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ARTICLE · FEBRUARY 2014

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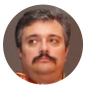
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Proceedings of
The Ninth International Conference on
Civil & Structural
Engineering Computing

Edited by
B.H.V. Topping

Egmond-aan-Zee - The Netherlands
2-4 September 2003

CIVIL-COMP PRESS

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Proceedings of the Ninth International Conference
on Civil and Structural Engineering Computing,
B.H.V. Topping (Editor),
Civil-Comp Press, Stirling, Scotland.

Paper 114

Design of Smart Beams for Suppression of Wind-Induced Vibrations

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Abstract

This paper presents the design of a vibration control mechanism for a beam with bonded piezoelectric sensors and actuators and an application of the arising smart structure for suppression of wind-induced vibrations. The mechanical modeling of the structure and the subsequent finite element approximation are based on the classical equations of motion, as they are derived from Hamilton's principle, in connection with simplified modeling of the piezoelectric sensors and actuators. Various control schemes have been implemented in structural control (LQR, LQG, H_2, and H_∞). The latter robust controllers are more flexible and lead to applicable design of smart structures. The numerical simulation shows that sufficient vibration suppression can be achieved by means of this general method.

Keywords: active structural control, smart beam, composite structure, piezoelectric layer, wind engineering.

1 Introduction

Due to the increasing demand of high structural requirements, the modeling and control of flexible structures have received considerable interest among the research community. One attractive way for vibration control of space and earth flexible structures is the incorporation of a ''smart'' material actuator, such as a piezoelectric material, into the structure [1], [4], [7], [9].

Piezoelectric materials respond to mechanical forces/pressures and generate an electric charge/voltage. This phenomenon is called the *direct piezoelectric effect*. Conversely, electric charge/field applied to the piezoelectric material induces mechanical stresses or strains, and this phenomenon is called the *converse piezoelectric effect.* In ''smart'' piezoelectric structures, the *direct* effect is used for structural measurements (*sensor*) and the *converse* effect is used for active vibration controls (*actuator*).

Although some people believe that this class of smart systems are not sufficient for control of strong excitations, like earthquakes, most people agree that applications on wind engineering are meaningful.
In this work a laminated beam with piezoelectric sensors and actuators is modelled by the finite element method. Furthermore, the problem of active control is studied using classical and robust optimal control (LQR and H_2, respectively) [2], [8], [10].

2 Modeling of the electromechanical system

2.1 Linear piezoelasticity

The constitutive relations for a piezoelectric material are given by

$$\{\sigma\} = [Q]\left(\{\varepsilon\} - [d]^T \{E\}\right), \tag{2.1}$$

$$\{D\} = [d][Q]\{\varepsilon\} + [\xi]\{E\}, \tag{2.2}$$

where $\{\sigma\}$ is the stress tensor, $\{\varepsilon\}$ is the strain tensor, $\{D\}$ is the electric displacement, $\{E\}$ is the electric field, $[Q]$ is the elastic stiffness matrix, $[d]$ is the piezoelectric matrix and $[\xi]$ is the permittivity matrix.
Equation (2.1) describes the inverse piezoelectric effect and equation (2.2) describes the direct piezoelectric effect.

2.2 Engineering smart beam model

2.2.1 Description of the smart beam

A slender beam with rectangular cross section having length L, width b and thickness h is considered. A pair of piezoelectric patches with thickness h_S and h_A is symmetrically bonded at the top (e.g. sensor) and the bottom (e.g., actuator) surfaces of the beam.

We derive engineering models for piezoelectric sensors and actuators and decouple electric and elastic parts of system (2.1)-(2.2), with the electric part being treated in a simplified way. We assume that the piezoelectric sensors and actuators are polarized in z-direction and exhibit transverse isotropic properties in xy-plane.
Under these assumptions the set of equations (2.1) and (2.2) is reduced as follows

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} - \begin{bmatrix} d_{31} \\ 0 \end{bmatrix} E_z \right) \tag{2.3}$$

$$D_z = Q_{11} d_{31} \varepsilon_x + \xi_{33} E_z \, . \tag{2.4}$$

The electric field intensity E_z can be expressed as

$$E_z = \frac{V_l}{h_l} \tag{2.5}$$

where V_l is the applied voltage across the l-layer and h_l is the thickness of the l-layer.

