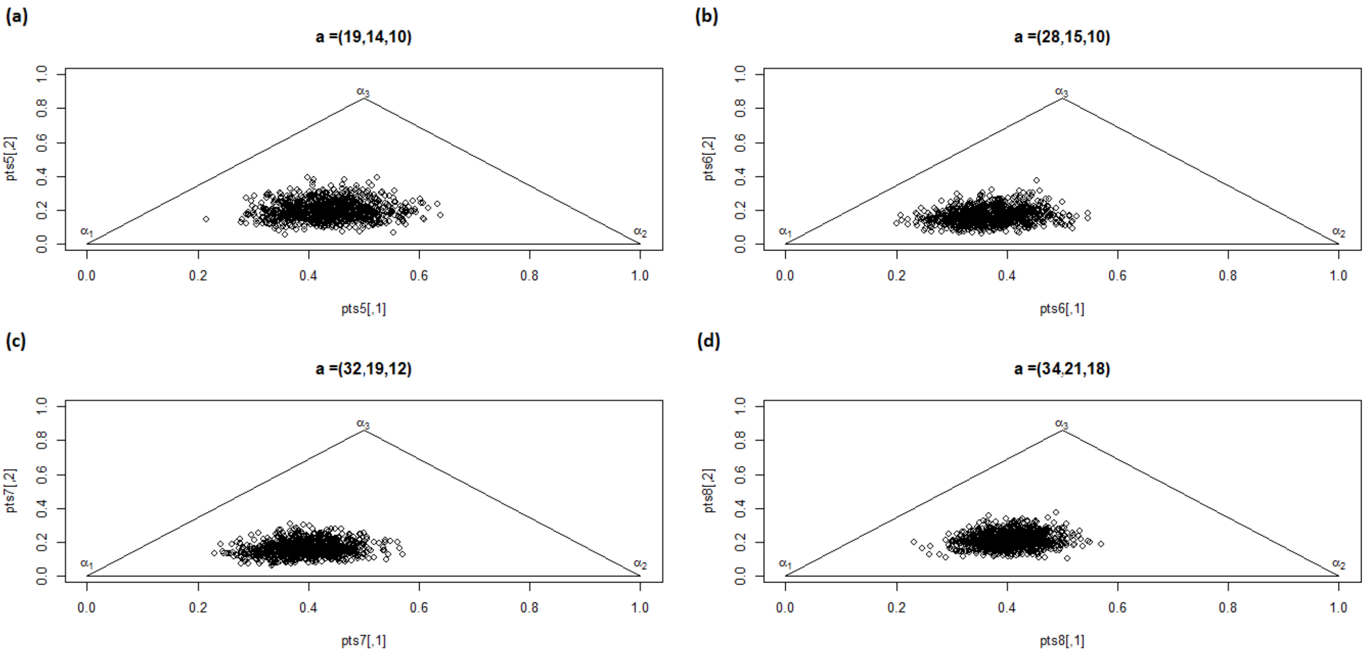

Figure 5.21: Dirichlet Distribution with hyperparameters: (a) α = (19,14,10), (b) α = (28,15,10), (c) α = (32,19,12), (d) α = (34,21,18)

In Figure 5.4 (a) archeologists have given in the sliders the values [3,3,4]. and the α hyperparameter is adjusted as follows $\alpha_1 = 38$, $\alpha_2 = 24$, $\alpha_3 = 22$. In the figures 5.4(b) and (c) archaeologists have given the values [8,1,1] and [8,0,1] respectively. It is clear from these figures that the values of a_1 are much higher from a_2 and a_3, with a significant difference from the other two. For this reason, the points of the triangle are closer to the vertex of a_1. Moreover, we can observe that the values of α-hyperparameter (α_1, α_2, α_3) are high and all points are aggregated into one point, always closer to a_1.

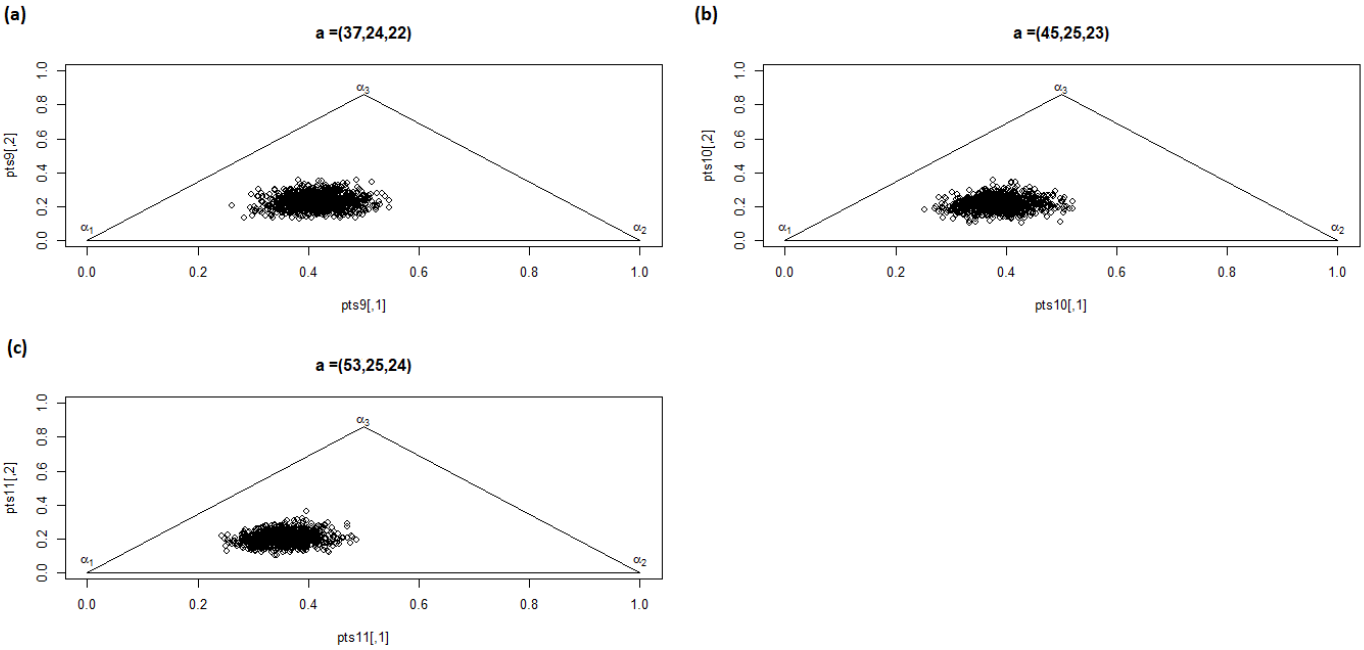

Figure 5.22: Dirichlet Distribution with hyperparameters: (a) α = (37,24,22), (b) α = (45,25,23), (c) α = (53,025,24)

In the second case, the hyperparameters for the distribution are α = (10,10,10). In the Figure 5.22 (a) the first archeologist has given in the sliders the values [5,3,2] and the α hyperparameter is adjusted as follows $\alpha_1 = 15$, $\alpha_2 = 13$, $\alpha_3 = 12$. In the figure 5.22 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist,α hyperparameter is adjusted as follows $\alpha_1 = 22$, $\alpha_2 = 15$, $\alpha_3 = 13$.In the figure 5.22 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 26$, $\alpha_2 = 18$, $\alpha_3 = 4$. In the figures 5.22 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1. With the same way the values of hyperparameter change after the input of every archaeologist as follow: [15,13,12], [22,15,13], [26,18,16], [28,23,19], [37,24,19], [41,28,21], [43,30,27], [46,33,31], [54,34,32], [62,34,33].

The Figure 5.5, Figure 5.6, Figure 5.7 represent the updating of the Dirichlet Distribution after every archaeologist's input in case 2.

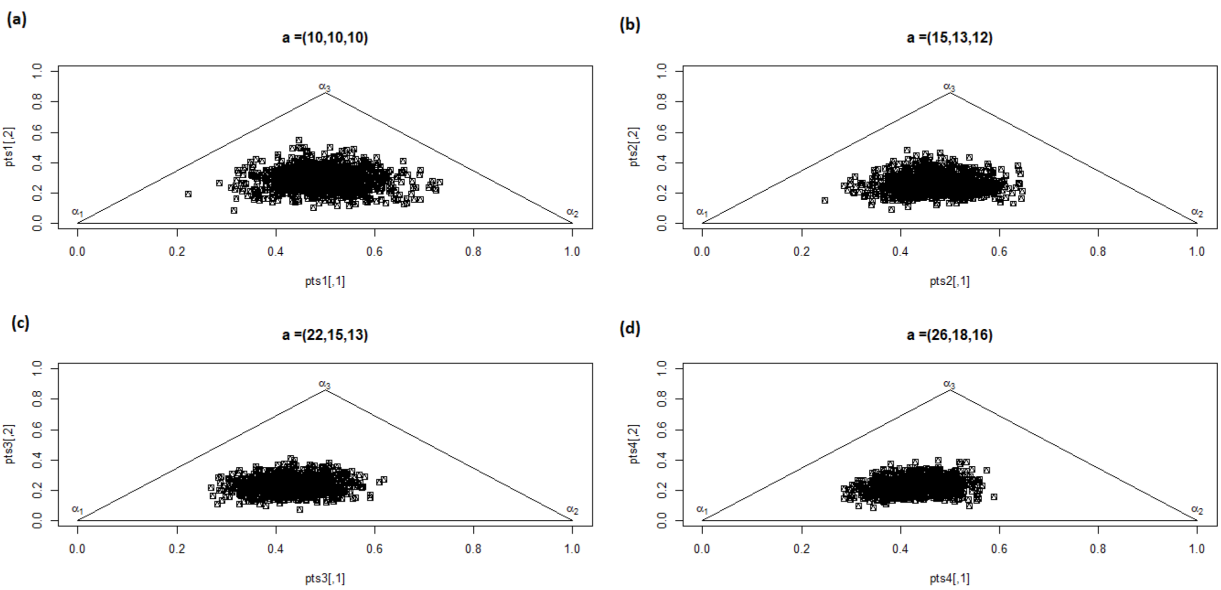

Figure 5.23: Dirichlet Distribution with hyperparameters: (a) α = (10,10,10), (b) α = (15,13,12), (c) α = (22,15,13), (d) α = (26,18,16)

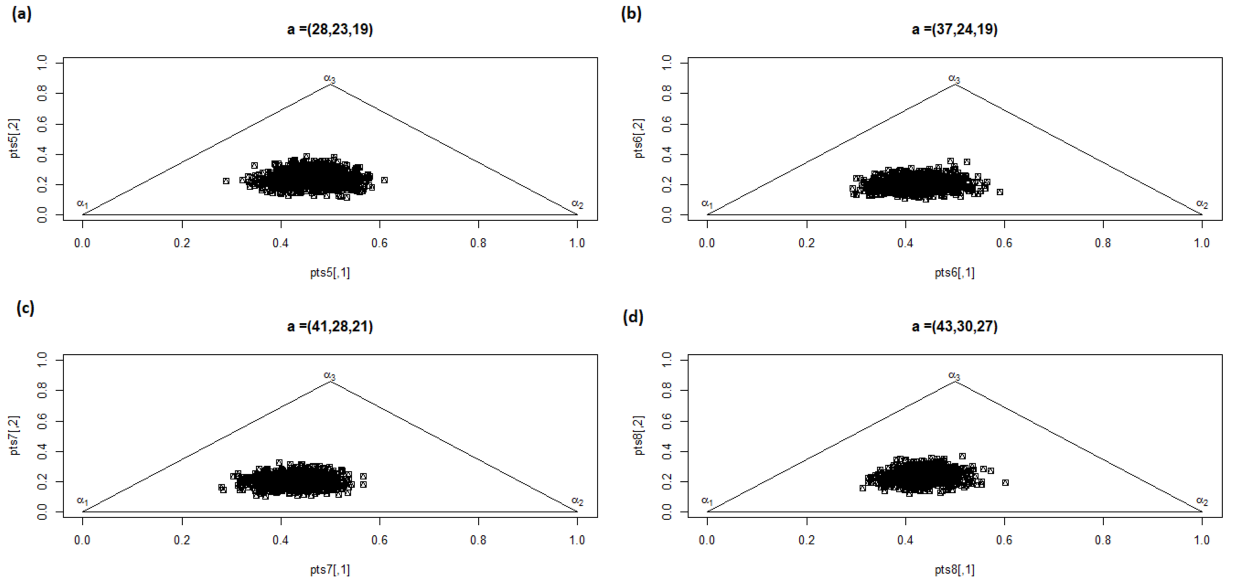

Figure 5.24: Dirichlet Distribution with hyperparameters: (a) α = (28,23,19), (b) α = (37,24,19), (c) α = (41,28,21), (d) α = (43,30,27)

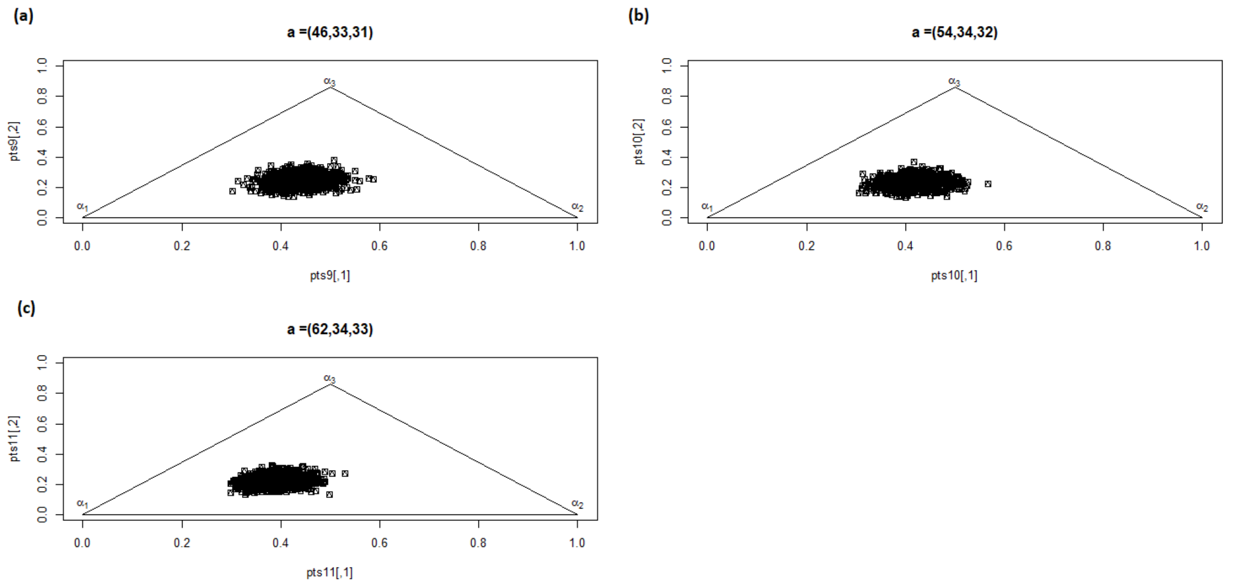

Figure 5.25: Dirichlet Distribution with hyperparameters: (a) α = (46,33,31), (b) α = (54,34,32), (c) α = (62,34,33)

In the second case, the hyperparameters for the distribution are α = (100,100,100). In the Figure 5.26 (a) the first archeologist has given in the sliders the values [5,3,2] and the α hyperparameter is adjusted as follows $\alpha_1 = 105$, $\alpha_2 = 103$, $\alpha_3 = 102$. In the figure 5.26 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist,α hyperparameter is adjusted as follows $\alpha_1 = 112$, $\alpha_2 = 105$, $\alpha_3 = 103$.In the figure 5.26 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 116$, $\alpha_2 = 108$, $\alpha_3 = 106$. In the figures 5.26 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1. With the same way the values of hyperparameter change after the input of every archaeologist as follow: [105,103,102], [112,105,103], [116,108,106], [118,113,109], [127,114,109], [127,114,109], [131,118,111], [133,123,121], [144,124,122], [152,124,123].

The Figure 5.26, Figure 5.27, Figure 5.28 represent the updating of the Dirichlet Distribution after every archaeologist's input in case 3.

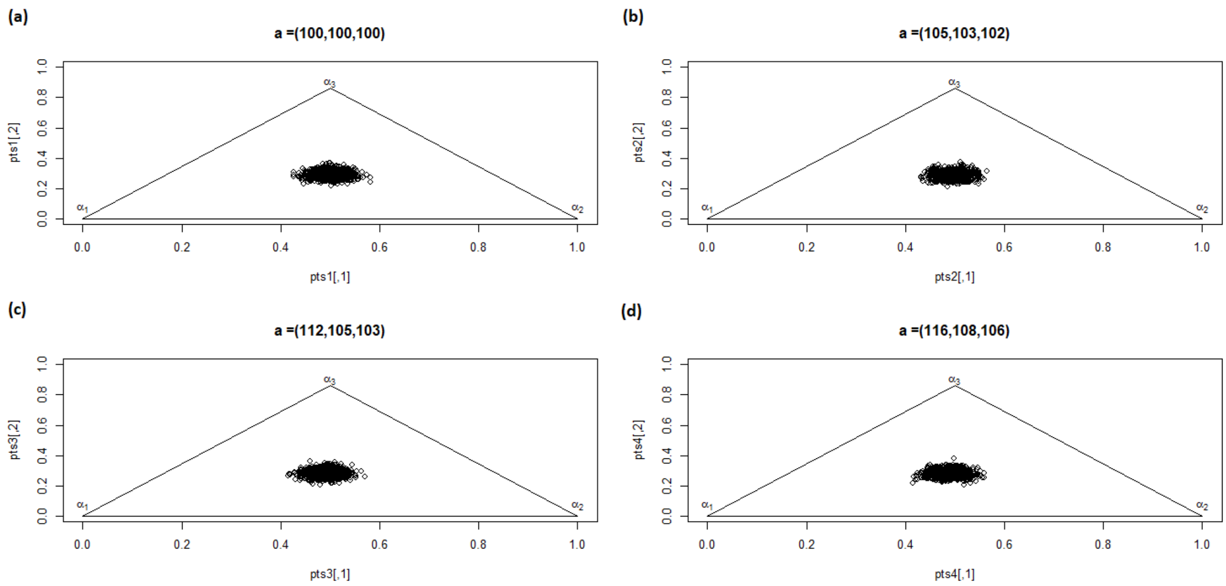

Figure 5.26: Dirichlet Distribution with hyperparameters: (a) α = (100,100,100), (b) α = (105,103,102), (c) α = (112,105,103), (d) α = (116,108,106)

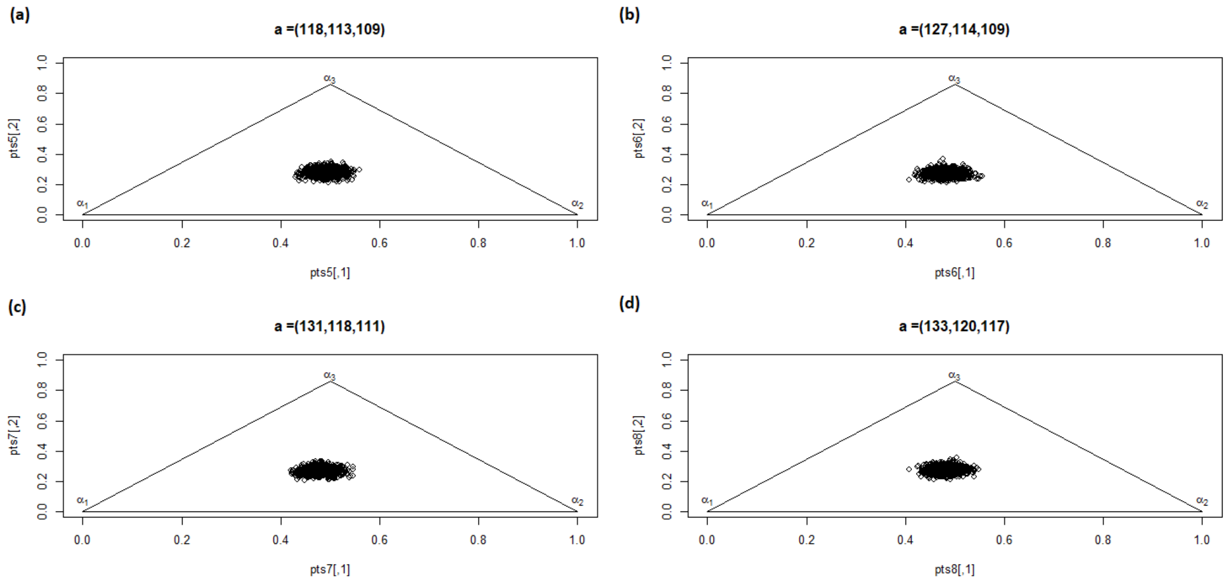

Figure 5.27: Dirichlet Distribution with hyperparameters: (a) α = (118,113,109), (b) α = (127,114,109), (c) α = (131,118,111), (d) α = (133,120,117)

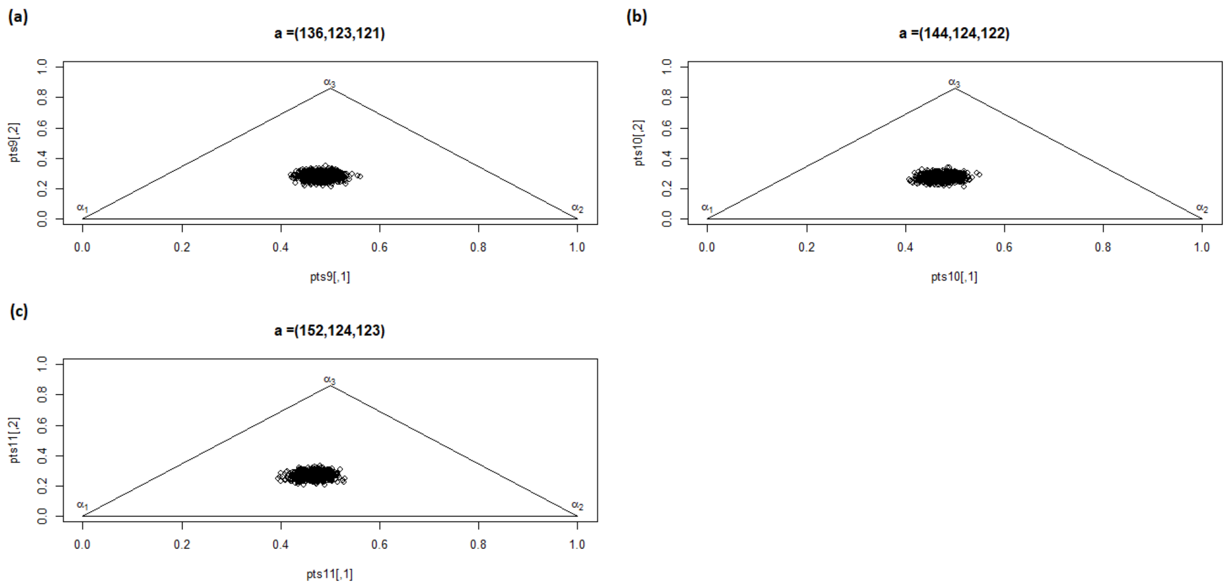

Figure 5.28: Dirichlet Distribution with hyperparameters: (a) α = (136,123,121), (b) α = (144,124,122), (c) α = (152,124,123)

5.3.3 The Counter c

As it is described in this work there are three points of interest of Zakros palace (Central Court, Tank Room and the Entrance of the Palace) taking part in our research. For each of these points of the Palace there are three possible hypotheses of how this point may have been in the past and these hypotheses are called reconstructions in archaeology. Using the Dirichlet distribution Dir(α), in this work, we are trying to treat the uncertainty concerning these points of interest. In the previous section 5.3.2, the definition of alpha hyperparameter and how the distribution changes depending on the initial value of alpha hyperparameter (α) were described. These distribution changes were observed through GUIs (Figures 5.21-5.28). In these GUIs, only the opinion of archaeologists about how likely each reconstruction is, concerning a point of interest, was taken into account. The input for these GUIs was the personal opinion of archaeologists about how certain is every reconstruction. This opinion of archaeologists is quantified through sliders, which are implemented in a User Interface as it is described in previous sections and as it is illustrated in Figures 5.17, 5.18, 5.19 and they can take values from 0 to 1.

