Design and active vibration control of composite beams with bonded piezoelectric sensors and actuators

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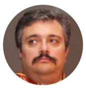

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Design and Active Vibration Control of Composite Beams with Bonded Piezoelectric Sensors and Actuators

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Introduction

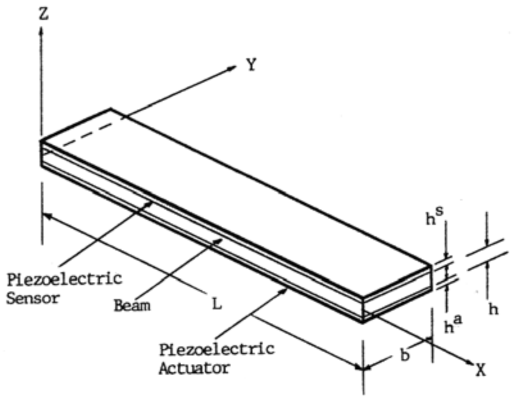
Piezoelectric materials respond to mechanical forces and generate an electric charge. This is the ***direct piezoelectric effect***. Conversely, electric charge applied to the piezoelectric material induces mechanical stresses or strains, and this is the ***converse piezoelectric effect***. In ‘‘smart’’ piezoelectric structures, the ***direct*** effect is used for structural measurements (***sensor***) and the ***converse*** effect is used for active vibration controls (***actuator***).

One attractive way for vibration suppression of flexible structures is bonding to the surface of host structures active piezoelectric sensors and actuators, which operate according to a chosen **control** law. The choice of the control technique is important in designing controllers, which ensure the suitable functioning of the structures under required conditions.

In this work a laminated beam with piezoelectric sensors and actuators is modelled by the **finite element method**. Furthermore, the problem of active control is studied using classical and robust optimal control (LQR and H_2). Numerical results show the effectiveness of the proposed methods.

Modeling of the electromechanical system

Smart beam model



The configuration of a beam with surface bonded sensors and actuators is shown in the Figure above. The **linear theory of piezoelectricity** is employed due to small structural vibrations. This theory assumes quasi-static motion indicating that the mechanical and electrical forces are balanced at any given instant. The linear constitutive equations of piezoelectric material are given

$$\{\sigma\} = [Q]\left(\{\varepsilon\} - [d]^T \{E\}\right) \tag{2.1}$$

$$\{D\} = [d][Q]\{\varepsilon\} + [\xi]\{E\} \tag{2.2}$$

$\{\boldsymbol{\sigma}\}_{6\text{x}1}$ –stress vector, $\{\boldsymbol{\varepsilon}\}_{6\text{x}1}$ –strain vector,

$\{\boldsymbol{D}\}_{3\text{x}1}$ – electric displacement, $\{\boldsymbol{E}\}_{3\text{x}1}$ – applied electric field,

$[\boldsymbol{Q}]_{6\text{x}6}$ – elastic stiffness matrix,

$[\boldsymbol{d}]_{3\text{x}6}$ – piezoelectric matrix, $[\boldsymbol{\xi}]_{3\text{x}3}$ – permittivity matrix.

Eq. **(2.1)** describes the ***inverse piezoelectric effect*** (actuator).

Eq. **(2.2)** describes the ***direct piezoelectric effect*** (sensor).

The equations for **piezoelectric** sensors/actuators (S/A) are based on the following **assumptions**

1. S/A are **thin** compared with the beam thickness.
2. **Poling** direction of the S/A is the positive ***z*-direction**.
3. Electric field **loading** of the S/A is uni-axial in the ***x*-direction**.
4. Piezoelectric material is **homogeneous**, **orthotropic** and **elastic**.