2.2.2 Sensor modeling

The direct piezoelectric equation (2.4) is used to calculate the output charge created by the strains in the beam. Since no electric field is applied to the sensor layer, we get

$$D_z = Q_{11} d_{31} \varepsilon_x . \tag{2.6}$$

The charge measured through the electrodes of the sensor is given by [1]

$$q(t) = \frac{1}{2}\left\{ \left(\int_{S_{ef}} D_z dS \right)_{z=h/2} + \left(\int_{S_{ef}} D_z dS \right)_{z=h/2+h_S} \right\} \tag{2.7}$$

where S_{ef} is the effective surface of the electrode of the sensor layer.

The current on the surface of the sensor is given by

$$i(t) = \frac{dq(t)}{dt} . \tag{2.8}$$

The current is converted into open-circuit sensor voltage output by

$$V^S = G_S i(t) . \tag{2.9}$$

where G_S is the gain of the current amplifier. The actuators are treated in a similar way.

2.2.3 Equations of motion

Let us assume that the composite beam is symmetric and that its centroidal and elastic axes coincide so that no bending-torsion coupling is considered. Furthermore we assume that the piezoelectric sensors and actuators are bonded perfectly on the host beam and they are much thinner than the host beam. The axial vibration of the beam centerline is considered negligible and the components of the displacement field $\{u\}$ based on the usual Timoshenko assumption can be expressed as:

$$\begin{aligned} u_1(\chi, y, z, t) &\approx z\psi(\chi, t), \\ u_2(\chi, y, z, t) &\approx 0, \\ u_1(\chi, y, z, t) &\approx w(\chi, t), \end{aligned} \tag{2.10}$$

where ψ is the rotation of the beam cross section about the positive y-axis and w is the transverse displacement of the point of the centroidal axis (y=z=0).

The strain-displacement relations are given by

$$\varepsilon_x = z\frac{\partial \psi}{\partial x}, \qquad \gamma_{xz} = \psi + \frac{\partial w}{\partial x} . \tag{2.11}$$

The simpler Euler-Bernoulli theory which considers zero transverse shear deformation γ_{xz} has also been tested, see [4], [5].

In order to derive the equations of motion of the beam, Hamilton's principle is used:

$$\delta \int_{t_1}^{t_2} (T - U + W) dt = 0 \tag{2.12}$$

where δ is the first variation operator, T is the kinetic energy, U is the potential energy and W is the work done by the external loads or moments.

The kinetic energy and the strain (potential) energy are known from classical structural analysis theories. If the only loading consists of moments induced by piezoelectric actuators and since the structure has no bending-twisting couple, then the first variation of the work has the form

$$\delta W = b\int_0^L M^A \delta\left(\frac{\partial \psi}{\partial x}\right)dx \tag{2.13}$$

where M^A is the moment per unit length induced by the actuator layer and is given by

$$M^A = \int_{-h/2-h_A}^{-h/2} z\sigma_x^A dz = \int_{-h/2-h_A}^{-h/2} zQ_{11}d_{31}E_z^A dz \tag{2.14}$$

and

$$E_z^A = \frac{V_A}{h_A}. \tag{2.15}$$

2.2.4 Finite element modeling

The model is composed of beam elements in bending, which have two mechanical degrees of freedom (d.o.f.) at each node: one translation d.o.f. $w_1(w_2)$ in direction z and one rotational d.o.f. $\psi_1(\psi_2)$. Using classical finite element interpolation functions and (2.12), the equations of motion for the discretized structure read [6]:

$$M\ddot{X} + D\dot{X} + KX = F_m + F_e \tag{2.16}$$

where vector $\boldsymbol{X}$ contains the states of the system (vertical transverse deflection and rotations of the nodes), $(\ddot{\ })$ stands for the second time derivative. $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}$ is the stiffness matrix. $\boldsymbol{F}_m$ is a mechanical point forces vector added *a posteriori* to the discretized system, $\boldsymbol{F}_e$ is the generalized electrical load vector provided by the applied voltages and proportional to them. $\boldsymbol{D}$ is a viscous damping matrix added *a posteriori* and $\dot{\mathbf{X}}$ is a velocity vector.