However, after a short discussion with the archaeologist, Dr Anna Simantiraki, it was clear that there are some evidence types which influence the opinion of archaeologists and these types are called influencing factors. These evidence or the so-called influencing factors which represent types of archaeological evidence of significance for the Minoan civilization are features (pits, walls, ditches, etc.), artefacts (tool, work of arts, etc.), comparisons (comparison with other structures of the same era or type of building), topography (area elevation, region characteristics, etc.), peer review (discussion with experts). After identifying which are the evidence types and after more discussions, we came to the conclusion that they should be taken into account in our system. More specifically, it was essential to show how these influencing factors form the knowledge of archaeologists and this could be done while they give their opinion as input in the User Interface. For this reason, except for the part of the User Interface with the sliders, where archaeologists insert how confident for every reconstruction they are, a part with checkboxes was implemented. Every checkbox correspond to an influencing factor and the archaeologists should check these which represent the influencing factors that led them to form their opinion about every reconstruction (Figures 5.17-5.19).

This part of our thesis, was important as it becomes clear that our system cannot take into account in the same way the opinions of all archaeologists. As it is logical, the opinion of archaeologists based on significant findings should be taken into account more seriously, than those who do not have findings to support their opinion. For example, if the opinion of an archaeologists is based on remains or wall which are included in “features” influencing factors, this opinion should have higher significance for our system than the opinion of an archaeologist who has not significant findings in order to form his knowledge. This influenced opinion of archaeologists is referred to in our text as counter c.

The counter c is a combination of the opinion of archaeologists about how confident they are of a reconstruction and the importance of the influencing factors that led to the formation of this opinion. In previous sections the User Interface which gives this information to our system is described in details. In Figures 5.4-5.8 the User Interface where archaeologists give their belief about every influencing factor is illustrated and in Figures 5.17-5.19 the User Interface where archaeologists give their input about every reconstruction of each point of interest in the Palace is illustrated. More specifically, through the user interface which implemented, using the Unity environment, archaeologists gave us the information about how significant every evidence type is according to their opinion and the information about their knowledge for every reconstruction of the three points of interests studied in this work. In the next paragraphs the input of this user interface results and how we exploit it in our system is described.

The first stage of the User Interface, which implemented for archaeologists and through which they give their opinion about how important every influencing factor can be, is separated into two parts. The first part includes a group of questions for every influencing factor. There are 8 questions for features, 4 for artefacts, 7 for comparative data, 5 for peer review as they are illustrated in Figures 5.4-5.7. The archeologists are offering a judgment on a 7-point scale, ranging from Never to Always for each statement (Figure 5.3), which correspond to integers from 1 to 7. For every influencing factor, a number is derived from the average of these statements. The second part contains a ranking number for every influencing factor from 1 to 5, because we have five evidence types and the number 1 represents the least important influencing factor and the number 5, the most important influencing factor (Figure 5.8). After completing these two parts of the questionnaire a number is obtained by adding the average derived from the first part for the questions included in each influencing factor and the ranking number for the corresponding evidence type of the second part. For example, after an archaeologist completing the filling of the questionnaire, if the average of the first part for "features" influencing factor is 3 and he/she believes that this factor is the most important, giving to this the value 5 in the second part, the final numerical number which is derived is 8 for the "feature" influencing factor. Then, after the input of another archaeologist (a second archaeologist), if the average of the first part for "features" influencing factor is 2 and he/she believes that this factor is an important influencing factor but not the most important among them, giving to this the value 4 in the second part, the final numerical number which is derived is 6 "feature" influencing factor, according to his/her answers. After the input of these two archaeologists, the total average is calculated. More specifically, the average in this example is (8 + 6)/2 = 7 and this number is the current value which represents how important this influencing factor is, which is referred to our work as weight and in the specific example is the weight of features. If for artefacts this result is 5, it is clear that because features have a higher value than artefacts, the former are more important as evidence than the latter.

The second step is for expert archaeologists to utilize a separate user interface giving their belief through sliders about whether the visualized reconstructions are accurate. There are three different reconstruction versions for each of the three points of interest. These points are in Central Court, in Tank Room and at the Entrance of the Palace. For each point, there are three possible reconstruction which show how this point of interest could be in the past. If archaeologists believe that the point, we are referring to, is likely to have been as visualized in the past, they move the slider to the right. Otherwise they move the slider to the left. Every slider takes values from 0 to 1, and we convert this value from 0 to 10, respectively. As it is mentioned above, we thought that these values are not enough for our system. As it is already explained, archaeologists can choose which influencing factors led to their opinion. This input for the influencing factors is given through checkboxes in the User Interface. The checkboxes can be checked, if an archaeologist was supported to this factor, or unchecked if this factor did not lead to any conclusion.

The first stage of the User Interface gives to our system the information about how important is every influencing factor which is represented from the final result derived for each influencing factor and is referred to this thesis as weight of influencing factors, denoted as w .The second stage of the User Interface gives to our system the information about how certain every archaeologist is about the reconstructions studied for each point of interest, which is denoted as θ, and which influencing factors formed this belief. The values, which are referred as weights (w_i, $where\ i = the\ evidence\ type$) corresponding to each of the influencing factors are numbers derived from the first stage of the User Interface. After archaeologists give their opinion about every reconstruction (Figures 5.17-5.19), the sum of weights of the selected influencing factors is calculated in order to identify how accurate is every archaeologist's input. Therefore, we calculate the total confidence of an archaeologist for a reconstruction as $w_f + w_a + w_c + w_t + w_p$, where w_f the weight of "features" factor, w_a the weight of "artefacts" factor, w_c the weight of "comparisons" factor, w_t the weight of "topography" factor and w_p the weight of "peer review" factor. As counter c is defined the sum of the weights of archaeologists input multiplied with the values of sliders. More specifically,

$$c_i = (w_f + w_a + w_c + w_t + w_p) \text{ x } \theta_i\ x\ 10\ (2)$$

$$\text{where } i = 1,2,3 \textit{ the possible reconstruction of interest point}$$

and θ_i is the confidence of archaeologists with every reconstruction for a specific point of interest, represented by the sliders and multiplied with 10. This counter c, gives to our system the knowledge of archaeologists depending on the evidence which form this knowledge. Combining the equations (1) and (2), the alpha hyperparameter is calculated from the equations (3) and (4):

$$a_i = a_i + c_i \ (3)$$

$$a_i = a_i + (w_f + w_a + w_c + w_t + w_p) \text{ x } \theta_i x 10 \ \ (4)$$

$$, where\ i = 1,2,3 \textit{ the possible reconstruction of interest point}$$

However, as it is described in section 5.3.2 there is an assumption about the selection of the initial value of alpha hyperparameter (α priori), based on how large it would be in relation to the values which represent the observed data. In our case study observed data is the new input of archaeologists in the User Interface (the first stage of User Interface is illustrated in Figures 5.4-5.4 and the second stage of the User Interface is illustrated in Figures 5.17-5.19). As the knowledge of archaeologists in section 5.3.2 was based only on the input of archaeologists interacting only in the sliders (Figures 5.17-5.19), which offer the information about how certain they are for each reconstruction ($\theta_1, \theta_2, \theta_3$). In this section, as the knowledge of archaeologists is based on the counter c (c_1, c_2, c_3), which takes into account the opinion of archaeologists and the influencing factors which led to this opinion, as it is described in the previous paragraphs. The goal of calculating the alpha hyperparameter through GUIs of the section 5.3.2 was to select a value for the initial alpha hyperparameter (α priori), considering what values the initial values of α (a_1, a_2, a_3) could be based on the values which represent the input of archaeologists for every reconstruction of every point of interest, ($\theta_1, \theta_2, \theta_3$). In case the alpha hyperparameter had an initially set of value a = (100,100,100) and the value of ($\theta_i x 10$) could be a value up to 10, which is high based on the knowledge of archaeologists. Therefore, the density of the distribution will change at a slow rate and there could be no conclusions about what is the most likely reconstruction according to archaeologists as the new values of alpha hyperparameter after the input of archaeologists would be to close with the initial values of alpha hyperparameter. On the other hand, if the initial value of α hyperparameter is small (a = (1,1,1)), after the input of every archaeologist though the User Interface (Figures 5.17-5.19) the distribution becomes denser at a faster rate, which is not appropriate for our case study, as after the input of the first archaeologist the new values of alpha hyperparameter will be far from the initial values, and the input of the next archaeologists will influence less the distribution, comparing with the change after the input of the first archaeologist.

In this section, we choose to take into account the significance of every influencing factor which leads to archaeologists' opinion for a reconstruction. In a previous example, two archaeologists give their input in the first stage of the User Interface which was designed to calculate the significance of each influencing factor (Figures 5.4-5.8). The results which were derived from the two archaeologists for the "features" factor, was 8 from the first archaeologist and 6 from the second. The average of these two results is 7 and represents the weight of the influencing factor "features", $w_f = 7$. After this step we want to calculate the counter c based on this w_f and the input of these two archaeologists θ_i, which is offered by archaeologists through the sliders (Figures 5.17-5.19) for the three reconstructions corresponding to a specific point of interest. From the equation (2) we can calculate the counter c as $c_i = w_f$ x $\theta_i \, x \, 10$. If the first archaeologist offers to the sliders of the User Interface (Figures 5.17-5.19) the values $\theta_1 = 0.5$, $\theta_2 = 0.3$, $\theta_3 = 0.2$ and the "features" factor is selected by the first archaeologist through the checkboxes ($w_f = 7$), the counter c is calculated as $c_1 = 7\, x0\,.5\, x\, 10 = 35$, $c_2 = 7\, x\, 0.3\, x\, 10 = 21$, $c_3 = 7\, x\, 0.2\, x\, 10 = 14$. If another archaeologist offers to the sliders of the User Interface (Figures 5.17-5.19) the values $\theta_1 = 0.5, \theta_2 = 0.3, \theta_3 = 0.2$ and only the "features" factor is selected by the second archaeologist through the checkboxes ($w_f = 7$), the counter c is calculated as $c_1 = 7\, x0\,.7\, x\, 10 = 49$, $c_2 = 7\, x\, 0.2\, x\, 10 = 14$, $c_3 = 7\, x\, 0.1\, x\, 10 = 7$.

In the first stage of the User Interface, archaeologists give their opinion about every influencing factor answering the question which was described above (Figures 5.4 – 5.7) and giving a ranking number based on how important each of these factors is according to their belief (Figure 5.8). A number is obtained by adding the average derived from the first part for the questions included in each influencing factor and the ranking number for the corresponding evidence type of the second part. However, as it is described in the previous paragraph, we want the alpha hyperparameter values to not be increased at a high rate neither at a slow rate, and for this reason, the results of the first stage of the User Interface (Figures 5.4 – 5.8) cannot be used in our system and they only determine the order of importance of the influencing factors, for instance, the "features" factor is more important than the "artefacts" factor. For this reason, we have to identify which can be the numbers, referred as weights (w), which represent the significance of every influencing factor. Every factor will take a number as its weight depends on its importance. For example, if in the next steps we decide that the most important type will take the value 2 as a weight, if the results of the questionnaire show that the most important factor is the "features", then this factor will take the value 2 as its weight, $w_f = 2$. However, if the results of the questionnaire show that the most important factor is the "artefacts", then this factor will take the value 2 as its weight, $w_a = 2$. In order to decide which would the numerical weight would be of the most significant influencing factors as well as the ones appearing less significant, we conducted certain statistical GUIs using the Dirichlet distribution, based on the R software.

In this section, we want to examine which is the right choice for the weight of the influencing factors w_i. As it is mentioned, alpha hyperparameter has the same number of elements as the number of the possible reconstructions for every point we study, $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. To achieve the goal of specifying the counter c, we implement GUIs for the different values of α priori which used to in section 5.3.2, which are $\alpha = (1,1,1)$, $\alpha = (10,10,10)$, $\alpha = (100,100,100)$. We continue to these GUIs with the assumption of the previous section that the values of $\alpha_1, \alpha_2, \alpha_3$ should have equal values $\alpha_1 = \alpha_2 = \alpha_3$, because no evidence exists without the input of archaeologists. For the GUIs of section 5.3.2, we chose three different combinations of $\alpha = (\alpha_1, \alpha_2, \alpha_3)$, which are $\alpha = (1,1,1)$, $\alpha = (10,10,10)$, $\alpha = (100,100,100)$. The order of the significance of each influencing factor derived through the results of the first stage of the User Interface (Figures 5.17 – 5.19), where archaeologists offered their opinion for each influencing factor. The results of the User Interface are the values of weights of every influencing factor denoted as w_i and the order of the significance of each influencing factor results by comparing these weight values with each other. For example, if $w_f > w_a$, where w_f is the weight of features and w_a the weight of artefacts, results that the "features" factor is more significant than the "artefacts" factor. In order to implement the following GUIs (Figures 5.29-5.53), we assumed that the archaeologists before they offer their opinion about each influencing factor in the second stage of the User Interface and as it is shown in Table 5.2, they offered their opinion in the first stage of User Interface, answering the questions for each influencing factor (Figures 5.4-5.7) and giving a ranking number for each one of them (Figure 5.8). For this example, the order of the significance of each influencing factor which was resulted, we assumed that it was the following starting from the most significant to the least significant: features, artefacts, absolute comparisons, peer review, topography. In the first GUI, we gave arbitrarily the following values to the weight of the factors: $w_f = 2.25, w_a = 2, w_c = 1.75, w_t = 1.5, w_p = 1.25$ and in the second the values of the importance of every factor were $w_f = 2, w_a = 1.75, w_c = 1.5, w_t = 1.25, w_p = 1$.

As it is described above, archaeologist give their knowledge about how certain they are for each reconstruction through the sliders implemented in the second stage of the User Interface (Figures 5.17-5.19). The values of the sliders are denoted as $\theta_1, \theta_2, \theta_3$ which correspond to the three possible reconstructions of a specific point of interest in the Palace. Furthermore, in the second stage of the User Interface, archaeologists can choose more than one influencing factor, which formed their belief (θ_i) about each reconstruction of a specific point of interest. If the findings of archaeologists which influenced their opinion belong to more than one influencing factor, they would check these influencing factors and the final value will be the sum of their values. For example, if the evidence at hand were wall remains and a comparison with a similar archaeological site, the archeologists would check the features and the absolute comparison. So, the final value will would be, for the first GUI, $w_f + w_c = 2.25 + 1.75 = 4$ and, for the second GUI, $w_f + w_c = 2 + 1.5 = 3.5$.

Archaeologists	**Slider 1 (θ_1)**	**Slider 2 (θ_2)**	**Slider 3 (θ_3)**	**Features**	**Artefacts**	**Absolute Comparison**	**Topography**	**Peers**
Archaeologist 1	0.5	0.3	0.2	YES	YES	NO	YES	YES
Archaeologist 2	0.7	0.2	0.1	YES	NO	YES	YES	YES
Archaeologist 3	0.4	0.3	0.3	YES	YES	YES	NO	NO
Archaeologist 4	0.2	0.5	0.3	YES	NO	NO	YES	YES
Archaeologist 5	0.9	0.1	0	YES	NO	NO	NO	YES
Archaeologist 6	0.4	0.4	0.2	YES	YES	YES	YES	NO
Archaeologist 7	0.2	0.2	0.6	YES	NO	YES	YES	YES

Archaeologist 8	0.3	0.3	0.4	YES	YES	NO	NO	NO
Archaeologist 9	0.8	0.1	0.1	YES	NO	YES	YES	YES
Archaeologist 10	0.8	0	0.1	YES	YES	NO	NO	YES

Table 5.2: Archaeologists' Input through the User Interface

The Table 5.2 shows the input of the sliders for every archaeologist and the input of the checkboxes for the three reconstruction for a specific point of interest. The values to the sliders take values from 0 to 1 and the values of checkboxes are YES (if the checkbox is selected) or NO (if the checkbox is not selected) which are recorded in database as 1 and 0, respectively.

In these GUIs we keep the same values for the sliders ($\theta_1, \theta_2, \theta_3$) as in the section 5.3.2, which are shown in the Table 5.2. More specifically, we assumed that ten archaeologists offered their opinion about each reconstruction through the User Interface (Figures 5.17 – 5.19) as values to the sliders ranging from 0 to 1 (Table 5.2 – Slider 1 (θ_1), Slider 2 (θ_2), Slider 3 (θ_3)), multiplied with the number 10 and the following values are derived respectively for each archaeologist: [5,3,2] , [7,2,1] , [4,3,3] , [2,5,3] , [9,1,0] , [4,4,2] , [2,2,6] , [3,3,4] , [8,1,1] , [8,0,1]. After giving the input about their confidence for every reconstruction of a specific point of interest through the sliders of the User Interface (Figures 5.17 – 5.19), archaeologists selected the factors which influenced their belief, through checkboxes in the same part of the User Interface for a specific point of interest of the Palace, as it is shown in Table 5.2. In this table "YES" shows that this influencing factor led to the formation of an archaeologist's belief and "NO" shows that this influencing factor did not lead to the formation of an archaeologist's belief. We assumed that the input values of each archaeologist for a point of interest are the ones that follow: [YES, YES, NO, YES, YES], [YES, NO, YES, YES, YES], [YES, YES, YES, NO, NO], [YES, NO, NO, YES, YES], [YES, NO, NO, NO, YES], [YES, YES, YES, YES, NO], [YES, NO, YES, YES, YES], [YES, YES, NO, NO, NO], [YES, NO, YES, YES, YES], [YES, YES, NO, NO, YES].

As we described above, we assume that the order of significance of each influencing factor, is the following starting from the most significant: features, artefacts, absolute comparisons, peer review, topography. For example, the input of the first archaeologist in the checkboxes was [YES, YES, NO, YES, YES] (Table 5.2 – Archaeologist 1), which means that he checked the first, the second, the fourth and the fifth checkboxes. More specifically, the factors which affected his opinion for every reconstruction were features, artefacts, peer review and topography. The input of the second archaeologist in the checkboxes was [YES, NO, YES, YES, YES] (Table 5.2 – Archaeologist 2), which means that he checked the first, the third, the fourth and the fifth checkboxes. More specifically, the factors which affected his opinion for every reconstruction were features, absolute comparisons, peer review and topography.

Firstly, the final sum of the weights of each influencing factor which was selected through the checkboxes which implemented in the second stage of the User Interface (Figures 5.17 – 5.19), which emerged from the evidence of the archaeologist who selected these influencing factors, will be calculated and then this result will be multiplied with every value of the sliders ($\theta_1, \theta_2, \theta_3$) which were selected . This result represents the counter c and it is calculated from the equation (2). For example, if the first archaeologists' opinion was based on wall remains on an archaeological site and comparison with the other Minoan palaces, they will check the box of feature and absolute comparison. First, our application calculates the sum of the evidence values. In this example, the sum is $w_f + w_c = 2.25 + 1.75 = 4$. This number will be multiplied with every value of the sliders and we will have the final counters. Therefore, according to the equation (2), the results will be $c_1 = (w_f + w_c)x\,\theta_1 x10 = (2.25 + 1.75)x\,5 = 4x5 = 20$, $c_2 = (w_f + w_c)x\,\theta_2 x10 = (2.25 + 1.75)x\,3 = 4x3 = 12$, $c_3 = (w_f + w_c)x\,\theta_3 x10 = (2.25 + 1.75)x\,2 = 4x2 = 8$.

As first case, the hyperparameters for the distribution are α = (1,1,1). The first archaeologist offers the values $\theta_1 = 0.5, \theta_2 = 0.3, \theta_3 = 0.2$ in the sliders of the User Interface which are multiplied with the value 10 and the values [5,3,2] are derived. Then the first archaeologist checks the checkboxes of features, artefacts, peer review and topography, same for all three reconstructions related to one point of interest, which is denoted as [YES, YES, NO, YES, YES], where NO represents the unchecked checkbox for the specific factor and YES represents the checked. After this input

$$a_1 = a_1 + c_1 = 1 + (2.25 + 2 + 1.5 + 1.25)x\;0.5\;x\;10 = 36,$$

$$a_2 = a_2 + c_2 = 1 + (2.25 + 2 + 1.5 + 1.25)x\;0.3\;x\;10 = 22,$$

$$a_3 = a_3 + c_3 = 1 + (2.25 + 2 + 1.5 + 1.25)x\;0.2\;x\;10 = 15$$

Where, a_1, c_1 relate to the first reconstruction , a_2, c_2 relates to the second reconstruction and a_3, c_3 relates to the third reconstruction of one specific point of interest.

The second archaeologist offers his/her opinion of each reconstruction through the sliders of the User Interface, giving the values $\theta_1 = 0.7, \theta_2 = 0.2, \theta_3 = 0.1$ and these values are multiplied with 10, the values 7,2,1 are derived, and checks the checkboxes features, absolute comparisons, peer review and topography, same for all three reconstructions related to one point of interest, which is denoted as [YES, NO, YES, YES, YES], where NO represents the unchecked checkbox for the specific factor and the YES the checked. These are added to the alpha hyperparameter came up from the input of the first archaeologist. After this input

$$a_1 = a_1 + c_1 = 36 + (2.25 + 1.75 + 1.5 + 1.25)x\ 0.7\ x\ 10 = 83.25,$$

$$a_2 = a_2 + c_2 = 22 + (2.25 + 1.75 + 1.5 + 1.25)x\ 0.2\ x\ 10 = 35.8,$$

$$a_3 = a_3 + c_3 = 15 + (2.25 + 2.75 + 1.5 + 1.25)x\ 0.1\ x\ 10 = 21.75$$

Where, a_1, c_1 relate to the first reconstruction , a_2, c_2 relates to the second reconstruction and a_3, c_3 relates to the third reconstruction of one specific point of interest.

The calculations of the previous paragraph show how the alpha hyperparameter changes (a_1, a_2, a_3) after the input of the first and second archaeologist in the User Interface (Table 5.2). With the same way the values of alpha hyperparameter (a_1, a_2, a_3) change after the input of the first until the tenth archaeologist as it is shown in the Table 5.2, deriving the values of alpha hyperparameter as follow (36,22,15), (83.25, 35.8, 21.75), (111.25, 56.8, 42.75), (121.25, 81.8, 57.75), (161.75, 86.3, 57.75), (195.75, 120.3, 74.75), (208.25, 132.8, 112.25).

The Figure 5.29, Figure 5.30, Figure 5.31 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 2.25, w_a = 2, w_c = 1.75, w_t = 1.5, w_p = 1.25$. In Figure 5.29 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.29 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 36$, $\alpha_2 = 22$, $\alpha_3 = 15$. In the figure 5.29 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 83.25$, $\alpha_2 = 35.8$, $\alpha_3 = 21.75$. In the figure 5.29 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 111.25$, $\alpha_2 = 56.8$, $\alpha_3 = 42.75$. In the figures 5.29 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

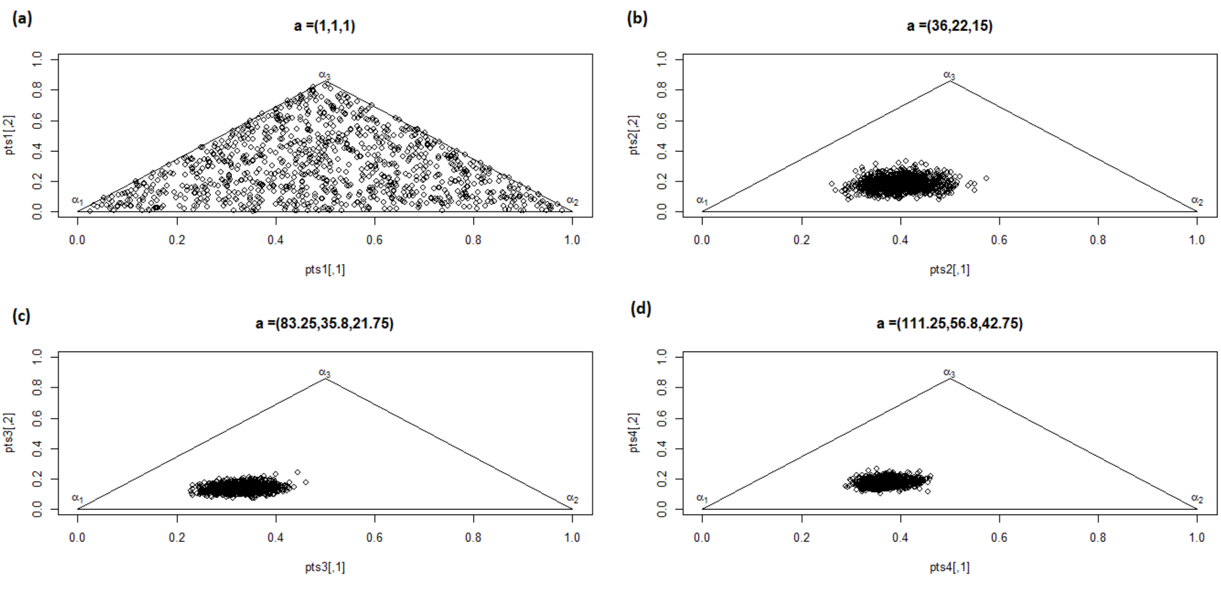

Figure 5.29: Dirichlet Distribution with hyperparameters: (a) α = (1,1,1), (b) α = (36,22,15), (c) α = (83.25, 35.8, 21.75), (d) α = (111.25, 56.8, 42.75).

