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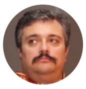

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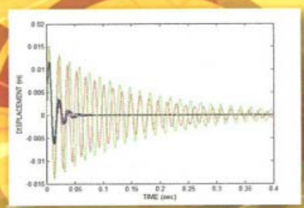

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NUMERICAL EXPERIMENTS ON SMART BEAMS AND PLATES

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Smart composite beams and plates with embedded piezoelectric sensors and actuators are considered. After a short presentation of the mechanical models and their discretization, we focus on problems of active structural control and identification. In particular we solve, using various algorithms, robust optimal control problems and damage identification tasks.

1. Introduction

The use of active control techniques in smart structures is an area of intensive research area. Vibration control of composite beams and plates including piezoelectric sensors and actuators is studied. A simplified model that decouples the multi-physics problem is adopted [1-4]. The control is based on linear feedback.

Since there are always differences between the physical plant, that is controlled, and the model on which the controller design is based (for instance, neglected higher frequency dynamics, damage, etc.) robustness is an important goal for any applicable controller [5-6]. The performance specifications, which the control system must fulfill and the class of uncertainties for which the control system must be robust against, determine the robust controller for any particular vibration control problem. In this study a vibration control problem in flexible structure (smart beam) is considered and the performance specification is stated in terms of a disturbance attenuation requirement for particular class of external disturbances acting on the structure.

In particular this contribution outlines H_2 and H_∞ robust controllers for the active vibration control of flexible structures using piezoelectric patches as sensors and actuators. The considered robust control design methodologies lead to linear time invariant feedback controllers. The controllers are designed to achieve optimal performance for a nominal model and maintain robust stability and robust performance for a given class of uncertainties. This is achieved by the solution of two algebraic Ricatti equations, while in classical structural control one such equation arises.

A more general nonlinear feedback controller can be constructed with the help of intelligent computational tools. Neural, fuzzy and hybrid control applications are briefly mentioned here. Finally, the existence of actuators and sensors with the corresponding wiring on the smart systems makes possible the consideration of structural health monitoring and damage identification schemes. Some existing results are mentioned at the end of this chapter.

This text is based on the cited, original publications of the authors; it's purpose is to demonstrate that smart systems design is a really multidisciplinary field with a large number of theoretical and practical questions, most of them open and suitable for further research.

2. Simplified modeling of composite smart structures

In the smart beam of Figure 1, the control actuators and the sensors are piezoelectric patches symmetrically bonded on the top and the bottom surfaces of the host beam. Both piezoelectric layers are positioned with identical poling directions and can be used as sensors or actuators. [7,8,9].

The linear theory of piezoelectricity is employed. Furthermore, quasi-static motion is assumed, which means that the mechanical and electrical forces are balanced at any given instant.

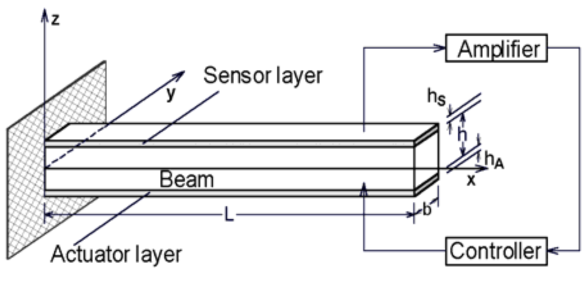

Figure 1. Laminated beam with piezoelectric sensors, actuators and the schematic control system.

The linear constitutive equations of the two coupled fields read:

$$\{\sigma\} = [Q]\left(\{\varepsilon\} - [d]^T \{E\}\right) \tag{1}$$

$$\{D\} = [d][Q]\{\varepsilon\} + [\xi]\{E\} \tag{2}$$

where $\{\sigma\}_{6x1}$ is the stress vector, $\{\varepsilon\}_{6x1}$ is the strain vector, $\{D\}_{3x1}$ is the electric displacement, $\{E\}_{3x1}$ is the strength of applied electric field acting on the surface of the piezoelectric layer, $[Q]_{6x6}$ is the elastic stiffness matrix, $[d]_{3x6}$ is the piezoelectric matrix and $[\xi]_{3x3}$ is the permittivity matrix. Eq. (1) describes the inverse piezoelectric effect (which is exploited for the design of the actuator). Eq. (2) describes the direct piezoelectric effect (which is used for the sensor). Additional assumptions are used for the construction of the simplified model: (a) Sensor and actuator (S/A) layers are thin compared with the beam thickness. (b) The polarization direction of the S/A is the thickness direction (z axis). (c) The electric field loading of the S/A is uniform uni-axial in the *x*-direction. (d) Piezoelectric material is homogeneous, transverse isotropic and elastic. Therefore, the set of equations (1) and (2) is reduced as follows

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} - \begin{bmatrix} d_{31} \\ 0 \end{bmatrix} E_z \right) \tag{3}$$

$$D_z = Q_{11} d_{31} \varepsilon_x + \xi_{33} E_z \tag{4}$$

The electric field intensity E_z can be expressed as

$$E_z = \frac{V}{h_A} \tag{5}$$

where V is the applied voltage across the thickness direction of the actuator and h_A is the thickness of the actuator layer.

Since, only strains produced by the host beam act on the sensor layer and no electric field is applied to it the output charge from the sensor can be calculated using eq. (4). The charge measured through the electrodes of the sensor is given by

$$q(t)=\frac{1}{2}\left\{\left(\int_{S_{ef}} D_z dS\right)_{z=h/2}+\left(\int_{S_{ef}} D_z dS\right)_{z=h/2+h_S}\right\} \tag{6}$$

where S_{ef} is the effective surface of the electrode placed on the sensor layer.
The current on the surface of the sensor is given by

$$i(t)=\frac{dq(t)}{dt}. \tag{7}$$

The current is converted into open-circuit sensor voltage output by

$$V^S=G_S i(t) \tag{8}$$

where G_S is the gain of the current amplifier.

Furthermore, we suppose that (5) bending-torsion coupling and the axial vibration of the beam centerline are negligible and (6) the components of the displacement field $\{u\}$ of the beam are based on the Timoshenko beam theory which, in turn, means that the axial displacement is proportional to z and to the rotation $\psi(x,t)$ of the beam cross section about the positive y-axis and that the transverse displacement is equals to the transverse displacement $w(x,t)$ of the point of the centroidal axis (y=z=0). The strain-displacement relationships read

$$\varepsilon_x=z\frac{\partial\psi}{\partial x},\qquad \varepsilon_{xz}=\psi+\frac{\partial w}{\partial x}. \tag{9}$$

The simpler Euler-Bernoulli theory which considers zero transverse shear deformation γ_{xz} has also been tested.
The kinetic energy of the beam with the layers can be expressed as

$$T=\frac{1}{2}\int_V \rho\{\dot{u}\}^T\{\dot{u}\}dV=\frac{b}{2}\int_0^L\int_{-h/2-h_A}^{h/2+h_s}\rho\left[(z\dot{\psi})^2+\dot{w}^2\right]dzdx \tag{10}$$

on the assumption that the host beam and piezoelectric patches identical densities. The strain (potential) energy is given by

$$U=\frac{1}{2}\int_V\{\varepsilon\}^T\{\sigma\}dV=\frac{b}{2}\int_0^L\int_{-\frac{h}{2}-h_A}^{\frac{h}{2}+h_S}\left[Q_{11}\left(z\frac{\partial\psi}{\partial x}\right)^2+Q_{55}\left(\psi+\frac{\partial w}{\partial x}\right)^2\right]dzdx \tag{11}$$

If the only loading consists of moments induced by piezoelectric actuators and since the structure has no bending-twisting couple then the first variation of the work has the form

$$\delta W=b\int_0^L M^A\delta\left(\frac{\partial\psi}{\partial x}\right)dx \tag{12}$$

where δ is the first variation operator, M^A is the moment per unit length induced by the actuator layer and is given by

$$M^A=\int_{-h/2-h_A}^{-h/2} z\sigma_x^A dz=\int_{-h/2-h_A}^{-h/2} zQ_{11}d_{31}E_z^A dz \qquad (E_z^A=\frac{V_A}{h_A}) \tag{13}$$

Using Hamilton 's principle the equations of motion of the beam are derived.