Under these assumptions the set of equations (2.1) and (2.2) is reduced as follows

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} - \begin{bmatrix} d_{31} \\ 0 \end{bmatrix} E_z \right) \tag{2.3}$$

$$D_z = Q_{11} d_{31} \varepsilon_x + \xi_{33} E_z \tag{2.4}$$

Eq. (2.4) is used to calculate the output charge on the sensor layer created by the strains in the beam. Since no electric field is applied to the sensor layer, we get

$$D_z = Q_{11} d_{31} \varepsilon_x . \tag{2.5}$$

The charge measured through the electrodes of the sensor is given by

$$q(t) = \frac{1}{2} \left\{ \left(\int_{S_{ef}} D_z dS \right)_{z = h/2} + \left(\int_{S_{ef}} D_z dS \right)_{z = h/2 + h_S} \right\} \tag{2.6}$$

(***S**$_{ef}$* is the effective surface of the electrode placed on the sensor layer)

The current $i(t) = \dfrac{dq(t)}{dt}$ on the surface of the sensor is converted into open-circuit sensor voltage output by $V^S = G_S i(t)$ (***G**$_S$* is the gain of the current amplifier)

Equations of motion

The equations of **motion** are derived based on the following **assumptions**

1. The **cross-section** of the beam is symmetric and its centroidal and elastic axes coincide so that no bending-torsion coupling is considered.
2. S/A are **bonded perfectly** on the host beam.
3. The **axial vibration** of the beam centerline is considered negligible.
4. The components of the displacement field **{*u*}** are based on the Timoshenko theory written as

$$\begin{aligned} u_1(x,y,z,t) &\approx z\psi(x,t), \\ u_2(x,y,z,t) &\approx 0, \\ u_3(x,y,z,t) &\approx w(x,t), \end{aligned} \tag{2.7}$$

where ψ is the rotation of the beam cross section about the positive ***y***-axis and ***w*** is the transverse displacement of the point of the centroidal axis (y=z=0).

The strain-displacement relations are given assuming they are small

$$\varepsilon_x = z\frac{\partial \psi}{\partial x}, \qquad \gamma_{xz} = \psi + \frac{\partial w}{\partial x}. \tag{2.8}$$

The dynamic equations of a piezoelectric continuum can be derived from the **Hamilton principle**.

$$\delta\int_{t_1}^{t_2}(T - U + W)dt = 0 \tag{2.9}$$

T is the kinetic energy, ***U*** is the potential energy and ***W*** is the work done by the external loads or moments.

The kinetic energy and the strain (potential) energy are known from classical structural analysis theories.

$$T = \frac{1}{2}\int_V \rho\{\dot{u}\}^T\{\dot{u}\}dV = \frac{b}{2}\int_0^L \int_{-h/2-h_A}^{h/2+h_s} \rho\left[(z\dot{\psi})^2 + \dot{w}^2\right]dzdx$$

$$U = \frac{1}{2}\int_V \{\varepsilon\}^T\{\sigma\}dV = \frac{1}{2}\int_V \left[\sigma_x\varepsilon_x + \tau_{xz}\gamma_{xz}\right]dV$$

$$= \frac{b}{2}\int_0^L \int_{-h/2-h_A}^{h/2+h_S} \left[Q_{11}\left(z\frac{\partial\psi}{\partial x}\right)^2 + Q_{55}\left(\psi + \frac{\partial w}{\partial x}\right)^2\right]dzdx$$

If the only loading consists of moments induced by piezoelectric actuators and since the structure has no bending-twisting couple, then the first variation of the work has the form

$$\delta W = b\int_0^L M^A \delta\left(\frac{\partial\psi}{\partial x}\right)dx \tag{2.10}$$

where $\boldsymbol{M^A}$ is the moment per unit length induced by the actuator layer and is given by

$$M^A = \int_{-h/2-h_A}^{-h/2} z\sigma_x^A dz = \int_{-h/2-h_A}^{-h/2} zQ_{11}d_{31}E_z^A dz \tag{2.11}$$

Finite element modeling

The model is composed of beam elements in bending, which have two mechanical degrees of freedom at each node: one translational $\boldsymbol{w_1}$ **(**$\boldsymbol{w_2}$**)** in z-direction and one rotational ψ_1 **(**ψ_2**)**. Using classical finite element interpolation functions and (2.9), the equations of motion for the discretized structure are

$$M\ddot{X} + D\dot{X} + KX = F_m + F_e \tag{2.12}$$

$\boldsymbol{X} = [\boldsymbol{w_1}\ \psi_1\ \boldsymbol{w_2}\ \psi_2]$ − states of the system,

$\boldsymbol{M}$ − mass matrix,

$\boldsymbol{K}$ − stiffness matrix.