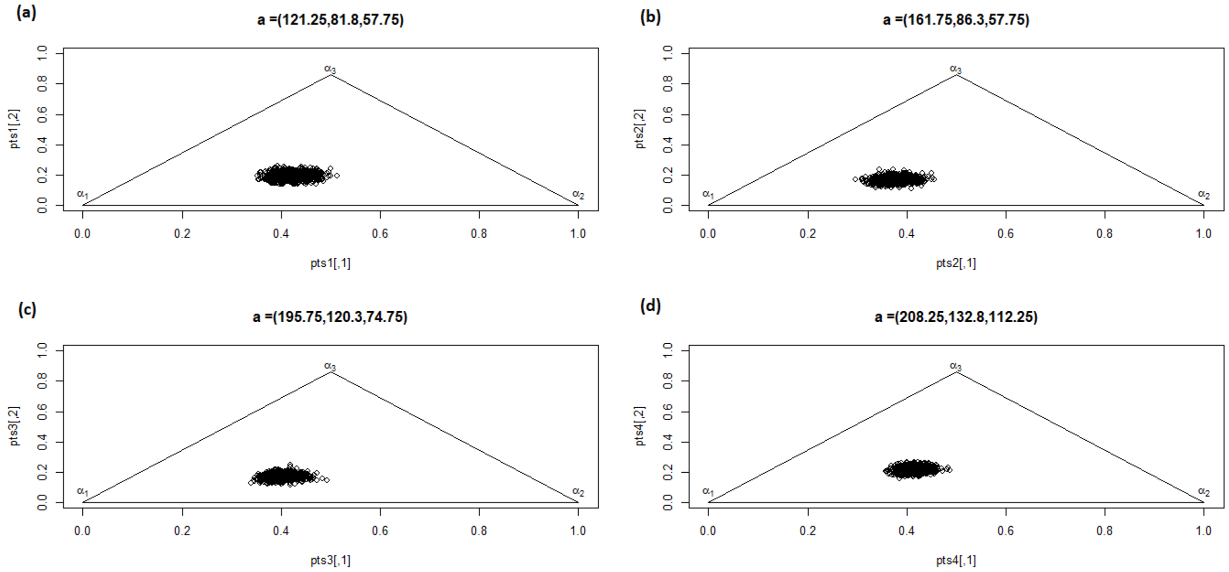

Figure 5.30: Dirichlet Distribution with hyperparameters: (a) α = (121.25, 81.8, 57.75), (b) α = (161.75, 86.3, 57.75), (c) α = (195.75, 120.3, 74.75), (d) α = (208.25, 132.8, 112.25).

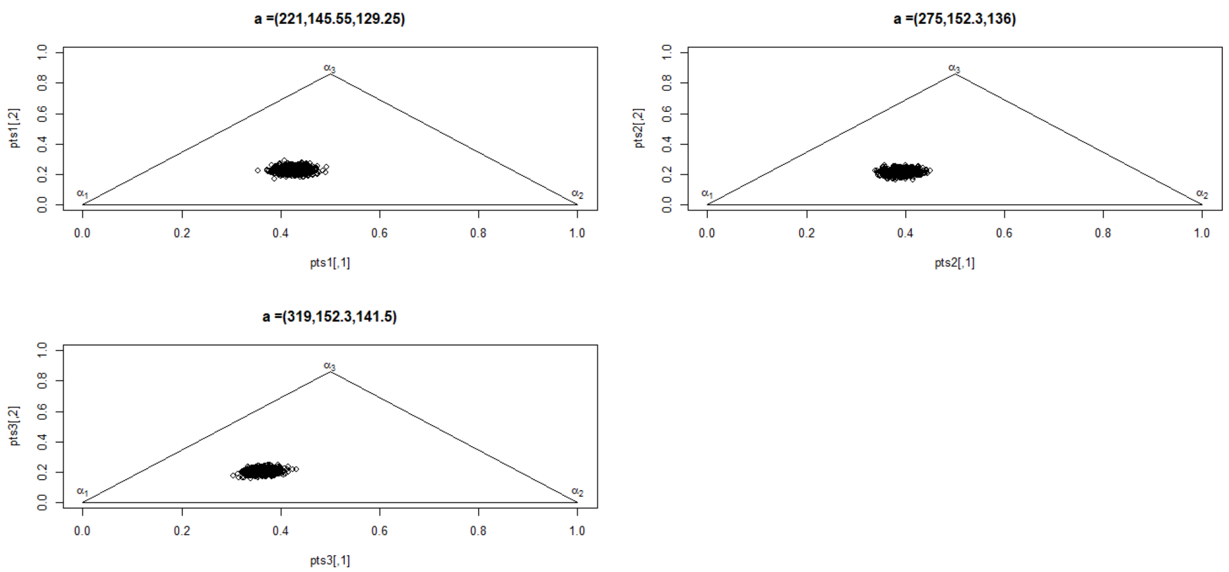

Figure 5.31: Dirichlet Distribution with hyperparameters: (a) α = (221, 145.55, 129.25), (b) α = (275, 152.3, 136), (c) α = (319, 152.3, 141.5).

The Figure 5.32, Figure 5.33, Figure 5.34 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 2, w_a = 1.75, w_c = 1.5, w_t = 1.25, w_p = 1$. In Figure 5.32 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.32 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 31$, $\alpha_2 = 19$, $\alpha_3 = 13$. In the figure 5.32 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 71.25$, $\alpha_2 = 30.5$, $\alpha_3 = 18.75$. In the figure 5.32 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 95.25$, $\alpha_2 = 48.5$, $\alpha_3 = 36.75$. In the figures 5.32 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

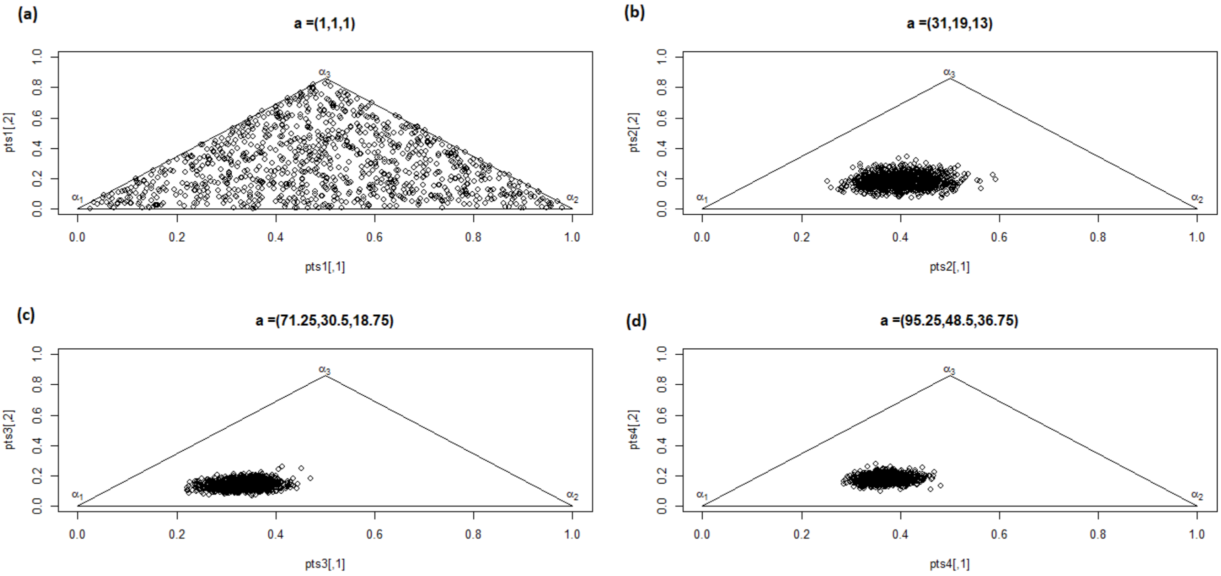

Figure 5.32: Dirichlet Distribution with hyperparameters: (a) α = (1, 1, 1), (b) α = (31, 19, 13), (c) α = (71.25, 30.5, 18.75), (d) α = (95.25, 48.5, 36.75).

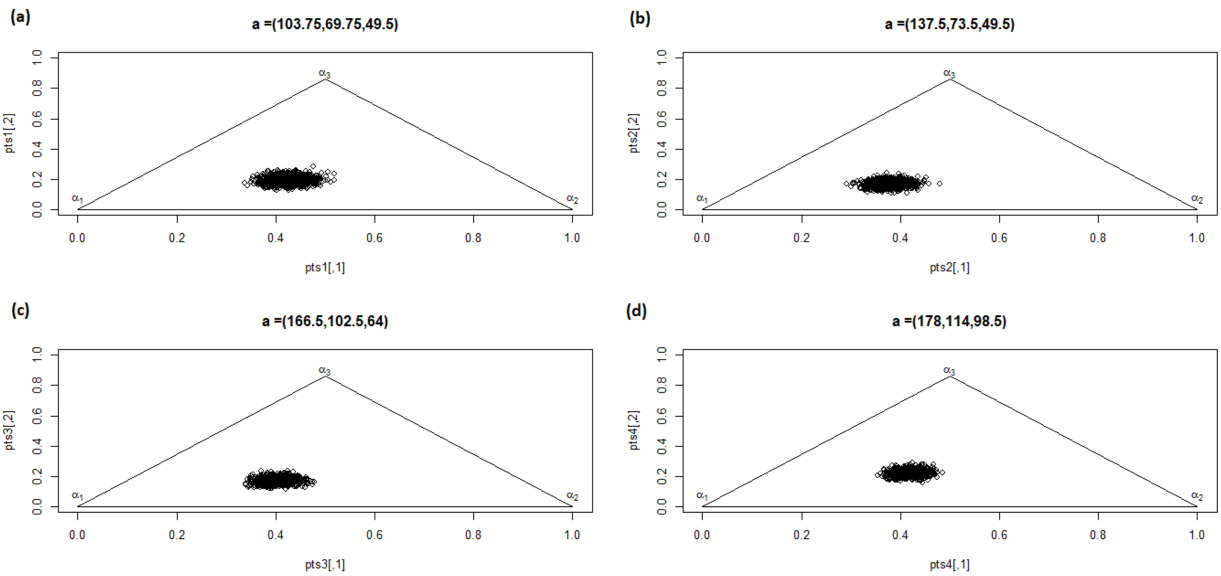

Figure 5.33: Dirichlet Distribution with hyperparameters: (a) α = (103.75, 69.75, 49.5), (b) α = (137.5, 73.5, 49.5), (c) α = (166.5, 102.5, 64), (d) α = (178, 114, 98.5).

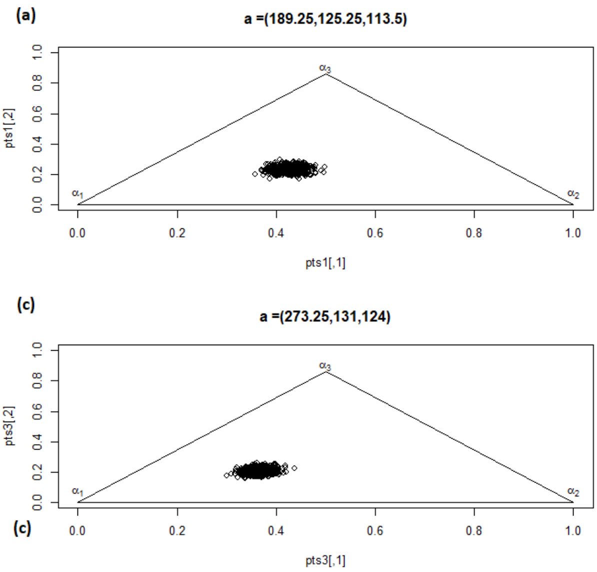

Figure 5.34: Dirichlet Distribution with hyperparameters: (a) α = (189.25, 125.25, 113.5), (b) α = (225.25, 131, 119.25), (c) α = (273.25, 131, 124).

The Figure 5.35, Figure 5.36, Figure 5.37 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 2.25, w_a = 2, w_c = 1.75, w_t = 1.5, w_p = 1.25$. In Figure 5.35 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 10$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.35 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 36$, $\alpha_2 = 22$, $\alpha_3 = 15$. In the figure 5.35 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 83.25$, $\alpha_2 = 35.8$, $\alpha_3 = 21.75$. In the figure 5.35 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 111.25$, $\alpha_2 = 56.8$, $\alpha_3 = 42.75$. In the figures 5.35 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

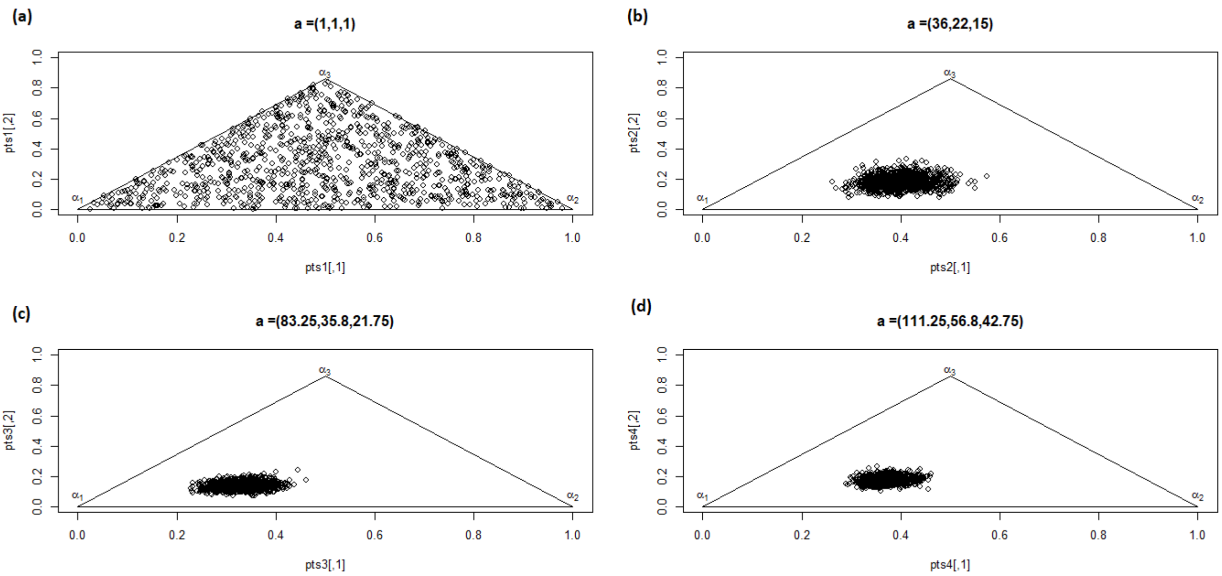

Figure 5.35: Dirichlet Distribution with hyperparameters: (a) α = (10, 10, 10), (b) α = (45, 31, 24), (c) α = (92.25, 44.5, 30.75), (d) α = (120.25, 65.5, 51.75).

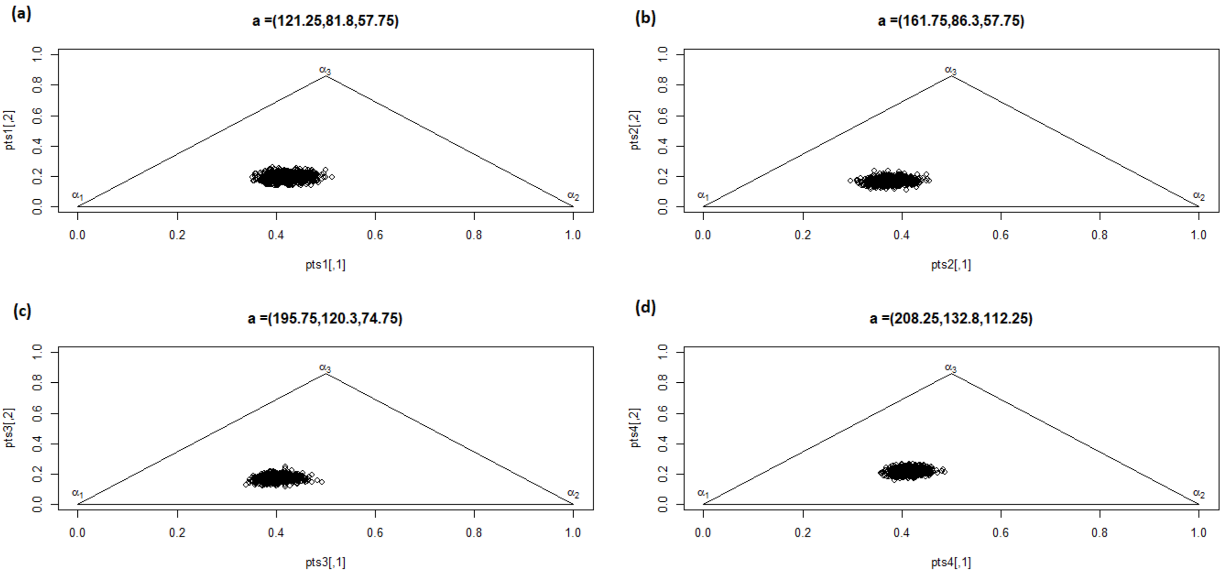

Figure 5.36: Dirichlet Distribution with hyperparameters: (a) α = (130.25, 90.5, 66.75), (b) α = (170.75, 95, 66.75), (c) α = (204.75, 129, 83.75), (d) α = (217.25, 141.5, 121.25).

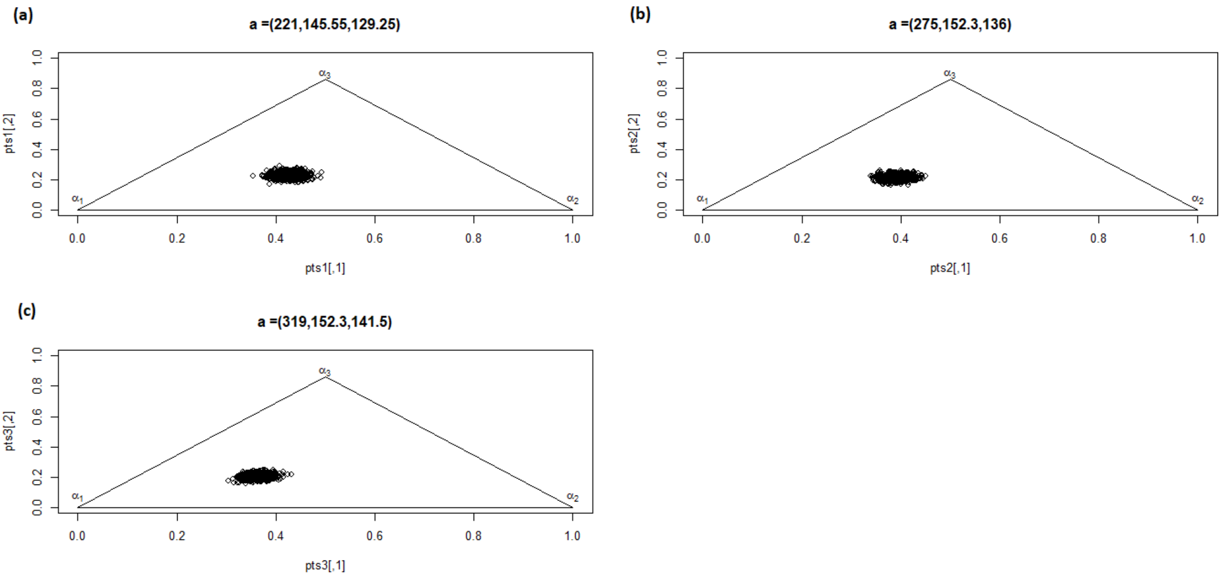

Figure 5.37: Dirichlet Distribution with hyperparameters: (a) α = (230, 154.25, 138.25), (b) α = (284, 161, 145), (c) α = (328, 161, 150.5).

The Figure 5.38, Figure 5.39, Figure 5.40 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 2, w_a = 1.75, w_c = 1.5, w_t = 1.25, w_p = 1$. In Figure 5.38 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 10$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.38 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 40$, $\alpha_2 = 28$, $\alpha_3 = 22$. In the figure 5.38 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 80.25$, $\alpha_2 = 39.5$, $\alpha_3 = 27.75$. In the figure 5.38 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 104.25$, $\alpha_2 = 57.5$, $\alpha_3 = 45.75$. In the figures 5.38 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

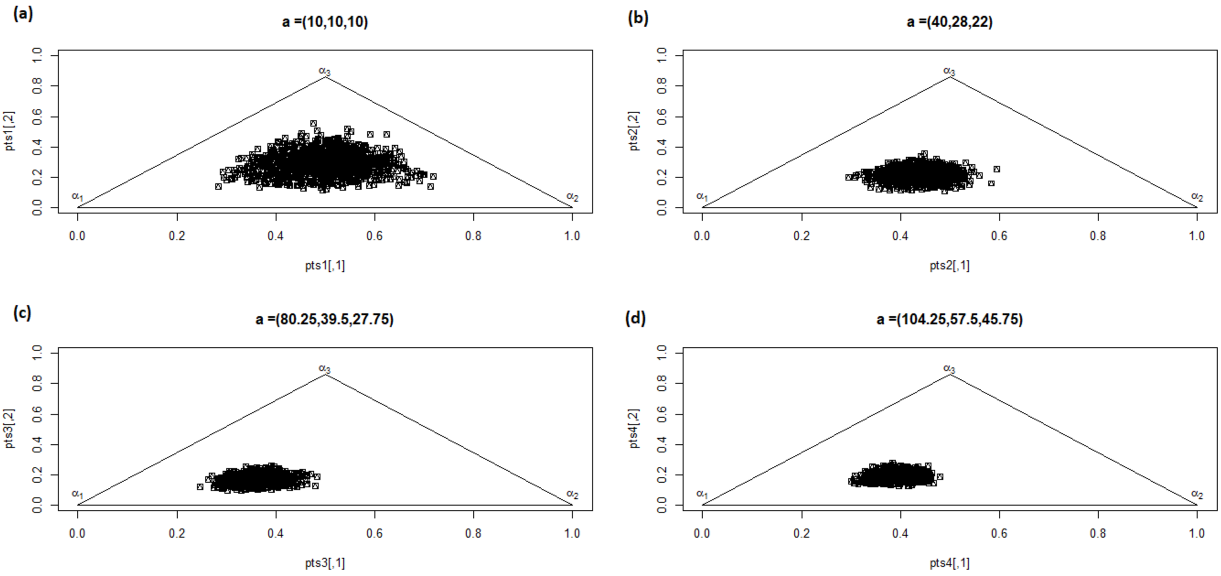

Figure 5.38: Dirichlet Distribution with hyperparameters: (a) α = (10, 10, 10), (b) α = (40, 28, 22), (c) α = (80.25, 39.5, 27.75), (d) α = (104.25, 57.5, 45.75).