For the finite element discretization beam finite elements are used, with two degrees of freedom at each node: the transversal deflection w_i and the rotation ψ_i. They are gathered to form the degrees of freedom vector $X_i=[w_i \quad \psi_i]$. After assembling the mass and stiffness matrices for all elements, we obtain the equation of motion in the form

$$\mathrm{M}\ddot{X}+\Lambda\dot{X}+\mathrm{K}X=F_m+F_e \tag{14}$$

where M and K are the generalized mass and stiffness matrices, F_e is the generalized control force vector produced by electromechanical coupling effects, Λ is the viscous damping matrix and F_m is the external loading vector. The computer implementation in MATLAB follows the lines of [10].

It should be mentioned here that bending theories for plates can be constructed analogously. Furthermore, a three-dimensional finite element model of a composite beam, without the simplifications introduced here, is presented in the Chapter by M. Betti et al. in the present Volume.

The main objective is to design robust control laws for the smart beam bonded with piezoelectric S/A subjected to external induced vibrations. For this purpose the following state space representation will be used:

$$\dot{x} = Ax + B_1 w + B_2 u \tag{15}$$

as it is common in control problems for general dynamical systems. Here

$$x = [X \quad \dot{X}]^T \ , \ A = \begin{bmatrix} 0 & I \\ -M^{-1}\mathrm{K} & -M^{-1}\Lambda \end{bmatrix}$$

x is the state vector, A is the state matrix, B_1 and B_2 are allocation matrices for the disturbances w (corresponding to external forces F_m) and control u (corresponding to F_e). The initial conditions are assumed to be zero. The identity matrix is denoted by I.

3. Controlled system

Let us consider that the measurements have the following form:

$$y = Cx + Du \tag{16}$$

The control law is a linear feedback of the form

$$u = Ky \tag{17}$$

where K is the unknown controller gain.
The objective in this study is to determine the vector of active control forces ***u(t)*** subjected to some performance criteria and satisfying the dynamical equations (15)-(17) of the structure, such that to reduce in an optimal way the external excitations and to meet the above mentioned requirements. The investigations may be implemented in the time domain as well as in the frequency domain. The problem for vibration suppression is solved by both LQR and H_2, H_{inf} optimal performance criteria. LQR is a state space method, while H_2 is a frequency domain approach.

3.1. *Linear Quadratic Regulator*

In this section the £$_2$ performance problem in the time domain is studied [11]. The following quadratic cost function is minimized

$$J = \frac{1}{2}\int_0^{\infty} (x^T Q x + u^T R u) dt \qquad \rightarrow \quad \min \tag{18}$$

The free parameters Q and R represent weights on the different states and control. They are the main design parameters. ***J*** represents the weighted sum of energy of the state and control. We require that Q be symmetric semi-positive definite and R be symmetric positive definite for a meaningful optimization problem. The problem (15), (18) is known as LQR problem and belongs to the powerful machinery of the optimal control.

Assuming full state feedback, the control law is given by

$$U = -K_{LQR}\, x, \tag{19}$$

with constant control gain

$$K_{LQR}=R^{-1}B^{T}P \tag{20}$$

The constant matrix P is a solution of the Riccati Equation

$$A^{T}P + PA + Q - PBR^{-1}B^{T}P = 0\ . \tag{21}$$

Under technical assumptions existence and uniqueness of the above controller is guaranteed. The closed loop system is given by

$$\dot{x} = (A - BK_{LQR})x + F_{m}\ . \tag{22}$$

LQR method is designed to satisfy specified requirements for steady state error, transient response, stability margins or closed loop pole location. An advantage of the linear quadratic formulation of the problem is the linearity of the control law, which leads to easy analysis and practical implementation. Another advantage is good disturbance rejection and good tracking. The gain and phase margins imply good stability.