3 Optimal control problem

3.1 Preparation

The second order system (2.16) can be written as the following state space one

$$\dot{x} = Ax + Bu + F, \qquad y = Cx \tag{3.1}$$

where $x^T = \left[X^T \dot{X}^T \right]$ is the state vector, $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$ is the system matrix, $B = \begin{bmatrix} 0 \\ M^{-1}F_e^* \end{bmatrix}$ with F_e^* the voltage factored-out electrical loads vector, $F = \begin{bmatrix} 0 \\ M^{-1}F_m \end{bmatrix}$.
Control input $\boldsymbol{u}$ is a column vector formed by the voltages applied to the actuators. The output vector $\boldsymbol{y}$ consists of measures formed from the state vector $\boldsymbol{x}$ by the output matrix $\mathbf{C}$.

3.2 Classical control (LQR, LQG)

Consider the linear system (3.1) and the quadratic cost function

$$J = \frac{1}{2}\int_0^\infty (x^T Qx + u^T Ru)dt\,. \tag{3.2}$$

The problem consists of minimizing the functional $\boldsymbol{J}$ with respect to the control input $\boldsymbol{u}$ subject to the linear system constraints (3.1). This is known as the linear quadratic regulator (LQR) problem.
This results in a linear controller of the feedback form

$$U = -Kx,\; K=R^{-1}B^TP \tag{3.3}$$

where the constant matrix P is a solution of the Algebraic Riccati Equation (ARE)

$$A^T P + PA + Q - PBR^{-1}B^T P = 0\,. \tag{3.4}$$

Under technical assumptions, which can be found in classical control theory treatises, existence and uniqueness of the above controller is guaranteed. The main design parameters for the controller are the weight matrices Q, determining the weight of each state, and R, determining the weight of each actuator voltage (control channel).
Without entering into details, we mention here that LQR problem requires the complete knowledge of the whole state for each time instance. This is a considerable deficiency for practical applications, since it requires expensive sensor instrumentation. If a limited number of measurements is available, as it is the case with the configuration of the smart beam treated here, a more realistic assumption can be used. From these measurements the whole system is first reconstructed, during the estimation phase, and then the optimal control problem is based on this reconstructed state vector. The assumption of Gaussian distribution for the unknown fields leads to the method known as LQG (linear quadratic Gaussian). Since a similar strategy of system estimation and control is used in the H_2 model of the next section, no further details of LQG are discussed here.

3.3 Robust modern control (H_2, $H_{infinity}$)

Modern control involves more complicated optimization problems than the one of (3.2). Robustness with respect to external disturbances or uncertainties of the system or of the loading is the key issue. One of this models, known as H_2 is tested here. Further models have been discussed, among others, in [2], [10]. The basic block diagram used in this paper is shown in Figure 1.

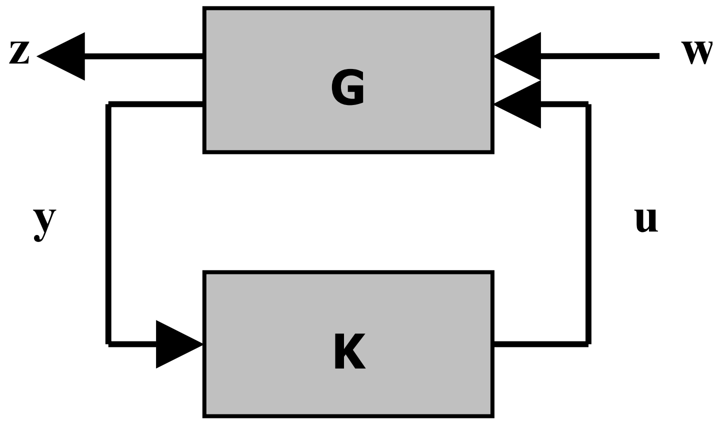

Figure 1: Closed-loop system diagram for H_2 control.

Here G is the generalized plant and K is the controller. Both G and K are real, rational and proper. The generalized plant G contains the plant plus all weighting functions, which may be used as scaling in order to make the numerical solution of the problem easier. The signal w contains all external inputs including disturbances, sensor noise and commands; the output z is an error signal; y is the measured variables; and u is the control input. The resulting closed-loop transfer function from w to z is denoted by T_{zw}.
A popular performance measure in optimal control theory is H_2 norm, defined in the frequency-domain for a stable transfer matrix $G(s)$ as [10]

$$\|G\|_2 = \left\{ \frac{1}{2\pi} \int_{-\infty}^{+\infty} tr\left[G^*(j\omega)G(j\omega)\right] d\omega \right\}^{\frac{1}{2}} . \tag{3.5}$$