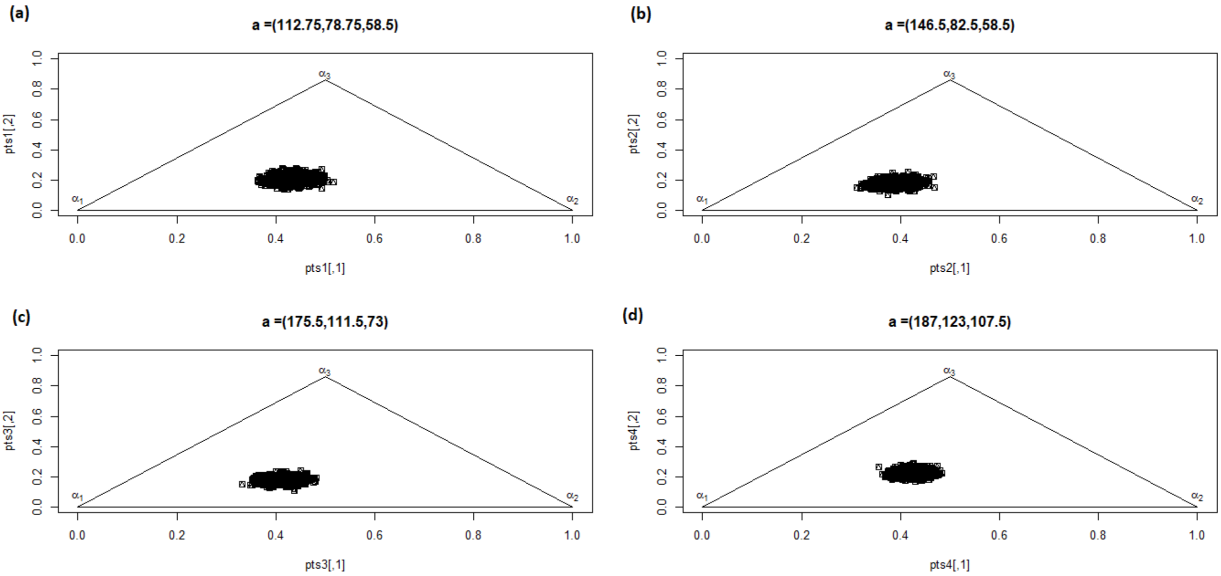

Figure 5.39: Dirichlet Distribution with hyperparameters: (a) α = (112.75, 78.75, 58.5), (b) α = (146.5, 82.5, 58.5), (c) α = (175.5, 111.5, 73), (d) α = (187, 123, 107.5).

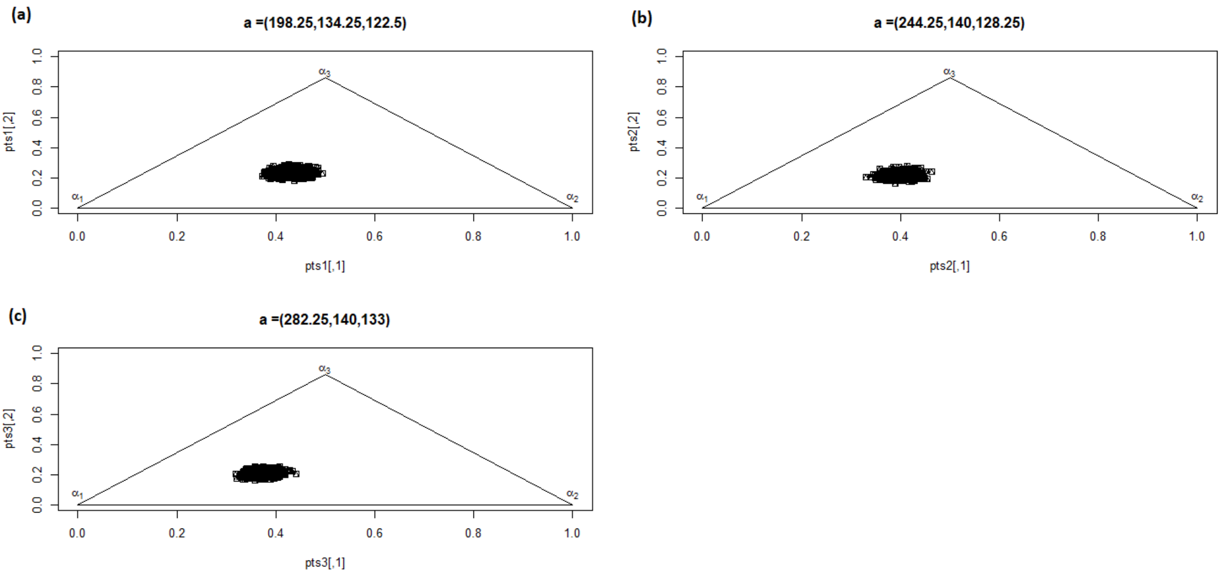

Figure 5.40: Dirichlet Distribution with hyperparameters: (a) α = (198.25, 134.25, 122.5), (b) α = (244.25, 140, 128.25), (c) α = (282.25, 140, 133).

The Figure 5.42, Figure 5.43, Figure 5.44 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 2.25, w_a = 2, w_c = 1.75, w_t = 1.5, w_p = 1.25$. In Figure 5.42 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 100$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.42 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 135$, $\alpha_2 = 121$, $\alpha_3 = 106.75$. In the figure 5.42 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 182.25$, $\alpha_2 = 134.5$, $\alpha_3 = 113.5$. In the figure 5.42 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 210.25$, $\alpha_2 = 155.5$, $\alpha_3 = 134.5$. In the figures 5.42 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

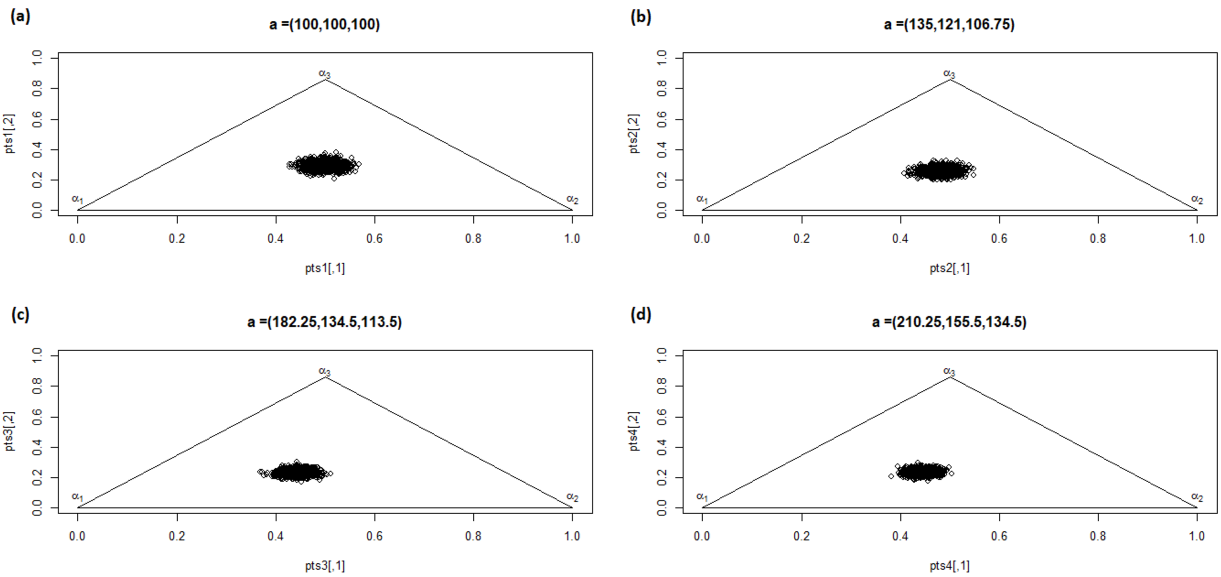

Figure 5.42: Dirichlet Distribution with hyperparameters: (a) α = (100, 100, 100), (b) α = (135, 121, 106.75), (c) α = (182.25, 134.5, 113.5), (d) α = (210.25, 155.5, 134.5).

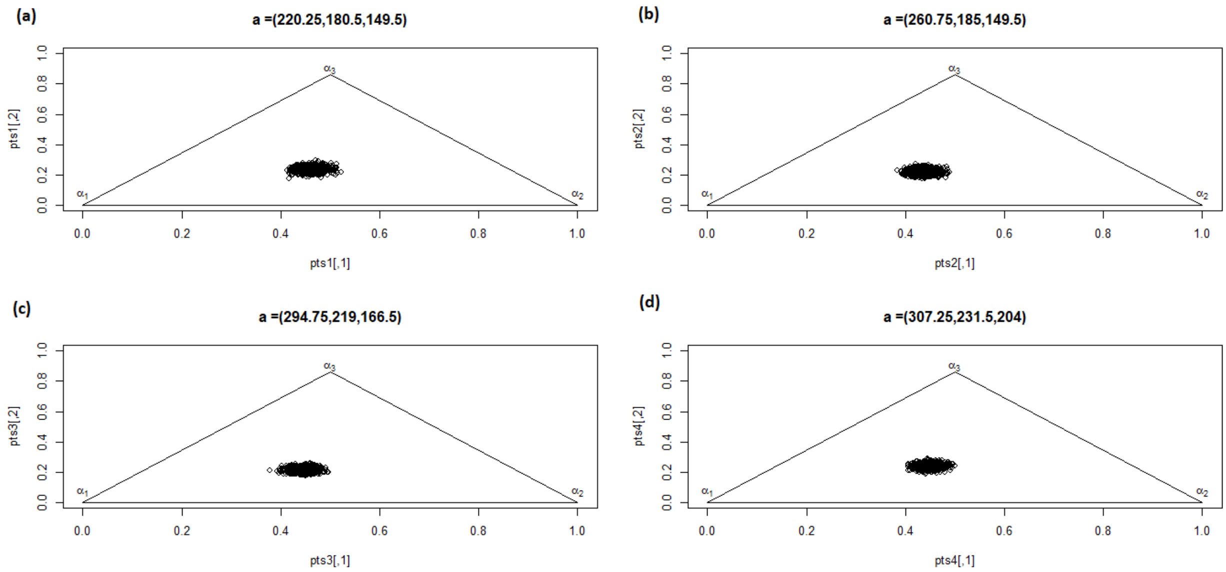

Figure 5.43: Dirichlet Distribution with hyperparameters: (a) α = (220.25, 180.5, 149.5), (b) α = (260.75, 185, 149.5), (c) α = (294.75, 219, 166.5), (d) α = (307.25, 231.5, 204).

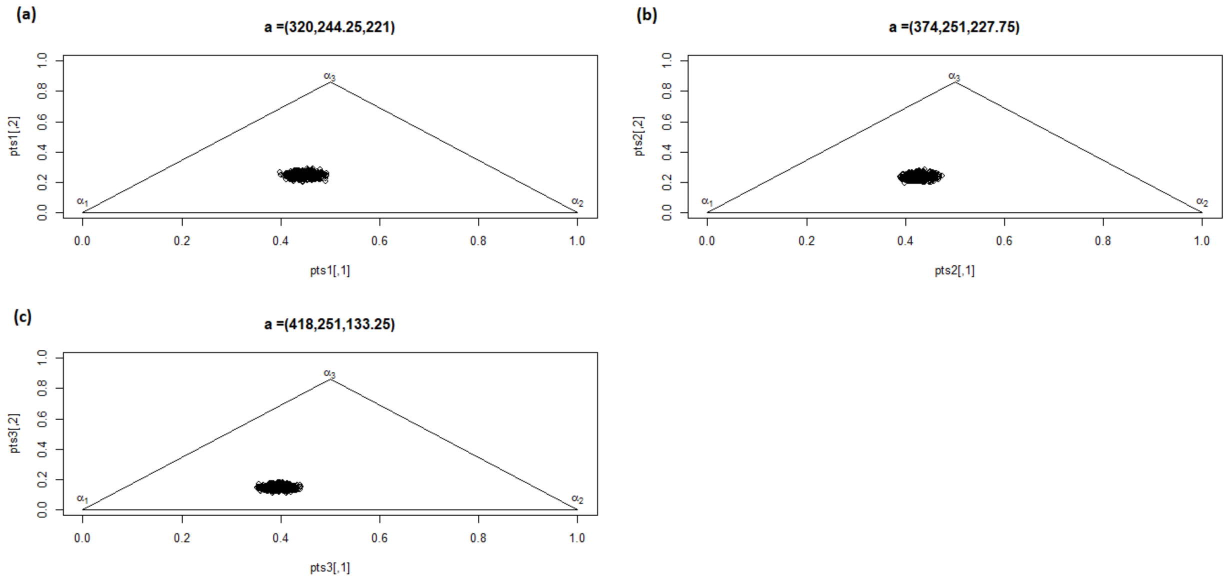

Figure 5.44: Dirichlet Distribution with hyperparameters: (a) α = (320, 244.25, 221), (b) α = (374, 251, 227.75), (c) α = (418, 251, 133.25).

The Figure 5.45, Figure 5.46, Figure 5.47 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 2, w_a = 1.75, w_c = 1.5, w_t = 1.25, w_p = 1$. In Figure 5.45 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 100$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.45 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 130$, $\alpha_2 = 118$, $\alpha_3 = 112$. In the figure 5.45 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 170.25$, $\alpha_2 = 129.5$, $\alpha_3 = 117.75$. In the figure 5.45 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 194.25$, $\alpha_2 = 147.5$, $\alpha_3 = 135.75$. In the figures 5.45 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

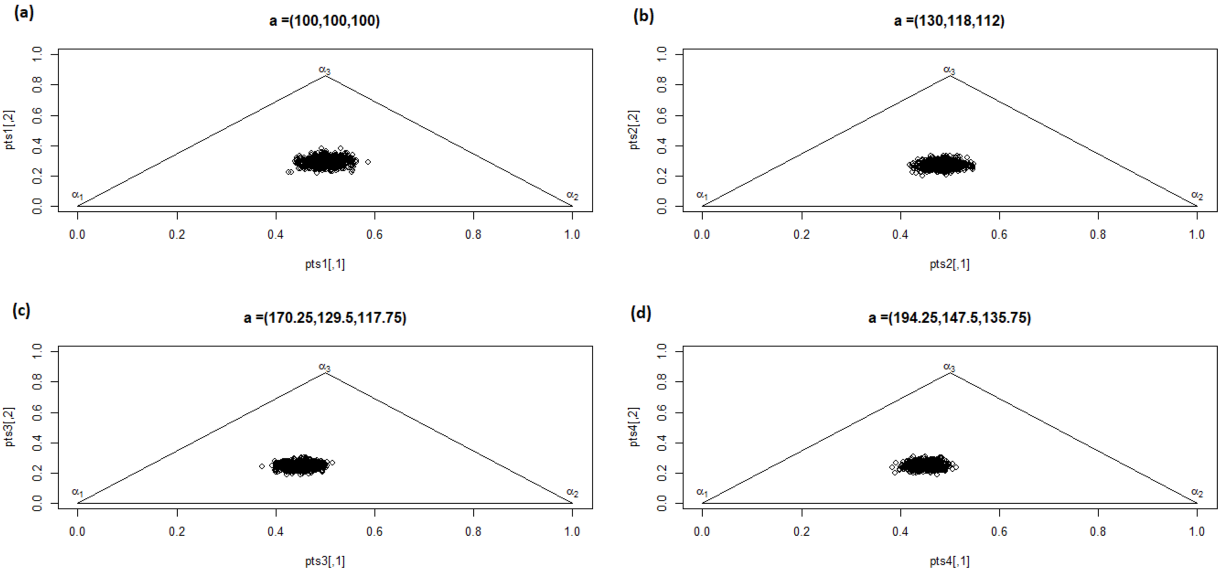

Figure 5.45: Dirichlet Distribution with hyperparameters: (a) α = (100, 100, 100), (b) α = (130, 118, 112), (c) α = (170.25, 129.5, 117.75), (d) α = (194.25, 147.5, 135.75).

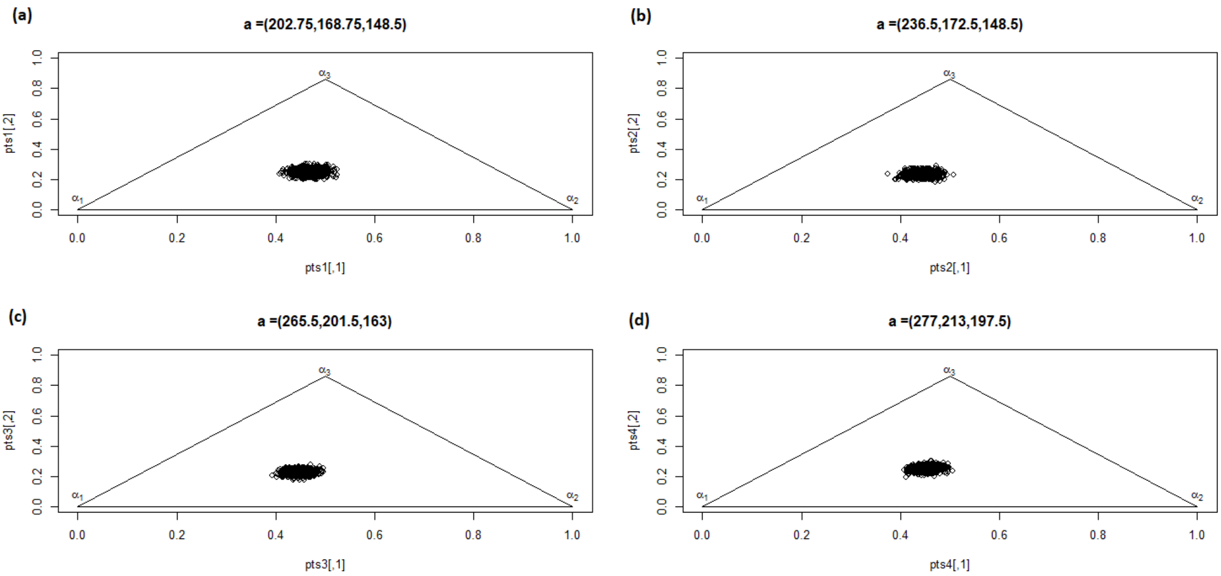

Figure 5.46: Dirichlet Distribution with hyperparameters: (a) α = (202.75, 168.75, 148.5), (b) α = (236.5, 172.5, 148.5), (c) α = (265.5, 201.5, 163), (d) α = (277, 213, 197.5).

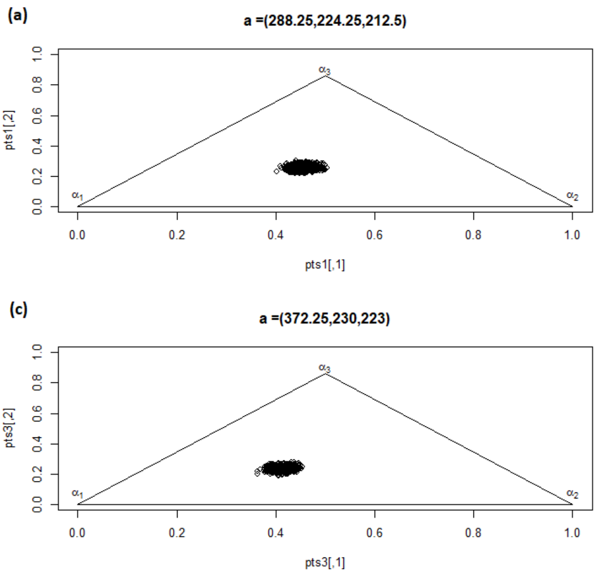

Figure 5.47: Dirichlet Distribution with hyperparameters: (a) α = (288.25, 224.25, 212.5), (b) α = (334.25, 230, 218.25), (c) α = (372.25, 230, 223).

The Figure 5.48, Figure 5.49, Figure 5.50 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 1, w_a = 0.8, w_c = 0.6, w_t = 0.4, w_p = 0.2$. In Figure 5.48 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.48 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 13$, $\alpha_2 = 8.2$, $\alpha_3 = 5.8$. In the figure 5.48 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 28.4$, $\alpha_2 = 12.6$, $\alpha_3 = 8$. In the figure 5.48 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 38$, $\alpha_2 = 19.8$, $\alpha_3 = 15.2$. In the figures 5.48 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

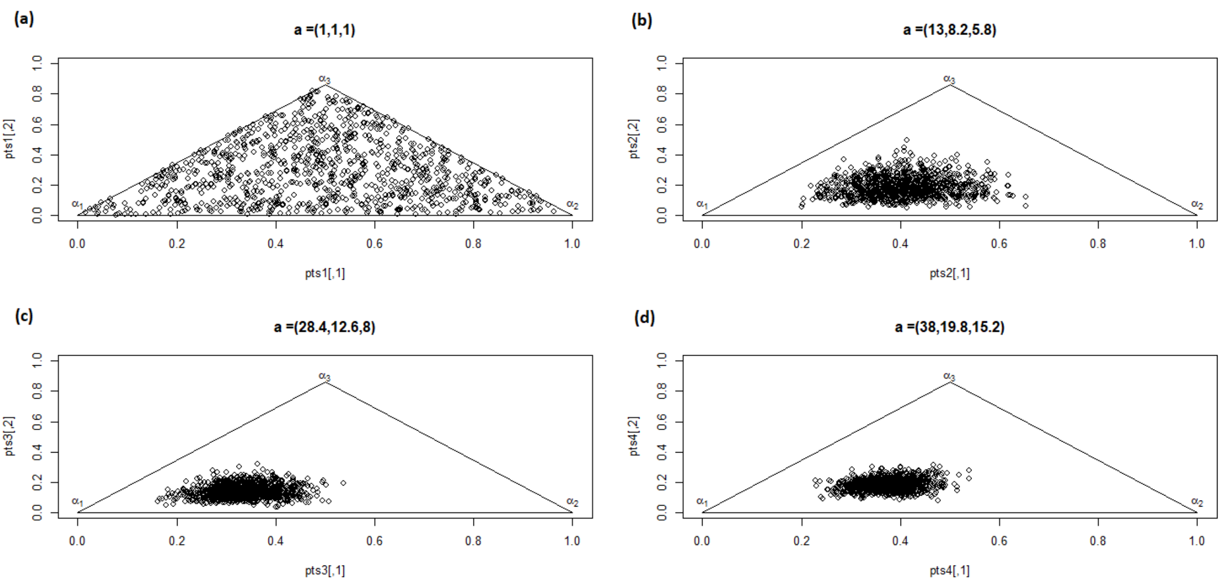

Figure 5.48: Dirichlet Distribution with hyperparameters: (a) α = (1, 1, 1), (b) α = (13, 8.2, 5.8), (c) α = (28.4, 12.6, 8), (d) α = (38, 19.8, 15.2).

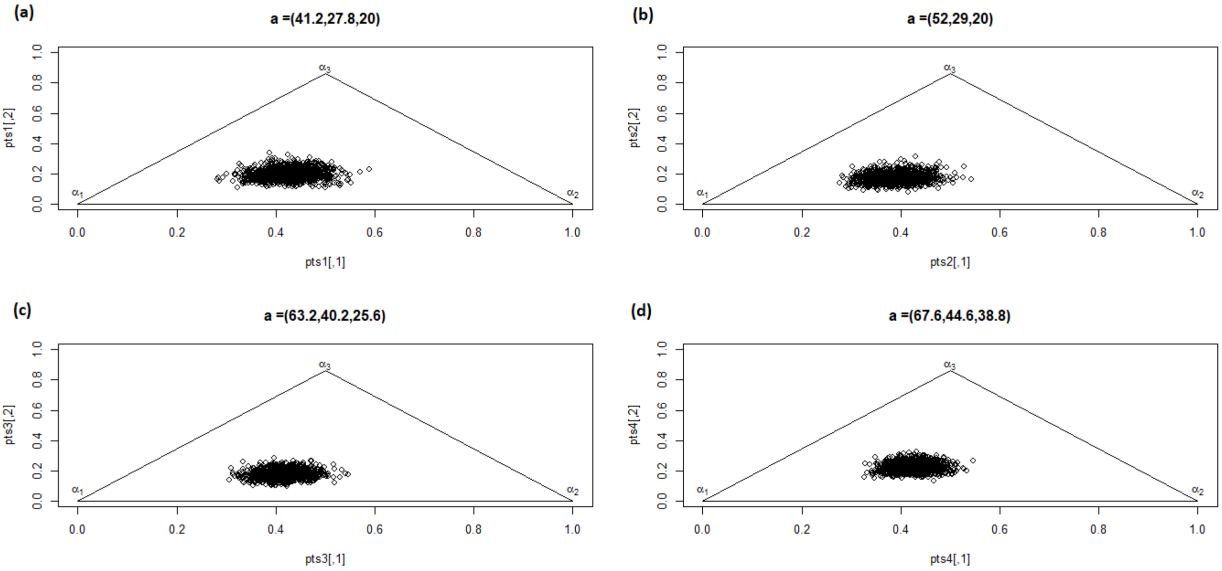

Figure 5.49: Dirichlet Distribution with hyperparameters: (a) α = (41.2,27.8,20), (b) α = (52, 29, 20), (c) α = (63.2, 40.2, 25.6), (d) α = (67.5, 44.6, 38.8).

