TECHNICAL UNIVERSITY OF CRETE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

MIMObit-assisted Antenna Selection

by

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Abstract

In this work, we study antenna selection criteria and algorithms for multiple-input multiple-output (MIMO) wireless systems using channel data produced by the specialized software MIMObit. We begin by summarizing conventional performance metrics for MIMO systems and presenting several antenna selection algorithms. We run these algorithms in MALAB, assuming that, when some antennas are not selected for operation, the channel coefficients of the selected ones do not change. However, this assumption (which is conventionally made in the literature) does not hold true in reality. To develop a more realistic solution for the antenna selection problem, we use MIMObit which offers an integrated platform that helps us simulate MIMO systems with custom created dipoles under different propagation environments and generate the corresponding channel coefficients. Using MIMObit, we utilize the actual channel coefficients after the antenna selection process and show that the MIMObit-assisted antenna selection approach always outperforms the conventional one. In some cases, the performance gain is noticeable.

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List of Abbreviations

BER	Bit Error Rate
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SNR	Signal to Noise Ratio
SVD	Singular Value Decomposition

Chapter 1

Single Transmit Antenna Systems

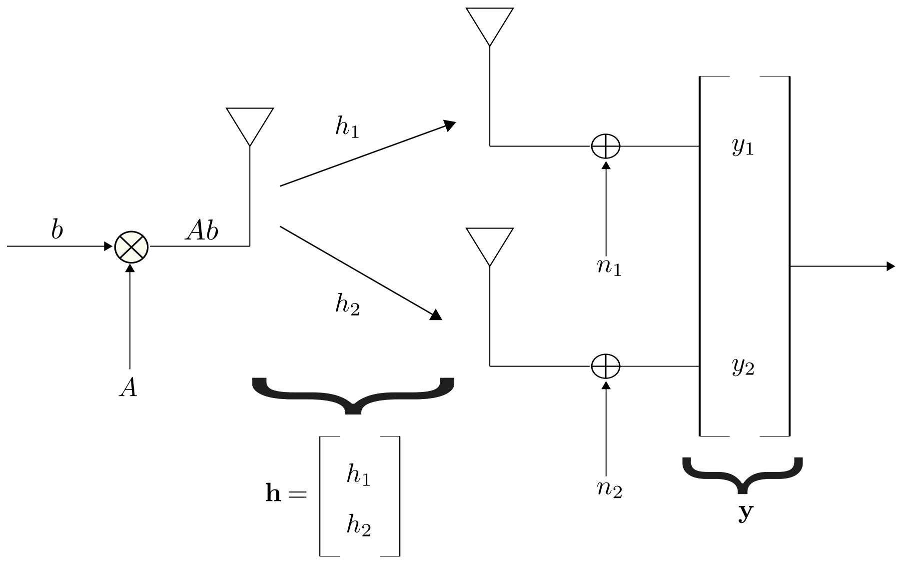

Figure 1.1: SIMO 1×2 system.

1.1 Single-Input Multiple-Output (SIMO)

A SIMO system consists of $N_{\mathrm{t}} = 1$ transmit antenna and N_{r} receive antennas. For instance, in Fig. 1.1, we present such a system with $N_{\mathrm{r}} = 2$ antennas at the receiver.

We use a SIMO system to transmit the uniformly distributed bit $b = \pm 1$. The transmitted signal is Ab where $A > 0$ is the signal amplitude. Assuming flat-fading propagation, the received signal is

$$\mathbf{y} = A\mathbf{h}b + \mathbf{n} \tag{1.1}$$

where $\mathbf{h} \in \mathbb{C}^{N_{\mathrm{r}}}$ is a $N_{\mathrm{r}} \times 1$ complex vector that contains the channel coefficients from the transmit antenna to the N_{r} receive antennas and $\mathbf{n} \sim \mathbb{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_{\mathrm{r}}})$ is an additive noise vector that follows the white (in time and space) complex Gaussian vector distribution with mean zero and variance σ^2. The optimal detector is

$$\mathrm{Re}(\mathbf{h}^H \mathbf{y}) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{1.2}$$

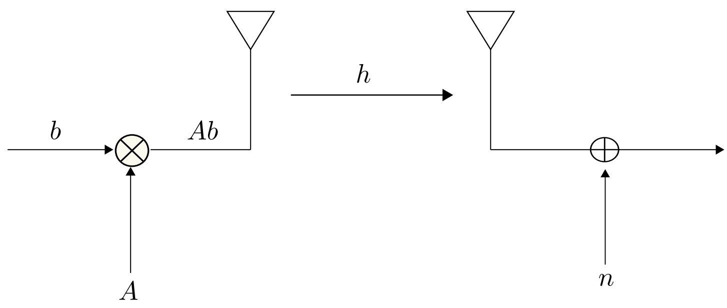

Figure 1.2: SISO system.

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2\|\mathbf{h}\|^2\frac{A^2}{\sigma^2}}\right). \tag{1.3}$$

The derivations of the optimal detector and its probability of error are provided in Chapter 6 (Appendix).

A special case of SIMO is the single-input single-output (SISO) system, where the receive antennas are $N_{\mathrm{r}} = 1$. Fig. 1.2 shows such a SISO system. Similarly to SIMO, the transmitted signal in a SISO system is Ab and the received signal is

$$y = Ahb + n \tag{1.4}$$

where $h \in \mathbb{C}$ is the channel coefficient and $n \sim \mathbb{CN}(0, \sigma^2)$ represents additive white (in time) complex Gaussian noise. The optimal detector in a SISO system simplifies to

$$\mathrm{Re}\left(h^* y\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{1.5}$$

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2|h|^2\frac{A^2}{\sigma^2}}\right). \tag{1.6}$$

As an illustration, in Fig. 1.3 we present the average probability of error versus the receive signal-to-noise ratio (SNR) per path for a SISO system and a SIMO system with $N_{\mathrm{r}} = 2, 3, 5, 7,$ and 10 receive antennas. The average probability of error is obtained over 10^6 channel realizations that follow the Rayleigh distribution. We observe that, while the SNR is increasing, the average probability of error is decreasing. We also observe that the average probability of error decreases with the number of receive antennas.

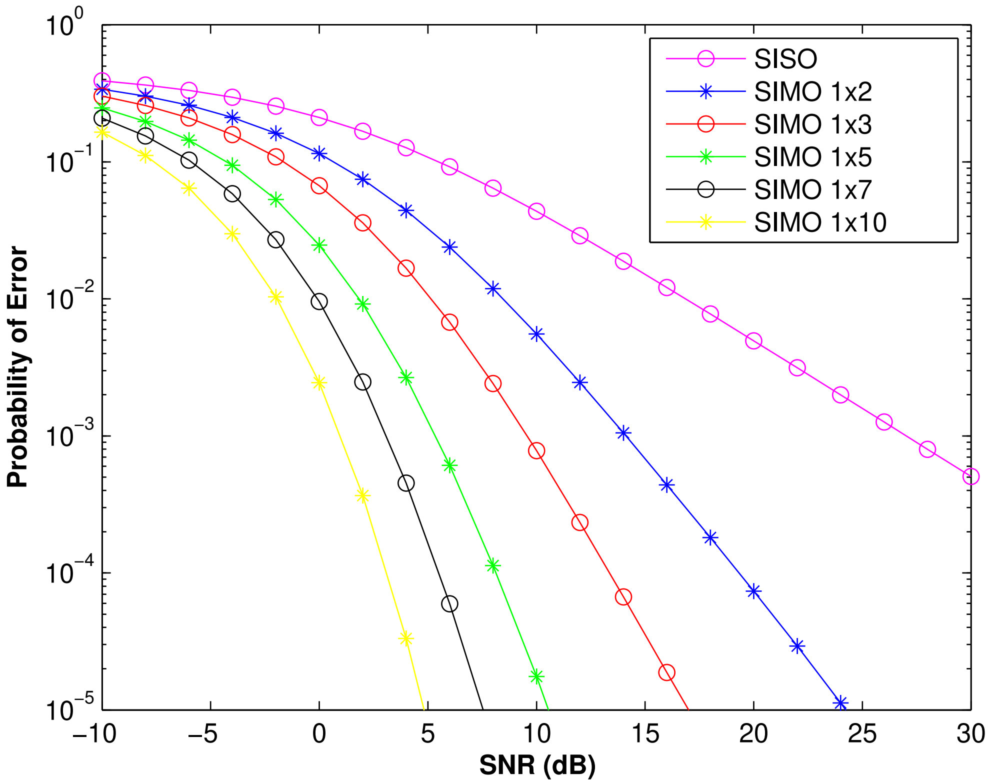

Figure 1.3: Average probability of error versus receive SNR per path for a SISO and a SIMO system with $N_{\mathrm{r}} = 2, 3, 5, 7$, and 10 receive antennas in Rayleigh fading.

Chapter 2

Multiple Transmit Antenna Systems

2.1 Multiple-Input Single-Output (MISO)

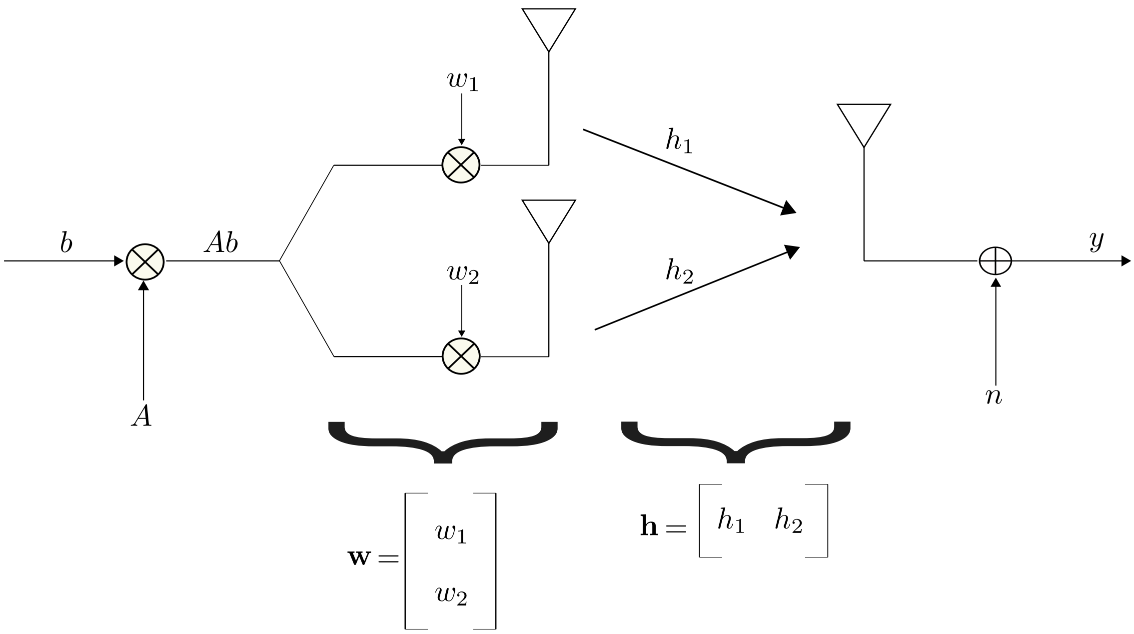

Figure 2.1: MISO 2×1 system.

A MISO system consists of N_t transmit antennas and $N_\mathrm{r} = 1$ receive antenna. For instance, in Fig. 2.1, we present such a system with $N_\mathrm{t} = 2$ antennas at the transmitter.

We use a MISO system to transmit the uniformly distributed bit $b = \pm 1$. The transmitted signal Ab, where $A > 0$ is the signal amplitude, is multiplied by a complex weight vector $\mathbf{w} \in \mathbb{C}^{N_\mathrm{t}}$ to feed the N_t transmit antennas. The norm of $\mathbf{w}$ is set to $\|\mathbf{w}\| = 1$ to guarantee that the transmitted signal power is A^2. The received signal is

$$y = A\mathbf{h}\mathbf{w}b + n \tag{2.1}$$

where $\mathbf{h} \in \mathbb{C}^{N_\mathrm{t}}$ is a $1 \times N_\mathrm{t}$ complex vector that contains the channel coefficients from the N_t transmit antennas to the receive antenna and $n \sim \mathbb{CN}(0, \sigma^2)$ represents additive white (in time) complex Gaussian noise.

For a fixed weight vector $\mathbf{w}$, (2.1) is equivalent to (1.4) for SISO by simply substituting

$h \in \mathbb{C}$ in (1.4) by $\mathbf{hw} \in \mathbb{C}$ in (2.1). Hence, the equations for the optimal detector and the probability of error for the SISO case apply directly to MISO, as described in (2.1). That is, for fixed $\mathbf{w}$, the optimal detector is

$$\mathrm{Re}\left((\mathbf{hw})^* y\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{2.2}$$

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2|\mathbf{hw}|^2 \frac{A^2}{\sigma^2}}\right). \tag{2.3}$$

Since $\mathbf{w}$ is a design parameter, we can optimize it to minimize the probability of error of the optimal detector. Since we have set the constraint $\|\mathbf{w}\| = 1$, the optimal weight vector is

$$\begin{aligned}\mathbf{w}_{\mathrm{opt}} &= \underset{\|\mathbf{w}\|=1}{\mathrm{argmin}}\, P_{\mathrm{e}} = \underset{\|\mathbf{w}\|=1}{\arg\max}\, |\mathbf{hw}| \\ &= \underset{\|\mathbf{w}\|=1}{\arg\max}\, \frac{|\mathbf{hw}|}{\|\mathbf{w}\|}. \end{aligned} \tag{2.4}$$

Using Cauchy-Scwartz Inequality,

$$\frac{|\mathbf{hw}|}{\|\mathbf{w}\|} \leq \frac{\|\mathbf{h}\|\|\mathbf{w}\|}{\|\mathbf{w}\|} = \|\mathbf{h}\|, \tag{2.5}$$

where equality exists if and only if $\mathbf{w}=\lambda\mathbf{h}^H$, $\lambda \in \mathbb{C} - \{0\}$. That is, equality holds if $\mathbf{w} = \dfrac{\mathbf{h}^H}{\|\mathbf{h}\|}$. Hence,

$$\mathbf{w}_{\mathrm{opt}} = \frac{\mathbf{h}^H}{\|\mathbf{h}\|}. \tag{2.6}$$

For the optimal weight vector $\mathbf{w}_{\mathrm{opt}}$ in (2.6), the optimal detector of (2.2) simplifies to equation

$$\mathrm{Re}(y) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{2.7}$$

and its probability of error becomes

$$P_e = Q\left(\sqrt{2\|\mathbf{h}\|^2 \frac{A^2}{\sigma^2}}\right). \tag{2.8}$$

As an illustration, in Fig. 2.2 we present the average probability of error versus the received signal-to-noise ratio (SNR) per path for a SISO system and a MISO system with $N_{\mathrm{t}} = 2, 3, 5, 7$, and 10 transmit antennas. The average probability of error is obtained over 10^6 channel realizations that follow the Rayleigh distribution. We observe that, while the SNR is increasing, the average probability of error is decreasing. We also observe that the average probability of error decreases with the number of transmit antennas.

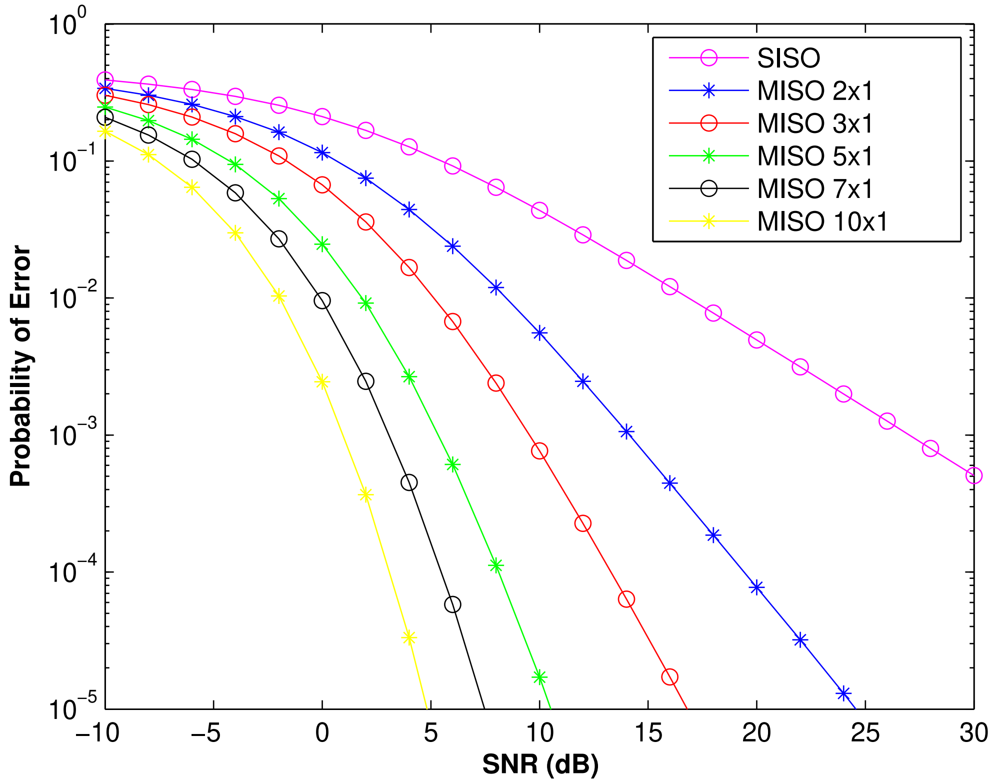

Figure 2.2: Average probability of error versus receive SNR per path for a MISO system with $N_\mathrm{t} = 2, 3, 5, 7$, and 10 transmit antennas in Rayleigh fading.