$\boldsymbol{D}$ − viscous damping matrix added *a posterior*,

$\boldsymbol{F}_m$ − mechanical point force added *a posterior* acting as external disturbance,

$\boldsymbol{F}_e$ − generalized electrical load provided by the applied voltages and acts as a control force.

The **main objective** is to design an **optimal control** law for the described smart beam

Optimal control problem

$$\dot{x} = Ax + Bu, \tag{3.1}$$

$$x^T = \begin{bmatrix} X^T \\ \dot{X}^T \end{bmatrix} \qquad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ M^{-1}F_e^* \end{bmatrix},$$

We will restrict our attention to a regulator type problem. For this purpose a **feedback** control law is used.

Let us consider that measurements are included in the output vector

$$y = Cx + Du \tag{3.2}$$

We consider the **steady state case** and the control law is a linear time invariant function of the outputs of the system

$$u = Ky \tag{3.3}$$

where ***K*** is the controller we have to determine.

The objective below is to determine the vector of active control forces ***u*(*t*)** subjected to some performance criteria and satisfying the dynamical equations (3.1)-(3.2) of the structure, such that to reduce in an optimal way the external excitations and to meet good tracking and disturbance rejection and to keep the controller within specified limits.

Depending on the type of the considered optimal performance criteria, **LQR** and **H_2** optimal control problems are solved.

Classical control (LQR)

First, the £$_2$ performance problem in the time domain is studied. The objective function is a quadratic functional of the plant states and control inputs.

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \quad \rightarrow \quad \min \tag{3.4}$$

$\boldsymbol{Q} \geq \mathbf{0}$ ($Q = Q^T$) and $\boldsymbol{R} > \mathbf{0}$ ($R = R^T$) represent weights on the different states and control channels and are the main design parameters. The problem (3.1)-(3.4) is known as the **linear quadratic regulator** (LQR) problem

Applying the Pontriagin's maximum principle the solution results in a constant control gain

$$\boldsymbol{u} = -K_{LQR}\, x, \qquad K_{LQR}=R^{-1}B^T P \tag{3.5}$$

The constant matrix $\boldsymbol{P}$ is a solution of ARE

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{3.6}$$

An advantage of the linear quadratic formulation of the problem is the **linearity** of the control law, which leads to easy analysis and practical implementation.

Another advantage is good disturbance rejection and tracking and good stability.

All these preferences are met when a complete knowledge of the whole state for each time instance is available. This is a considerable deficiency for practical applications.

Robust control (H_2)

The major problem with LQR solution is the lack of robustness. Robustness with respect to external disturbances or uncertainties of the system or of the loading has been the main reason why the authors started studying techniques dealing with feedback properties and frequency domain machinery.

We assume that the exogenous signals are fixed or have fixed power spectrum.

Let us divide the system **inputs** in two groups:

- **exogenous** input ***w*** that lumps external disturbances, sensor noise, and command signals, which cannot be manipulated
- **control** input ***u*** that is the output of the controller and becomes the input to the actuators driving the plant.

The plant **outputs** are also categorized in two groups:

- **measurements** ***y*** that are fed back to the controller
- **regulated** outputs ***z*** we are interesting in controlling.

Then the plant (3.1)-(3.2) can be represented in the more general state space form as

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x \qquad\quad + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{3.7}$$

Let $\boldsymbol{T_{zw}}$ denotes the linear time invariant system from $\boldsymbol{w}$ to $\boldsymbol{z}$ and $\boldsymbol{\tilde{T}_{zw}}$ its transfer matrix.

We get as a performance criterion the minimization of the $\boldsymbol{H_2}$ norm of $\boldsymbol{\tilde{T}_{zw}}$

$$\left\|\widetilde{T}_{zw}\right\|_2 = \left(\frac{1}{2} \int_{-\infty}^{+\infty} trace[\widetilde{T}_{zw}(j\omega) * \widetilde{T}_{zw}(j\omega)] d\omega \right)^{1/2} \tag{3.8}$$

over all internally stabilizing controllers $\boldsymbol{K}$.