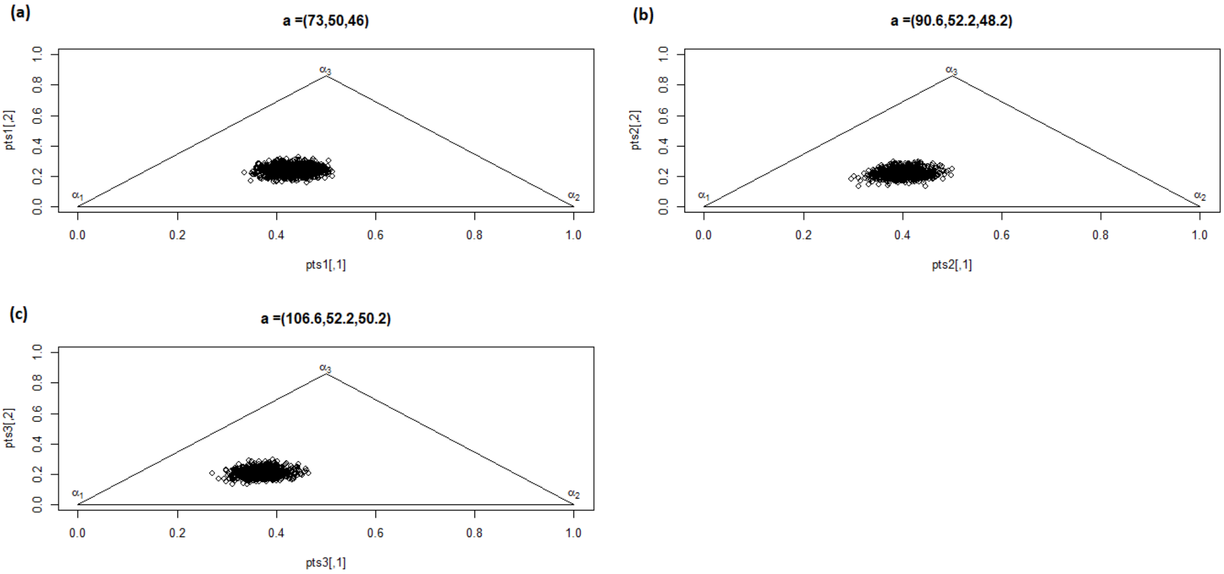

Figure 5.50: Dirichlet Distribution with hyperparameters: (a) α = (73, 50, 46), (b) α = (90.6, 52.2, 48.3), (c) α = (106.6, 52.2, 50.2).

The Figure 5.51, Figure 5.52, Figure 5.53 represent the updating of the Dirichlet Distribution after every archaeologist's input, and for these GUIs the values of the weights for each influencing factor are assigned as $w_f = 0.5, w_a = 0.4, w_c = 0.3, w_t = 0.2, w_p = 0.1$. In Figure 5.51 (a) the α hyperparameter has the initial equal values $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and as we can see if every vertex of the triangle is one of the possible options for reconstruction, then there aren't evidence and indication for which is more possible. However, in the Figure 5.51 (b) the first archeologist offered his opinion through the sliders and the values [5,3,2] were derived and the α hyperparameter is adjusted as follows $\alpha_1 = 7$, $\alpha_2 = 4.6$, $\alpha_3 = 3.4$. In the figure 5.51 (c) the values of alpha hyperparameter comes if we add the input of the third archaeologist [7,2,1] to the results which came from the input of the previous archaeologist, α hyperparameter is adjusted as follows $\alpha_1 = 14.7$, $\alpha_2 = 6.8$, $\alpha_3 = 4.5$. In the figure 5.51 (d) the values of alpha hyperparameter comes if we add the input of the fourth archaeologist [4,3,3] to the results which came from the input of the previous archaeologist α hyperparameter is adjusted as follows $\alpha_1 = 19.5$, $\alpha_2 = 10.4$, $\alpha_3 = 8.1$. In the figures 5.51 (c) and (d) the values of a_1 are higher so the points of the triangle are closer to the vertex of a_1.

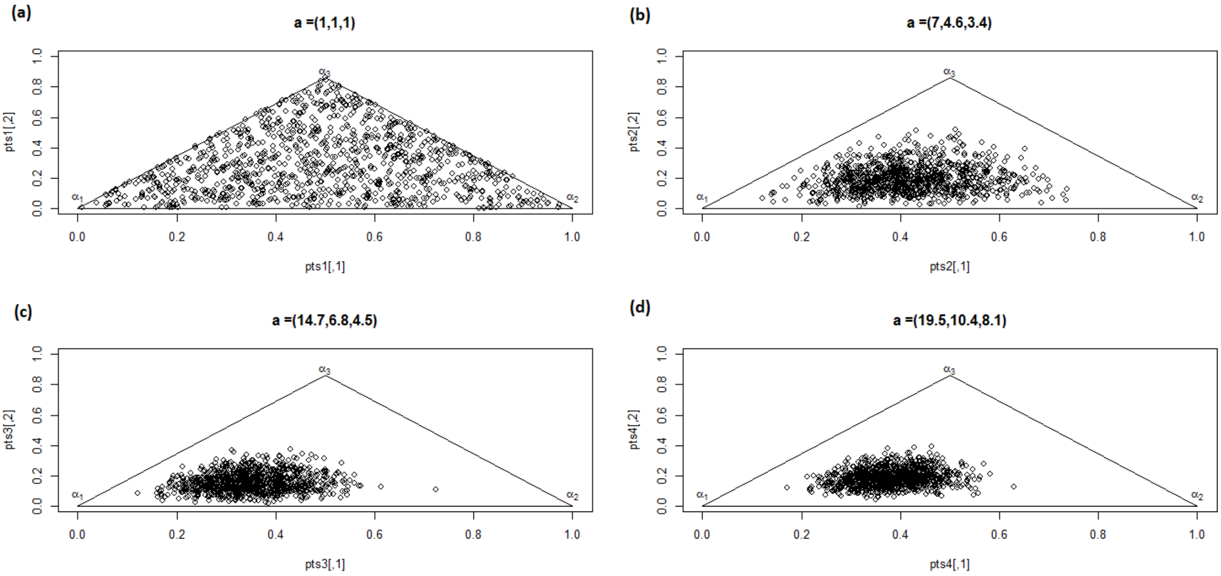

Figure 5.51: Dirichlet Distribution with hyperparameters: (a) α = (1, 1, 1), (b) α = (7, 4.6, 3.4), (c) α = (14.7, 6.8, 4.5), (d) α = (19.5, 10.4, 8.1).

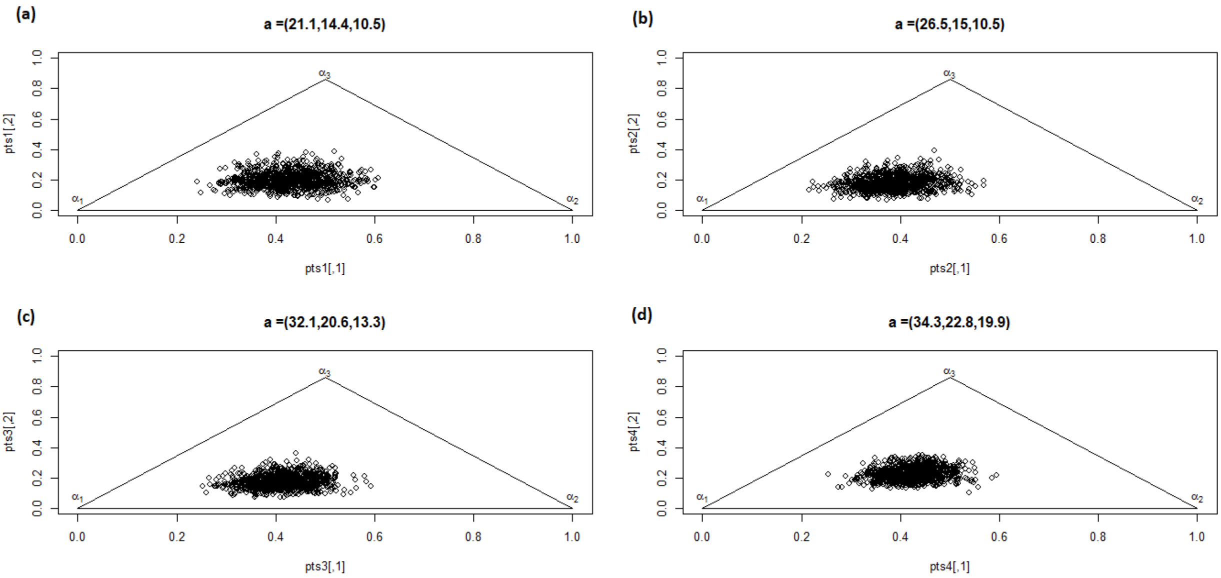

Figure 5.52: Dirichlet Distribution with hyperparameters: (a) α = (21.1, 14.4, 10.5), (b) α = (26.5, 15, 10.5), (c) α = (32.1, 20.6, 13.3), (d) α = (34.3, 22.8, 19.9).

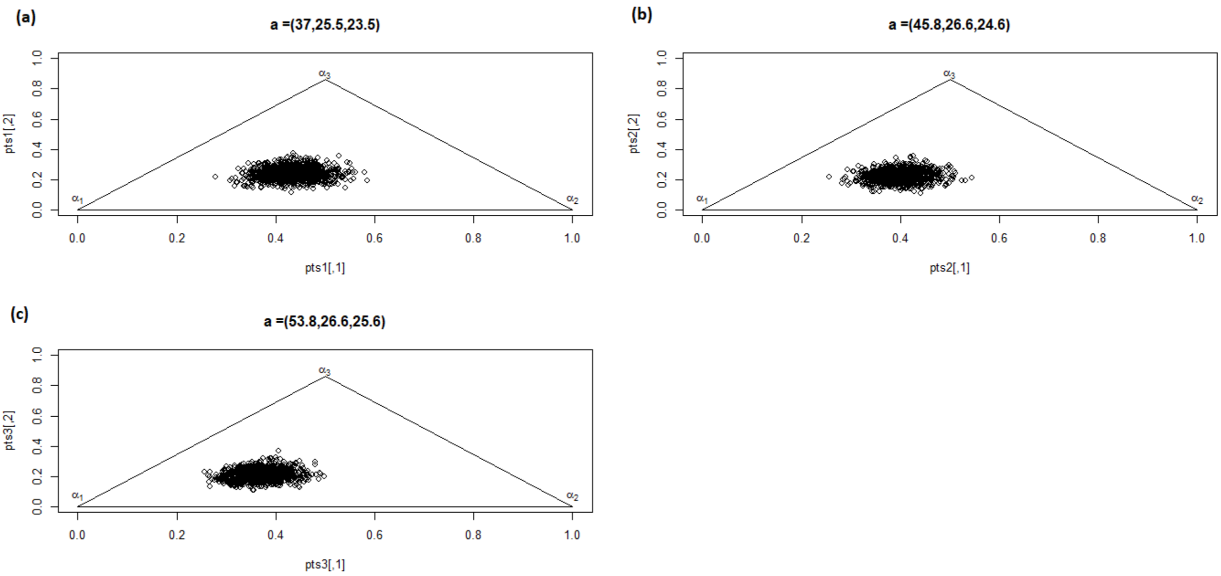

Figure 5.53: Dirichlet Distribution with hyperparameters: (a) α = (37, 25.5, 23.5), (b) α = (45.8, 26.6 24.6), (c) α = (53.8, 26.6, 25.6).

Using the above GUIs, we observed the changes of Dirichlet Distribution after the input of every archaeologist, from the first until the tenth archaeologist (Table 2), in the User Interface of our system which is described in the previous sections (Figures 5.17-5.19). Through these observations, we decided which are the most appropriate of the initial values of alpha hyperparameter (a_1, a_2, a_3) and the weights of influencing factors *(w_f, w_a, w_c, w_p, w_t)*. For our case study, the appropriate values are $a_1 = a_2 = a_3 = 10$ and $w_f = 2, w_a = 1.75, w_c = 1.5, w_p = 1.25$ and $w_t = 1$. After the decision about which should be the initial values of the alpha hyperparameter and the values which should be assigned to the weights of influencing factors, we had to calculate and visualize the PDF (Probability Density Function) and the samples of the Dirichlet distribution, which are the output of our system.

5.4 System Output

The Dirichlet distribution was used as mathematical model for the treatment of uncertainty for three interest points of Zakros Palace. This distribution, often denoted by *Dir(a)*, is a family of continuous multivariate probability distributions that take on positive real number values based on parameter *a*. This hyperparameter is often referred as the concentration parameter because it determines the spread of the realization from the distribution. This means that higher alpha gives a denser distribution whereas a lower alpha gives a sparser distribution. As it is described above, in section 5.3.1, dense distribution means that the most of archaeologists have a similar opinion among themselves and therefore gave similar values on the sliders of the User Interface. If the distribution is sparse, means that there were many different opinions about the three possible reconstruction of a point of interest.

Moreover, after a discussion with the archaeologist, Dr Anna Simantiraki, it was decided that our system cannot take into account in the same way the opinions of all archaeologists. As it is logical, the opinion of archaeologists based on significant findings should be taken into account more seriously, than those who do not have findings to support their opinion. For example, if the opinion of an archaeologists is based on remains or wall which are included in “features” influencing factors, this opinion should have higher significance for our system than the opinion of an archaeologist who has not significant findings in order to form his knowledge. This influenced opinion of archaeologists is referred to in our text as counter c. This influenced counter c, is based on the weight of influencing factors $(w_f, w_a, w_c, w_p, w_t)$ which led to the opinion of archaeologists and the confidence of each archaeologist (θ_ι) for every reconstruction of one specific point of interest. As there are three points of interest in this work, which are studied, the counter c has the values (c_1, c_2, c_3).

In the previous sections, the appropriate initial values of alpha hyperparameter (a_1, a_2, a_3) and the values which correspond to the weight of every influencing factor $(w_f, w_a, w_c, w_p, w_t)$ were decided as $a_1 = a_2 = a_3 = 10$ and $w_f = 2, w_a = 1.75, w_c = 1.5, w_p = 1.25$ and $w_t = 1$ respectively, through the observations of the GUIs of sections 5.3.2 and 5.3.3. These values were decided as through the GUIs of the previous sections we observed that using these values, the after every archaeologist’s input, the distribution becomes denser at a smooth rate, as if the rate was fast (for initial values of alpha hyperparameter $a_1 = a_2 = a_3 = 1$) after the input of the first archaeologist the new values of alpha hyperparameter would be far from the initial values, and the input of the next archaeologists will influence less the distribution, comparing with the change after the input of the first archaeologist. While if the rate was slow (for initial values of alpha hyperparameter $a_1 = a_2 = a_3 = 100$) after the input of the first archaeologist the new values of alpha hyperparameter would be close to the initial values, and the input of the next archaeologists would change the distribution slightly.

After this decision about which should be the initial values of the alpha hyperparameter and the values which should be assigned to the weights of influencing factors, we had to calculate and visualize the PDF (Probability Density Function) and the samples of the Dirichlet distribution, which are the output of our system.

In our work, it was necessary for archaeologists to visualize the uncertainty, in order to give them a knowledge about other archaeologists' input and to update the belief of archaeologists in real time, as new evidence can be found and their belief can be changed.

The application of the Dirichlet distribution in our case study has certain useful priorities. First of all, the Dirichlet distribution, as mention in previous section, is the conjugate prior for the likelihood of multinomial distribution. In order to update the posterior of multinomial distribution with Dirichlet prior, the update of Dirichlet prior is needed, by adding the observation counts to the Dirichlet hyperprior and the number of observations directly reveals the confidence on the expected value of the parameters. Therefore, the state of knowledge about uncertain can represented and it is known the way the given data to be updated.

The first step of the output of our system is the visualization of Probability Density Function (Pdf) of the Dirichlet distribution as it is described in details in section 5.4.1 and an example is illustrated in Figure 5.54. After archaeologists have introduced their opinion on which is the importance of every influencing factor (first stage of the User Interface, Figures 5.4-5.8) and how it could be one of the parts of the palace that is examined in the past (second stage of the User Interface, Figures 5.17-5.19), they can see the visualization of Pdf, through which they can understand what is the most probable version based on the opinion of all archaeologists who have added their own knowledge (θ_i) and which influencing factors formed this knowledge . This step offers to archaeologists the ability to observe how the distribution is configured and which is the point of the simplex (triangle) with the higher density (symbolized by a more intense red color in the distribution) which is the most likely combination of probabilities for each reconstruction. The density changes in real time depending on the new input of archaeologists.

A usual question to ask regarding any distribution is how to sample from it and in this chapter, we describe how we generate random values from Dirichlet Distribution, which is the next step of our system. In subchapters 3.4.6 the methods for the generation of random values from Dirichlet Distribution, Gamma Distribution and Normal Distribution are described, so as to understand this section. When we say we sample from a distribution, we mean that we choose some discrete points with likelihood defined by the distribution's probability density function.

A key step in this thesis is the interaction of archaeologists with the samples of Dirichlet Distribution, which distribution is based on the insertion of their view through the setting panel, as described in 5.2.2(Figures 5.17-5.19). To achieve this, we implemented an algorithm to generate Dirichlet samples using Gamma random variables. In the following subsections describe the steps we took to generate these samples and create the user interface in order archaeologists to interact with the samples.

Generating samples from the Dirichlet distribution using Gamma-distributed random variables is more computationally efficient than other methods. In Chapter 3 of this thesis, the theoretical background was described and a procedure for the generating the 3-dimentional Dirichlet Distribution with parameters (a_1, a_2, a_3), using a source of Gamma-distributed random variables is developed. This method has two steps. The first step is the generation of gamma realizations (Gamma-distributed random variables): for $i = 1, \ldots, k$, draw a number z_i from $Gamma(\alpha_i, 1) = \frac{{z_i}^{a_i - 1} e^{-z_i}}{\Gamma(\alpha_i)}$. The second is the normalization of them to form a pmf: for $i = 1, \ldots, k$ set $q_i = \frac{z_i}{\sum_{j=1}^{k} z_j}$. Then q is a realization of Dir(α).

These samples are the core of our system, as archaeologists can have a clear picture of what the general opinion of archaeologists is and what is the degree of uncertainty of each version of a point. In our thesis we sampled from a random vector $x = (x_1, x_2, x_3)$ from the 3-Dimensional Dirichlet Distribution with parameters (a_1, a_2, a_3). As mentioned earlier, we are talking about three-dimensional distribution as we have three versions for each point, we want to create its reconstruction. In our system archaeologists see the random values as points in a triangle and hovering them they can see multiple visualization of uncertainty for a place in the Palace.

5.4.1 Probability Density Function (Pdf)

In this view of the system, we mainly focus on the Probability Density Function (PDF) of the Dirichlet distribution. As we described in 5.2.1, archaeologists are asked to provide a numerical assessment of their opinion in relation to the possible reconstructions concerning three interest points in the Palace of Zakros. They enter their belief through the setting panel (Figures 5.17, 5.18, 5.19). In the first part of this panel, they enter values in the sliders and in the second part of the panel they choose the existing evidence types on which they based their opinion. When archaeologists enter their opinion through the setting panel and they press the button RUN, the updated distribution is calculated. After these actions, users can see the visualization of the Dirichlet distribution.

For example, in Figure 5.54 we can see the PDF for the three possible versions of the wall at the West Wing. In this example the three possible reconstructions are: the wall has not windows, the wall has windows with shutters or the wall has windows with leather. These are places on the vertices of the simplex represented by a triangle. The lower left part of the screen shows the scaling of probabilistic density as a multi-colored bar. The more the probabilistic density increases, the more each point tends towards the red color.

The point with higher density is the most likely combination of probabilities. As we can see in the Figure it is clear that most of the archaeologists were more positive with the version of windows with shutters. If we consider that the point with the higher density signified by a red peak has values (0.2, 0.6, 0.2) for the beliefs no windows, windows with shutters and windows with leather, respectively. It is apparent that the value 0.6 signifies a higher probability of the 'windows with shutters' hypothesis in relation to remaining two hypotheses. Users can actually see in this screen, that most archaeologists believe that the probability about the reconstruction of the wall without windows is 0.2, the probability about the reconstruction of the wall with windows covered by shutters is 0.6 and the probability about the reconstruction of the wall covered by leather is 0.2.

As we can see, the two hypotheses with associated smaller probability values in relation to specific reconstructions, are not excluded or zeroed in relation to their probability of occurrence. If additional archaeologists enter a different opinion, the calculations change and the point with higher probabilistic density can change. For example, if archaeologists start to believe that the more likely reconstruction is the reconstruction of wall without windows, the distribution will change and the point with higher density will be more closely to "no windows". Again, the remaining two hypotheses will not be excluded. This is very close to the way archaeologists consider evidence and potentially update their views in time based on new evidence or a new way of thinking.

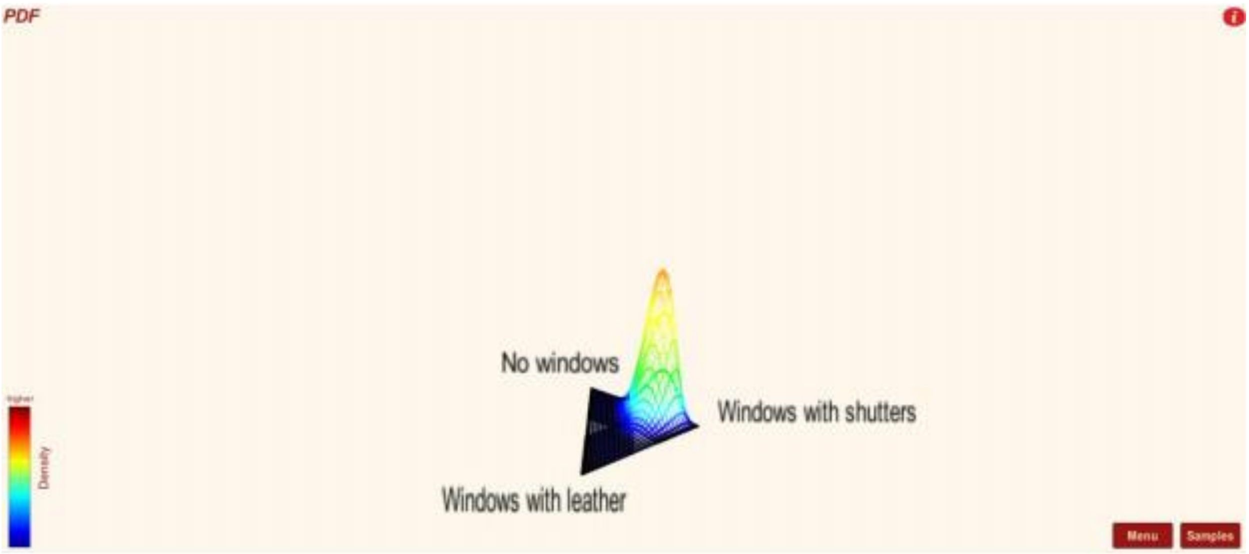

Figure 5.54: PDF

5.4.2 Samples from Dirichlet Distribution

In the previous subsection, we described how archeologists communicate the level of their uncertainty concerning possible interpretations of a how a specific point (Figures 5.17-5.19), representing a spatial location in and around the Palace of Zakros used to be formed in the past. When archaeologists click on the button “RUN”, the PDF will appear on the screen according to the values representing their belief of all archaeologists who gave their opinion on the possible reconstructions of each point. In the next step archaeologists can press the button “Samples”. After this action, the system creates random values from Dirichlet Distribution and these values appear as points on a triangle, the 2-Simplex. After that, when archaeologists are hovering the points, they can see different color visualizations of the uncertainty of the possible reconstructions in the right part of the screen. For example, for the case of figure 5.40, the three possible reconstructions for the specific point of the palace are to be all housing, to be partially roofed or to have no shed. As we can see in this figure the views of the archaeologists have shaped the distribution accordingly and as we see the samples concentrate on the reconstruction “no shed”. Wherever archaeologists hover, the reconstruction of no shed will have more intense shade of green (therefore, more probable) than the two other potential reconstructions. However, depending on which sample archaeologists hover, the exact shade of the reconstructions will change. As shown in Figures 5.56, the points of the 2-Simplex are more concentrated because the opinions of the expert archaeologists were more or less in agreement. In Figures 5.56 and 5.57, the distribution of points is spatially apart which means that the archeologists who provided input had diverse opinions.

If they believe that certain of these visualizations are more plausible, they support them by pressing with their mouse at the specific point of interest which supports this hypothesis. The Figure 5.55, Figure 5.56 and Figure 5.57 show the 2-Simplex for each of the three points of uncertainty.