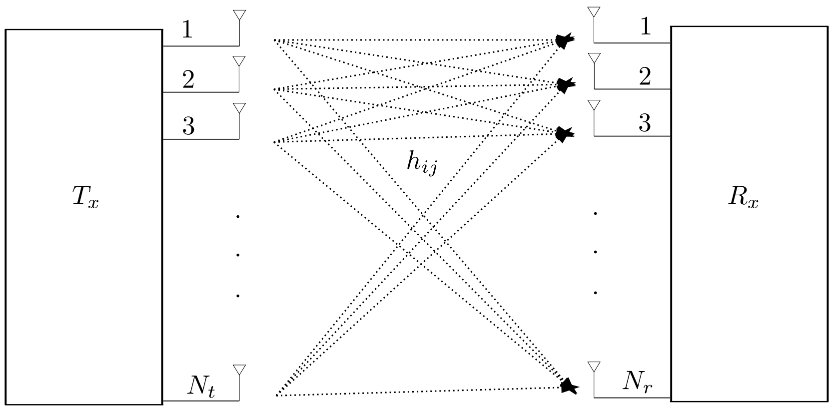

Figure 2.3: MIMO $N_\mathrm{t} \times N_\mathrm{r}$ system.

2.2 Multiple-Input Multiple-Output

A $N_\mathrm{t} \times N_\mathrm{r}$ MIMO system consists of N_t transmit antennas and N_r receive antennas. Such a system is presented in Fig. 2.3. If we denote by $h_{ij} \in \mathbb{C}$ the channel coefficient from the jth transmit antenna to the ith receive antenna, $i = 1, \ldots, N_\mathrm{r}$, $j = 1, \ldots, N_\mathrm{t}$, then we can

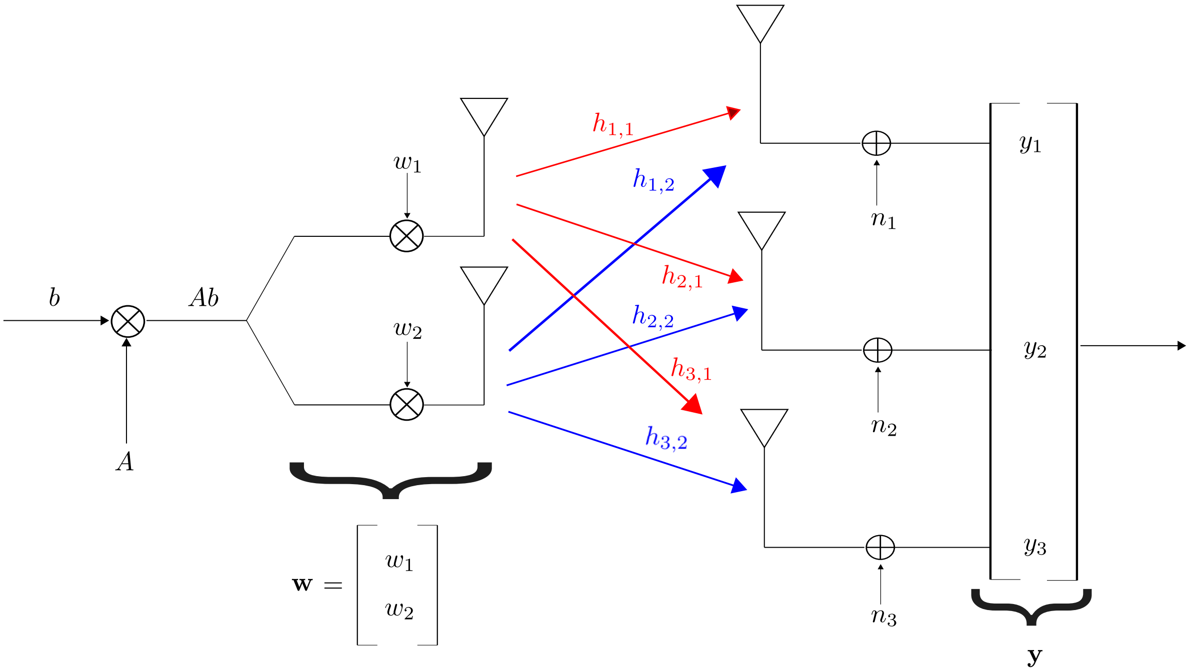

Figure 2.4: MIMO 2 × 3 system utilizing beamforming.

define the MIMO channel matrix as

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1N_\mathrm{t}} \\ h_{21} & h_{22} & \dots & h_{2N_\mathrm{t}} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_\mathrm{r}1} & h_{N_\mathrm{r}2} & \dots & h_{N_\mathrm{r}N_\mathrm{t}} \end{bmatrix}.$$

In MIMO systems, we may utilize all $N_\mathrm{t}N_\mathrm{r}$ communication paths to transmit the same information carried by a single symbol (bit). Alternatively, we may transmit simultaneously more than one symbols at the cost of fewer degrees of freedom per symbol transmission. In the sequel, we present and study these two alternatives. The first one is termed MIMO-beamforming, while the second one is termed MIMO-multiplexing.

2.2.1 MIMO-beamforming

We consider a MIMO system and use it to transmit the uniformly distributed bit $b = \pm 1$. The transmitted signal Ab, where $A > 0$ is the signal amplitude, is multiplied by a complex weight vector $\mathbf{w} \in \mathbb{C}^{N_\mathrm{t}}$ to feed the N_t transmit antennas. The norm of $\mathbf{w}$ is set to $\|\mathbf{w}\| = 1$ to guarantee that the transmitted signal power is A^2. In Fig. 2.4, we present such a MIMO system with $N_\mathrm{t} = 2$ antennas at the transmitter and $N_\mathrm{r} = 3$ antennas at the receiver.

Assuming flat-fading propagation, the received signal is

$$\mathbf{y} = A\mathbf{H}\mathbf{w}b + \mathbf{n} \tag{2.9}$$

where $\mathbf{n} \sim \mathbb{CN}(\mathbf{0}, \sigma^2\mathbf{I}_{N_\mathrm{r}})$ is an additive noise vector that follows the white (in time and space) complex Gaussian vector distribution with mean zero and variance σ^2. For a fixed weight vector $\mathbf{w}$, (2.9) is equivalent to (1.1) for SIMO by simply substituting $\mathbf{h} \in \mathbb{C}^{N_\mathrm{r}}$ in (1.1) by $\mathbf{Hw} \in \mathbb{C}^{N_\mathrm{r}}$. Hence, the equations for the optimal detector in (1.2) and the probability of error in (1.3) for the SIMO case apply directly to MIMO beamforming, as described in (2.9). That is, for fixed $\mathbf{w}$, the optimal detector is

$$\mathrm{Re}\left((\mathbf{Hw})^H y\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{2.10}$$

and its probability of error is

$$P_\mathrm{e} = Q\left(\sqrt{2\|\mathbf{Hw}\|^2 \frac{A^2}{\sigma^2}}\right). \tag{2.11}$$

Since $\mathbf{w}$ is a design parameter, we can optimize it to minimize the probability of error of the optimal detector. Due to the constraint $\|\mathbf{w}\| = 1$, the optimal weight vector is

$$\mathbf{w}_\mathrm{opt} = \underset{\|\mathbf{w}\|=1}{\arg\max} \|\mathbf{Hw}\|^2 = \underset{\|\mathbf{w}\|=1}{\arg\max} \frac{(\mathbf{Hw})^H (\mathbf{Hw})}{\|\mathbf{w}\|^2} = \underset{\|\mathbf{w}\|=1}{\arg\max} \frac{\mathbf{w}^H \mathbf{H}^H \mathbf{Hw}}{\|\mathbf{w}\|^2}. \tag{2.12}$$

The solution to (2.12) is the normalized eigenvector corresponding to the maximum eigenvalue of the positive semidefinite matrix $\mathbf{H}^H\mathbf{H}$, because of the quadratic form

$$\lambda_\mathrm{min}(\mathbf{H}^H\mathbf{H}) \leq \frac{\mathbf{w}^H \mathbf{H}^H \mathbf{Hw}}{\|\mathbf{w}\|^2} \leq \lambda_\mathrm{max}(\mathbf{H}^H\mathbf{H}) \tag{2.13}$$

where the right equality is achieved if and only if $\mathbf{w}$ is the normalized eigenvector corresponding to the maximum eigenvalue of $\mathbf{H}^H\mathbf{H}$. We note that $\mathbf{H}$ can be expressed as the product of three matrices using the singular value decomposition (SVD), i.e.,

$$\mathbf{H} = \mathbf{U\Sigma V}^H \tag{2.14}$$

where $\mathbf{U}$ and $\mathbf{V}$ are $N_\mathrm{r} \times N_\mathrm{r}$ and $N_\mathrm{t} \times N_\mathrm{t}$, respectively, unitary matrices, which means that

$$\mathbf{UU}^H = \mathbf{U}^H\mathbf{U} = \mathbf{I}_{N_\mathrm{r}}, \mathbf{VV}^H = \mathbf{V}^H\mathbf{V} = \mathbf{I}_{N_\mathrm{t}}, \tag{2.15}$$

and $\mathbf{\Sigma}$ is a $N_\mathrm{r} \times N_\mathrm{t}$ diagonal matrix whose diagonal elements are nonnegative and are called the singular values of $\mathbf{H}$. Without loss of generality (w.l.o.g), we can assume that

$$\sigma_1 \geqslant \sigma_2 \geqslant \ldots \geqslant \sigma_r > 0 \tag{2.16}$$

where

$$r = \mathrm{rank}(\mathbf{H}) \leq \min(N_\mathrm{t}, N_\mathrm{r}). \tag{2.17}$$

Using (2.14), we obtain

$$\mathbf{H}^H\mathbf{H} = (\mathbf{U\Sigma V}^H)^H\mathbf{U\Sigma V}^H = \mathbf{V\Sigma}^H\mathbf{U}^H\mathbf{U\Sigma V}^H = \mathbf{V\Sigma}^H\mathbf{\Sigma V}^H = \mathbf{V\Lambda V}^H. \tag{2.18}$$

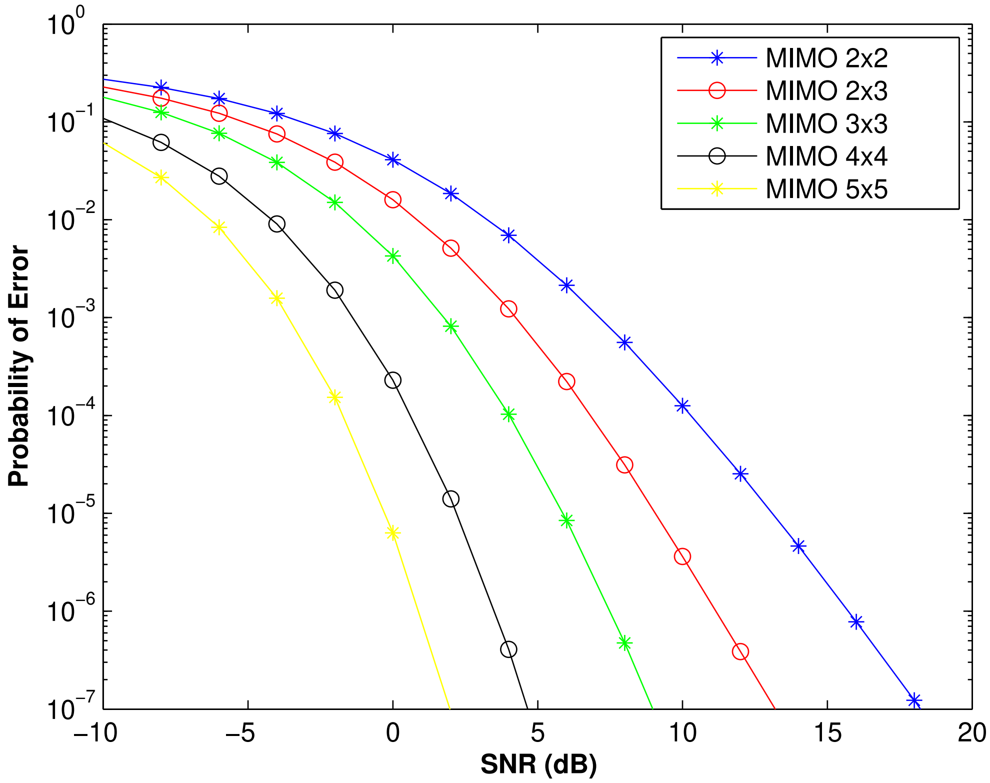

Figure 2.5: Average probability of error versus receive SNR per path for MIMO systems utilizing beamforming in Rayleigh fading.

Since $\mathbf{V}$ is unitary and $\mathbf{\Lambda} = \mathbf{\Sigma}^H \mathbf{\Sigma}$ is diagonal, (2.18) represents the eigenvalue decomposition of $\mathbf{H}^H\mathbf{H}$, that is, the ith column of $\mathbf{V}$ is its normalized eigenvector that corresponds to its eigenvalue λ_i, $i = 1, 2, \ldots, N_\mathrm{t}$. Note that $\lambda_i = {\sigma_i}^2$ if $1 \le i \le r$, while $\lambda_i = 0$ if $r + 1 \le i \le N_\mathrm{t}$. Due to (2.16), the solution to (2.12) is the first column of $\mathbf{V}$, i.e.,

$$\mathbf{w}_\mathrm{opt} = \mathbf{v}_1. \tag{2.19}$$

Subsequently, by substituting (2.19) into (2.11), the minimum probability of error (that is, the probability of error of the optimal weight vector in (2.19)) becomes

$$\begin{aligned} P_\mathrm{e} &= Q\left(\sqrt{2\|\mathbf{H}\mathbf{w}_\mathrm{opt}\|^2 \frac{A^2}{\sigma^2}}\right) = Q\left(\sqrt{2\|\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H\mathbf{v}_1\|^2 \frac{A^2}{\sigma^2}}\right) = Q\left(\sqrt{2\|\mathbf{u}_1\sigma_1\|^2 \frac{A^2}{\sigma^2}}\right) \\ &= Q\left(\sqrt{2(\mathbf{u}_1\sigma_1)^H(\mathbf{u}_1\sigma_1)\frac{A^2}{\sigma^2}}\right) = Q\left(\sqrt{2\sigma_1\mathbf{u}_1^H\mathbf{u}_1\sigma_1\frac{A^2}{\sigma^2}}\right) = Q\left(\sqrt{2{\sigma_1}^2\frac{A^2}{\sigma^2}}\right). \end{aligned} \tag{2.20}$$

As an illustration, in Fig. 2.5, we present the average probability of error versus the receive signal-to-noise ratio (SNR) per path for $2\times2, 2\times3, 3\times3, 4\times4$, and 5×5 MIMO systems. The average probability of error is obtained over 10^6 channel realizations that follow the Rayleigh distribution. We observe that, while the SNR is increasing, the average probability of error is decreasing. We also observe that the average probability of error

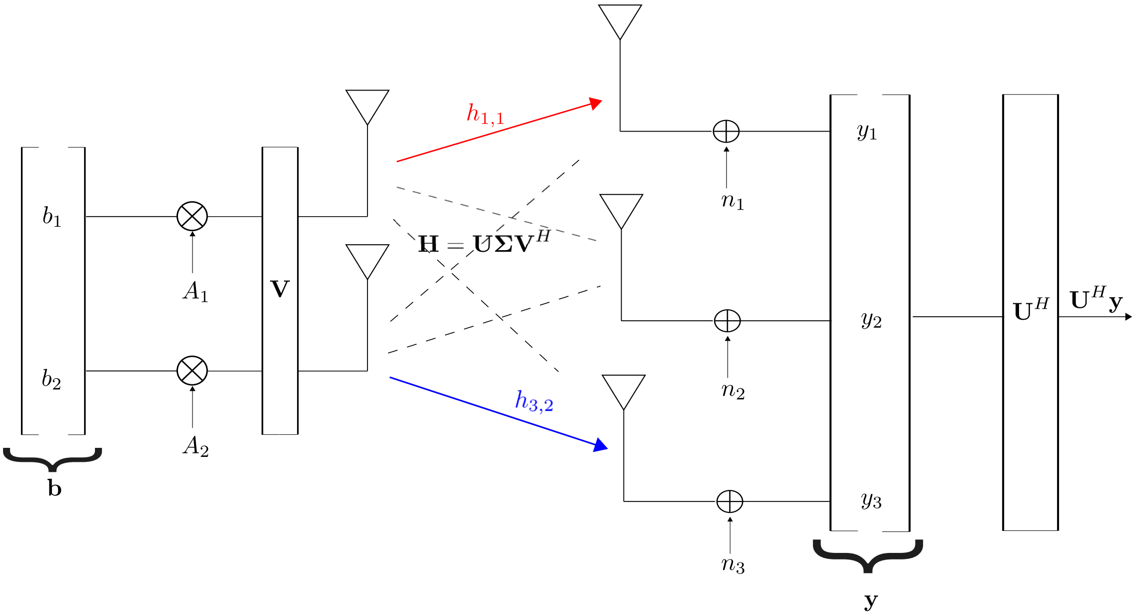

Figure 2.6: MIMO 2×3 system utilizing multiplexing.

decreases with the total number of receive and transmit antennas.