All these preferences are met when a complete knowledge of the whole state for each time instance is available. If a limited number of measurements are available and they are supposed to be corrupted by some measurement errors the effectiveness of LQR deteriorates. In this case, first the system is reconstructed by the available measurements, and then the optimal control problem is based on this reconstructed system.

3.2. *H_2 Control*

The major problem with LQR is the lack of robustness. Too much emphasis on optimality and not enough attention to the model uncertainty leads to control that fail to work in real environment. Robustness with respect to external disturbances or uncertainties of the system or loading is the main reason why the authors started studying techniques dealing with feedback properties in frequency domain. We assume that the exogenous signals are fixed or have fixed power spectrum. Since the vibration control is stated in terms of a disturbance attenuation request for a particular class of external disturbances, the H_2 robust control methodology is particularly suited. Unlike the standard LQG approach which is based on a nominal model [12], the H_2 technique is based on an uncertain system model.

Let us divide the system inputs in two groups: exogenous input ***w*** and command signals ***u*** that are the output of the controller and becomes the input to the actuators driving the plant. The plant outputs are also categorized in two groups: the measurements ***y*** that are fed back to the controller and the regulated outputs ***z*** we are interesting in controlling. The plant (15)-(17) can be represented in the more general state space form as

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x \qquad + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{23}$$

Suppose that the measures ***y*** are corrupted and the regulated outputs ***z*** are controlled. ***w***, ***u***, ***y***, and ***z*** are continuous-time signals.
Let T_{zw} denote the linear time invariant system from ***w*** to ***z*** and $\tilde{T}_{zw}$ is its transfer function. We get as a performance criterion the minimization of the H_2 norm of $\tilde{T}_{zw}$,

$$\left\|\tilde{T}_{zw}\right\|_2 = \left(\frac{1}{2} \int_{-\infty}^{+\infty} trace[\tilde{T}_{zw}(j\omega) * \tilde{T}_{zw}(j\omega)] d\omega \right)^{1/2} \tag{24}$$

over all internally stabilizing controllers *K*. The H_2 norm of $\tilde{T}_{zw}$ minimizes the worst case root mean square value of the regulated variables when the disturbances are unit intensity white processes. This circumstance allows for a state space solution to the frequency domain optimization problem.
Under some assumptions it can be shown that there exists a unique controller K_2 which minimizes $\tilde{T}_{zw}$ with the following transfer matrix representation [6]

$$K_2 = \left[\begin{array}{c|c} A - B_2 B_2^T X - YC_2^T C_2 & YC_2^T \\ \hline -B_2^T X & 0 \end{array} \right]$$

where *X* and *Y* are the solutions of the two Riccati equations

$$\begin{aligned} A^T X + XA - XB_2 B_2^T X + C_1^T C_1 &= 0 \quad , \\ AY + YA^T - YC_2^T C_2 T + B_1 B_1^T &= 0 \end{aligned} \tag{25}$$

for a stable matrix *A*. Applications on smart beams have been presented in [8].

3.3. *Uncertainty Modelling and Robust Control*

Uncertainty denotes the difference between the model and the reality. The H_∞ approach begins with an uncertain system model for the plant to be controlled. In this section we will consider an uncertainty introduced by varying the nominal plant parameters. Disk-shaped regions on the real axis approximate the variations in the structure system. A multiplicative uncertainty as shown in Figure 2 is assumed.

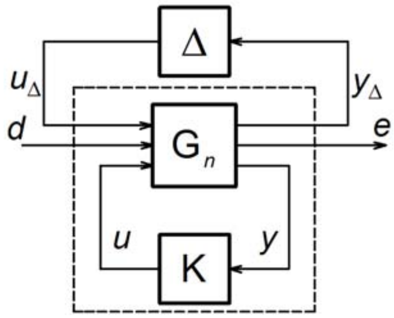

Figure 2. Introduction of uncertain in the dynamical system.