In general, the 2-Simplex visualizations demonstrate the probability of a hypothesis occurring signifying visually that the other hypotheses places on the vertices of the triangle are also possible. It is a very good way of visualizing the collective view of expert archaeologists which gets updated as more views are added, or when views change. The spatial location in the triangle of the points, showcase a tendency that a hypothesis could be more accurate without excluding the remaining hypotheses. Also, it showcases how variant the views of the expert archeologists are about a potential reconstructed option.

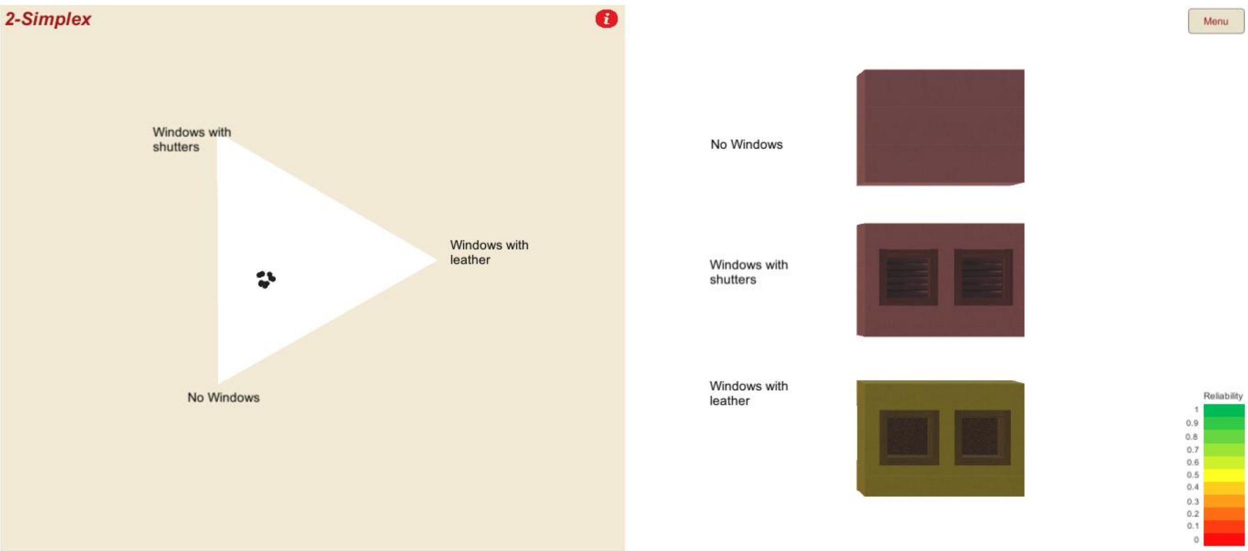

Figure 5.55: 2-Simplex for Uncertainty Point 1

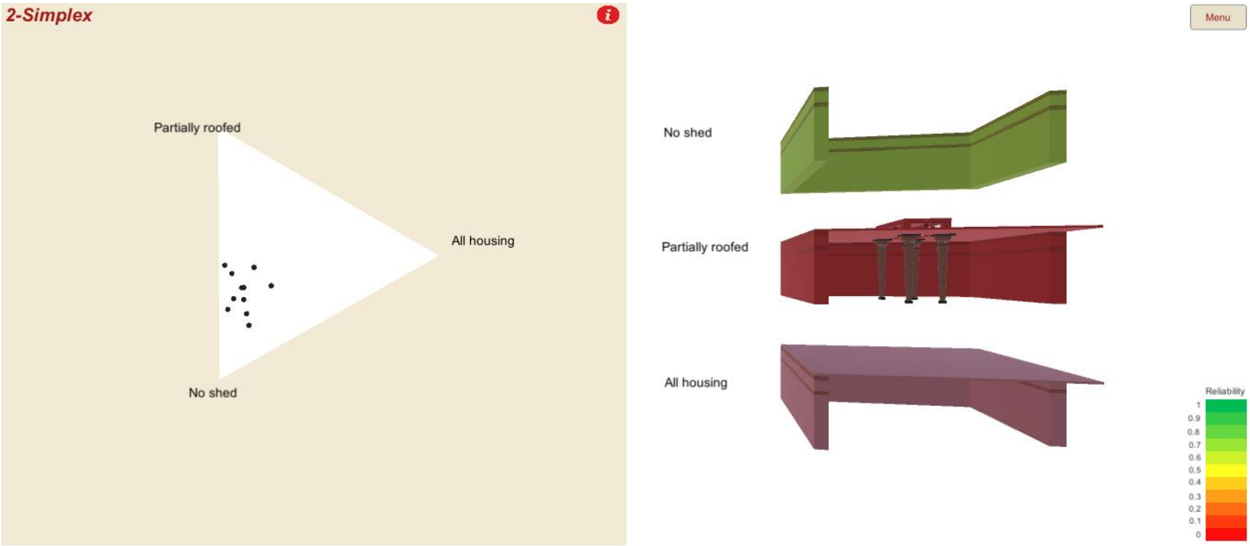

Figure 5.56:2-Simplex for Uncertainty Point 2

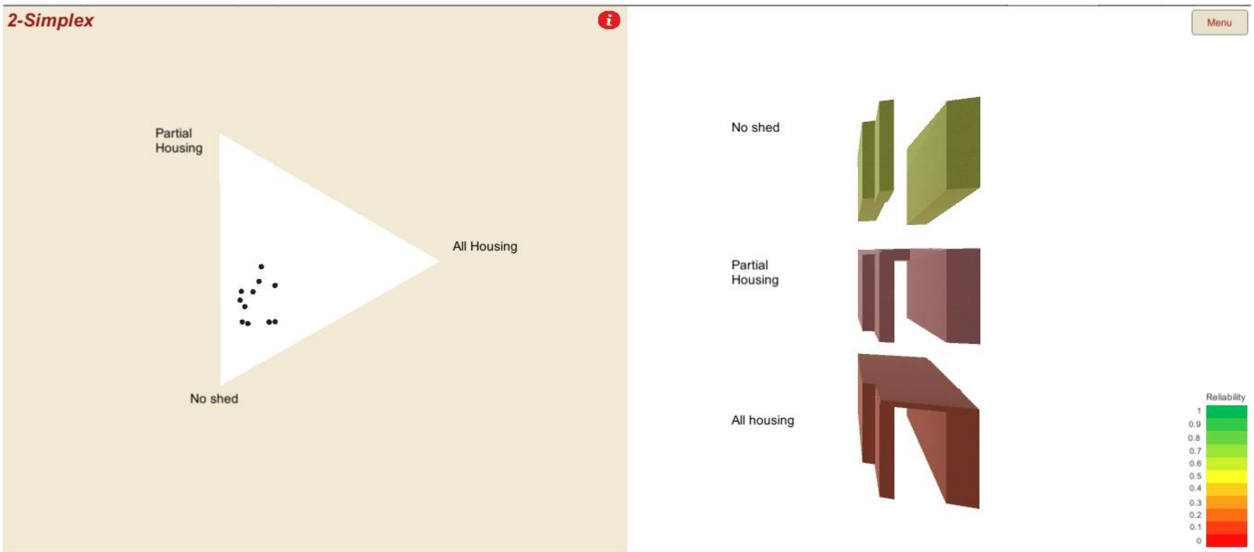

Figure 5.57:2-Simplex for Uncertainty Point 3

6 Visual Reconstruction of the Palace of Zakros

The purpose of this chapter is to present the 3D visualization of the 3D reconstructions based on archaeological uncertainty calculations and the interaction of the user with the reconstructed Palace of Zakros. The generation of the 3D model is analyzed in detail in section 6.1. In section 6.2, we present the interactive elements offered by the uncertainty visualization system to the user. First of all, the user has the opportunity to navigate inside the palace, as described in the section 6.2.1. In section 6.2.2, the user is presented with information about every part of the building, while navigating the archeological site. Finally, in section 6.2.3, we present the interaction capabilities of the user with certain objects found in the Palace of Zakros, such as vases and pithos.

6.1 Generation of the 3D model

6.1.1 The environment

When the users initially enter the scene, they will come across a top view of the Palace of Zakros and its surrounding environment. The environment consists of a terrain. The terrain includes the mountain near the remnants of the palace, the hills behind the palace to which the settlements stretched and the bay near the palace. Also, a few trees were placed in the surrounding area to complement the visualization of the natural environment. Figure 6.1 shows the three-dimensional visualization of the environment.

We created the surrounding areas around the palace based on photographs given to us by the expert archaeologist, Dr. Simantiraki with whom we closely collaborated throughout this thesis. Figures 6.2 and Figure 6.3 depict the natural environment around the Palace of Zakros. These figures helped us to create the ground, the trees, the bay and the other elements of the environment.

Figure 6.1: The 3D environment.

Figure 6.2: Aerial photography of the actual site of the Palace of Zakros, Crete, Greece (Platon, N. 1974).

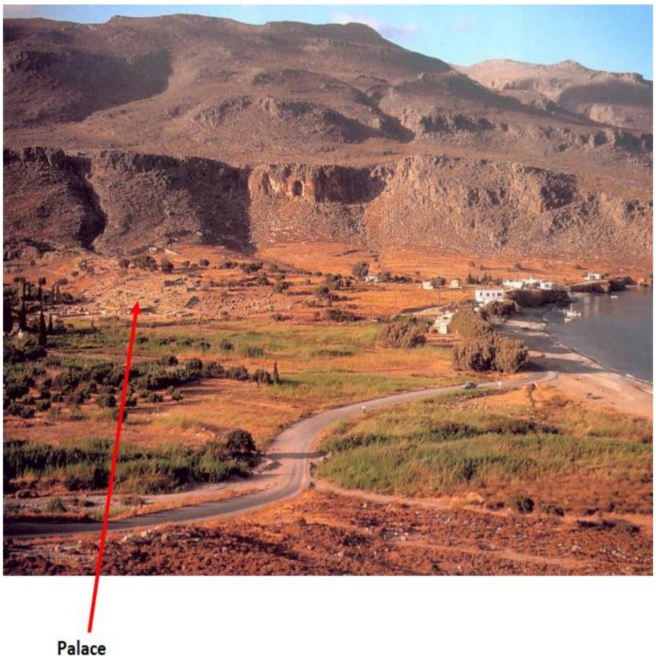

Figure 6.3: The general view of the bay of Zakros (Platon, N. 1974).

To make the visualization of the area more plausible, some of the settlements were created following a hypothetical drawing of how the palace used to be, which was the work of the painter K. Iliakis (Platon, N. 1974). Figure 7.4 shows the palace and the settlements that were built around it, as depicted in the work by K. Iliakis.

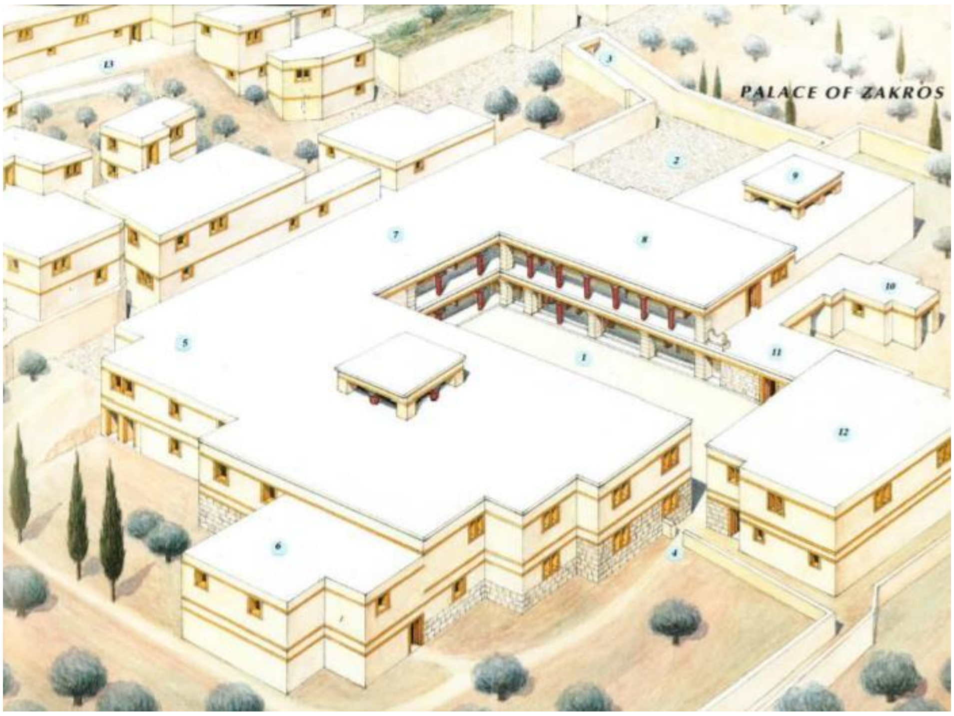

Figure 7.6: A hypothetical drawing of the Palace of Zakros, Crete, Greece (Platon, N. 1974).

Initially, the point of view is outdoors. The interface includes two buttons on the top right of the screen, as shown in Figure 6.5. The first button is the “Palace” and the second is the “Info”. If the users press the button “Palace”, they will be transferred inside the palace so that it can be navigated. The details of the user’s navigation capability are listed in Section 6.2.1. If the users click on the Info button, a window containing general information about the palace and the surrounding area pops up. The users can press the “X” button to close the information window. Figure 6.6 depicts the information window.

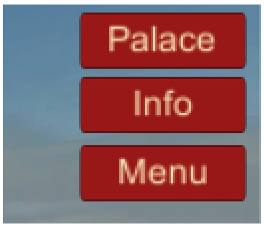

Figure 6.5: The UI buttons.

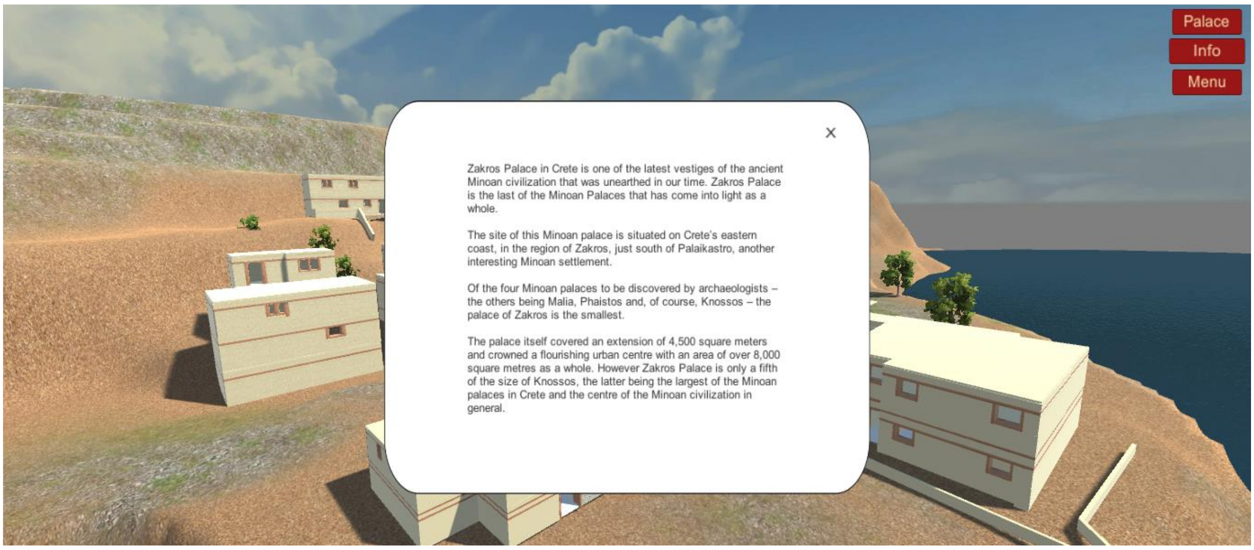

Figure 6.6: The information window.

6.1.2 The 3D reconstruction of the Zakros Palace

To create the 3D reconstruction of the Palace of Zakros, we used the architectural designs present on Platon's book, Zakros the Discovery of a Lost Palace (Platon, N. 1974). Certain of the floor plans are shown in Figures 6.1-6.8.

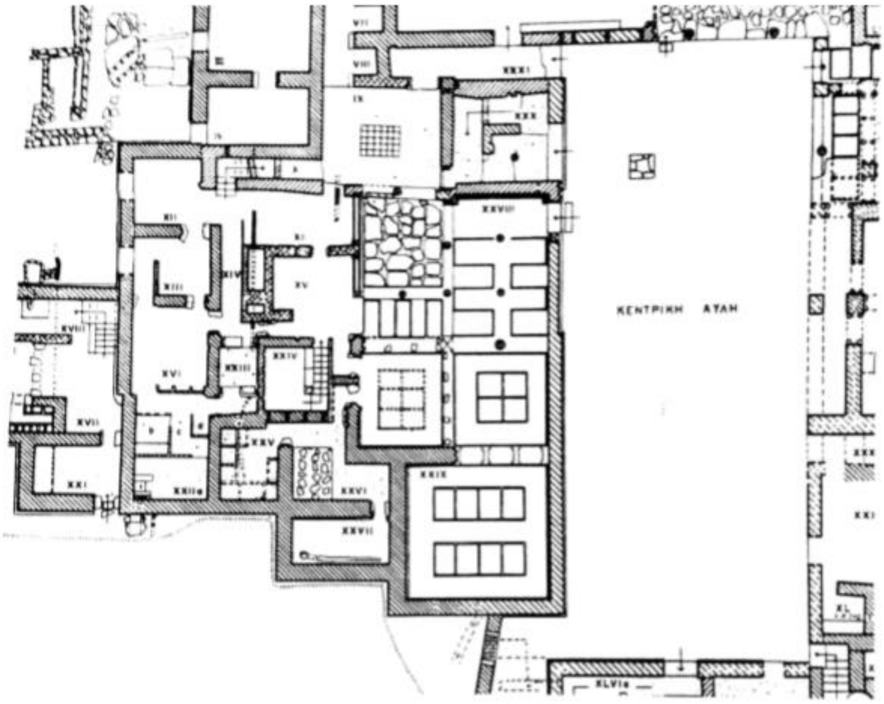

Figure 6.7: Top view of the West Wing (Platon, N. 1974).

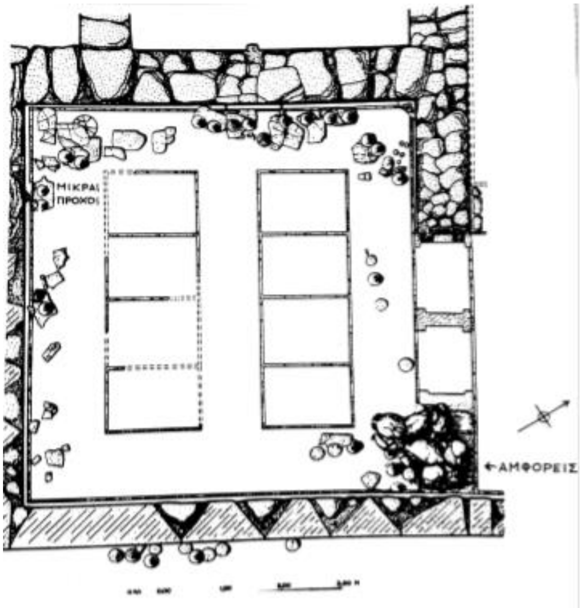

Figure 6.8: Symposium Floor Plan (Platon, N. 1974).

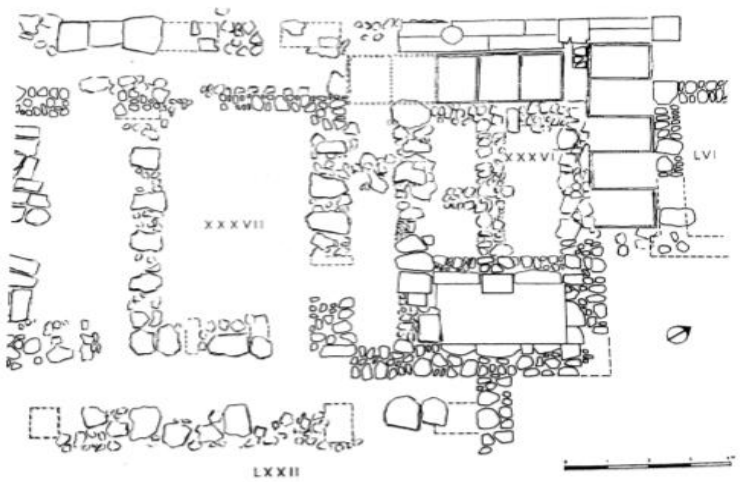

Figure 6.9: Floor plan of royal apartments (Platon, N. 1974).

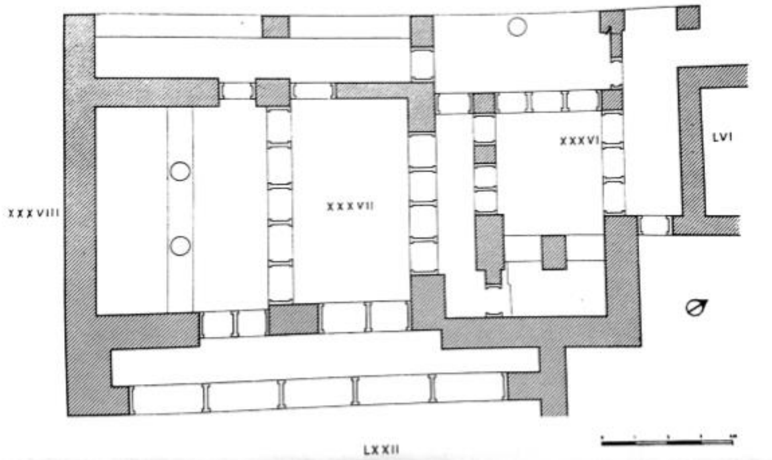

Figure 6.10: Floor plan of a possible reconstruction of royal apartments (Platon, N. 1974).

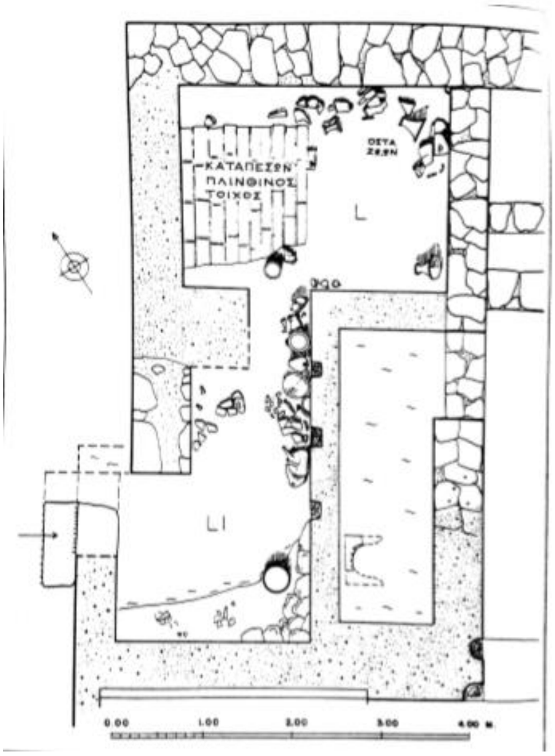

Figure 6.11: Floor plan of auxiliary rooms (Platon, N. 1974).

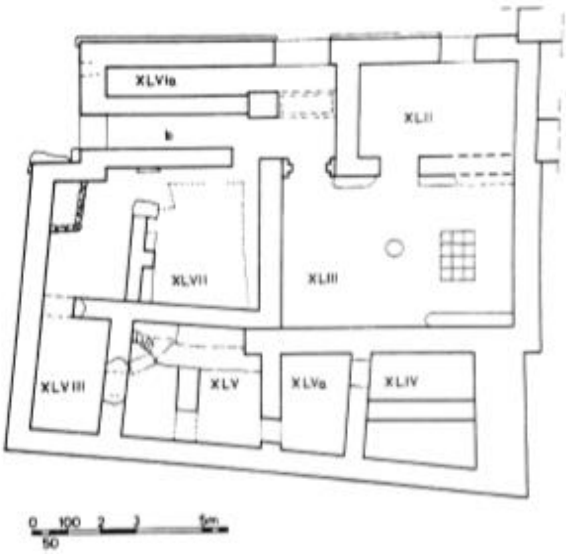

Figure 6.12: Plan the laboratory sector of southern wings (Platon, N. 1974).