The H_2 norm of $\boldsymbol{\tilde{T}_{zw}}$ minimizes the worst case root mean square value of the regulated variables when the disturbances are unit intensity white processes.

It can be shown that there exists an unique controller $\boldsymbol{K_2}$, which minimizes $\boldsymbol{\tilde{T}_{zw}}$ and depends on the solutions of two ARE

$$A^T X + XA - XB_2 B_2^T X + C_1^T C_1 = 0$$

$$AY + YA^T - YC_2^T C_2 T + B_1 B_1^T = 0 \tag{3.9}$$

The controller $\boldsymbol{K_2}$ has a separation structure. From the measurements the whole system is first reconstructed in an optimal way using Kalman – Bucy filter during the estimation phase, and then the optimal control problem is based on this reconstructed state vector.

Numerical results

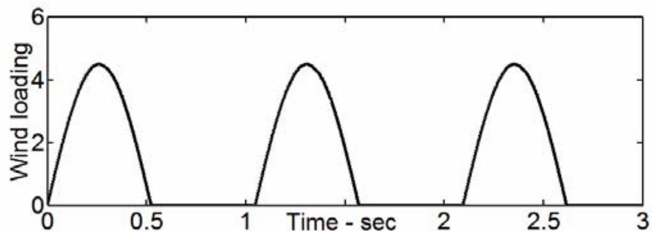
For numerical simulations a slender cantilever beam with four finite elements is considered. A pair of piezoelectric patches is bonded symmetrically at the top and the bottom surfaces of each beam element.

The disturbances influence the displacements and the rotations.

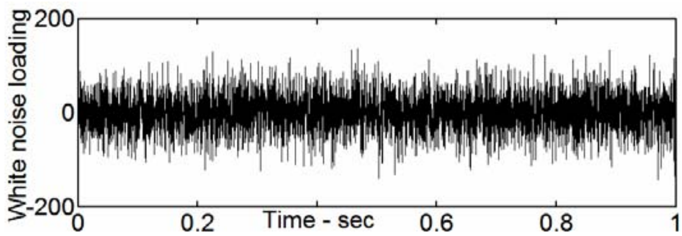
The rotation of every node is controlled by the strains of the two adjoining piezoelectric actuators.

Three kinds of dynamic **loading** are used as disturbances:

- Instantaneous transverse constant force distributed in the free end of the beam.
- Periodic sinusoidal loading pressure acting on every node on one side of the structure simulating a strong wind.



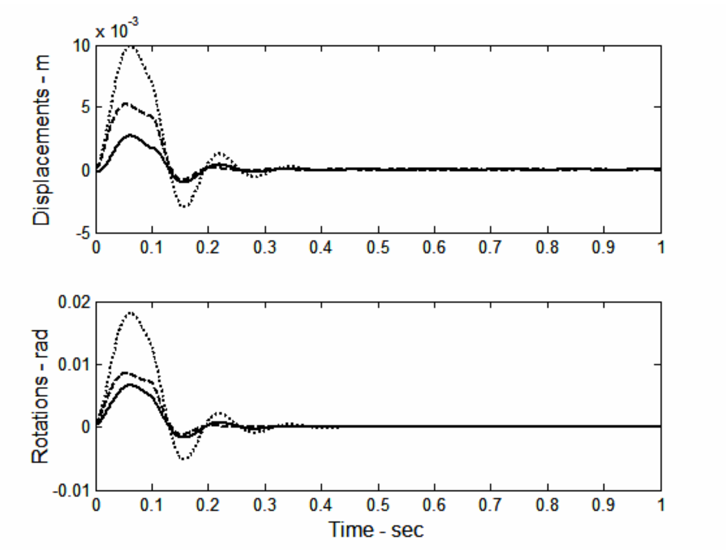
- Random white noise with zero mean acting along the transverse direction.