2.2.2 MIMO-multiplexing (closed-loop)

In MIMO-multiplexing, we consider a MIMO system and use it to transmit a vector $\mathbf{b}$ of information symbols. The transmitted signal is $\mathbf{Ab}$, where $\mathbf{A}$ is a $N_\mathrm{t} \times N_\mathrm{t}$ matrix that contains the signal amplitudes and $\mathbf{b}$ is a $N_\mathrm{t} \times 1$ symbol vector. In Fig. 2.6, we present such a MIMO system with $N_\mathrm{t} = 2$ antennas at the transmitter and $N_\mathrm{r} = 3$ antennas at the receiver.

Assuming flat-fading propagation, the received signal is

$$\mathbf{y} = \mathbf{HAb} + \mathbf{n} = \mathbf{U\Sigma V}^H\mathbf{Ab} + \mathbf{n} \tag{2.21}$$

where $\mathbf{H} = \mathbf{U\Sigma V}^H$ (through SVD) is the channel matrix that contains the channel coefficients from the N_t transmit antennas to the N_r receive antennas and $\mathbf{n} \sim \mathbb{CN}(\mathbf{0}, \sigma^2\mathbf{I}_{N_\mathrm{r}})$ is an additive noise vector that follows the white (in time and space) complex Gaussian vector distribution with mean zero and variance σ^2.

In closed-loop MIMO multiplexing, $\mathbf{H}$ (hence, $\mathbf{V}$) is available at the transmitter. Hence the transmitter w.l.o.g can pre-process $\mathbf{Ab}$ by $\mathbf{V}$ (i.e., it can transmit $\mathbf{VAb}$ instead of $\mathbf{Ab}$). Then, the received signal becomes

$$\mathbf{y} = \mathbf{U\Sigma V}^H\mathbf{VAb} + \mathbf{n} = \mathbf{U\Sigma Ab} + \mathbf{n}. \tag{2.22}$$

Similarly, at the receiver we w.l.o.g can post-process $\mathbf{y}$ by $\mathbf{U}^H$ to obtain

$$\widetilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y} = \mathbf{U}^H(\mathbf{U\Sigma Ab}) + \mathbf{U}^H\mathbf{n} = \mathbf{\Sigma Ab} + \widetilde{\mathbf{n}} \tag{2.23}$$

where

$$\widetilde{\mathbf{n}} = \mathbf{U}^H\mathbf{n} \sim \mathbb{CN}(\mathbf{0}, \sigma^2\mathbf{I}_{N_\mathrm{r}}). \tag{2.24}$$

Since the $N_\mathrm{r} \times N_\mathrm{t}$ matrix $\mathbf{\Sigma}$ has r nonzero diagonal elements, according to (2.23) we obtain r parallel independent complex Gaussian channels. For any $i = 1, 2, \ldots, r$, the ith channel is a SISO one with received symbol given by

$$\widetilde{y}_i = A_i\sigma_i b_i + \widetilde{n}_i \tag{2.25}$$

where A_i is the amplitude that the transmitter uses for symbol b_i, σ_i is the ith singular value of $\mathbf{H}$, and $\widetilde{n}_i \sim \mathbb{CN}(0, \sigma^2)$.

The significance of the above is that it shows that, by appropriate pre-processing at the transmitter and post-processing at the receiver, it is possible to decompose a MIMO channel into r SISO channels (each of which consists of a scaled version of the transmitted symbol plus noise), hence we can transmit up to r symbols simultaneously.

The overall capacity of the MIMO system is the sum of the capacities of the individual channels, that is,

$$C = \sum_{i=1}^{r} \log_2\left(1 + \frac{p_i\sigma_i^2}{\sigma^2}\right) \qquad \text{bits/symbol} \tag{2.26}$$

where $p_i = {A_i}^2$ is the power allocated to the ith parallel channel (assuming $E\{|b_i|^2\} = 1$), $i = 1, 2, \ldots, r$. If we are allowed to optimize the allocation of the total power $P = p_1 + p_2 + \cdots + p_r$ among the r symbols, then it can be proven that the optimal power vector is

$$\mathbf{p} = \begin{bmatrix} \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_1^2}\right)^+ \\ \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_2^2}\right)^+ \\ \vdots \\ \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_r^2}\right)^+ \end{bmatrix} \tag{2.27}$$

subject to the power constraint

$$\sum_{i=1}^{r} p_i = P, \qquad p_i \geqslant 0, \qquad i = 1, \ldots, r. \tag{2.28}$$

To find the parameter $\frac{1}{\lambda}$, we use the first power constraint of (2.28) to obtain

$$\frac{1}{\lambda} = \frac{P + \frac{\sigma^2}{\sigma_1^2} + \frac{\sigma^2}{\sigma_1^2} + \cdots \frac{\sigma^2}{\sigma_r^2}}{r} \tag{2.29}$$

and substitute the solution of (2.29) into the power vector of (2.27).If any of the power

Algorithm 1 Waterfilling

procedure Waterfilling

Input: $\mathbf{n} \leftarrow \frac{\sigma^2}{\sigma_1^2}, \frac{\sigma^2}{\sigma_2^2}, \cdots, \frac{\sigma^2}{\sigma_r^2}, \quad P_{\text{total}} \leftarrow P$

$S \leftarrow$ sum of all elements in $\mathbf{n}$

$\frac{1}{\lambda} \leftarrow \frac{P_{\text{total}} + S}{r}$

$\mathbf{p} \leftarrow \left(\frac{1}{\lambda} - \mathbf{n}\right)^+$

Loop: check for zero elements in $\mathbf{p}$

If true

set elements in $\mathbf{n}$ corresponding to zero power equal to zero

$Z \leftarrow$ number of zero elements in $\mathbf{n}$

$S \leftarrow$ sum of all elements in $\mathbf{n}$

$\frac{1}{\lambda} \leftarrow \frac{P_{\text{total}} + S}{r - Z}$

$\mathbf{p} \leftarrow \left(\frac{1}{\lambda} - \mathbf{n}\right)^+$

goto line 6

else

Output: $\frac{1}{\lambda}$

elements is negative and -due to the $()^+$ operation- turns to be zero, we ignore it and repeat from the beginning the allocation process for only the remaining channels until no power element is negative before the $()^+$ operation. This is a repetitive process called waterfilling.

By implementing the above waterfilling algorithm, we get the final value of $\frac{1}{\lambda}$ and, by substituting it in (2.27), we calculate the optimal power vector. In general, the transmitter allocates more power to the stronger parallel channels, taking advantage of the better channel conditions, and less or even no power to the weaker ones. Then, we use the optimal power vector in (2.26) to calculate the system capacity.

In Fig. 2.8, we present an example of the implementation of the waterfilling algorithm, assuming that we have $r = 3$ parallel channels in which we want to allocate a given power. If P units of water are filled into the vessel, $\frac{1}{\lambda}$ is the height of the water surface. Note that there is a channel 3 where the bottom of the vessel is above the water and no power is allocated to the channel and eventually $P_3 = 0$. This means that channel 3 is too poor for it to be worthwhile to transmit information.

As an illustration, in Fig. 2.7, we present the average closed-loop capacity versus the receive signal-to-noise ratio (SNR) per path for $2 \times 2, 2 \times 3, 3 \times 3, 4 \times 4$, and 5×5 MIMO system using the waterfilling algorithm. The average capacity is obtained over 10^6 channel realizations that follow the Rayleigh distribution. We observe that, while the SNR is in-

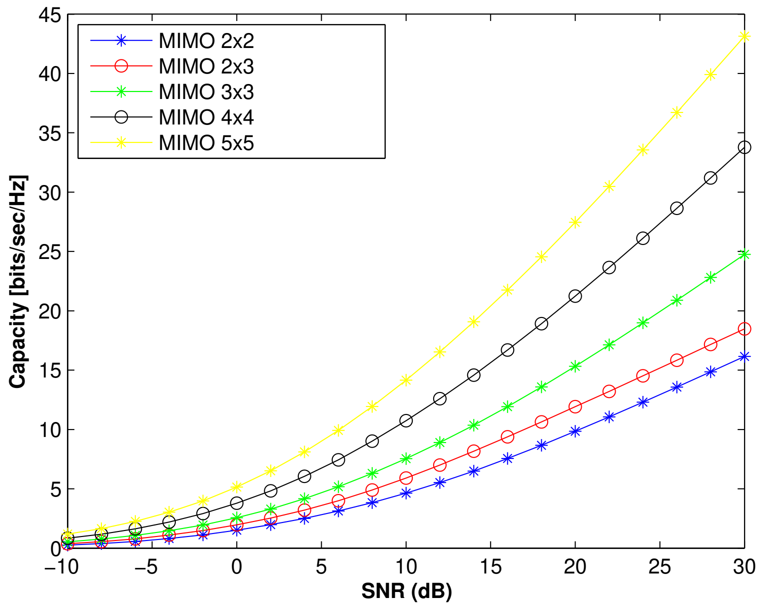

Figure 2.7: Average Capacity versus receive SNR per path for MIMO systems utilizing closed-loop multiplexing in Rayleigh fading.

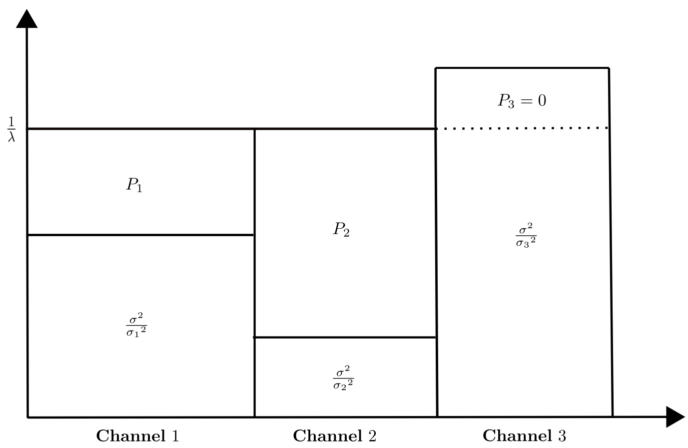

Figure 2.8: Waterfilling for parallel channels.

creasing, the average capacity is increasing too. We also observe that the average capacity increases with the total number of the receive and transmit antennas and is higher than the average capacity gained without using the waterfilling algorithm.

2.2.3 MIMO-multiplexing (open-loop)

In open-loop MIMO-multiplexing, the channel matrix $\mathbf{H}$ is not available at the transmitter. In such a case, it can be proven that, for a specific power allocation $p_1, p_2, \ldots, p_t$ among the N_t transmit antennas, the system capacity is

$$C = \log_2 \left| \mathbf{I}_{N_r} + \frac{1}{\sigma^2} \mathbf{H} \begin{bmatrix} p_1 & & 0 \\ & \ddots & \\ 0 & & p_{N_t} \end{bmatrix} \mathbf{H}^H \right| \qquad \text{bits/symbol} \tag{2.30}$$

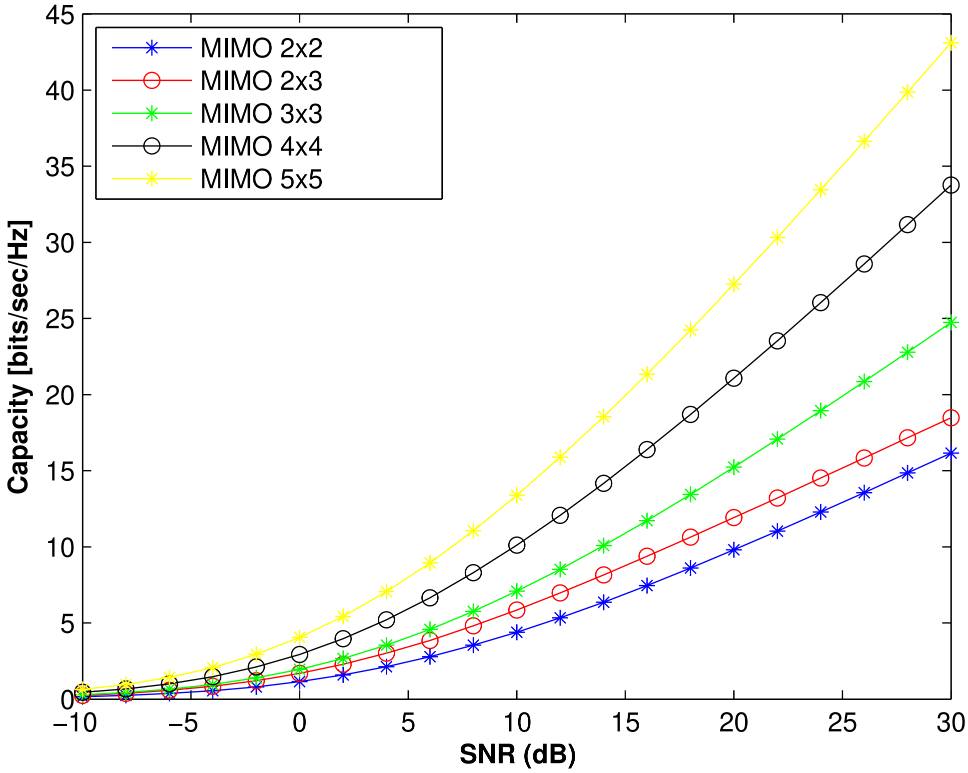

Figure 2.9: Average Capacity versus receive SNR per path for MIMO systems utilizing open-loop multiplexing in Rayleigh fading.

where p_i is the power allocated to the ith transmit antenna, $i = 1, 2, \ldots, N_t$. If we are allowed to allocate the total transmit power $P = p_1 + p_2 + \ldots + p_{N_t}$ among the N_t transmit antennas, then it can be proven that the optimal allocation is $p_1 = p_2 = \cdots = p_{N_t} = \dfrac{P}{N_t}$ and the capacity is

$$C = \log_2 \left| \mathbf{I}_{N_r} + \frac{P}{N_t \sigma^2} \mathbf{H}\mathbf{H}^H \right| \qquad \text{bits/symbol.} \tag{2.31}$$

We note that

$$\frac{1}{N_\mathrm{t}} \mathbf{H}\mathbf{H}^H \to \mathbf{I}_{N_\mathrm{r}} \tag{2.32}$$

as N_t gets large and N_r is fixed. That is, in the limit of large N_t, the capacity is

$$C = N_r \log_2 \left(1 + \frac{P}{\sigma^2} \right) \qquad \text{bits/symbol} \tag{2.33}$$

where the term $\log_2 \left(1 + \dfrac{P}{\sigma^2} \right)$ is the SISO system capacity.

As an illustration, in Fig. 2.9, we present the average open-loop capacity versus the receive signal-to-noise ratio (SNR) per path for $2 \times 2, 2 \times 3, 3 \times 3, 4 \times 4$, and 5×5 MIMO systems. The average capacity is obtained over 10^6 channel realizations that follow the Rayleigh distribution. We observe that, while the SNR is increasing, the average capacity is increasing too. We also observe that the average capacity increases with the total number of the receive and transmit antennas.