Let us suppose that the three actual physical parameters M, Λ, and K in the eq. (14) are lie within known intervals. In particular, the actual mass M is within p_{M} percentages of the nominal mass $\overline{\mathrm{M}}$, the actual damping value Λ is within p_Λ percentages of the nominal value $\overline{\Lambda}$, and the spring stiffness K is within p_{K} percentages of its nominal value of $\overline{\mathrm{K}}$. Further, real perturbations are introduced:

$$\Delta_{\mathrm{M}} = \delta_{\mathrm{M}} I, \qquad \Delta_\Lambda = \delta_\Lambda I, \qquad \Delta_{\mathrm{K}} = \delta_{\mathrm{K}} I, \tag{26}$$

which are assumed to be unknown within the values $(-1 \le \delta_{\mathrm{M}}, \delta_\Lambda, \delta_{\mathrm{K}} \le 1)$. Thus, the actual physical parameters of the system take the form

$$\mathrm{M} = \overline{\mathrm{M}}(I + p_{\mathrm{M}}\Delta_{\mathrm{M}}),\ \Lambda = \overline{\Lambda}(I + p_\Lambda \Delta_\Lambda), \mathrm{K} = \overline{\mathrm{K}}(I + p_{\mathrm{K}}\Delta_{\mathrm{K}}) \tag{27}$$

The uncertainty in the matrices M^{-1}, Λ and K can be represented by matrix functions called upper linear fractional transformations (LFT) in the perturbations Δ_{M}, Δ_Λ and Δ_{K} respectively [6]

$$\mathrm{M}^{-1} = F_U\left(\begin{bmatrix} -p_{\mathrm{M}} I & \overline{\mathrm{M}}^{-1} \\ -p_{\mathrm{M}} I & \overline{\mathrm{M}}^{-1} \end{bmatrix}, \Delta_{\mathrm{M}}\right), \quad \Lambda = F_U\left(\begin{bmatrix} 0 & \overline{\Lambda} \\ p_{\Lambda} I & \overline{\Lambda} \end{bmatrix}, \Delta_{\Lambda}\right),$$

$$\mathrm{K} = F_U\left(\begin{bmatrix} 0 & \overline{\mathrm{K}} \\ p_{\mathrm{K}} I & \overline{\mathrm{K}} \end{bmatrix}, \Delta_{\mathrm{K}}\right). \tag{28}$$

Thus, the considered control design problem will be formulated in a LFT framework. The LFT in (28) have a nominal mapping that are perturbed by Δ_{M}, Δ_{Λ}, Δ_{K} while the other members of the matrices describe how the perturbations affect the nominal maps. This way the system can be rearranged as a standard one via "pulling out the Δ's". For this purpose, we first isolate the uncertainty parameters and denote the inputs of Δ_{M}, Δ_{Λ}, Δ_{K} as y_{M}, y_{Λ}, y_{K} and their outputs as u_{M}, u_{Λ}, u_{K}. The outputs $u_{\Delta} = [u_{\mathrm{M}}, u_{\Lambda}, u_{\mathrm{K}}]$ from the perturbations are added to the system's inputs and the inputs $y_{\Delta} = [y_{\mathrm{M}}, y_{\Lambda}, y_{\mathrm{K}}]$ to the perturbations are added to the system's outputs (see Figure 2). The model for the uncertain system is finally obtained in the following matrix form

$$\begin{bmatrix} \dot{x} \\ y_{\Delta} \\ y \end{bmatrix} = G \begin{bmatrix} x \\ u_{\Delta} \\ u \end{bmatrix}, \quad G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}, \tag{29}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ -p_{\mathrm{M}} I & -p_{\Lambda}\overline{\mathrm{M}}^{-1} & -p_{\mathrm{K}}\overline{\mathrm{M}}^{-1} \end{bmatrix}, C_1 = \begin{bmatrix} \mathrm{M}^{-1}\mathrm{K} & -\overline{\mathrm{M}}^{-1}\overline{\Lambda} \\ 0 & \overline{\Lambda} \\ \overline{\mathrm{K}} & 0 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} -p_{\mathrm{M}} I & -p_{\Lambda}\overline{\mathrm{M}}^{-1} & -p_{\mathrm{K}}\overline{\mathrm{M}}^{-1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} \overline{\mathrm{M}}^{-1}H \\ 0 \\ 0 \end{bmatrix}, \; D_{21} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},$$

and represents an LFT of the natural uncertainty parameters δ_{M}, δ_{Λ}, δ_{K}. The matrix H is a distribution matrix defining the locations of the control forces. The matrix G in eq. (29) is known from the nominal parameters of the system. The system model uncertainty matrix in eq. (29), denoted by Δ, is a structured matrix.