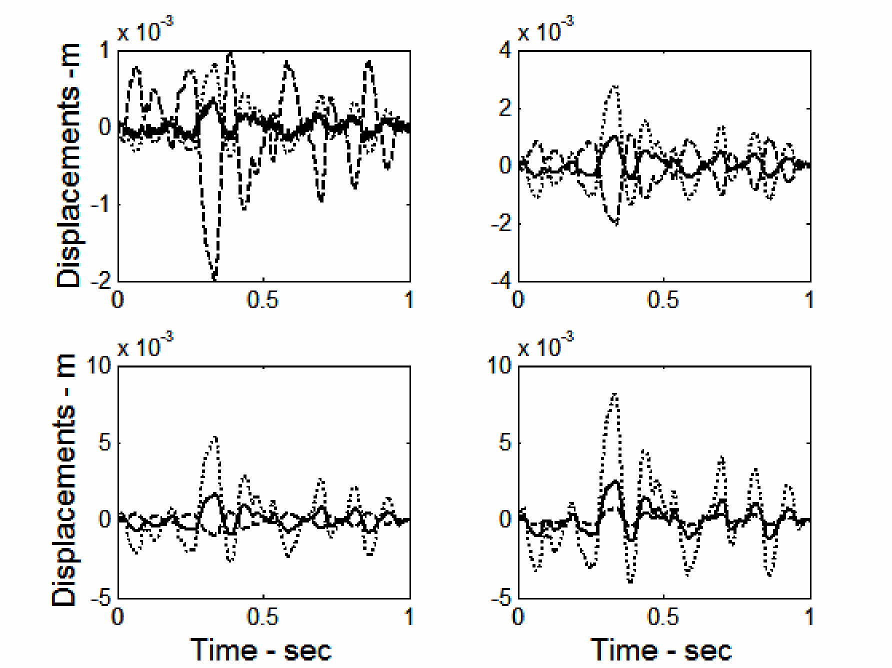
We limit our discussion to the control of the **transient response**. The responses of the open-loop and closed-loop systems are compared based on the reduction of the magnitude of the maximum transverse displacements and rotations.

All simulation cases illustrate asymptotic stability of both control strategies. Combination of states available for measure is used to create the LQR control gain as well as for organizing the control and filter gains for $\mathbf{H_2}$ approach. This leads to heavy penalizing of the control in the LQR performance criterion ($R = r.I_{4x4}$, $r = 0.000001$).

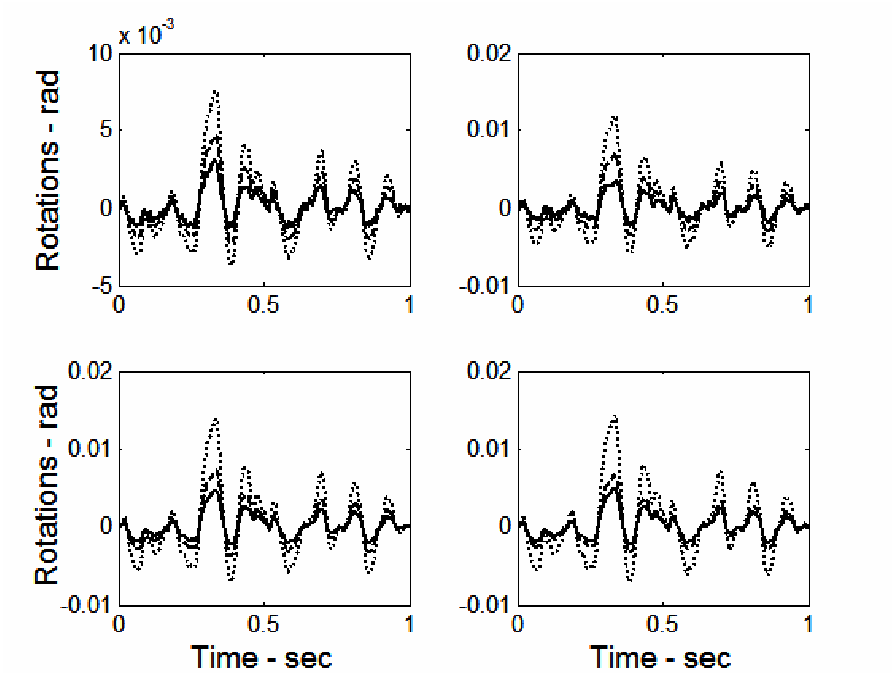
Applying an impulsive constant load we observe that the two control laws very quickly suppress the vibrations. Better results are obtained with $\mathbf{H_2}$ control law. The plots of the displacements and rotations for the free (dot), LQR (dash) and $\mathbf{H_2}$ (solid) controlled beam tip are displaced in the Figure below.