Chapter 3

Transmit Antenna Selection

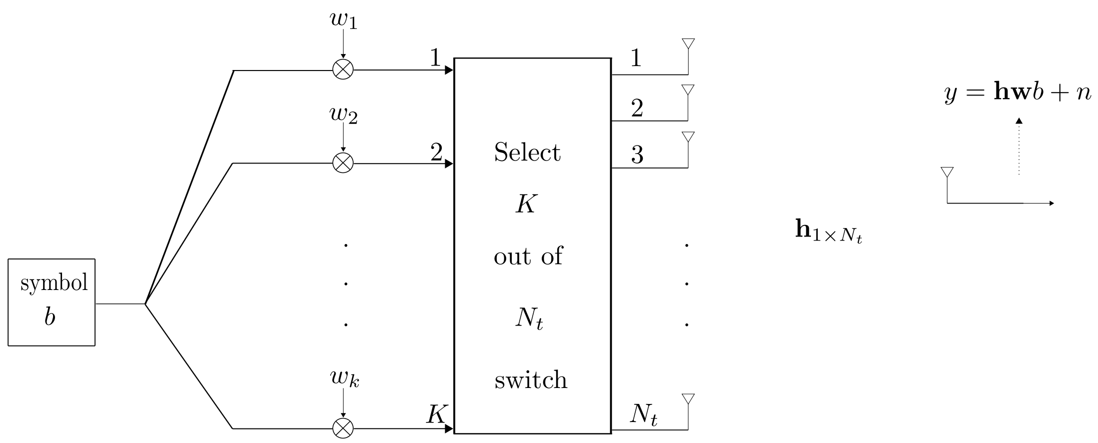

Figure 3.1: MISO system with transmit antenna selection.

3.1 Transmit Antenna Selection - One receive antenna

We consider the straightforward case of a MISO system with $N_\mathrm{r} = 1$ receive antenna and N_t transmit antennas among which we wish to select K transmit antennas. Such a system is presented in Fig. 3.1.

The transmitted symbol is $b \in \mathbb{C}$ and, if all N_t transmit antennas are used, the received signal is

$$y = \mathbf{h}\mathbf{w}b + n \tag{3.1}$$

where $\mathbf{w} \in \mathbb{C}^{N_\mathrm{t}}$ is a $N_\mathrm{t} \times 1$ beamforming vector, $\mathbf{h} \in \mathbb{C}^{N_\mathrm{t}}$ is a $1 \times N_\mathrm{t}$ complex vector that contains the channel coefficients from the transmit antennas to the $N_\mathrm{r} = 1$ receive antenna, and $n \sim \mathbb{CN}(0, \sigma^2)$ represents additive white (in time) complex Gaussian noise.

We select K out of N_t available transmit antennas by selecting the K indices corresponding to the selected antennas. Then, the received signal is

$$\widetilde{y} = \widetilde{\mathbf{h}}\widetilde{\mathbf{w}}b + n \tag{3.2}$$

where $\widetilde{\mathbf{h}}$ has size $1 \times K$ and $\widetilde{\mathbf{w}}$ has size $K \times 1$.

Before the antenna selection, the optimal detector is

$$\mathrm{Re}\left((\mathbf{h}\mathbf{w})^* y\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{3.3}$$

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2|\mathbf{h}\mathbf{w}|^2 \frac{A^2}{\sigma^2}}\right). \tag{3.4}$$

After K antenna elements are selected, the optimal detector is

$$\mathrm{Re}\left((\widetilde{\mathbf{h}}\widetilde{\mathbf{w}})^* \widetilde{y}\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{3.5}$$

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2|\widetilde{\mathbf{h}}\widetilde{\mathbf{w}}|^2 \frac{A^2}{\sigma^2}}\right). \tag{3.6}$$

In Chapter 2, we proved that for such a signal model the optimum $\mathbf{w}$ that minimizes the probability of error is $\mathbf{w}_{\mathrm{opt}} = \dfrac{\mathbf{h}^H}{\|\mathbf{h}\|}$. Therefore, after antenna selection, the optimum beamformer becomes

$$\widetilde{\mathbf{w}}_{\mathrm{opt}} = \frac{\widetilde{\mathbf{h}}^H}{\|\widetilde{\mathbf{h}}\|}. \tag{3.7}$$

Subtituting (3.7) into (3.6), we obtain the minimum probability of error for a fixed set of K selected antennas which is equal to

$$P_{\mathrm{e}} = Q\left(\sqrt{2\|\widetilde{\mathbf{h}}\|^2 \frac{A^2}{\sigma^2}}\right). \tag{3.8}$$

Based on the above, we conclude that the optimal set of selected antennas that minimizes the probability of error is the one that consists of the indices that correspond to the K largest (in magnitude) elements of $\mathbf{h}$.

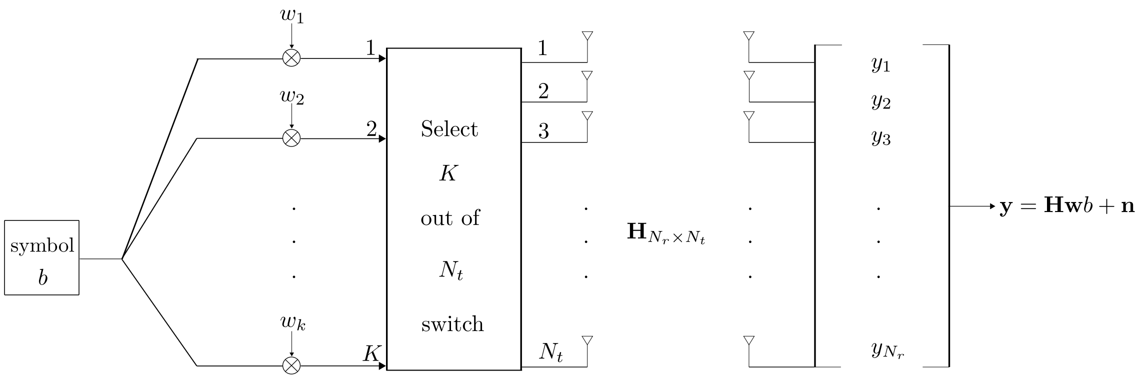

Figure 3.2: MIMO system with transmit antenna selection.

3.2 Transmit Antenna Selection - N_{r} receive antennas

We consider the case of a MIMO system with N_{t} transmit antennas and N_{r} receive antennas. Such a system is presented in Fig. 3.2.

The transmitted symbol is $b \in \mathbb{C}$ and, if all N_{t} transmit antennas are used, the received signal is

$$\mathbf{y} = \mathbf{H}\mathbf{w}b + \mathbf{n} \tag{3.9}$$

where $\mathbf{w} \in \mathbb{C}^{N_{\mathrm{t}}}$ is a $N_{\mathrm{t}} \times 1$ beamforming vector and $\mathbf{n} \sim \mathbb{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_{\mathrm{r}}})$ is an additive noise vector that follows the white (in time and space) complex Gaussian vector distribution with mean zero and variance σ^2. Furthermore, $\mathbf{H}$ is a $N_{\mathrm{r}} \times N_{\mathrm{t}}$ complex matrix that contains the channel coefficients from the N_{t} transmit antennas to the N_{r} receive antennas.

Before the antenna selection, the optimal detector is

$$\mathrm{Re}\left((\mathbf{H}\mathbf{w})^H \mathbf{y}\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{3.10}$$

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2\|\mathbf{H}\mathbf{w}\|^2 \frac{A^2}{\sigma^2}}\right). \tag{3.11}$$

After K antenna columns from matrix $\mathbf{H}$ are selected, a $\mathbf{H}_{N_{\mathrm{r}} \times K}$ is formed and the received signal is

$$\widetilde{\mathbf{y}} = \widetilde{\mathbf{H}}\widetilde{\mathbf{w}}b + \mathbf{n} \tag{3.12}$$

where $\widetilde{\mathbf{w}}$ is a $K \times 1$ beamformer that operates on the K selected transmit antennas. Then, the optimal detector is

$$\mathrm{Re}\left((\widetilde{\mathbf{H}}\widetilde{\mathbf{w}})^H \mathbf{y}\right) \underset{\hat{b}=-1}{\overset{\hat{b}=1}{\gtrless}} 0 \tag{3.13}$$

and its probability of error is

$$P_{\mathrm{e}} = Q\left(\sqrt{2\left\|\widetilde{\mathbf{H}}\widetilde{\mathbf{w}}\right\|^2 \frac{A^2}{\sigma^2}}\right). \tag{3.14}$$

In Chapter 2, we proved that, without antenna selection, the optimum $\mathbf{w}$ that minimizes the probability of error in (3.11) is $\mathbf{w}_{\mathrm{opt}} = \mathbf{v}_1$ where $\mathbf{v}_1$ is the principal right singular vector of $\mathbf{H}$. Therefore, after antenna selection, the optimum beamformer that minimizes the probability of error in (3.14) becomes

$$\widetilde{\mathbf{w}}_{\mathrm{opt}} = \widetilde{\mathbf{v}}_1 \tag{3.15}$$

where $\widetilde{\mathbf{v}}_1$ is the principal right singular vector of $\widetilde{\mathbf{H}}$. Substituting (3.15) into (3.14), we obtain the minimum probability of error for a fixed set of K selected antennas which is equal to

$$P_{\mathrm{e}} = Q\left(\sqrt{2\widetilde{\sigma_1}^2 \frac{A^2}{\sigma^2}}\right) \tag{3.16}$$

where $\widetilde{\sigma_1}$ is the principal singular value of $\widetilde{\mathbf{H}}$.

Based on the above, the optimal selection of the K transmit antennas that minimizes the probability of error, simplifies to the selection of K columns of $\mathbf{H}$ that form the $N_{\mathrm{r}} \times K$ submatrix with the maximum principal singular value. The indices of the selected columns are the indices of the optimally selected antennas. Then, for these optimal indices, the optimal beamforming vector is given by the principal right singular vector of the selected submatrix.

Chapter 4

MIMObit Simulations

Neben's MIMObit is a software tool that models the propagation of electromagnetic waves under various types of environments in MIMO systems. It is appropriate to analyze MIMO systems and signal processing algorithms.

Using MIMObit, we are able to model the systems introduced in Chapters 2 and 3. The produced channel coefficients vary according to a distribution depending on the propagation environment and the antenna system used in MIMObit.

In this thesis, we simulate the communication between one transmitter and one receiver, each of them being equipped with one or more dipole antennas, based on whether we want to simulate a SIMO, MISO, or MIMO system model. The dipoles used are designed in MIMObit. We also calculate the error probability and the capacity regarding SIMO, MISO, and MIMO systems using the channel coefficients produced by MIMObit and Matlab and compare the results.

4.1 Basics Overview

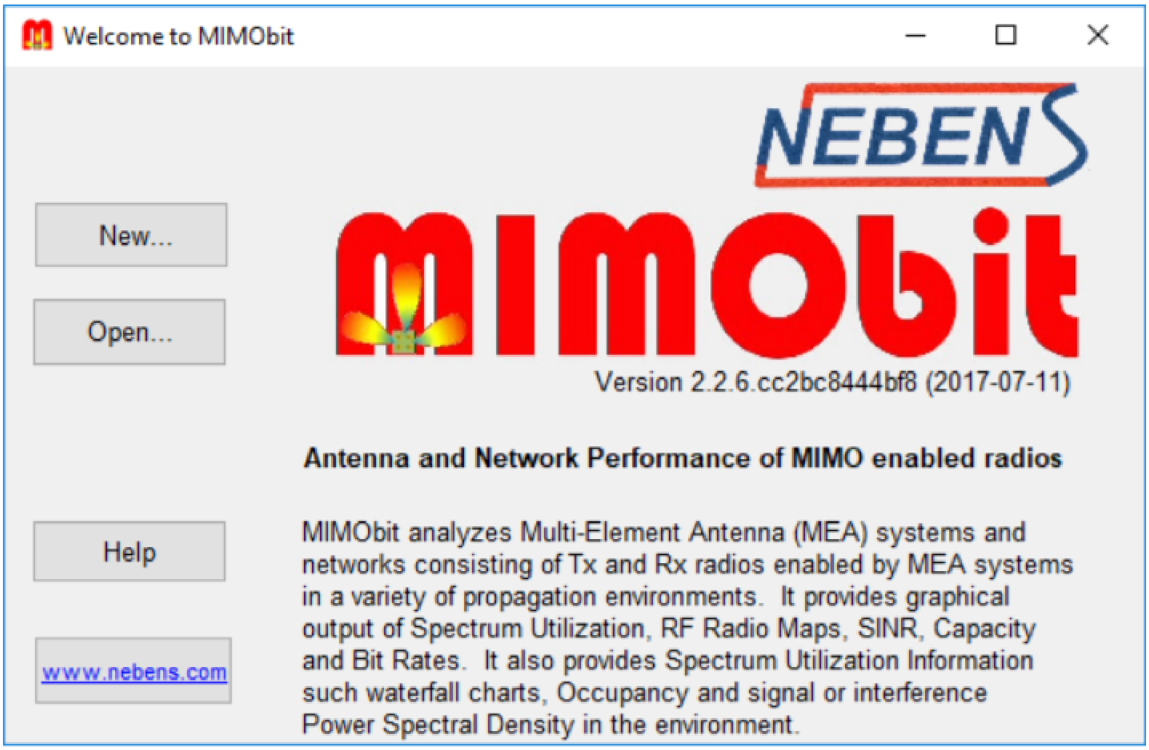

Figure 4.1: MIMObit initial menu.

The first window that appears after MIMObit is launched is illustrated in Fig. 4.1. From this window, the user is able to create a new configuration file or load an existing one. The configuration file is an important component, since it contains the settings of several fundamental parameters that can affect the simulation results.

To run a simulation in MIMObit, we need to define five components.

- **Transmitter**
- **Propagation environment**
- **Receiver**
- **Frequency**
- **Time**

Any parameter that is not mentioned in each of the above components is considered to have its default value. For more details about the functionality of certain parameters, we refer the reader to the MIMObit's manual.

In the window illustrated in Fig. 4.2, we can access and inspect or alter almost every simulation parameter and finally run the simulation. Only the most common parameters appear in this graphical environment. There exist several hidden parameters that can only be edited by opening the configuration file manually in a text editor.

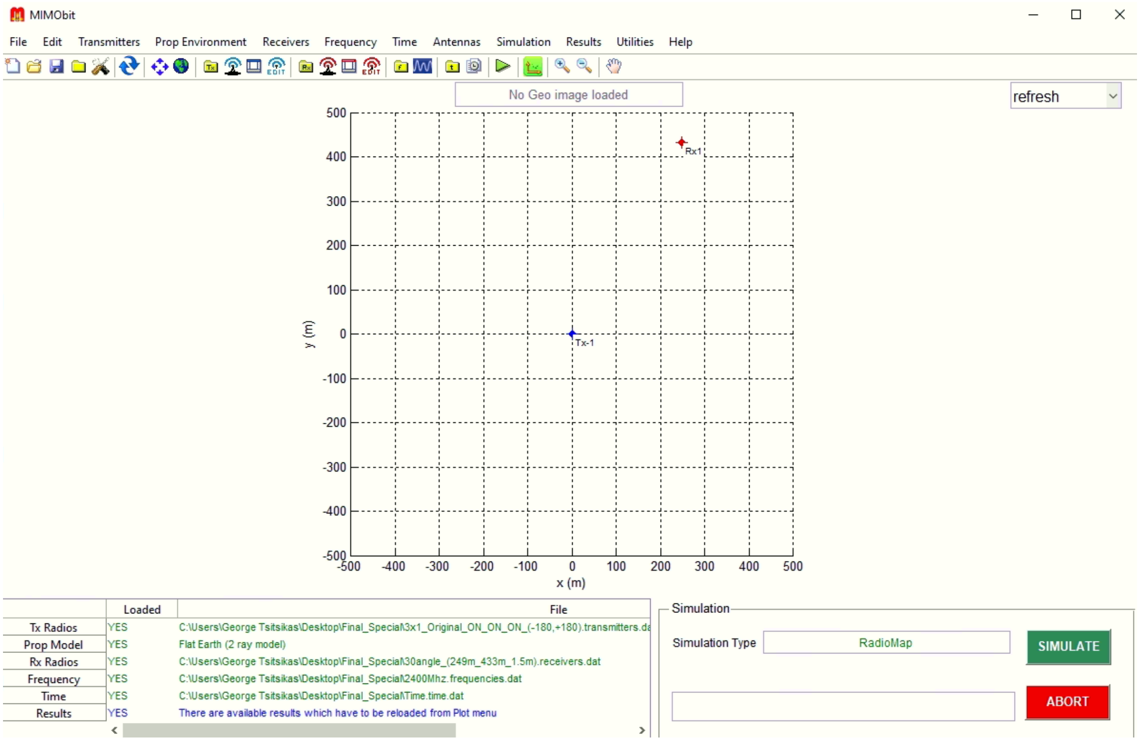

Figure 4.2: MIMObit user interface.