If the transfer matrix in terms of state-space data is denoted by $G(s)$ where

$$G(s) = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = C(sI - A)^{-1} B \tag{3.6}$$

$\|G\|_2$ can, in principle, be computed from the definition (3.1). An alternative useful characterization is the following. If L_c denotes the controlability Gramian of (A,B) and L_o the observability Gramian of (C,A) then

$$AL_c + L_c A^T + BB^T = 0 \quad , \quad A^T L_o + L_o A + C^T C = 0$$

and

$$\|G\|_2^2 = tr(CL_c C^T) = tr(B^T L_o B) . \tag{3.7}$$

Note that this computation involves the solution of a linear equation and can be done in a finite number of steps.
Now we will discuss the assumptions on G that are used in H_2 theory. Then we will show how to choose K which minimizes the H_2 norm of $T_{zw.}$ To be admissible K is constrained to provide internal stability. In our application state models of G and K

are considered. Then internal stability means that the states of *G* and *K* go to zero from all initial values when *w=0*.

The realization of the transfer matrix *G* is taken to be of the form

$$\begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \tag{3.8}$$

After normalization, the system becomes

$$\begin{bmatrix} A & B_1 & B_2\left(D_{12}^T D_{12}\right)^{-\frac{1}{2}} \\ C_1 & 0 & D_{12}\left(D_{12}^T D_{12}\right)^{-1} \\ \left(D_{21}D_{21}^T\right)^{-\frac{1}{2}} C_2 & \left(D_{21}D_{21}^T\right)^{-\frac{1}{2}} D_{21} & 0 \end{bmatrix} \tag{3.9}$$

Further technical assumptions which guarantee the existence of the solution can be found in the literature and are not discussed here.

The solution of the H_2 problem uses a state estimator and feeds back the estimated states. The controller and estimator gains are computed from two algebraic Riccati equations (ARE).

Finally the feedback control is given by

$$u = K_c \hat{x}, \tag{3.10}$$

where $\hat{x}$ is the optimal estimate of *x*. The problem is to find an admissible controller K_c which minimizes $\left\|T_{zw}\right\|_2$

Using Pontriagin's maximum principle for controller and observer, we obtain the two Hamiltonian matrices

$$H_2 = \begin{bmatrix} A - B_2\left(D_{12}^T D_{12}\right)^{-1} D_{12}^T C_1 & -B_2\left(D_{12}^T D_{12}\right)^{-1} B_2^T \\ -C_1^T\left[I - D_{12}\left(D_{12}^T D_{12}\right)^{-1} D_{12}^T\right]C_1 & -A^T + C_1^T D_{12}\left(D_{12}^T D_{12}\right)^{-1} B_2^T \end{bmatrix} \tag{3.11}$$

$$J_2 = \begin{bmatrix} A^T - C_2^T\left(D_{21}^T D_{21}\right)^{-1} D_{21} B_1^T & -C_2^T\left(D_{21}D_{21}^T\right)^{-1} C_2 \\ -B_1\left[I - D_{21}^T\left(D_{21}D_{21}^T\right)^{-1} D_{21}\right]B_1^T & -A + B_1 D_{21}^T\left(D_{21}D_{21}^T\right)^{-1} C_2 \end{bmatrix} \tag{3.12}$$

that belong to *dom(Ric)*. Let $X_2=Ric(H_2)$ and $Y_2=Ric(J_2)$ are the corresponding positive definite solutions of the two ARE. Then the controller and observer gains are

$$K_2 = -\left(D_{12}^T D_{12}\right)^{-1}\left(B_2^T X_2 + D_{12}^T C_1\right) \tag{3.13}$$

$$L_2 = -\left(Y_2 C_2^T + B_1 D_{21}^T\right)\left(D_{21}D_{21}^T\right)^{-1} \tag{3.14}$$

and the systems are

$$G_c = \begin{bmatrix} A + B_2 K_2 & I \\ C_1 + D_{21} K_2 & 0 \end{bmatrix}, \qquad G_l = \begin{bmatrix} A + L_2 C_2 & B_1 + L_2 D_{21} \\ I & 0 \end{bmatrix}. \tag{3.15}$$

The unique optimal controller is

$$K_c = \begin{bmatrix} A + B_2 K_2 + L_2 C_2 & -L_2 \\ K_2 & 0 \end{bmatrix}. \tag{3.16}$$