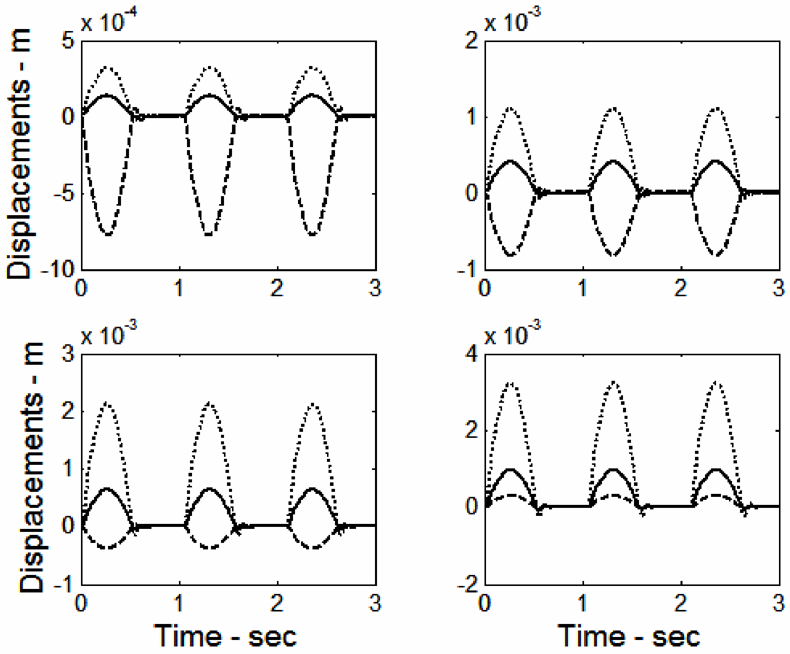
When $\mathbf{H_2}$ controller is applied without amplification of the control channels the vibrations near the free end can be efficiently suppressed. Worse results for the vertical displacement appearing near the fixed end are due to the fact that $\mathbf{H_2}$ robust controller includes an estimation of the structural system from incomplete measurements and insufficient accuracy of the simplified finite element model in higher vibrational modes. This deficiency can be improved amplifying the first control channel and accepting the fact that effectiveness of the suppression of the displacements of the free end then will be slightly less.



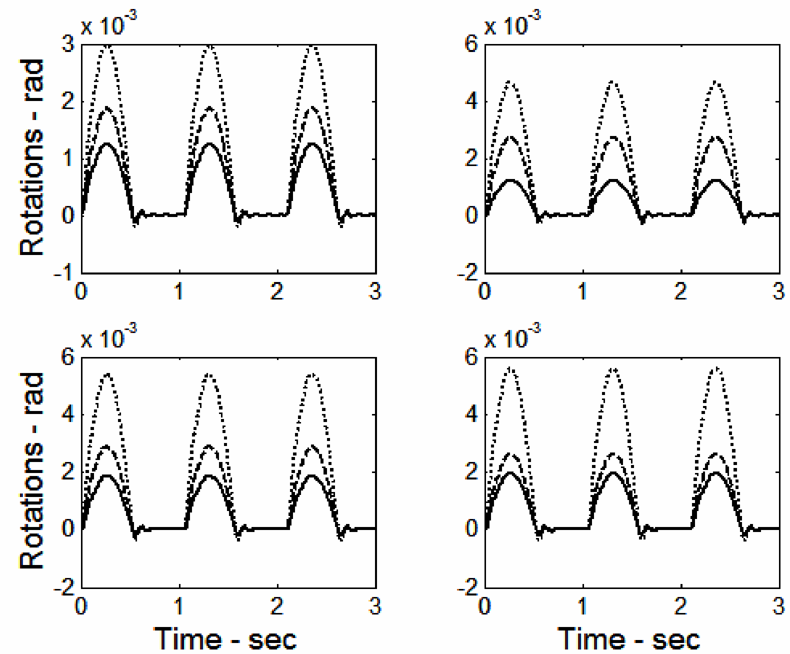
Displacements for the four nodes of uncontrolled (dot) and controlled H_2 law without (dash) and with (solid) amplifier due to random loading.