Transmitter

This component, illustrated in Fig. 4.3, refers to the transmitter's properties. Some of these properties include the location of the transmitter in the three-dimensional space, which is specified by the notation (x, y, z). In this notation, x refers to length, y refers to width, and z refers to height in meters. Other properties refer to the the total available power which is distributed among the different antenna ports as well as the bandwidth over which we transmit our signal. In our simulations, the total available power of the transmitted signal is uniformly distributed over the signal bandwidth.

Propagation environment

This component refers to the way that the electromagnetic waves are transmitted through the physical environment until they reach the receiver. MIMObit software includes different propagation environments to choose from. There are available both random and deterministic propagation environments. Random propagation environments produce different channel coefficients every time we run a new simulation under the same communication setup, whereas the deterministic propagation environments produce the same. In our simulations we used the deterministic ones. Those propagation environments are the Line of Sight (LOS) and the Flat Earth (2-Ray) model.

Receiver

This component, illustrated in Fig. 4.4, refers to the receiver's properties. Some of these properties include the location of the receiver in the three dimensional space, which is specified by the notation (x, y, z). In this notation, x refers to length, y refers to width, and z refers to height in meters. Other properties refer to the gain and the resolution bandwidth of the receive antennas.

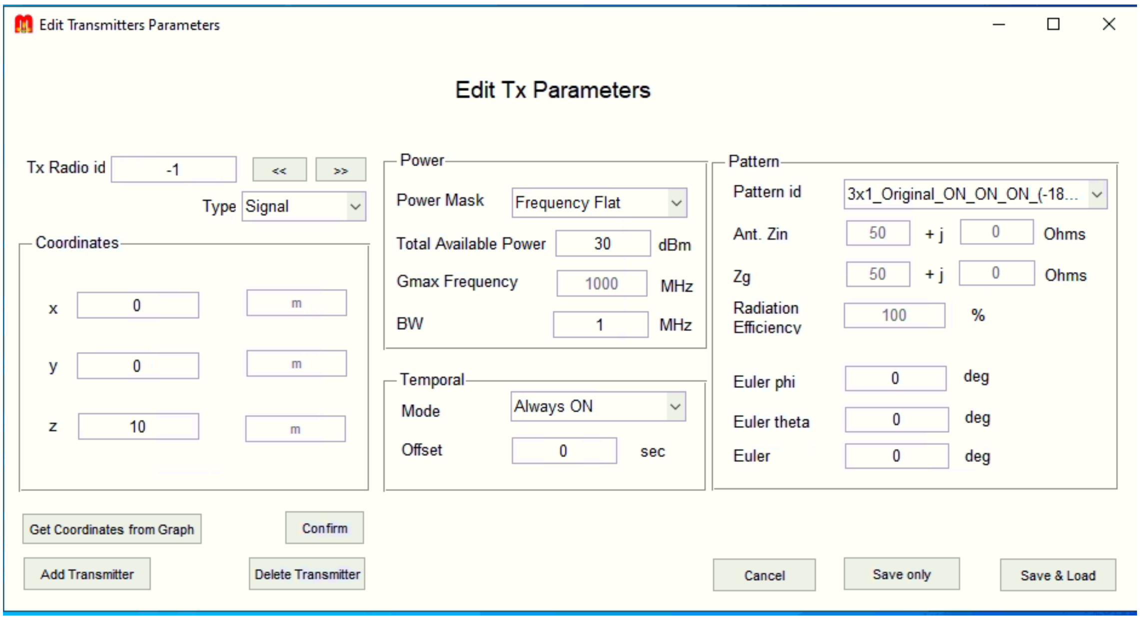

Figure 4.3: MIMObit transmitter setup.

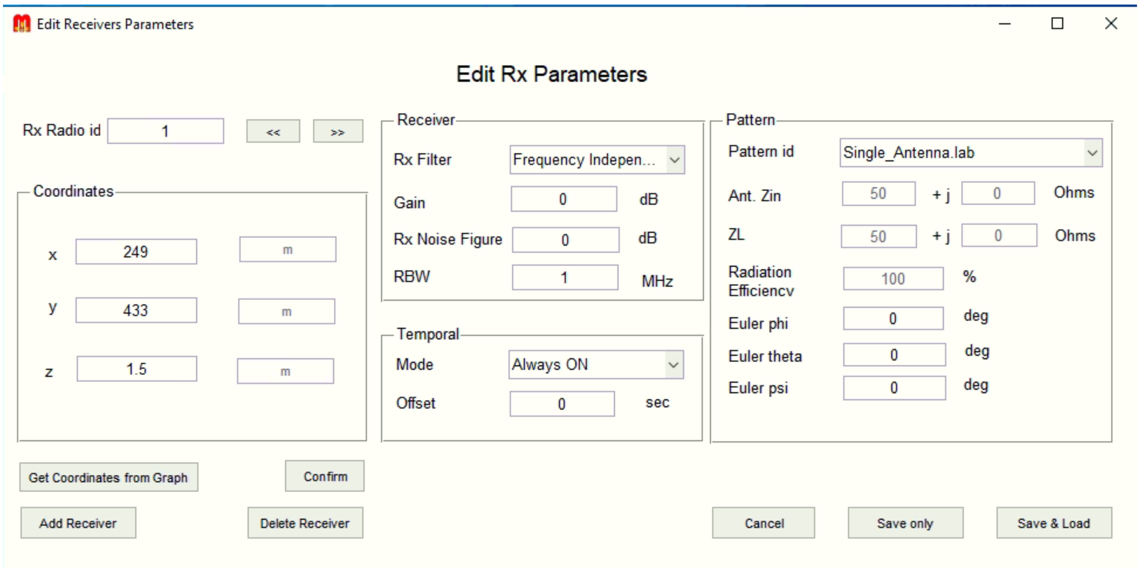

Figure 4.4: MIMObit receiver setup.

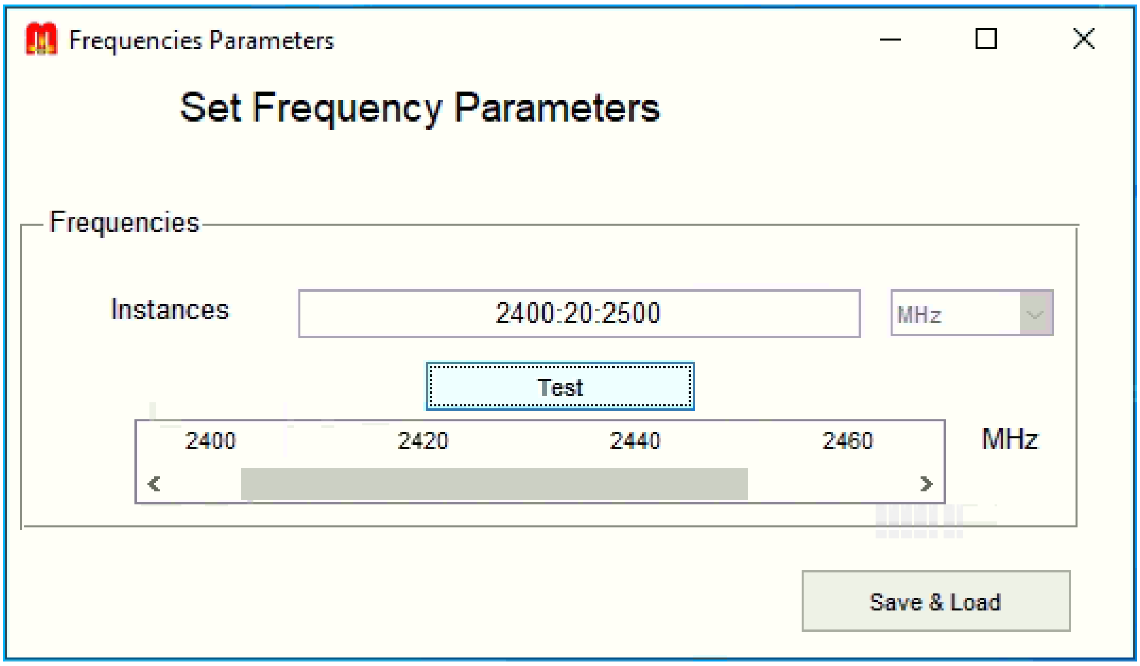

Figure 4.5: MIMObit frequency setup.

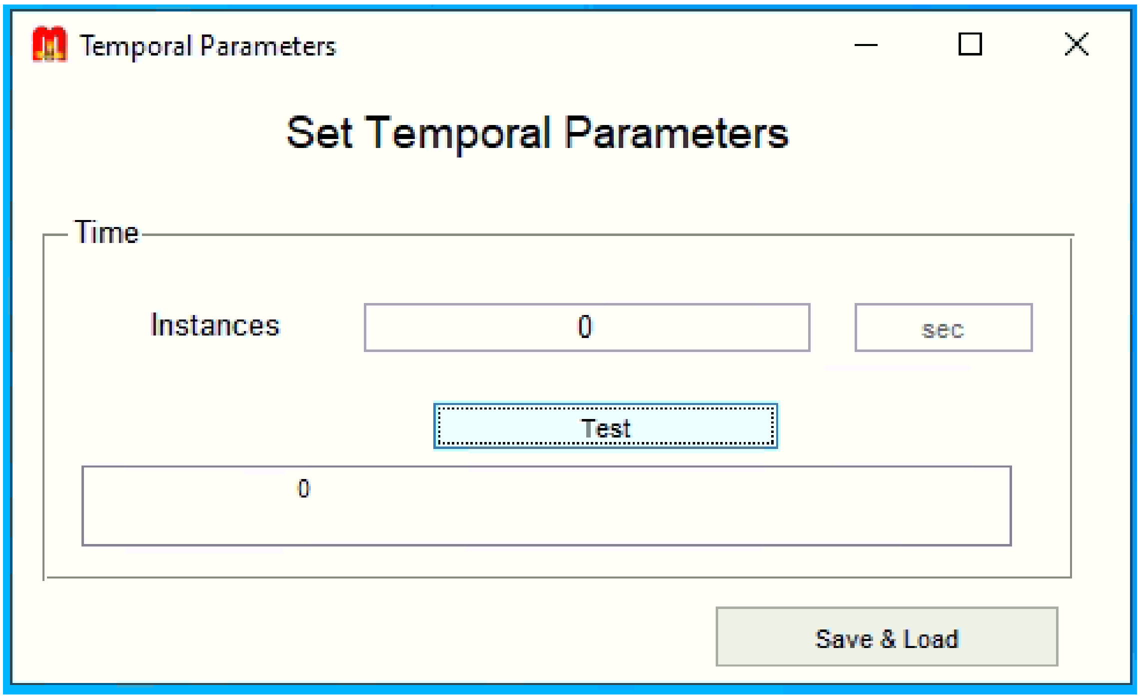

Figure 4.6: MIMObit time setup.

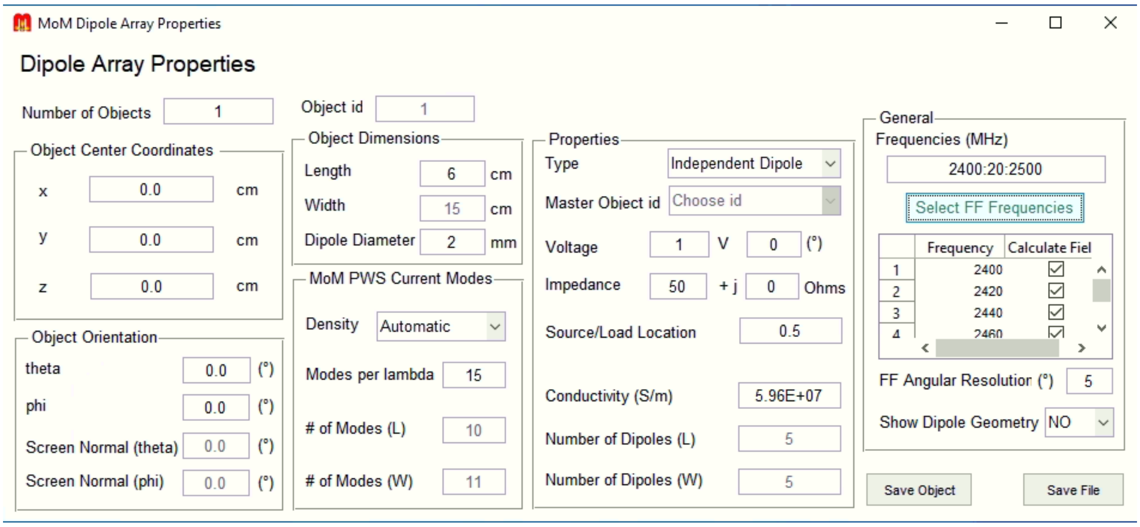

Figure 4.7: MIMObit dipole array setup.

Frequency

This component, illustrated in Fig. 4.5, refers to the specific frequencies at which we simulate the communication system.

Time

This component, illustrated in Fig. 4.6, refers to the ability of the user to make modifications on the temporal behavior of any Tx or Rx radio. For our simulations, we used the value 0.

4.1.1 Dipole Antennas

To be able to simulate communication systems in MIMObit, we need to design custom dipoles and use them at the transmitter and the receiver. The process of creating custom dipoles is illustrated in Fig. 4.7, where we choose the number of dipoles of our transmitter or receiver as well as their exact location. The property referring to dipole's length specifies the frequency at which the dipole is operating. In other words, keeping in mind that

$$c = 3 \cdot 10^8 \text{m/s} \tag{4.1}$$

by using the fundamental equation

$$c = \lambda f, \tag{4.2}$$

where c is the speed of light and λ is the wavelength, we can specify the operating frequency at which we get the maximum radiated power. Also, since we are using dipoles, the variable "Length" in MIMObit's "Dipole Array" component equals to the half of dipole's length. For instance, in order to specify a frequency of 2500Mhz we need $\lambda = 0.12$m, so the Length = 6 cm. By following the above procedure, a ".lab" file is created and then is loaded into the "Pattern id" section of Rx or Tx Parameters of Fig. 4.3 and Fig. 4.4.

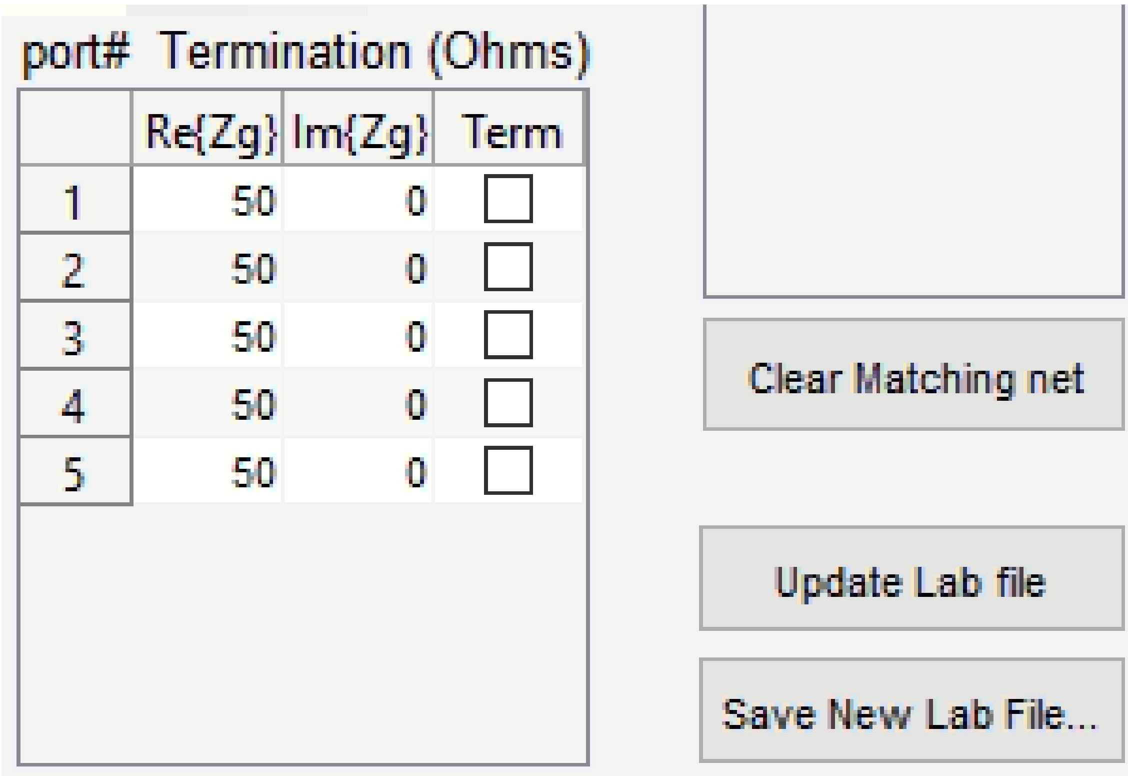

Figure 4.8: MIMObit antenna selection setup.