Moreover

$$\min \|T_{zw}\|_2^2 = \|G_c B_1\|_2^2 + \left\| \left(D_{12}^T D_{12}\right)^{\frac{1}{2}} K_2 G_l \right\|_2^2 = tr\left(B_1^T X_2 B_1\right) + tr(D_{12}^T D_{12} K_2 Y_2 K_2^T)$$

The controller K_c has the well-known separation structure. It implies reducing the output feedback problem to a combination of the full information and the output estimation problems. The controller equations can be written in standard observer form as

$$\dot{\hat{x}} = A\hat{x} + B_2 \left(D_{12}^T D_{12}\right)^{-\frac{1}{2}} u + L_2 \left[\left(D_{21} D_{21}^T\right)^{-\frac{1}{2}} C_2 \hat{x} - y \right]$$

$$u = K_2 \hat{x}$$

For general thoughts about active structures the reader may consult the review article [9]. Both Timoshenko and Euler-Bernoulli technical beam theories have been tested cf. [1]. The H_2 control design technique provides better robustness and allows for the control objectives to be conveniently defined in time domain [4]. Another theory of robust optimal control, based on the H_{infinity} norm is discussed in Reference [7].

4 Numerical results

A cantilever beam with four finite element nodes under ambient vibration and sinusoidal, wind-type loading is considered. Results for uncontrolled (green) and controlled (blue) vertical displacements for all nodes are given in Figures 2-6.

The effect of using less measurements on the performance of the control scheme is shown in Figures 2,3. There all four elements have independent controllers (piezoelectric actuators), while the number of measurements, which are based on the available sensors, varies between four and two.

Vibrations near the free end (bottom-right plot) can efficiently be suppressed. Worse results appearing near the fixed end (upper-left plot) are due to the fact that H_2 robust controller includes an estimation of the structural system from incomplete measurements and insufficient accuracy of the simplified finite element model in higher vibrational modes (compare Figures 5 and 6).

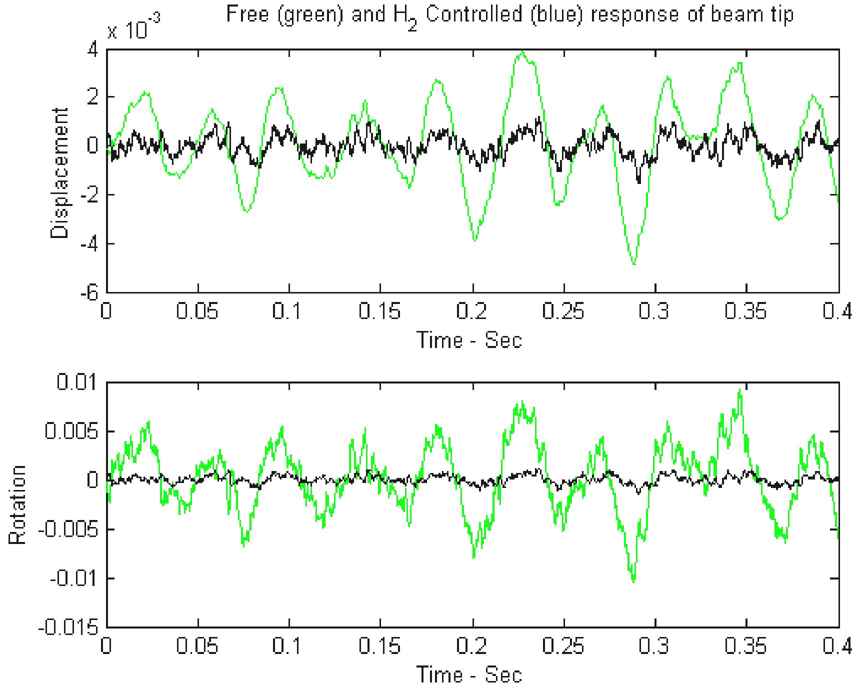

Figure 2: Vibration of the free end subjected to ambient vibration loading. Four controls and four sensors.