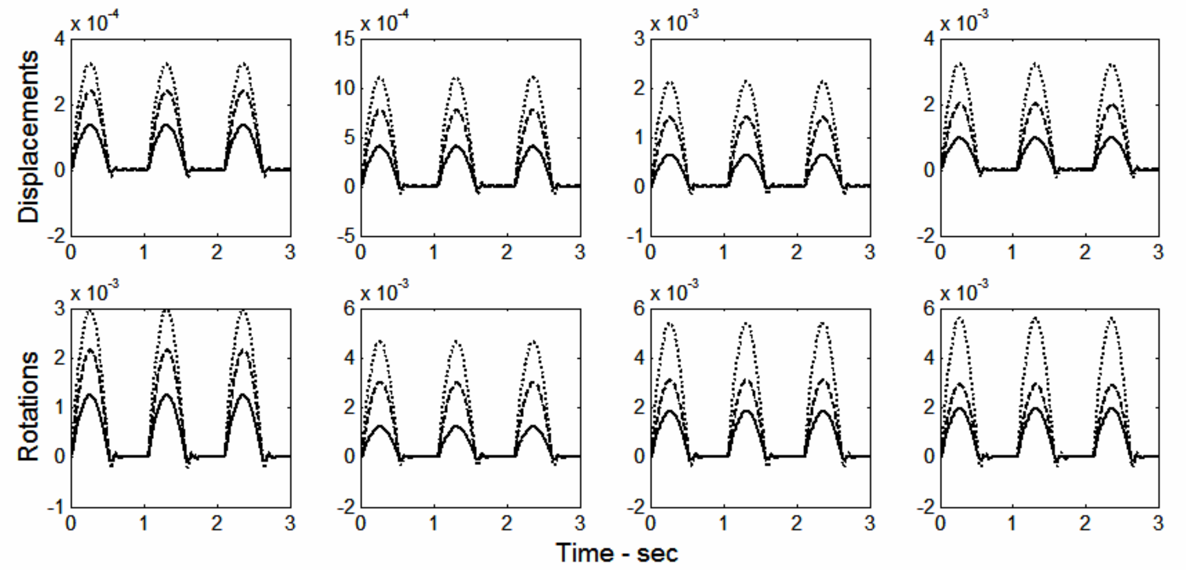
Rotations for the four nodes of uncontrolled (dot) and controlled H_2 law without (dash) and with (solid) amplifier due to random loading.