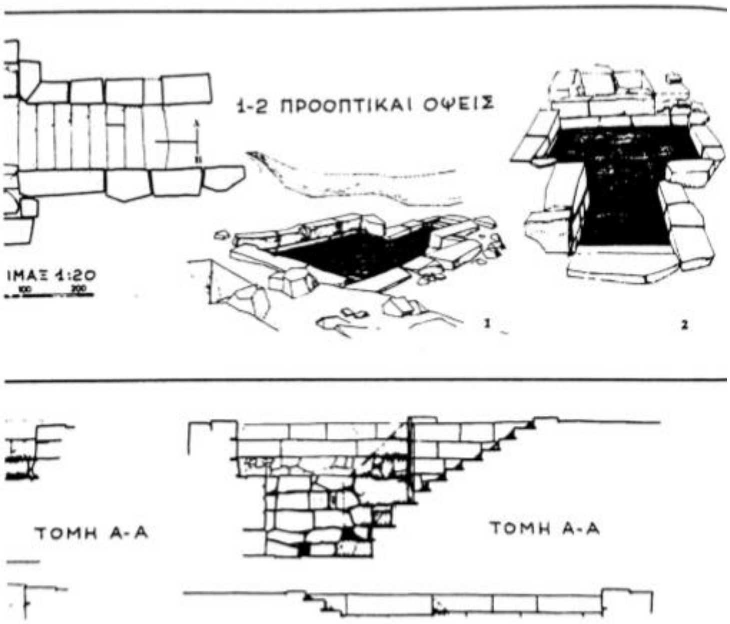

Figure 6.13: A fountain of the palace (Platon, N. 1974).

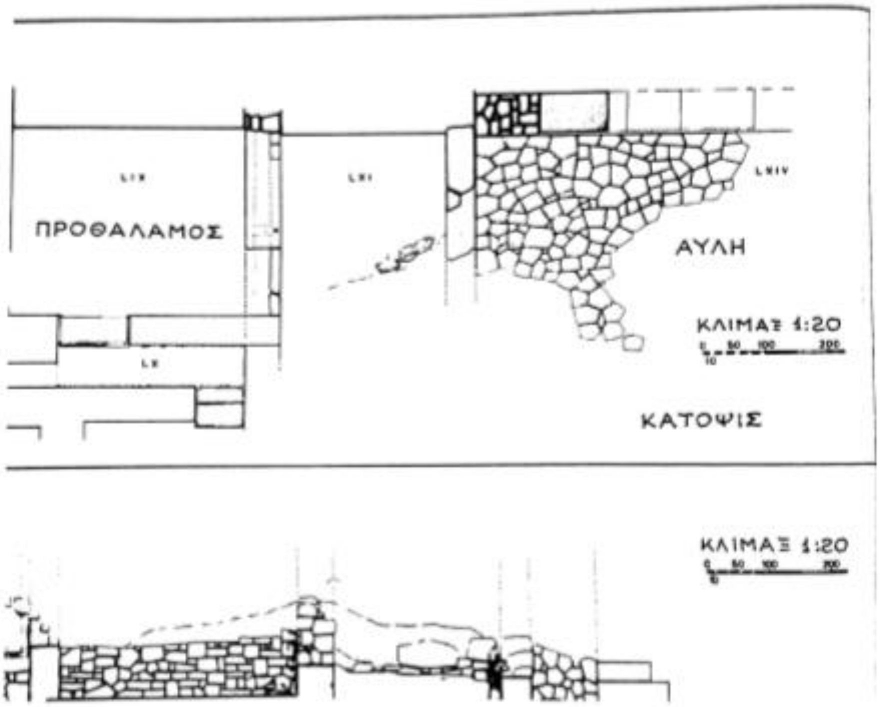

Figure 6.14: Plan of the bath and the adjoining rooms (Platon, N. 1974).

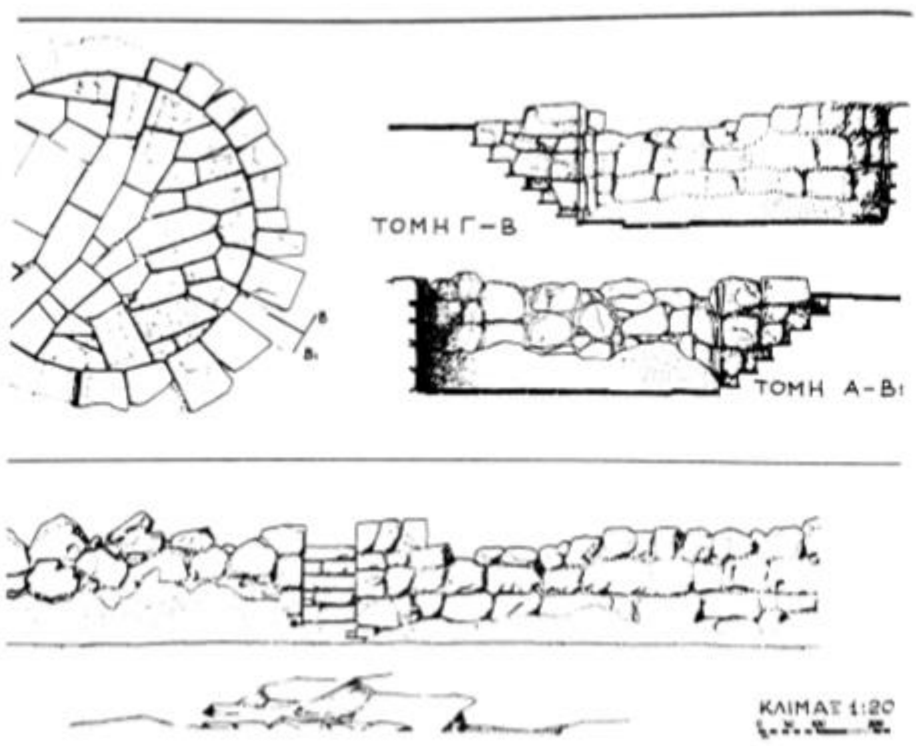

Figure 6.15: Plan of the circular tank of the large square room (Platon, N. 1974).

The top view of the reconstruction of the Palace of Zakros is shown in Figure 6.16.

Figure 6.16: The plan of the reconstructed Palace.

6.1.3 The reconstructed objects of the Palace

Apart from the reconstruction of the building, we created a few objects in 3D which represent objects actually unearthed during the excavations of the Palace of Zakros (Platon, N. 1974). The first object was a pithos, a large earthenware storage jar (Figure 6.17). There were many in the Palace, which were used for storage. The second was an amphora which was placed in the purifying basin of the sanctuary (Figure 6.18). The third was a cup of Holy society which was placed in the treasury of the sanctuary (Figure 6.19). The next one was a conical vase (Figure 6.20) and the last one was a receptacle which was placed in a storage of the West Wing (Figure 6.21).

Figure 6.17: Pithos, a large earthenware storage jar.

Figure 6.18: Amphora which was placed in the purifying basin of the sanctuary.

Figure 6.19: Cup of Holy society, placed in the treasury of the sanctuary.

Figure 6.20: Conical vase.

Figure 6.21: Receptacle which was placed in a storage of the West Wing.

6.2 User Interaction

In this project, the user has the ability to interact with the 3D reconstruction. Section 6.2.1 describes the user's navigation capability in the 3D model of the palace. Section 6.2.2 describes the information of the Palace of Zakros as displayed to the user and user interaction with it. Finally, in Section 6.2.3 is describes the interaction of the user with the objects of the Palace.

6.2.1 Navigation inside the Palace

The users can navigate the 3D reconstruction of the Palace of Zakros by pressing the arrow keys on the keyboard. The palace is large in extent, so it is possible for the users to move into the space by selecting the area of the Palace they wish to navigate through the dropdown menu. The users can see a button named “Wings” (Figure 6.21). If they press this button, four dropdown menus are displayed to the users, one dropdown menu for every wing of the palace (Figure 6.22). Every dropdown menu includes as options the areas belonging to each wing. For example, if the users press the dropdown menu named “North Wing”, five options appear, e.g., North Wing, Kitchen, Storage area, Auxiliary rooms, Stairwell (Figure 6.24). We select the option of the “North Wing”, in order to be able to turn to that wing. These menus serve as navigation aids towards the specified spatial locations. It is certainly possible to reach every part of the 3D reconstructed model by free navigation.

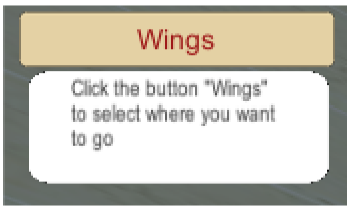

Figure 6.21: The button Wing.

Figure 6.22: The user's option after pressing the "Wings".

Figure 6.23: Options of the DropDown "North Wing".

6.2.2 The points with uncertainty

Guided by Dr Simantiraki's input, we selected three points in the Palace of Zakros to visualize their multiple reconstructions signifying different hypotheses of how they existed in the past. Besides offering these varied reconstruction options, the system offers the experts' confidence in each reconstructed option visualized employing a green to red visualization scheme demonstrating the relative experts' uncertainty for each reconstruction. For these points, the goal was to visualize the possible interpretations of how they could have existed in the past, as well as the associated uncertainty level associated to each of them.

For the visualization of uncertainty, we used a green to red colour visualization scheme. The color visualization uses a 10-value scale of equal intervals between Red and Green. The greater the level of uncertainty of a point the closer to the red appears. The higher the confidence in a reconstruction, it is more closely visualized in green. We preferred colour visualization and not transparency visualization (more transparent means more uncertain) because the building was complex and the user would be confused if the transparency scheme would have been adopted.

The user can reach the spatial locations for which varied reconstructions are offered in two ways. The first is navigating the Palace and reaching these points based on the 3D navigation. The second is by pressing the button named "Uncertainty Points" and selecting the exact spatial location to be teleported, signified by Point1, Point 2, Point3. By hovering on the buttons, the user can actually see text in relation to which these locations are as shown in Figure 6.25. This ability is clear in Figure 6.21 and 6.22.

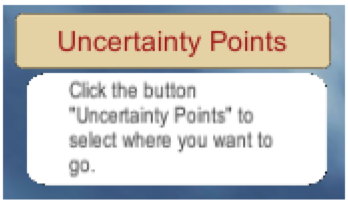

Figure 6.24: Hover the button "Uncertainty Menu".

Figure 6.25: Hover the Buttons.

When the user approaches the location of an interest point based on free navigation, the user needs to approach a specified invisible area so that the multiple reconstructions get activated. When approaching, the user views the most probable reconstruction visualized in colour as the colourways of the rest of the building. When approaching, the colour is going to change based on the green-red visualization scheme (the greener will be visualized first). Two arrows are then going to appear left and right which the user can press to view the remaining two reconstructed interpretations for the same spatial location.

The first point of uncertainty is the Wall of the West Wing. The three possible interpretations are a wall without windows, a wall that has shutters on the windows or a wall that has windows and is covered with leather. The walls were created in Unity3D, in the same way as the rest of the palace. Shutters and leather for windows were created in SketchUp. In order to make the leather appear realistic, we used a leather material from the Asset Store of the Unity3D platform. Figure 6.27 shows how the shutters were created and how they were incorporated into the Palace. Respectively, Figure 6.28 also shows the design of the material that covered the window.

The second point of uncertainty is in the Basin Hall. This room either has no shelter at all, be partially roofed or completely roofed. Figure 6.29 shows the second interpretation as represented in rendering. Figure 6.29, 6.30, 6.31 show the three possible interpretations of the Basin Hall.

The third point of uncertainty is at the Entrance of the Palace. The first interpretation of this point is the corridor not having a shelter, to be partial roofed or the corridor to be covered. Figures 6.32, 6.33, 6.34 show the three possible interpretations of the Palace's entrance.

In this thesis, the Dirichlet Distribution was used to study the uncertainty level associated with each reconstruction and how to calculate these based on archeologists' beliefs and confidence concerning the possible hypotheses of how the specific structures of interest existed in the past. As we have seen in Chapter 5, the Dirichlet distribution does not come up with one number for uncertainty but with a distribution of probabilities. In order to define the colour gradient of each structural interpretation in the green to red colour scheme, we used the uncertainty values for each interpretation which are most likely to apply. Specifically, this value is the mean of the Dirichlet distribution, each one associated with a specific Dirichlet distribution associated with each interpretation (out of three for each spatial interest point). The mean is calculated by the mathematical formula: $E_i = \frac{a_i}{A}$, where $A = \sum_i \quad A$.

Figure 6.26: Uncertainty Visualization for the first reconstruction of Point 1 (Wall of the West Wing).

Figure 6.27: Uncertainty Visualization for the second reconstruction of Point 1 (Wall of the West Wing).

Figure 6.28: Uncertainty Visualization for the third reconstruction of (Wall of the West Wing).

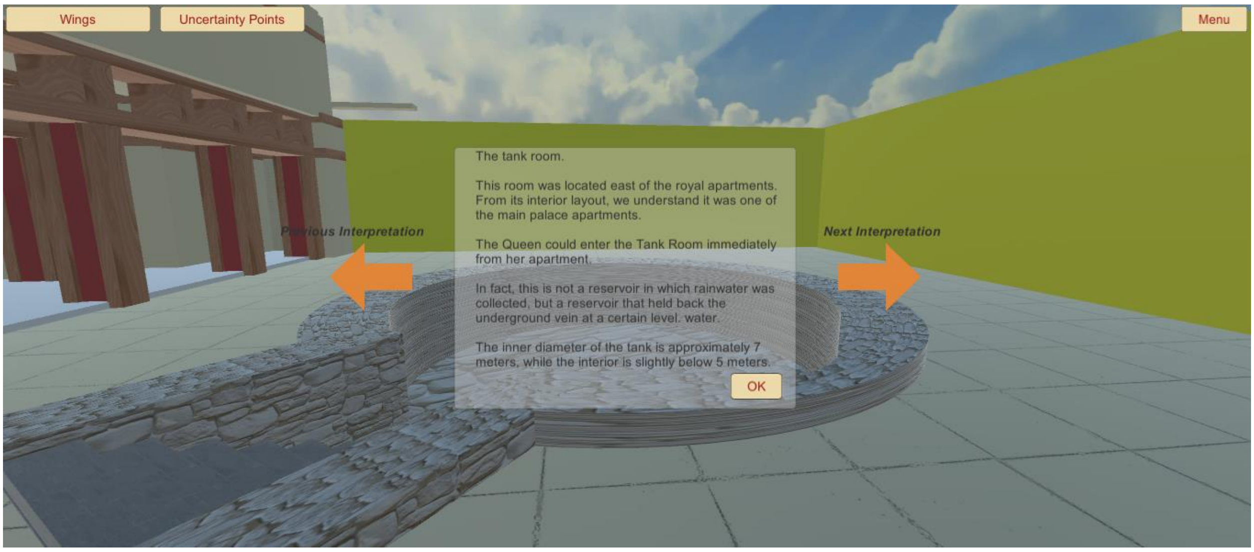

Figure 6.29: Uncertainty Visualization for the first reconstruction of Point 2 (Basin Hall).

Figure 6.30: Uncertainty Visualization for the second reconstruction of Point 2 (Basin Hall).

Figure 6.31: Uncertainty Visualization for the third reconstruction of Point 2 (Basin Hall).

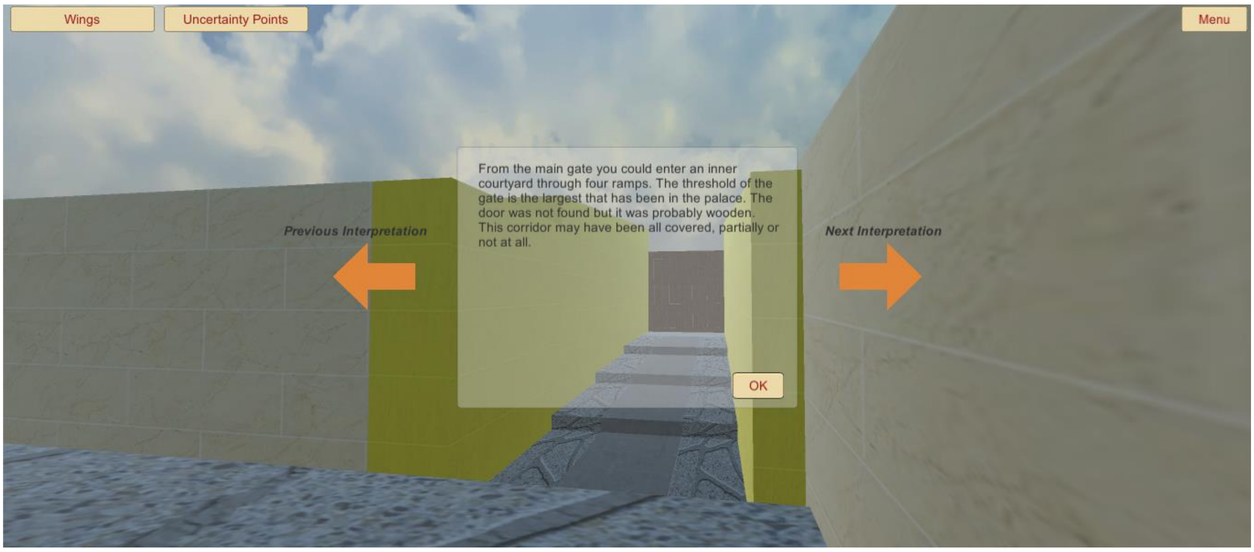

Figure 6.32: Uncertainty Visualization for the first reconstruction of Point 3 (Entrance of the Palace).

Figure 6.33: Uncertainty Visualization for the second reconstruction of Point 3 (Entrance of the Palace).

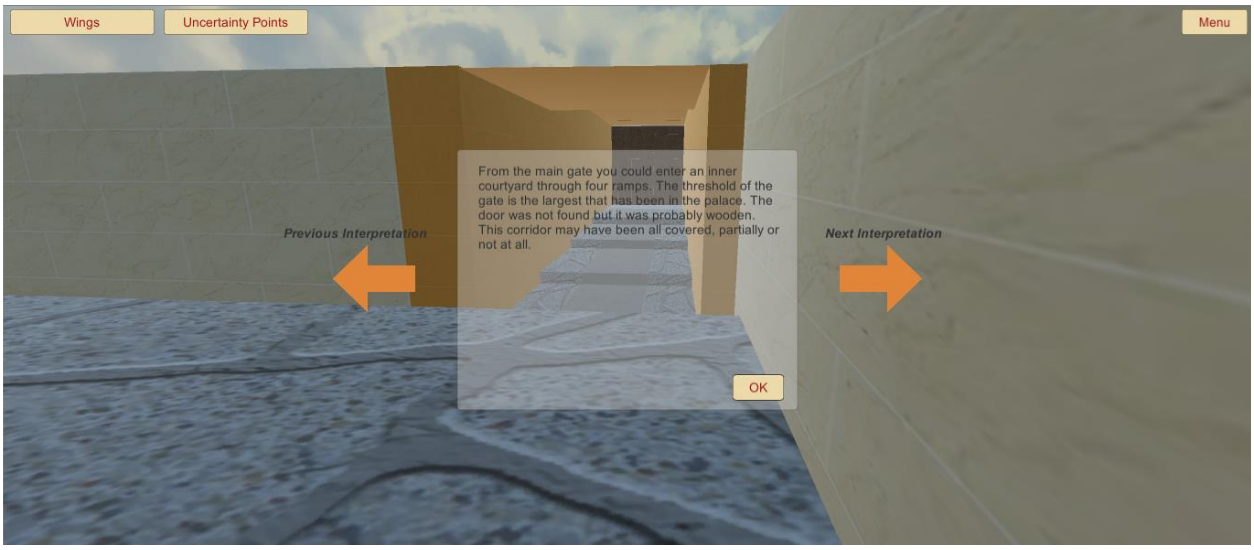

Figure 6.34: Uncertainty Visualization for the third reconstruction of Point 3 (Entrance of the Palace).

6.2.3 Information

As users navigate the Palace of Zakros, information about where they are located are displayed in front of them. The user has the ability, by pressing a button, to remove this information from the viewing window. When the information window is removed, a small red icon is displayed on the bottom right corner which, if pressed, the information will re-appear (Figure 6.36). For example, if users are in the central court area information about this area of the Palace will appear, as shown in Figure 6.35.

Figure 6.35: Information about the central court.

Figure 6.36: Closed information window.

6.2.4 Objects

An important feature of the system is the inclusion in the Palace of certain objects unearthed in the Palace of Zakros during excavations, modeled in 3D. The user can view them in the scene and interact with them. If the cursor is placed over an object, the user can see which object that is. If the user clicks on an object, information about this object is visible. For example, Figure 6.37 shows a Pithos. When the user hovers over this object, the object's name is displayed over it. If the user clicks on the object, information appears about as shown in Figure 6.38. When the "o" is pressed as input from the keyboard, the information window is removed.

Figure 6.37: Mouse over the object “Pithos”.

Figure 6.38: Pithos’ information.

As we mentioned Dr Anna Simantiraki helped as with the details of the Palace and she gave us the multiple options of reconstruction of every point. After the end of the project, she interacted with the system and the input and out were very clear for her. Under the circumstances it wasn’t necessary to change something in the visualization system. However, when she saw the multiple reconstruction options in the 3D environment of Unity, she gave more details for them in order to be more valid the reconstructions and she gave us more reconstruction options. For example, we hadn’t three different options for the reconstruction of the Entrance of the Palace because we hadn’t enough evidence of our own. Thus, she gave us the third reconstruction option to have no roof above the entrance. Also, when she saw the general image of the 3D reconstructed Palace of Zakros to do more abruptly which wasn’t very clear from the picture of the book.

6.3 Database Implementation

6.3.1 Questionnaire

For the purposes of this project a database was required to save the answers of archaeologists as data. Having stored the answers in a database, we use the averages of the answers of the archaeologists for every section, for example an average for the answers of the “features”, “artefacts” etc. and we give them as the relative weight for every type of evidence that have been identified.

In this project, as it is described in Chapter 4.3, the phpMyAdmin has been used in combination with XAMPP software tools. The XAMPP tool used as it offers an Apache web server which processes and delivers web content to a computer and it’s the most popular web server online. It also offers a MySQL database server in order to connect with the phpMyAdmin software tool that’s handling the MySQL database over the web.

For this part of the system there are four rows in the table. Every row has the values that are given in the User Interface from archaeologists for every influencing factor. The tables have a unique ID for every record in the table and a column for every statement as we can see the structure in Figure 6.39. The answers are saved in columns fcomp1, fcomp2 etc. and they are of type int(11) because the answer “Never” has the value 1, the answer “Almost Never” has the value 2 etc. Figure 6.40 illustrates some examples of records. It is clear from the second picture that for the first archaeologist the answer in the four questions is “Never”, as the value for fcomp1, fcomp2, fcomp3, fcomp4 is 1.

#	Name	Type	Collation	Attributes	Null	Default	Comments	Extra	Action
1	**id6**	int(11)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
2	**fcomp1**	int(11)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
3	**fcomp2**	int(11)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
4	**fcomp3**	int(11)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
5	**fcomp4**	int(11)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More

Figure 6.39: Table Structure.

fcomp1 ▲ 1	fcomp2	fcomp3	fcomp4
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
3	1	2	1
3	1	1	3

Figure 6.40: Records Examples.

In order to transfer the values from Unity to database, an SQL query has been written in PHP code (see Figure 6.41).

```
$id6 = $_POST["id6Post"];
$fcomp1 = $_POST["fcomp1Post"];
$fcomp2 = $_POST["fcomp2Post"];
$fcomp3 = $_POST["fcomp3Post"];
$fcomp4 = $_POST["fcomp4Post"];
$fcomp5 = $_POST["fcomp5Post"];
$fcomp6 = $_POST["fcomp6Post"];
$fcomp7 = $_POST["fcomp7Post"];
$fcomp8 = $_POST["fcomp8Post"];


//Make Connection
$conn = new mysqli($servername, $username, $password, $dbName);

//Check Connection
if(!$conn){
	die("Connection Failed. ". mysqli_connect_error());
}
else echo("Connection Success");

$sql = "INSERT INTO questionnaire (id6 , fcomp1 , fcomp2 , fcomp3 , fcomp4 , fcomp5 , fcomp6, fcomp7, fcomp8)
	VALUES ('".$id6."','".$fcomp1."','".$fcomp2."','".$fcomp3."','".$fcomp4."','".$fcomp5."','".$fcomp6."','".$fcomp7."','".$fcomp8."')";
$result = mysqli_query($conn ,$sql);



if(!result) echo "there was an error";
else echo "Everything ok.";
```

Figure 6.41: Insert Query for Features Table.