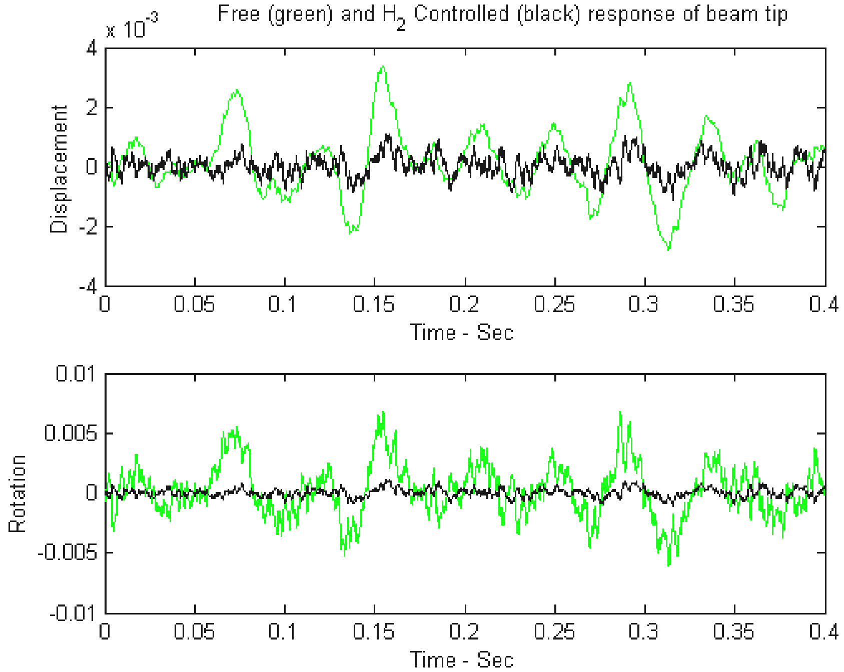

Figure 3: Vibration of the free end subjected to ambient vibration loading. Four controls and one sensor.

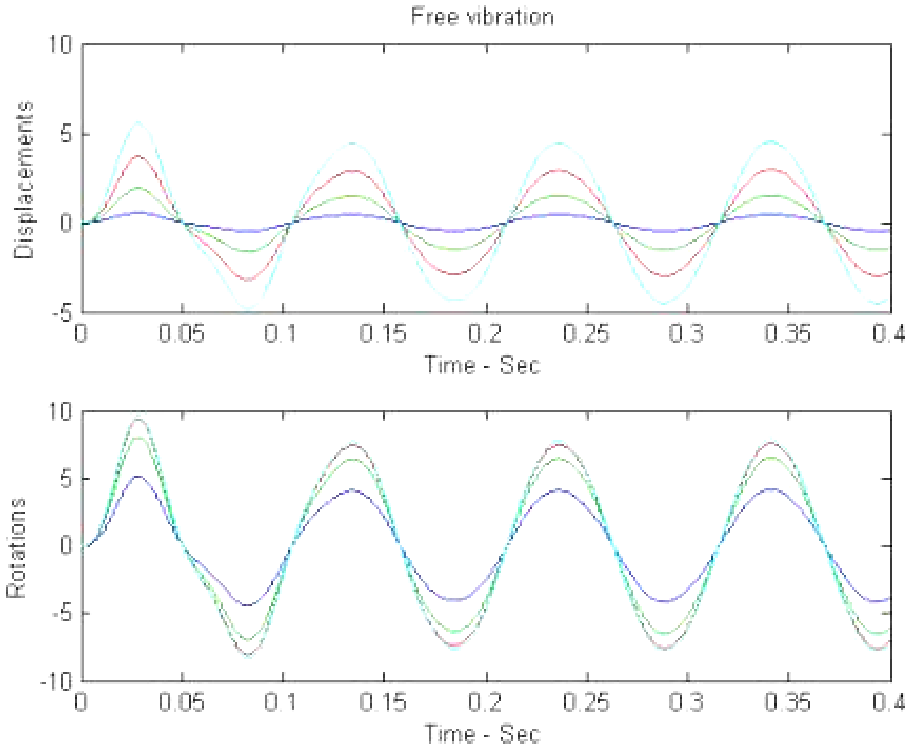

Figure 4: The forced vibration of the beam under sinusoidal loading.