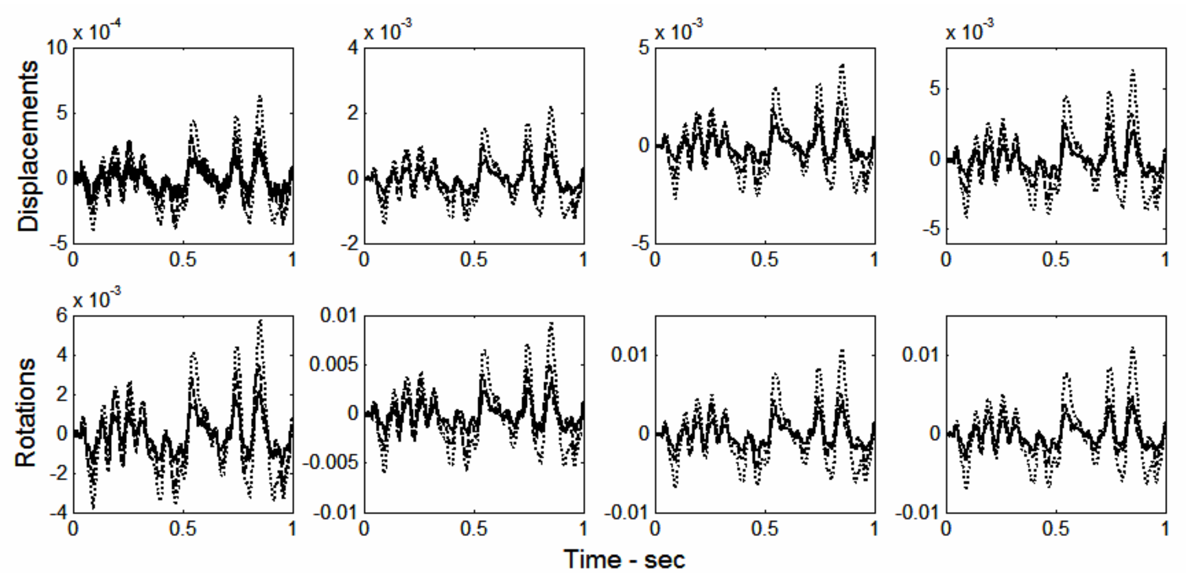
Displacements for the four nodes of uncontrolled (dot) and controlled H_2 law without (dash) and with (solid) amplifier due to sinusoidal loading.



Rotations for the four nodes of uncontrolled (dot) and controlled H_2 law without (dash) and with (solid) amplifier due to sinusoidal-like loading.

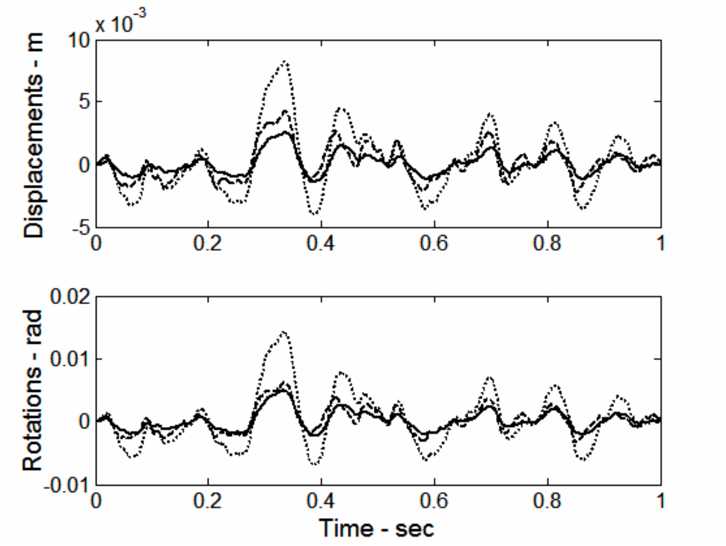


Responses of the free (dot) vibrating beam and controlled beam with LQR (dash) and H_2 (solid) control due to impulsive-like wind loading.



The vertical displacements and rotations for all nodes for uncontrolled (dot), LQR (dash) and H_2 (solid) regulated beam due to white noise loading.

The response of the free end of the beam is shown in the Figure.



The response of the free end of the beam subjected to random loading.

Conclusions

Mathematical formulation and the computational model for the active vibration control of a slender beam bonded with piezoelectric sensors and actuators are presented.

The problem of active control is studied by using the classical **LQR** and the robust $\mathbf{H_2}$ optimal approaches.

The comparison between the two proposed control laws shows that both strategies are effective. The second method is preferred due to the robustness properties.

The numerical simulations show that the proposed methods are usable for vibration suppression of a laminated beam subjected to different kind of loading.

A detailed investigation of the dynamical response of active beams and other structures will give us confidence in order to propose concrete industrial applications. Among others, suppression of wind vibrations and noise reduction in lightweight (e.g., aluminium) facades in architecture and civil engineering can be achieved by means of the proposed method.

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