4.2 Antenna Selection using MIMObit

In order to be able to specify which antennas of our communication system are transmitting each time, we modify the ".lab" file which has been created and loaded into the transmitter's or the receiver's components. As illustrated in Fig. 4.8, we need to change the impedance from 50 Ohms, which is the default impedance value assigned during creating the custom dipoles, to 0 Ohms. In other words, at any port that we want to be terminated, we assign the Re$\{Zg\}$ to zero and then click on the "Term" option. Finally, to update these changes we need to select the "Update Lab file" option.

Our Objective

When all the available transmit antennas are operating, we obtain a channel vector $\mathbf{h}$ or channel matrix $\mathbf{H}$ in the case of a SIMO/MISO or MIMO, respectively, system. Each element of $\mathbf{h}$ or column of $\mathbf{H}$ represents the channel coefficients generated between the corresponding transmit antenna and the receive antenna(s). There are two ways to implement the antenna selection process. The conventional way is to assume that, when some transmit antennas are terminated (i.e., not operating), the channel coefficients that correspond to the remaining antennas (that are still operating) do not change. In other words, we assume that we obtain a sub-vector of the original vector $\mathbf{h}$ or a sub-matrix of the original matrix $\mathbf{H}$ with the same elements or columns which correspond to the antennas that were selected for operation.

However, in reality, the above assumption does not hold true. Depending on the termination load we use for the nonselected antennas, the channel coefficients that correspond to the selected antennas may change. This is an observation we made in MIMObit when we used a termination load different than 50 Ohms.

Hence, a second way to select the transmit antennas is to utilize MIMObit and obtain a solution with lower error probability and higher capacity. Specifically, for each possible combination of selected antennas, we terminate the nonselected ones and repeat the transmission process to obtain the channel vector or matrix that corresponds to this combination. Then, we select the combination with the best performance (error probability or capacity) metric.

In the sequel, we present 14 transmit antenna selection cases which we simulated in MIMObit. In each case, we study how the change of the channel coefficients, during the antenna selection process, affects the performance in terms of BER or capacity based on the theory presented in the previous chapters. In all cases, the termination load for each nonselected antenna was set to 0 Ohm and the carrier frequency was set to 2400 MHz. Moreover, both the transmitter and the receiver were equipped with dipoles of length 6 cm (maximum radiated power at 2500 MHz).

Case 1

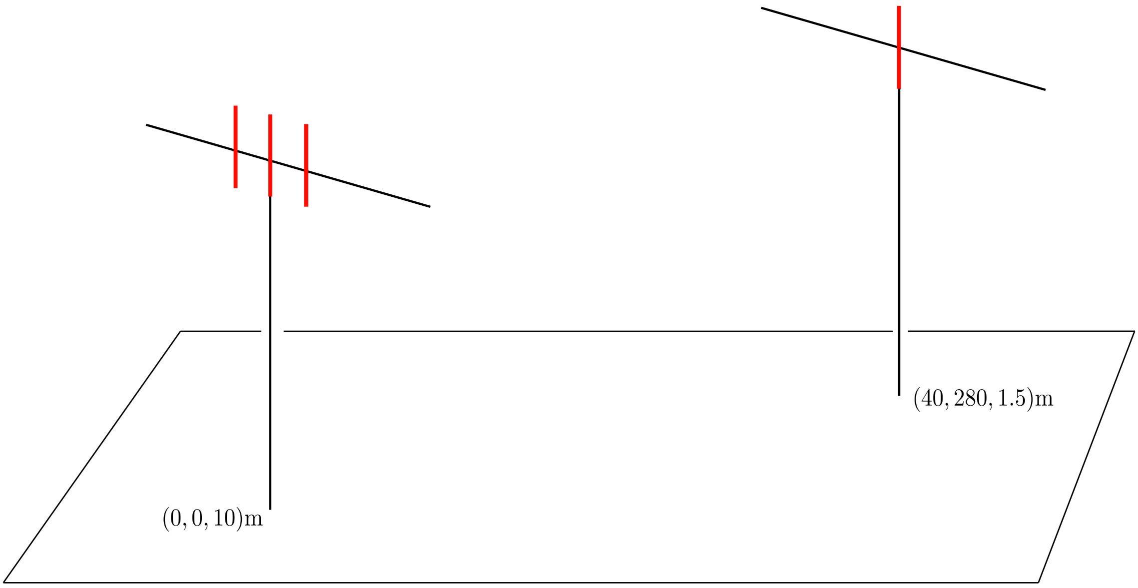

Figure 4.9: MISO 3×1, 8^o angle, 20cm interelement distance.

In the first case, we study a 3×1 MISO system, illustrated in Fig. 4.9, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(40, 280, 1.5)$m. The dipoles of the transmitter are located at $(-20, 0)$cm for Tx1, $(0, 0)$cm for Tx2, and $(+20, 0)$cm for Tx3. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 8 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 1 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

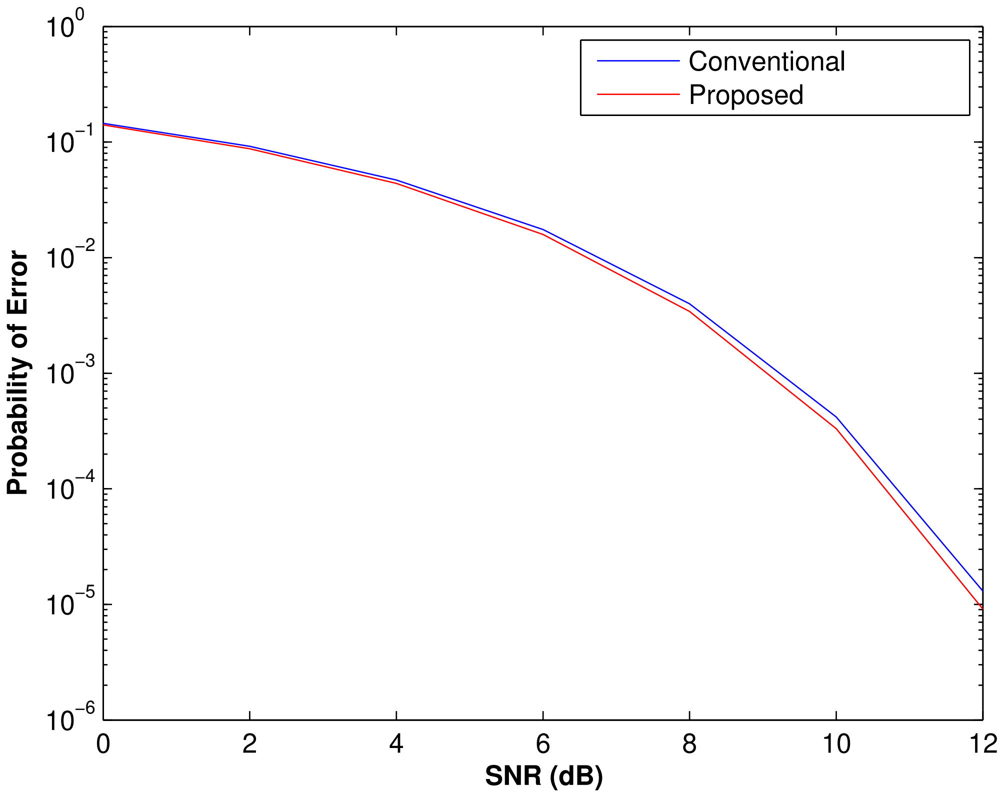

Figure 4.10: Probability of error versus SNR for $K = 1$ selected antenna in a 3×1 MISO system with 8^o angle and 20cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
2	3

.

In the above table, we observe that the conventional method would select Tx2, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx2 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other two are not selected, is Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.10, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 2

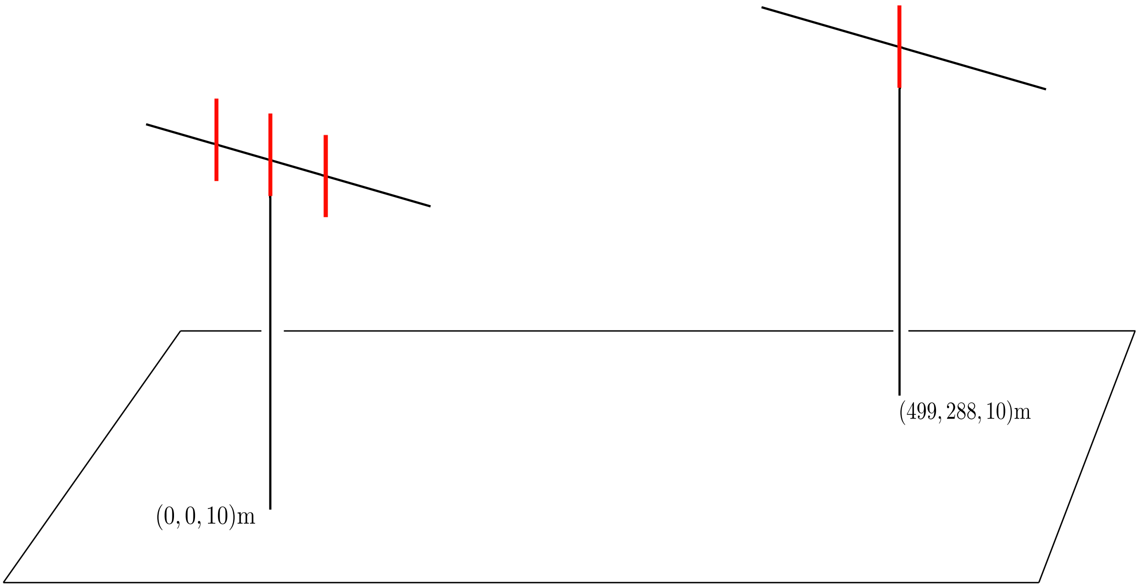

Figure 4.11: MISO 3×1, 60^o angle, 50cm interelement distance.

In this case, we study a 3×1 MISO system, illustrated in Fig. 4.11, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-50, 0)$cm for Tx1, $(0, 0)$cm for Tx2, and $(+50, 0)$cm for Tx3. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 1 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

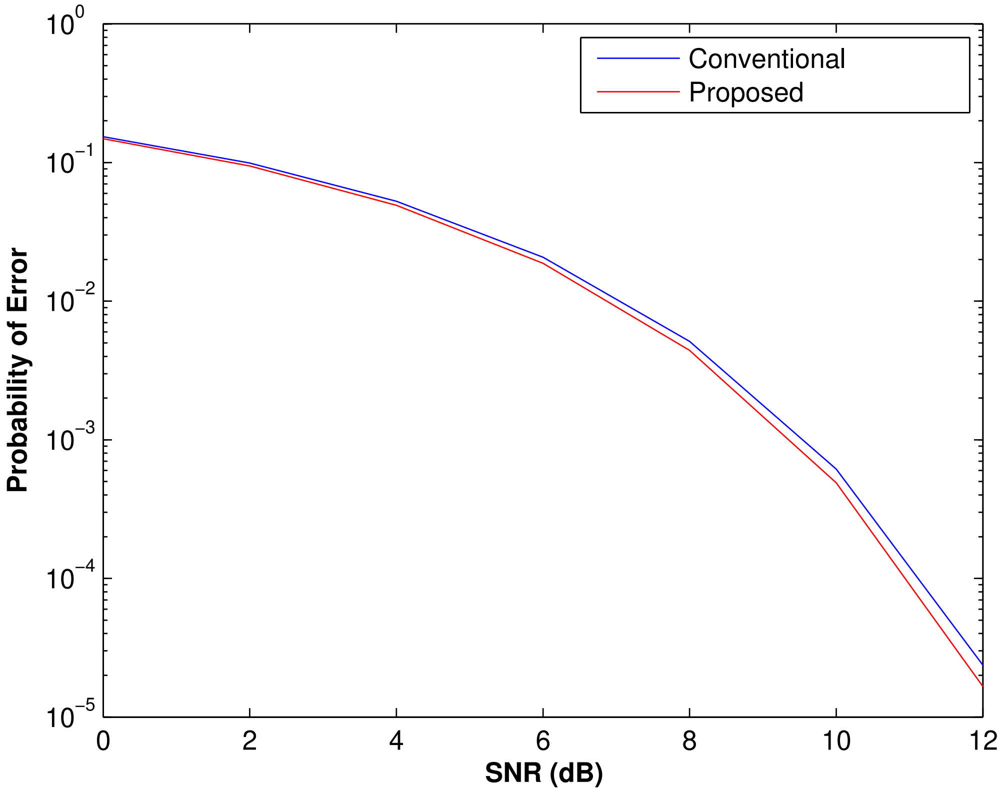

Figure 4.12: Probability of error versus SNR for $K = 1$ selected antenna in a 3×1 MISO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
1	2

.

In the above table, we observe that the conventional method would select Tx1, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx1 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other two are not selected, is Tx2.

The above observation leads to different performance, as illustrated in Fig. 4.12, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 3

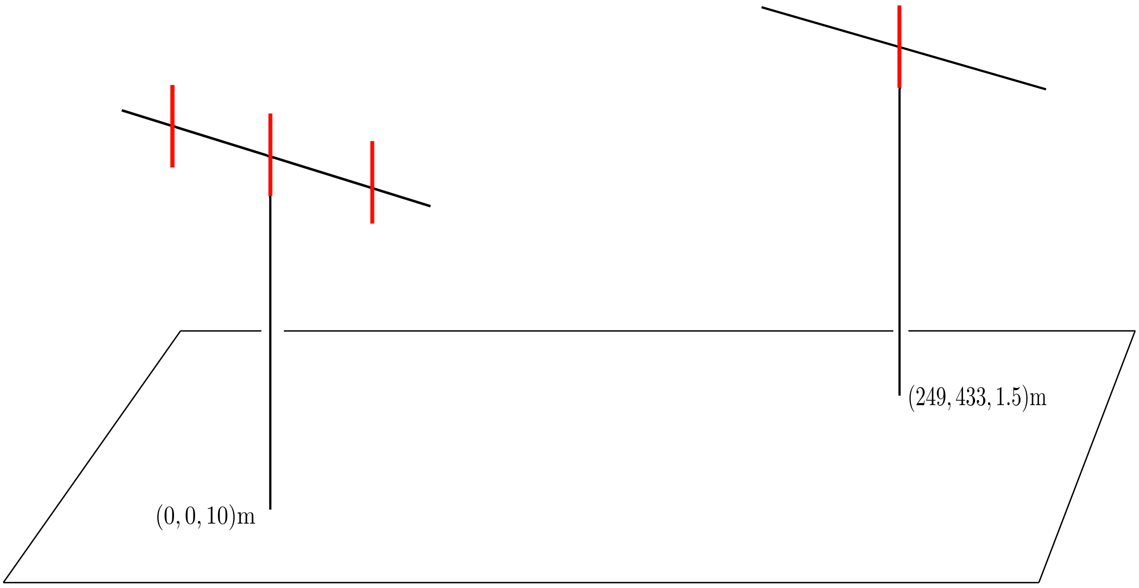

Figure 4.13: MISO 3×1, 30^o angle, 180cm interelement distance.

In this case, we study a 3×1 MISO system, illustrated in Fig. 4.13, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(249, 433, 1.5)$m. The dipoles of the transmitter are located at $(-180, 0)$cm for Tx1, $(0, 0)$cm for Tx2, and $(+180, 0)$cm for Tx3. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 30 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 1 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the LOS model.