$$\Delta = diag\begin{bmatrix} \Delta_M & \Delta_\Lambda & \Delta_{\mathrm{K}} \end{bmatrix}. \tag{30}$$

It is H_∞ norm bounded, $\|\Delta\|_\infty \leq 1$, has a block diagonal structure and influences on the input/output connection between the control u and the output y in a way that can be represented as a feedback by the upper LFT

$$y = F_U(G, \Delta)u\,. \tag{31}$$

Further we consider the perturbed system (29). The performance criterion is to keep the errors as small as possible in some sense for all perturbed models. The performance specifications will be specified in some requirements on the closed loop frequency response of the transfer matrix between the disturbances and the errors, within the H_∞ theory. The robust stability and robust performance criteria can be treated in a unified framework using LFT and the structured singular value (SSV) μ_Δ. We shall consider the real parametric uncertainty with norm-bounded dynamical uncertainty.

For the robust stability analysis the controller K can be viewed as a known system component and absorbed into an interconnection structure P together with the plant G_n marked by a dashed line in Figure 2. According to the Nyquist criterion, if the matrices P and Δ are stable then the interconnection system is stable if and only if $\det(I - P\Delta) \neq 0$ [6]. For the robust stability we are interested in finding the smallest perturbation Δ, real and norm bounded $\|\Delta\|_\infty <$ 1 (that is ensured by means of eq. (24)) in the sense of maximal singular value $\overline{\sigma}(\Delta)$, such that destabilizes the closed loop framework i.e.

$$\det(I - P\Delta) = 0 \tag{32}$$

The matrix function **SSV** is defined as

$$\mu_\Delta(P) = \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in D, \det(I - P\Delta) = 0\}} \tag{33}$$

SSV μ_Δ is bounded by the spectral radius $\rho(P)$ of the matrix P as lower bound and by is the maximal singular value $\overline{\sigma}(P)$ of the matrix P as follows

$$\rho(P) \le \mu_\Delta(P) \le \overline{\sigma}(P) \tag{34}$$

The interconnection system is well-posed and internally stable for all norm bounded perturbations Δ if and only if

$$\sup_{\omega \in R} \mu_\Delta(P(j\omega)) < 1 \tag{35}$$

Hence, the peak value on the μ_Δ plot of the frequency response determines the size of the perturbations for which the loop is robustly stable. The quantity

$$\frac{1}{\max\limits_{\omega} \mu_\Delta[P(j\omega)]} \tag{36}$$

is a stock of stability with respect to the structured uncertainty influenced P.

The robust stability is not the unique feature required for the system with parameter perturbations. Often, exogenous influences acting on the system lead to errors in tracking and regulating. Therefore, we need to test the robust performance of the system.

The nominal performance of a system is characterized by using the H_∞ norm of some transfer matrix, here we take the weighted sensitivity transfer matrix of the closed loop. We assume that for good performance the following relationship is satisfied

$$\left\| W_p (I + GK)^{-1} \right\|_\infty < 1 \tag{37}$$

The weighting matrix W_p is taken such that to suppress the influence of the disturbance on the output. Further details and an application of damage-induced uncertainties of smart beams are given in [13].

3.4. *H_infinity Control*

To obtain a best possible performance in the face of the uncertainties a robust H_∞ optimal control is considered. The implementation of H_∞ control theory is motivated by the inability of the H_2 theory to directly accommodate plant uncertainties. Let us present the considered uncertain system (23) by the diagram in Figure 3.