In order to implement the questionnaire and to connect the database with the Unity the following scripting was necessary. First of all, we had to initialize the URL of php file as we can see in Figure 6.42 which has the appropriate query for the insertion of data in the database.

```
string CreateURL = "http://localhost/thesisDB/Features.php";
```

Figure 6.42: Php file URL.

The necessary values for every page of our questionnaire were these for back and next page, the values of every dropdown (Figure 6.43) and a bool variable which has the value 1 if the archaeologist has done with the specific section of the questionnaire.

```
public Button back;
public Button nextpage;

public Dropdown d1;
public Dropdown d2;
public Dropdown d3;
public Dropdown d4;
public Dropdown d5;
public Dropdown d6;
public Dropdown d7;
public Dropdown d8;


static int id;
static int featComp_1;
static int featComp_2;
static int featComp_3;
static int featComp_4;
static int featComp_5;
static int featComp_6;
static int featComp_7;
static int featComp_8;

bool endOfPage6;
```

Figure 6.43: Necessary variables.

When archeologists want to continue to the next page/section of the questionnaire there was the code of Figure 6.44 in this class which posts the values of every statement in Database. The saved values for the statements were the values of the Dropdown plus 1 because the Unity starts the values from 0 and we wanted the smallest value to be the 1.

```
void NextPage()
{
    if (endOfPage6 == false)
    {
        id = mainMenu.archId;

        WWWForm form = new WWWForm();
        form.AddField("id6Post", id);
        form.AddField("fcomp1Post", featComp_1);
        form.AddField("fcomp2Post", featComp_2);
        form.AddField("fcomp3Post", featComp_3);
        form.AddField("fcomp4Post", featComp_4);
        form.AddField("fcomp5Post", featComp_5);
        form.AddField("fcomp6Post", featComp_6);
        form.AddField("fcomp7Post", featComp_7);
        form.AddField("fcomp8Post", featComp_8);

        WWW www = new WWW(CreateURL, form);

        page_7.SetActive(true);
        page_6.SetActive(false);
        endOfPage6 = true;
    }
}

void Answers()
{

    featComp_1 = d1.value + 1;
    featComp_2 = d2.value + 1;
    featComp_3 = d3.value + 1;
    featComp_4 = d4.value + 1;
    featComp_5 = d5.value + 1;
    featComp_6 = d6.value + 1;
    featComp_7 = d7.value + 1;
    featComp_8 = d8.value + 1;

    print("featComp_1" + d1.value + "featComp_2" + d2.value + "featComp_3" + d3.value + "featComp_4" + d4.value + "featComp_5" + d5.value + "featComp_6" + d6.value + "featComp_4" + d6.value + "featComp_6" + d6.value);


}
```

Figure 6.44: Post of Results in Database.

The Figure 6.45 shows the code which was implemented for the back which can be press from archaeologists in order to change page.

```
void Back()
{
    page_5.SetActive(true);
    page_6.SetActive(false);
}
```

Figure 6.45: Back Button.

6.3.2 Archaeologists' belief

For this part of the system there is a table in the database, a table for each part of the palace we are considering. As we mentioned in previous sections, we study the uncertainty for three points of the Zakros palace. Therefore, there are three tables in the database, having the same structure as it is shown in Figure 6.46. In the Setting Panel, archaeologists can give their belief about how likely every scenario is through three sliders. These values are float numbers between 0 and 1. Moreover, they can choose the type of evidence which helped them to form this belief. The checkboxes give the value 0 if the checkbox isn't selected and the value 1 if it is selected. The tables have a unique ID for every record in the table, three columns for every slider and five columns for the checkboxes. Figure 6.47 illustrates some examples of records. More specific, the figure shows that the first archaeologist believes that the first reconstruction is more possible than the other two (th1 > th2 > th3). The factors which led him to this opinion were features, absolute comparison with other similar archaeological sites, topography and peer review, as these factors are given the value 1.

#	Name	Type	Collation	Attributes	Null	Default	Comments	Extra	Action
1	**idAr2**	int(11)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
2	**th1**	float			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
3	**th2**	float			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
4	**th3**	float			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
5	**feaut**	tinyint(4)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
6	**art**	tinyint(4)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
7	**absComp**	tinyint(4)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
8	**top**	tinyint(4)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More
9	**peer**	tinyint(4)			No	*None*			Change Drop Primary Unique Index Spatial Fulltext More

Figure 6.46: Table Structure.

th1	th2	th3	feat	art	absComp	top	peer
0.5	0.3	0.2	1	0	1	1	1
0.5	0.3	0.2	1	0	1	1	1
0.5	0.3	0.2	1	0	1	1	1
0.0404	0.94796	0.01164	1	1	1	1	1
0.314043	0.357721	0.328236	1	1	1	1	1
0.337156	0.314297	0.348547	1	1	1	1	1
0.705379	0.476689	0.2	1	1	1	1	1
0.5	0.3	0.2	1	1	1	1	1
0.871979	0.3	0.2	1	1	1	1	1
0.5	0.3	0.2	1	1	1	1	1
0.5	0.3	0.2	1	1	1	1	1
0.5	0.3	0.2	1	1	1	1	1
0.5	0.3	0.2	1	1	1	1	1
0.706145	0.525766	0.274976	0	1	0	1	1
0.88134	0.471807	0.745385	0	1	0	1	1
0.630346	0.494392	0.797675	0	1	0	1	1

Figure 6.47: Records Examples.

In order to transfer the values from Unity to database, an SQL query has been written in PHP code (see Figure 6.48). The values of the three sliders are inserted in the database through insert query as ‘th1’, ‘th2’ and ‘th3’. The values of checkboxes for the influencing factors are inserted in the database as ‘feat’, ‘art’, ‘absComp’, ‘top’ and ‘peer’.

```
$idAr2 = $_POST["idAr2Post"];
$th1 = $_POST["theta1"];
$th2 = $_POST["theta2"];
$th3 = $_POST["theta3"];
$feat = $_POST["feaut"];
$art = $_POST["art"];
$absComp = $_POST["absComp"];
$top = $_POST["top"];
$contComp = $_POST["contComp"];
$peer = $_POST["peer"];

//Make Connection
$conn = new mysqli($servername, $username, $password, $dbName);

//Check Connection
if(!$conn){
        die("Connection Failed. ". mysqli_connect_error());
}
else echo("Connection Success");


$sql = "INSERT INTO windows (th1 , th2 , th3 , feat , art , absComp , top , contComp , peer)
        VALUES ('".$th1."','".$th2."','".$th3."','".$feat."','".$art."','".$absComp."','".$top."','".$contComp."','".$peer."')";
```

Figure 6.48: Insert Query for Setting Panel.

For the implementation of the setting panel the code of Figure 7.49 was necessary in order to calculate the values the importance of every influencing factor. As we mentioned these are features, artefacts, comparisons, peer review and topography.

```
for (int i=1;i<=4;i++) {

    for (int j=i-1;j>=0;j--) {
        print("values[j]" + values[j] + "values[i]" + values[i]);
        if (values[j] > values[i])
        {
            float tmp;
            tmp = values[j];
            values[j] = values[i];
            values[i] = tmp;

            break;
        }
    }

}
for (int i = 0; i <= 4; i++)
{
    for (int j=0; j<=5; j++) {
        if (values[i]== valuesStart[j]) {
            valuesInt[i] = j;
        }
    }
}
```

Figure 6.49: Order of influencing factors.

```
if (valuesInt[0] == 4) {
    ad5On = 1.0f;
} else if (valuesInt[0] == 3) {
    ad4On = 1.0f;
} else if (valuesInt[0] == 2) {
    ad3On = 1.0f;
} else if (valuesInt[0] == 1) {
    ad2On = 1.0f;
} else if (valuesInt[0] == 0) {
    ad1On = 1.0f;
}

if (valuesInt[1] == 4)
{
    ad5On = 1.25f;
}
else if (valuesInt[1] == 3)
{
    ad4On = 1.25f;
}
else if (valuesInt[1] == 2)
{
    ad3On = 1.25f;
}
else if (valuesInt[1] == 1)
{
    ad2On = 1.25f;
}
else if (valuesInt[1] == 0)
{
    ad1On = 1.25f;
}
```

Figure 6.50: Weight of Influencing Factors.

The Figure 6.50 shows how we give the appropriate weight to each influencing factor. We check which of these factors has the lowest value and we give to the appropriate variable the corresponding value. For example, if the less significant type of evidence is the 'feature' we give to the variable ad1On the value 1.0f, if the less significant type of evidence is the 'artefacts' we give to the variable ad2On the value 1.0f etc.

```
if (t1.isOn)
{
    feaut = 1;
    ad1 = ad1On;
}
else {
    feaut = 0;
    ad1 = 0;
}
```

Figure 6.51: Example with 'feature' influencing factor.

We use the example of 'feature' influencing factor, if an archaeologist has checked the checkbox of this factor, the Boolean value of the variable becomes true (1) and the weight of this influencing factor is the value of ad1On.

```
sumEv = ad1 + ad2 + ad3 + ad4 + ad5 ;
```

Figure 6.52: Summary of the values of evidence.

When archaeologists press the button “RUN” the data will be saved to our database. The Figure 6.53 shows the URL of php script with the Insert query. These data are the values of the sliders and the values of the checkboxes for the evidence types.

```
string CreateURL = "http://localhost/thesisDB/giveUncertainty.php";
```

Figure 6.53: Php file URL.

We have created three sliders with the help of Unity and the value is multiplied by 10 because sliders give float values (Figure 6.54 and 6.55). The code in Figure 6.37 shows the post of values in database.

```
public Slider slider1;
public Slider slider2;
public Slider slider3;
```

Figure 6.54: Slider’s variables.

```
sl1 = slider1.value * 10;
sl2 = slider2.value * 10;
sl3 = slider3.value * 10;
```

Figure 6.55: Slider values multiplication.

In Figure 6.54 there are the three variables of sliders, one for each alternative reconstruction option. As we mentioned, archaeologists give their opinion about how sure they are about these options. In our system, these three values are multiplied with 10 (Figure 6.55), in order to have higher values.

```
a1 = sl1 * sumEv;
a2 = sl2 * sumEv;
a3 = sl3 * sumEv;


if (pussed == true)
{
    finala1 = finala1 + a1;
    finala2 = finala2 + a2;
    finala3 = finala3 + a3;

    print("sl1" + sl1);
    print("sl2" + sl2);
    print("sl3" + sl3);
    print("feaut" + feaut);
    print("art" + art);
    print("absComp" + absComp);
    print("top" + top);
    print("peer" + peer);

    WWWForm form = new WWWForm();

    id = mainMenu.archId;
    form.AddField("idAr2Post", id);
    form.AddField("theta1", slider1.value.ToString());
    form.AddField("theta2", slider2.value.ToString());
    form.AddField("theta3", slider3.value.ToString());
    form.AddField("feaut", feaut);
    form.AddField("art", art);
    form.AddField("absComp", absComp);
    form.AddField("top", top);
    form.AddField("peer", peer);

    WWW www = new WWW(CreateURL, form);
    pussed = false;
```

Figure 6.56: Post of the values in the database.

The post of archaeologists’ input was necessary for our project. For this reason, we opened the connection with database through the statements *WWWForm form = new WWWForm();* and *WWW www = new WWW(CreateURL,* *form);*. In the form we add the fields of the inputs and we insert them in the database. More specifically, we add the values of the three sliders and the five checkboxes of the user input.

6.3.3 Samples from Dirichlet distribution

As we have described in subsection 6.2, archaeologists insert their belief about whether the visualized reconstructions are accurate through three sliders in a setting panel and they select the evidence types which formed their belief in the accuracy of the visualized reconstruction. This information is inserted in tables of a database as data and it is necessary to manage and use them so as to generate samples from Dirichlet Distribution.

Figure 6.57 illustrates the Url which connects the database with Unity and Figure 6.58 shows the code of this php script which selects the data of archaeologists input that have been saved.

```
WWW itemsData = new WWW("http://localhost/thesisDB/getUncertainty.php");
```

Figure 6.57: Php file URL.

```
$sql2 = "SELECT count(*) as total from archuncertainty";
$sql = "SELECT idAr2 , th1, th2 , th3 , feat , art , absComp , top , contComp , peer FROM archuncertainty";
```

Figure 6.58: Select Query.

These data are a very important part of our system, as they shape the distribution and consequently the samples which are illustrated in the figures 6.59, 6.60, 6.61. In order to sample this distribution, the writing of some lines of code based on theoretical background was essential. In Chapter 3 and 4 our system and the method of sampling are described in details. Repeating what we have said in previous chapters referring to the mathematics of our research, in order to generate samples from the Dirichlet Distribution we have to follow a simple procedure: Sample values $y_1, \dots, y_L$ such that each $y_i \sim Gamma(a_i)$ and then normalize to get a probability distribution, where $Gamma(a, \beta)$ is the Gamma distribution. Based on this method we proceeded to produce the code.

Firstly, we created the function *GetDataValue(string data, string index)* of Figure 6.44, which gives the records of the tables saved in the database. The Figure 6.45 shows how we use these data in order to generate the samples of Dirichlet Distribution. As it is described in details in previous sections, Dirichlet Distribution has one hyperparameter, alpha hyperparameter. As we have mentioned, for each point of the Zakros Palace we study, there are three different versions of how this point might have been in the past. For this reason, the parameter alpha (α) is a vector of length 3. In our case study the updated Dirichlet distribution has parameters $a_1 = a_1 + c_1, a_2 = a_2 + c_2, a_2 = a_2 + c_2$, c_i is the counter C, described in details in the chapter 5.3.3, with length $i = 3$. Therefore, the scripts in the Figure 6.45 show how we calculate the parameters a_1, a_2, a_3. Figure 6.46 shows the generation of samples from Dirichlet Distribution using Gamma random variables, as defined above.

```
string GetDataValue(string data, string index)
{
    string value = data.Substring(data.IndexOf(index) + index.Length);
    if (value.Contains("|")) value = value.Remove(value.IndexOf("|"));
    return value;
}
```

Figure 6.59: Select Query.

```
yield return itemsData;
string itemsDataString = itemsData.text;
print(itemsDataString);

items = itemsDataString.Split(';');
tot = GetDataValue(items[0], "total:");
print("tot:" + tot);
total = int.Parse(tot);
print("total:" + total);


for (int i = 1; i <= total; i++)
{
    items = itemsDataString.Split(';');

    value = GetDataValue(items[i], "idAr2:");
    print("idAr2:" + value);
    id = int.Parse(value);
    print("idAr2:" + id);

    value = GetDataValue(items[i], "th1:");
    a1 = a1 + (float.Parse(value)) * 10;
    print("th1:" + a1);

    value = GetDataValue(items[i], "th2:");
    a2 = a2 + (float.Parse(value)) * 10;
    print("th2:" + a2);

    value = GetDataValue(items[i], "th3:");
    a3 = a3 + (float.Parse(value)) * 10;
    print("th3:" + a3);
}

a1 = a1 - 1;
a2 = a2 - 1;
a3 = a3 - 1;

Sample1();
```

Figure 6.60: Necessary Values for Sampling.

```
float U = Random.Range(0.0f, 1.0f);
float X1, X2;

//gamma sample
float d = (a1 - 0.33333f);
float c = (1 / Mathf.Sqrt(9 * d));
float v = 0;

do
{
    //normal sample
    do
    {
        U1 = Random.Range(-1.0f, 1.0f);
        U2 = Random.Range(-1.0f, 1.0f);

        S = Mathf.Pow(U1, 2) + Mathf.Pow(U2, 2);
    } while (S >= 1);

    log = Mathf.Log(S, 2.71828183f);
    X1 = U1 * Mathf.Sqrt((-2 * log) / S);
    X2 = U2 * Mathf.Sqrt((-2 * log) / S);

    v = Mathf.Pow(1 + c * X1, 3);
    print("v" + v);

} while (v <= 0 || Mathf.Log10(U) >= (0.5 * Mathf.Pow(X1, 2) + d - d * v + d * Mathf.Log(v)));

gammaRVa1 = d * v;
```

Figure 6.61: Sampling from Dirichlet Distribution.

7. Conclusions and Future Work

7.1 Main Contributions

This research examines archaeological theory and inferential process and provides insight into computer visualization. It describes how the two areas, of archaeology and computer graphics, have formatted a useful, but often tumultuous, relationship through the years. By uncertainty, we define an archaeological expert's level of confidence in an interpretation deriving from gathered evidence. Archaeologists and computer scientists have urged caution in the use of 3D for archaeological reconstructions because the availability of other possible hypotheses is not always being acknowledged. This thesis presents a complete visualization system for demonstrating archeological uncertainty in 3D reconstructions, by providing multiple reconstructed options of specific spatial locations, driven by an uncertainty mathematical model based on Bayesian probabilities. Archaeologists also insert their belief about whether the visualized reconstructions are accurate. They select the evidence types which formed their belief in the accuracy of the visualized reconstruction. Therefore, it is now possible different parts of a building to be associated with varied probabilities in relation to whether the actual reconstruction is accurate and at which degree. In this sense, the visualization offers multiple reconstructed options in relation to how an architectural structure existed in the past.

Many findings and evidence can give information about how the Palace of Zakros was existed in the past and which was its structure. For example, the remnants of the walls indicate the positioning of the structure of the Palace. The discussions with the archeologist Dr. Anna Semantraki-Grimshaw., who helps us to interpret the evidence unearthed during the excavation, certain elements of the palace are of high uncertainty in relation to their function or exact positioning. These are the wall of Western façade, the central court and the room of the tank. In the first case, there is uncertainty about the existence of windows on the wall. In the second case, in the central court a stone structure could be used as an altar or there could be a tree. In relation to the room of the tank, it is controversial whether this was an outdoor space or not. Certainly, there exist other spots in the Palace of Zakros with uncertainty. Some of the evidence contradict each other. This can lead to alternative scenarios concerning the positioning as well as the function of parts of palace affecting the reconstruction of the Palace of Zakros.

In our system archaeologists have to give their belief about which is the most likely option of reconstruction. The goal of this project is to develop a user interface based on which archaeologists will input their belief about a scenario and this belief will be the input of a probabilistic mathematical model. In the first step the archaeologists will give their belief about every different alternative scenario for specific locations of the Palace where there is uncertainty, by answering questions about how confident they are about every hypothesis, which will form the necessary input for the mathematical model. Then they will be able to visualize these different hypotheses based on distinct visualization outputs, each one depending on the results of the mathematical model.

The importance among different evidence types for archaeological interpretation identified through the design, distribution, and analysis of a questionnaire to expert archaeologists. This interpretation is derived from the relative importance of findings that have been uncovered from excavations, such as artifacts, textual evidence, ceramics, etc. A belief quantification model is analyzed and evaluated, in relation to archaeological uncertainty.

This thesis analyses and evaluates, in relation to archaeological uncertainty, a belief quantification model. By uncertainty we define an archaeological expert's level of confidence in an interpretation of how an archaeological site was in the past. The thesis examines another way to evaluate the belief of archaeologists in a reconstruction, by using the Dirichlet distribution.

Multiple uncertainty levels are visualized, related to how an archaeological structure may have existed in the past, at the same time, based on dynamic, mathematical uncertainty calculations, bringing the element of the archaeological expert's uncertainty in the visualization. In a single 3D archaeological reconstruction, we can now differentiate through diverse visualization the reconstructed parts for which there is strong evidence that they existed as visualized, as opposed to the ones that there is less evidence and knowledge about their past form. The test case we used was the remaining structure of the Palace of Zakros in Crete, Greece.

This thesis presents a complete visualization system for demonstrating archeological uncertainty in 3D reconstructions, by providing multiple reconstructed options of specific spatial locations, driven by an uncertainty mathematical model based on Bayesian probabilities. Expert archaeologists offer their opinion to statements through a questionnaire in order to identify the relative importance among a set of archeological evidence which relates to Minoan archeology. Evidence or the so-called influencing factors which were selected to represent types of archeological evidence of significance for the Minoan civilization are features (pits, walls, ditches, etc.), artefacts (tool, work of arts, etc.), comparisons (comparison with other structures of the same era or type of building), topography (area elevation, region characteristics, etc.), peer review (discussion with experts). Archaeologists also insert their belief about whether the visualized reconstructions are accurate. They select the evidence types which formed their belief in the accuracy of the visualized reconstruction. Based on this input, the output of the system offers a preview of the Probability Density Function (PDF) of the Dirichlet-Distribution, where archaeologists can see the density of the points and the point with higher density is the most likely combination of probabilities. The density of the points but also their spatial location and concentration in space (in a triangle, for which the three vertices represent three differently reconstructed options for the same spatial location) also reveals which reconstructed option is most probable. The system creates random values (random sampling) from this Distribution and these values appear as points on a triangle. Every point of the triangle is a different combination of probabilities. When archaeologists hover a point, they can simultaneously view all reconstructed options for the same spatial location, associated to a specific combination of probabilities. The color from red to green employed signifies a low to high probability for the range of reconstructed options offered, for the same spatial location. It is significant to note that when numerous expert archeologists provide input in relation to the significance of evidence types and their belief in the accuracy of the visualized reconstructions, their data is combined and the distribution, especially if the all agree, converges to one of the three reconstructed options. Even if they don't agree, it is spatially evident in the Dirichlet distribution's simplex the convergence of their opinions (whether the concentration of points approaches a vertex of the simplex meaning one of the three reconstructed options is highly probable, or whether the distribution of points is more in the middle ground of the simplex meaning the archaeologists interacting with the system have diverse opinions).

7.2 Evaluation of the visualization system

While we created the visualization of the system for archaeological uncertainty two evaluations were done with the help of archaeologist Dr. Anna Simandiraki-Grimshaw. The first were after the creation of the questionnaire. Initially the questionnaire had the same statements as Sifniotis had in her work. However, Dr. Anna Simandiraki-Grimshaw explained that in a Minoan archaeological structure the influencing factors aren't the same. After a long discussion we came to the conclusion that the evidence types which form archaeologists about how a Minoan archaeological site existed in the past were Features, Artefacts, Comparisons, Topography and Peer Review. So, we removed statements for Biofacts and Textual evidence which are not valid for our test case.

The next evaluation was at the end of visualization system. The archaeologist interacted with the system and the input and out were very clear for her. Under the circumstances it wasn't necessary to change something in the visualization system. However, when she saw the multiple reconstruction options in the 3D environment of Unity, she gave more details for them in order to be more valid the reconstructions and she gave us more reconstruction options. For examples, we hadn't three different options for the reconstruction of the Entrance of the Palace because we hadn't enough evidence of our own. Thus, she gave us the third reconstruction option to have no roof above the entrance. Also, when she saw the general image of the 3D reconstructed Palace of Zakros to do more abruptly which wasn't very clear from the picture of the book.

7.3 Future Work

The goals we have set for this work successfully achieved. Representing uncertainty in archaeological reconstructions is a niche research area. There is vast amount of further work to be done in order to be extended this work.

7.3.1 More number of reconstruction options

In this thesis we were limited to a small number of interpretations for each point of uncertainty. We gave three alternative reconstruction options for each point examined in the Palace of Zakros. It could represent more than three options about how these parts of the archaeological site existed in the past. The three reconstruction points gives as a 2-Simplex as we have described with details in previous Chapters. We have led to this decision because our main goal was to present Dirichlet Distribution in this work. So, the three interpretations for every point were enough to achieve our goal. Also, because the fact that 2-Simplex needs 3-dimensions, more than three reconstruction options would need more than three dimensions. This would be difficult but very interesting in its implementation.

7.3.2 Advanced visualization techniques

Due to this thesis gave more attention at the computational model and the visualization system, we limited ourselves to the techniques Sifniotis had proposed in her work. However, we used only color visualization technique.

Transparent visualization technique wasn't appropriate as we have examined in this work because of the complexity of the building. So, further work is needed for more visualization approaches. It would be interesting to see a new technique of visualizing the uncertainty as well as to allow the user to choose which technique he / she wants to apply. More specifically it would be interesting in the visualization system user can see the visualization of uncertainty with more than one visualization techniques. This is important because different users can react differently to different techniques and they can understand with different ways the uncertainty.

7.3.3 More details in 3D Reconstruction

Due to the limited time for thesis termination, we did not spend much time on the details of the building. So it would be better a more detailed reconstruction of the Palace of Zakros. In this work would be necessary the help of more archaeologists in order to combine all the evidence and give more details from their excavations and their research. Also, more objects like vases, axes would be interesting to be reconstructed because there are many texts and pictures in sources for them.

8 References

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