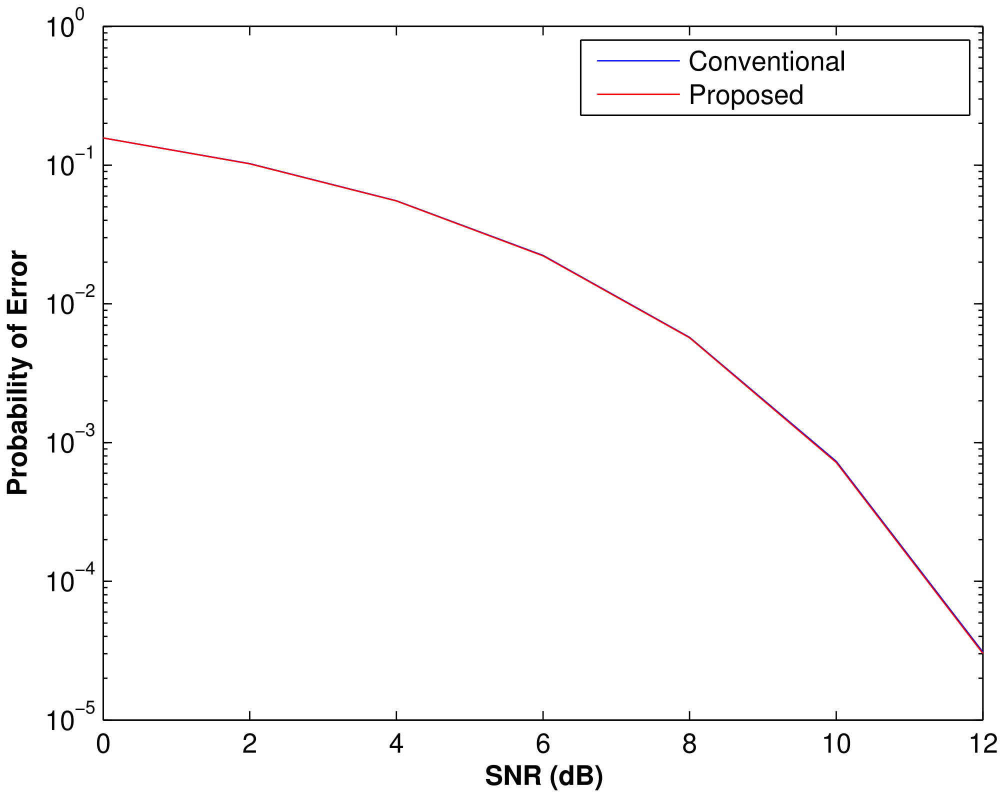

Figure 4.14: Probability of error versus SNR for $K = 1$ selected antenna in a 3×1 MISO system with 30^o angle and 180cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
2	3

.

In the above table, we observe that the conventional method would select Tx2, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx2 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other two are not selected, is Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.14, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 4

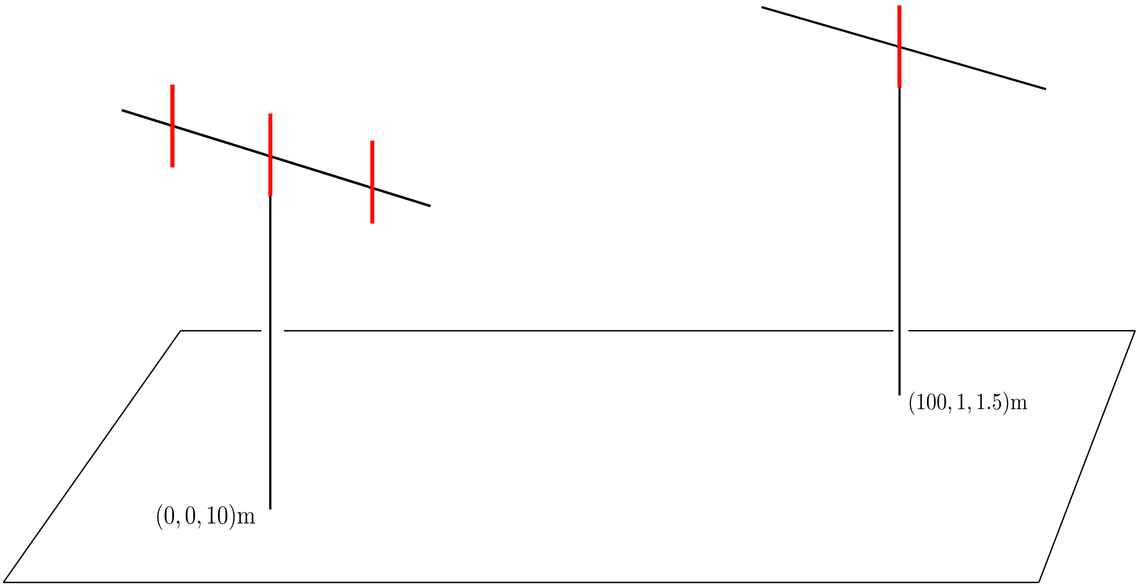

Figure 4.15: MISO 3×1, 1^o angle, 180cm interelement distance.

In this case, we study a 3×1 MISO system, illustrated in Fig. 4.15, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(100, 1, 1.5)$m. The dipoles of the transmitter are located at $(-180, 0)$cm for Tx1, $(0, 0)$cm for Tx2, and $(+180, 0)$cm for Tx3. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 1 degree off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 1 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the LOS model.

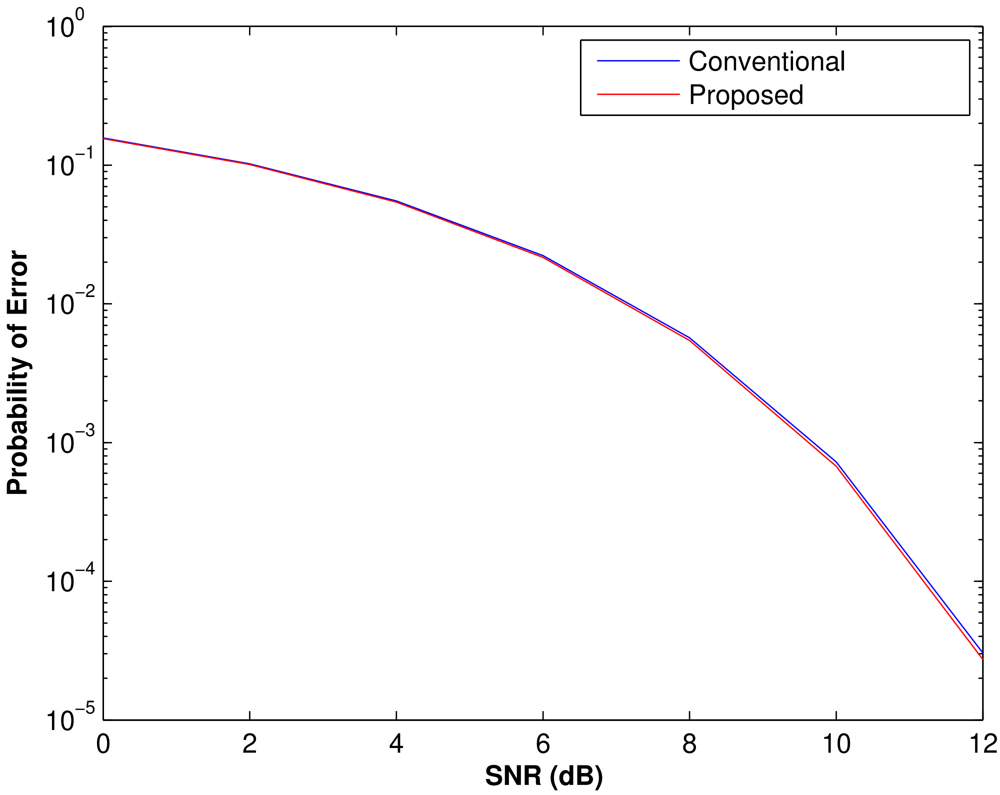

Figure 4.16: Probability of error versus SNR for $K = 1$ selected antenna in a 3×1 MISO system with 1^o angle and 180cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
2	3

.

In the above table, we observe that the conventional method would select Tx2, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx2 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other two are not selected, is Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.16, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 5

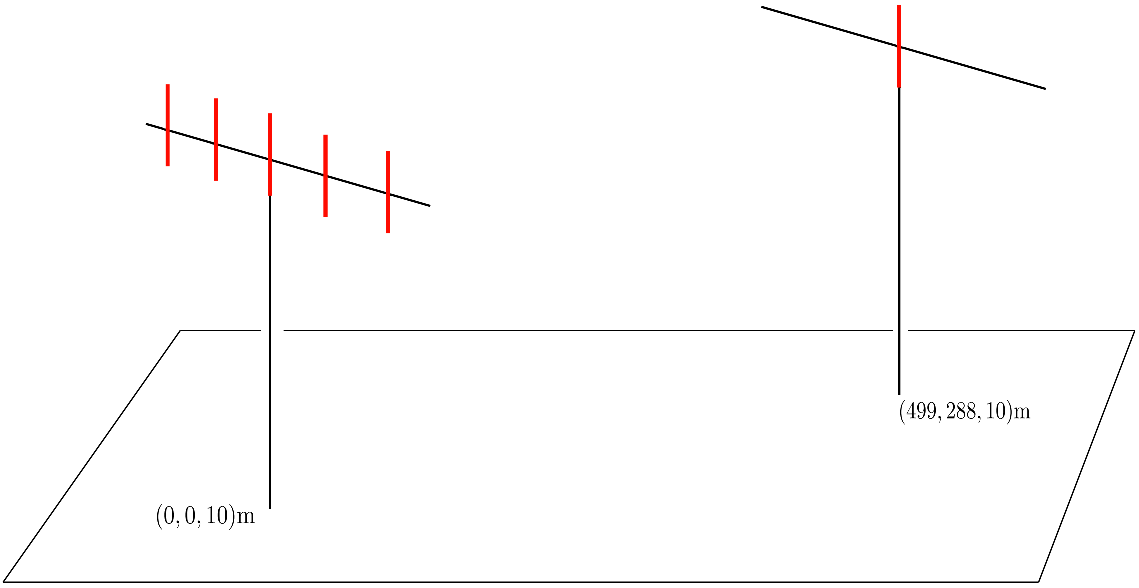

Figure 4.17: MISO 5×1, 60^o angle, 50cm interelement distance.

In this case, we study a 5×1 MISO system, illustrated in Fig. 4.17, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our first objective is to select 1 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other four antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

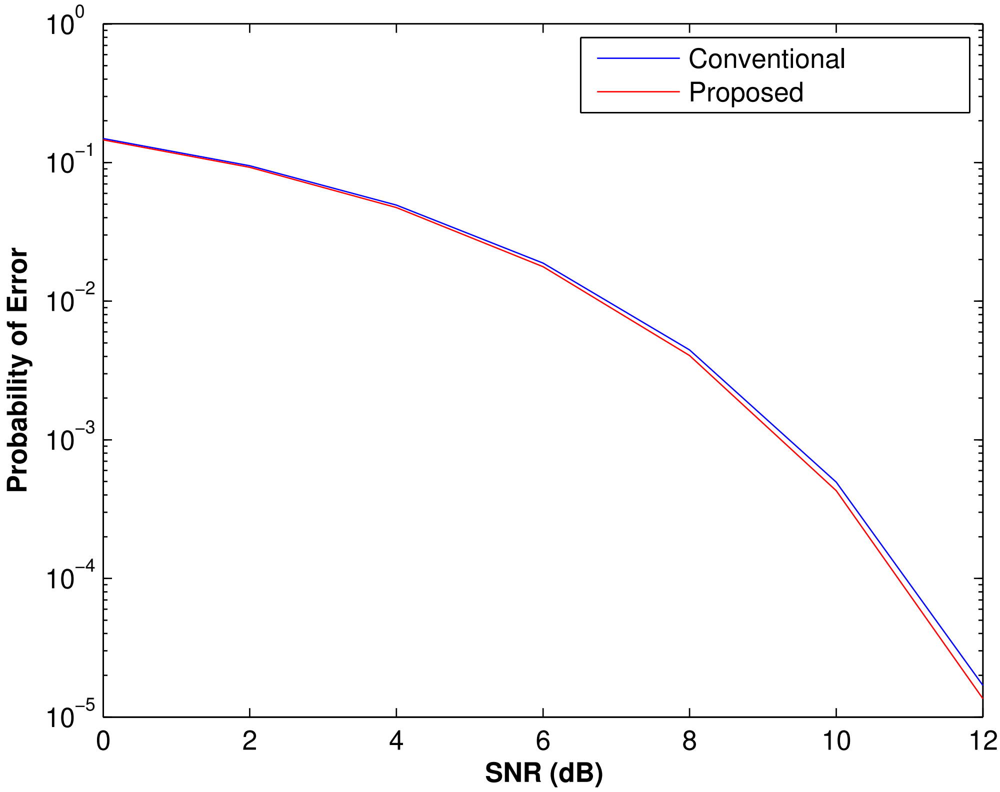

Figure 4.18: Probability of error versus SNR for $K = 1$ selected antenna in a 5×1 MISO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
1	2

.

In the above table, we observe that the conventional method would select Tx2, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx2 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other four are not selected, is Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.18, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

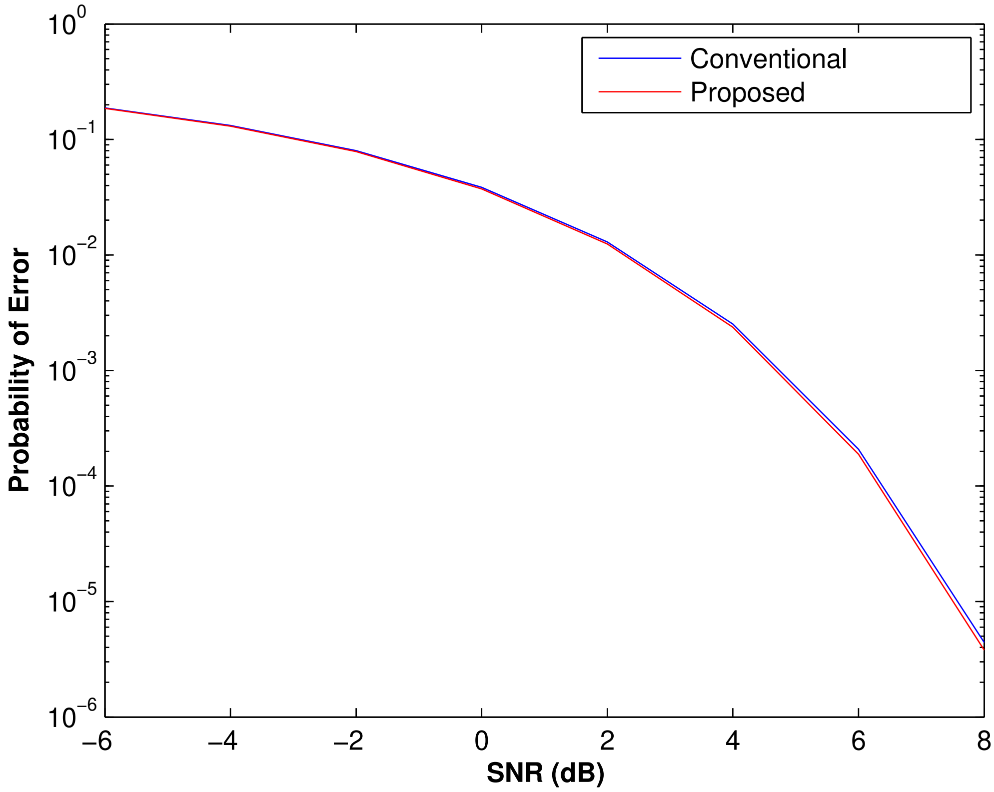

Figure 4.19: Probability of error versus SNR for $K = 3$ selected antennas in a 5×1 MISO system with 60^o angle and 50cm interelement distance.

Our second objective is to select 3 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 3 selected transmit antennas, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

Selected Antenna Index	
Conventional	Proposed
Combination $1-2-3$	Combination $1-2-4$

In the above table, we observe that the conventional method would select the combination of Tx1, Tx2 and Tx3, because, when all transmit antennas operate, three strongest channel coefficients are the ones between Tx1, Tx2 and Tx3 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the combination of antennas that actually provides the strongest channels after they are selected, while the other two are not selected, is Tx1, Tx2 and Tx4.