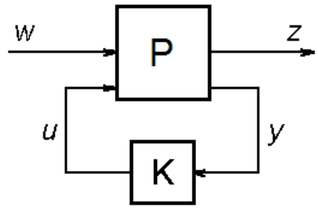

Figure 3. General framework for the H$_\infty$ control problem.

where the exogenous input $w = [u_\Delta \quad d]^T$ includes all signals coming to the system and the error $z = [y_\Delta \quad e]^T$ includes all signals characterizing the system response. Therefore, the system can be represented by the equation

$$\begin{bmatrix} z \\ y \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} \tag{38}$$

The aim of this section is to design an admissible controller, which stabilizes internally the system and minimizes the H$_\infty$ norm of the closed loop transfer matrix from *w* to *z*. The closed loop transfer matrix from *w* to *z* is given as a lower LFT in *K*

$$z = F_L(P,K)w \tag{41}$$

Then the optimal H_∞ control design problem can be formulated as:

$$\|F_L(P,K)\|_\infty = \max_\omega \overline{\sigma}(F_L(P,K)(j\omega)) \to \quad \min \tag{42}$$

The transfer matrix $F_L(P,K)$ contains measures of nominal performance and stability robustness. Its H$_\infty$ norm gives a measure of the worst case response of the system over an entire class of input disturbances. The optimal H$_\infty$ controller as just defined is not unique for a MIMO system (in contrast with the standard H$_2$ theory, in which the optimal controller is unique). Knowing the optimal H$_\infty$ norm is useful theoretically since it sets a limit on what we can achieve. In practice a suboptimal solution may be useful too: For given $\gamma > 0$, find an admissible controller $K_s(s)$ such that the H$_\infty$ norm of the closed loop transfer matrix is less than γ. Theoretically the optimal controller leads to a difficult, possibly nonconvex optimization problem for which many theoretical and algorithmic questions remain open.

3.5. *Nonlinear and Intelligent Control*

The advantage of classical control, which covers the study of all previously outlined methods, is the availability of mathematical tools for the design of the controller and the study of it's properties, like stability, robustness etc. Nevertheless, one should mention that most beneficial properties are based on the knowledge of the whole dynamical system, which is usually nonrealistic. At this stage an estimator, like a Kalman-filter one, is introduced. The quality and reliability of this estimator defines the effectiveness of the whole control system. Furthermore, a serious disadvantage is the adoption of a linear feedback. Nonlinear control laws may be more suitable. The tools provided by the classical control for the design of nonlinear controllers are less developed. Therefore nonlinear controllers are mainly based on intelligent and soft computing tools. Without details, we mention several possibilities of using intelligent control in smart structures.

1. *Neural networks* can be trained to approximate every nonlinear mapping. They can be used for the approximation of the inverse dynamical mapping of a system. Subsequently the trained network is used to suppress vibration of the system. For this application a large number of representative measurements, or data from modeling, is required for the training and testing of the neural network system.
2. *Fuzzy inference* rules systematize existing experience and can be used for the rational formulation of nonlinear controllers. The feedback is based on fuzzy inference and may be arbitrary nonlinear and complicated. Knowledge or experience on the controlled system is required for the application of this technique. Since the linguistic rules are difficult to be explained and formulated for multi-input, multi-output systems, most applications are based on multi-input, single-output controllers.
3. *Hybrid techniques* that combine the best of every world have also been proposed. For example the required details of a fuzzy inference system can be tuned by means of examples and neural networks of genetic optimization.

4. Inverse and Identification Problems

Nondestructive evaluation techniques are often based on dynamic excitation and changes of the response due to an internal defect [14, 15]. It is generally accepted that the suitability of the method is case-dependent and that the interpretation of the results strongly depends on the experience of the user. Output error minimization provides a suitable vehicle for an objective study of the arising inverse problems [14, 16]. Unfortunately this approach requires the integration of highly sophisticated structural analysis and optimization software and, in addition, may lead to nonclassical nonconvex optimization problems with the possibility of many local minima. Investigations on crack identification problems for two-dimensional elasticity problems (plane stress model) led to meaningful results. Beyond classical optimization or the powerful but expensive genetic optimization, inversion techniques based on neural networks and filter algorithms have also been proposed and tested [14, 17, 18]. Recently, an extension to defect identification problems for plates in bending has been attempted. First results, using genetic optimization, demonstrate that this approach is useful [19].