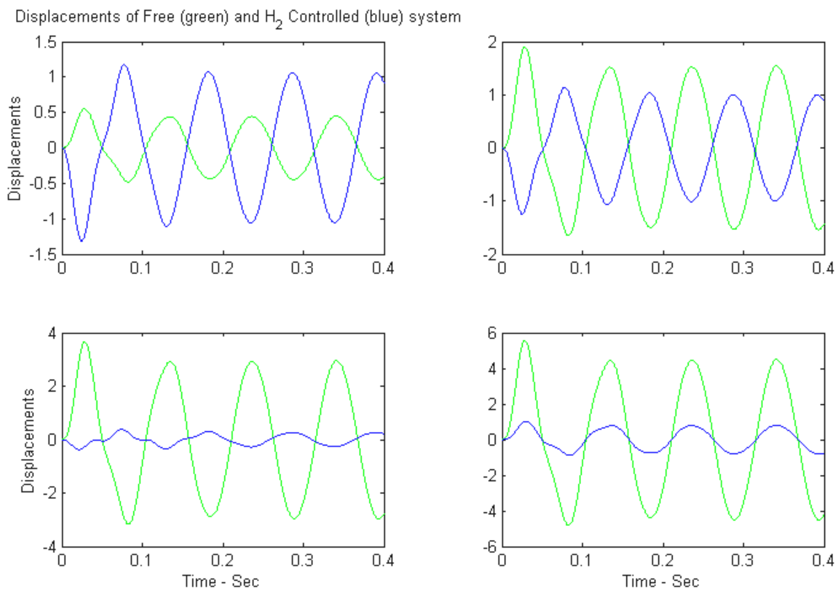

Figure 5: Vibration of the beam without and with control, from near-support node to the free node.

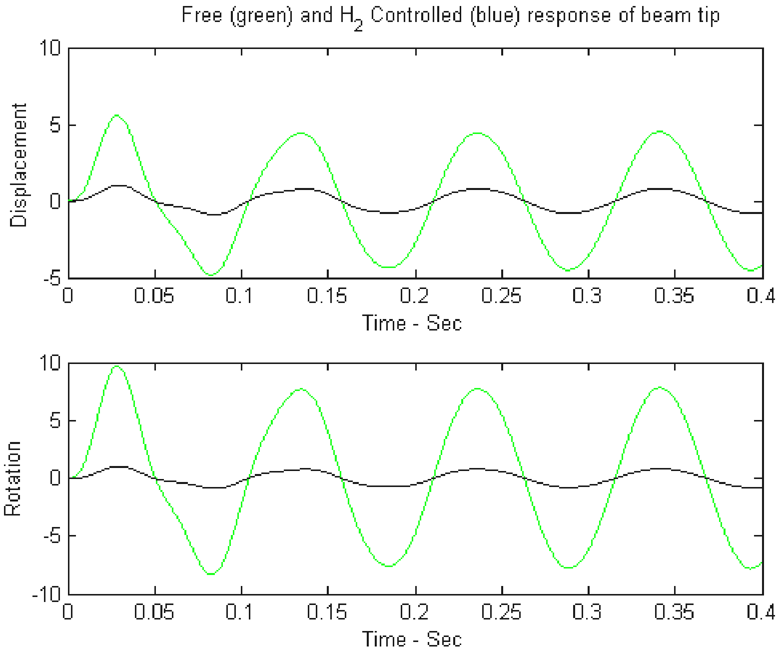

Figure 6: Sufficient vibration suppression at the end of the beam.

For demonstration purposes we assume a smart beam with length equal to 0,8 m; width 0,02 m and height equal to 0,01 m, respectively.

Material constants are taken from the literature to be equal with: Modified Elastic modulus (GPa) 73, Density (kg/m^3) 2700. Without having experimental data, a viscous damping coefficient equal to 0.0001 is considered.

A sinusoidal loading, which approximates the wind loading on a structure is used (It corresponds to the technical recommendation with ρ=0.125 N/m; v_m=28.0; g=2.504; c_f=1.0; $p_w=0.5*\rho*v_m^2(1+g)*c_f\sin(t)$ according to reference [3]).

A detailed investigation of the dynamical response of active beams and other structures will give us confidence in order to propose concrete industrial applications. Among others, suppression of wind vibrations and noise reduction in leightweight (e.g., aluminium) facades in architecture and civil engineering can achieved by means of the proposed method.

Acknowledgements

The work reported here has been partially supported by the European Union Research and Training Network (RTN) “Smart Systems. New Materials, Adaptive Systems and their Nonlinearities. Modelling, Control and Numerical Simulation”, with contract number HPRN-CT-2002-00284.

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