Two general classes of problems can be identified in this area:

1. *Structural health monitoring*, where one tries to identify changes of the structural system related to possible damages, cracks etc, and secondly one tries to correlate these changes with concrete sources. Ambient or service loads and corresponding measurements are used for this task. The usefulness of having an instigator of structural integrity and a warning for possibly dangerous changes is obvious.
2. *Parameter and defect identification* is a more complicated task, since one tries to find, in addition, the cause and size of the structural changes. To this end, usually additional test loadings are required, focusing on specific parts of interest in the structure under investigation.

The challenge is that smart systems have already integrated sensors and actuators. Therefore one is able to introduce suitably designed test loadings and use the measurements in order to solve both above mentioned problems. The design of the experiments and the post processing of huge amounts of measurements is, by no means, a trivial task. Current research effort focused on the study of the problem for specific structural systems, like beams and plates in bending etc.

5. Representative numerical results

5.1. *Vibration Suppression of a Smart Piezocomposite Beam*

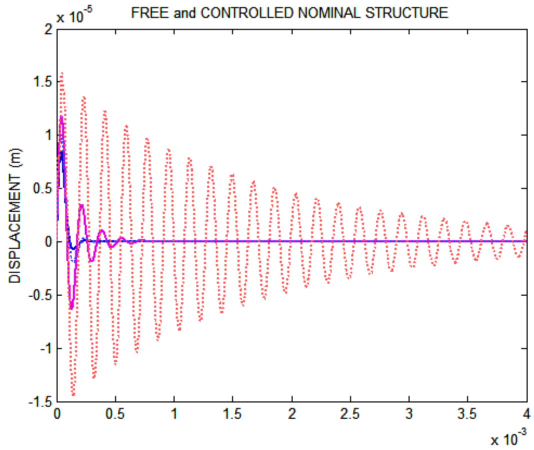

Figure 4. Vibration of a composite cantilevered beam. Free vibration with and without control, with red (grey) dotted and magenta (grey) solid lines respectively. Controlled vibration with and without damage, with blue (dark) dotted and blue (dark) solid lines.

The effectiveness of a control scheme applied on a composite cantilevered beam for the case of a beam without and with a small damage is schematically shown in Figure 4. A suitable design of the controller makes the controlled system robust and less sensitive to changes of it's mechanical parameters, which is possible due to damage, cracks, delaminations or fatigue of the composite. Details can be found in [13].

5.2. *Damage Identification for a Plate in Bending using Genetic Algorithms*

For a plate in bending we consider a dynamical loading and the introduction of a small damage. A suitable error norm transforms the defect identification problem to an optimization problem. Typical contours of the error are shown in Figures 5a. The appearance of local minima and one global minima, exactly at the position of the real defect, is observed. Therefore genetic optimization is used for the solution of the inverse problem. One of the results is documented in Figure 5b. More details can be found in [19].

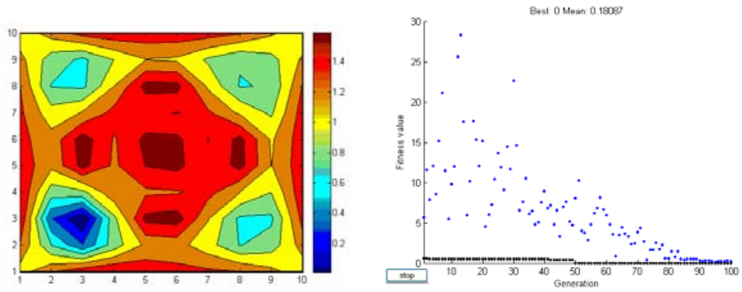

Figure 2. Error function for a damage at position (3,3) (a) and corresponding defect identification using genetic optimization (b).

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