The above observation leads to different performance, as illustrated in Fig. 4.19, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 6

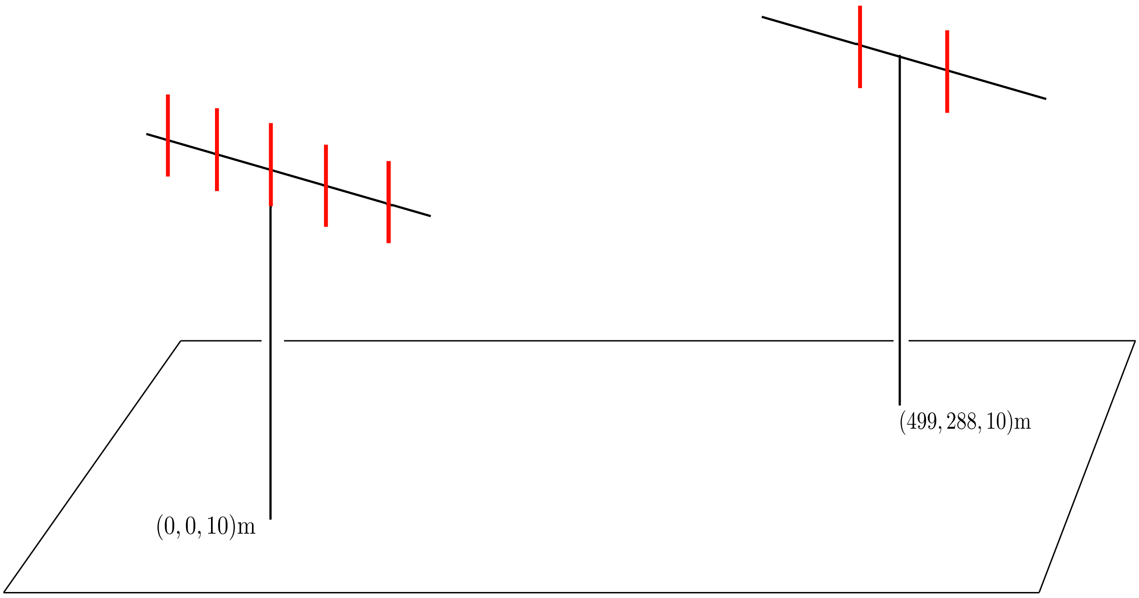

Figure 4.20: MIMO 5×2, 60^o angle, 50cm interelement distance.

In this case, we study a 5×2 MIMO system, illustrated in Fig. 4.20, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipoles of the receiver are located at $(-50, 0)$cm for Tx1 and $(+50, 0)$cm for Tx2. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our first objective is to select 1 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other four antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

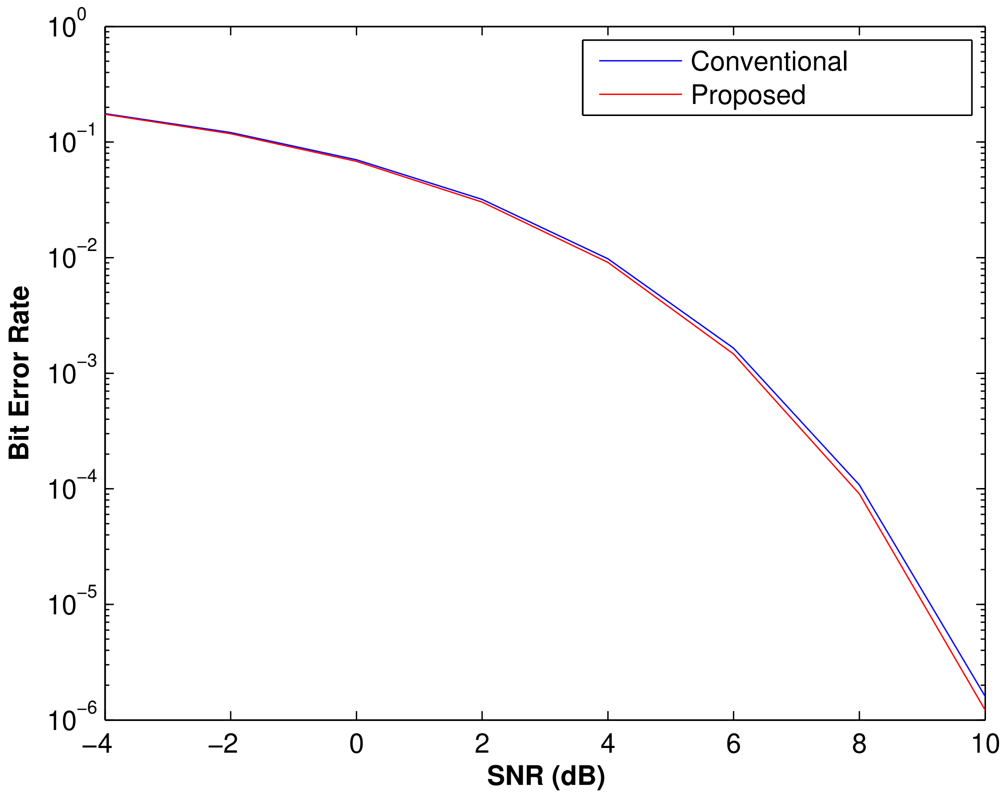

Figure 4.21: Probability of error versus SNR for $K = 1$ selected antenna in a 5×2 MIMO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
1	2

.

In the above table, we observe that the conventional method would select Tx1, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx1 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other four are not selected, is Tx2.

The above observation leads to different performance, as illustrated in Fig. 4.21, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 7

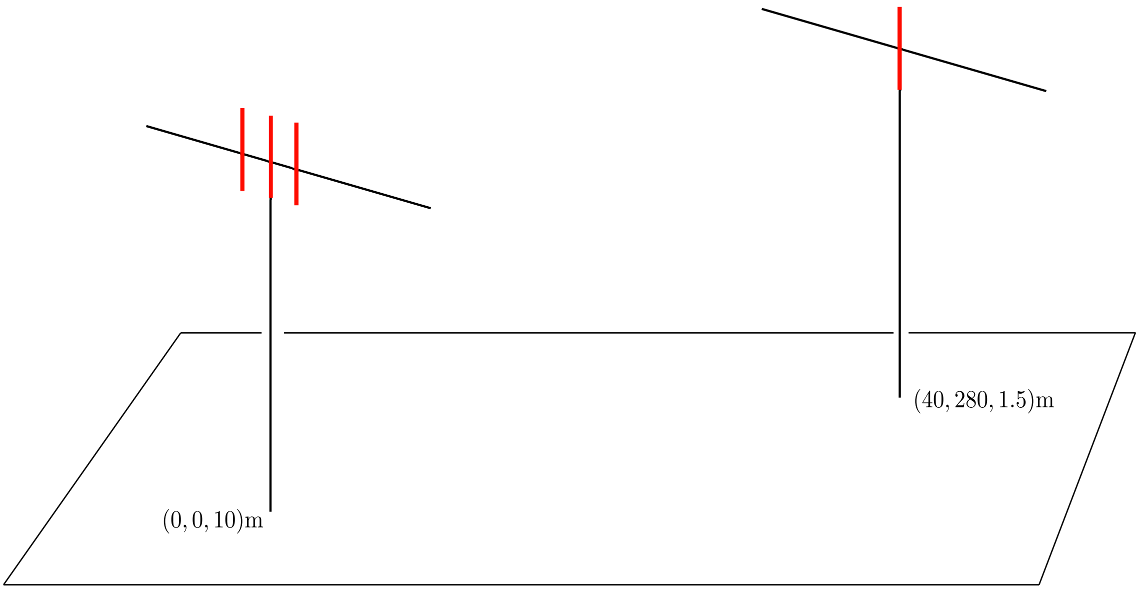

Figure 4.22: MISO 3×1, 8^o angle, 5cm interelement distance.

In this case, we study a 3×1 MISO system, illustrated in Fig. 4.22, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(40, 280, 1.5)$m. The dipoles of the transmitter are located at $(-5, 0)$cm for Tx1, $(0, 0)$cm for Tx2, and $(+5, 0)$cm for Tx3. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 8 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our first objective is to select 1 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other two antennas terminated (nonselected). In this case, the termination of the nonselected antennas was implemented by changing the impedance from 50 0hms to 50000 Ohms. For the propagation environment, we choose the LOS model.

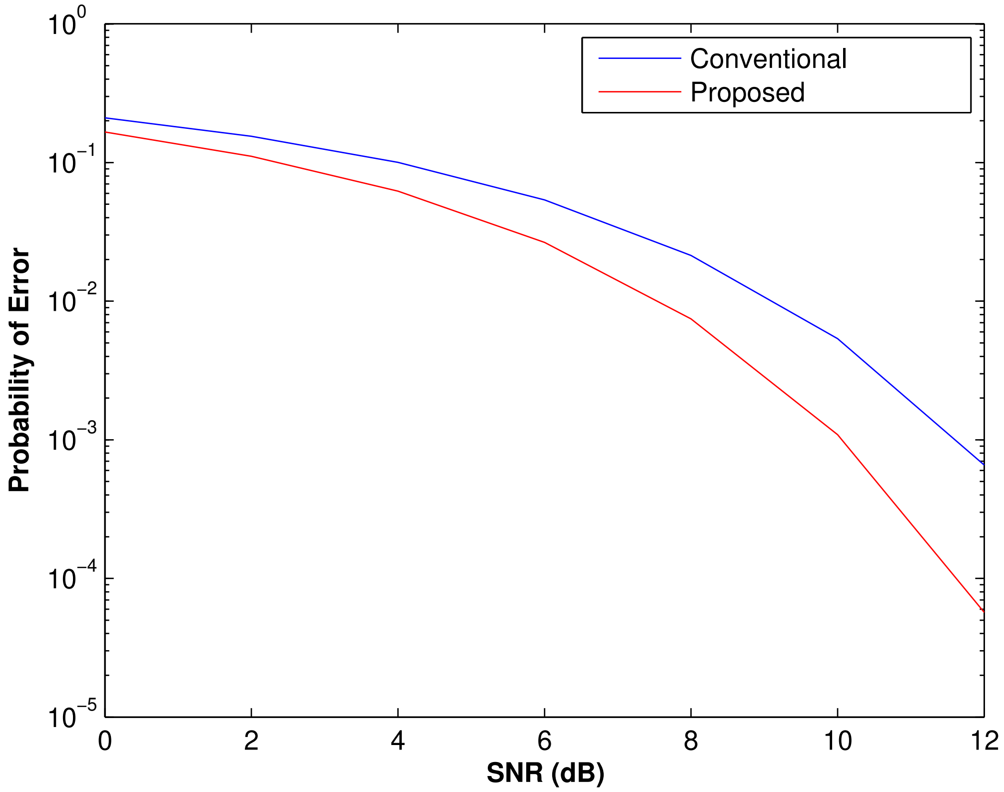

Figure 4.23: Probability of error versus SNR for $K = 1$ selected antenna in a 3×1 MISO system with 8^o angle and 5cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
2	3

.

In the above table, we observe that the conventional method would select Tx2, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx2 and the receiver. However, if we perform the antenna selection process as described in the second way above, based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other two are not selected, is Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.23, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

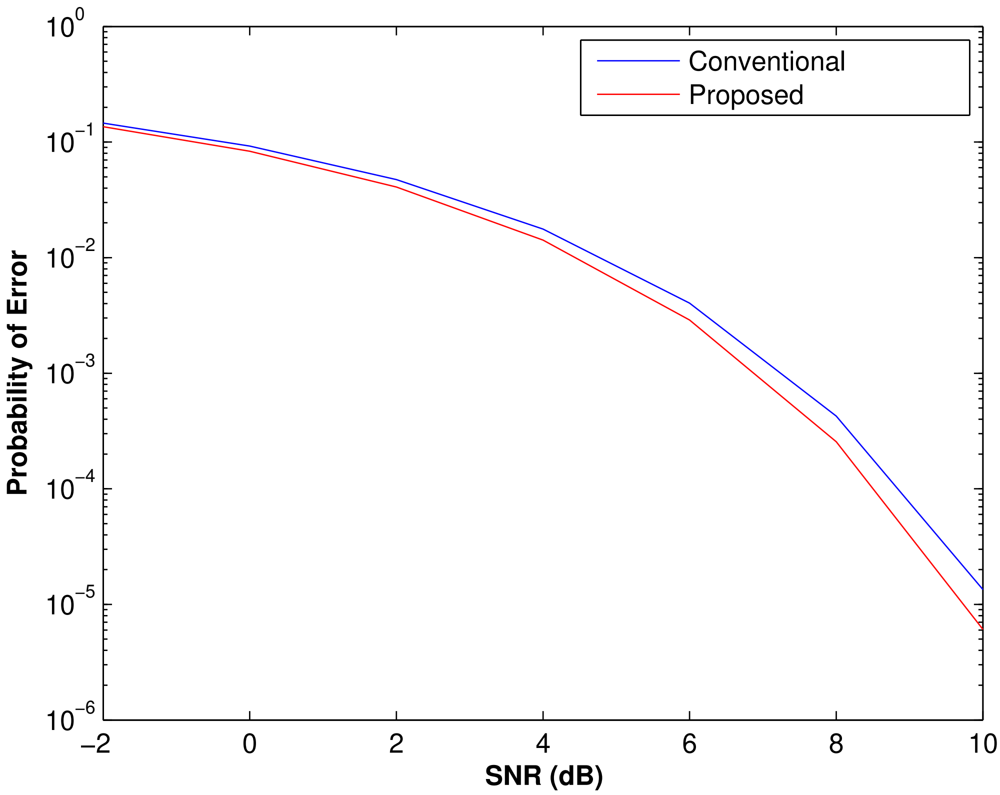

Figure 4.24: Probability of error versus SNR for $K = 2$ selected antennas in a 3×1 MISO system with 8^o angle and 5cm interelement distance.

Our second objective is to select 2 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 2 selected transmit antennas, each time with the other one antenna terminated (nonselected). In this case, the termination of the nonselected antennas was implemented by changing the impedance from 50 Ohms to 50000 Ohms. For the propagation environment, we choose the LOS model.

Selected Antenna Index	
Conventional	Proposed
Combination 2-3	Combination 1-3

.

In the above table, we observe that the conventional method would select the combination of Tx2 and Tx3, because, when all transmit antennas operate, the two strongest channel coefficients are the ones between Tx2 and Tx3 and the receiver. However, if we perform the antenna selection process as described in the second way above, based on MIMObit, we conclude that the combination of antennas that actually provides the strongest channels after they are selected, while the other antenna is not selected, is Tx1 and Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.24, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 8

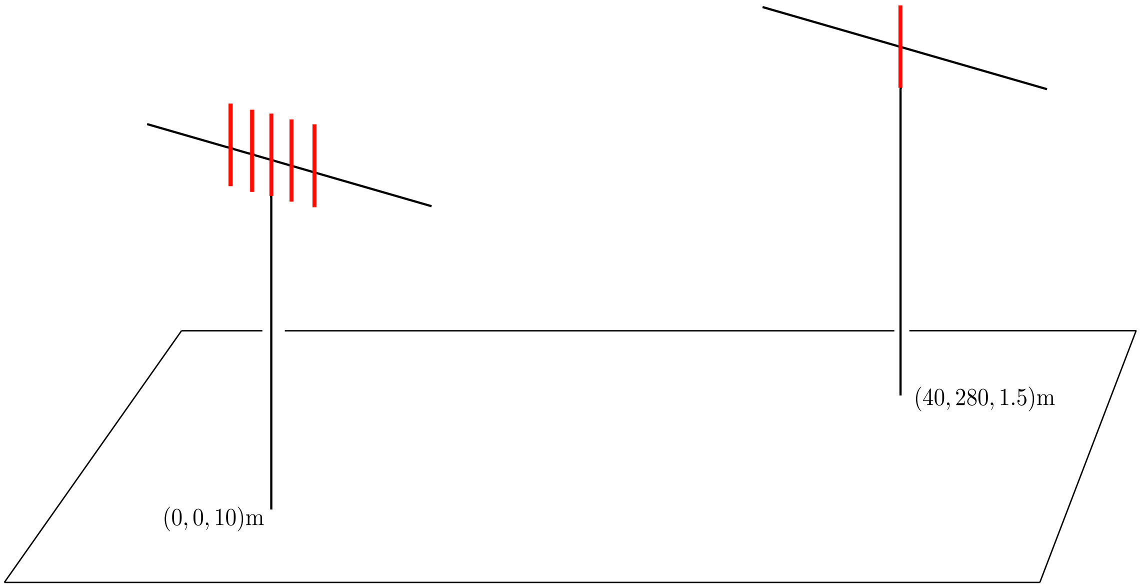

Figure 4.25: MISO 5×1, 8^o angle, 5cm interelement distance.

In this case, we study a 5×1 MISO system, illustrated in Fig. 4.25, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(40, 280, 1.5)$m. The dipoles of the transmitter are located at $(-10, 0)$cm for Tx1, $(-5, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+5, 0)$cm for Tx4, and $(+10, 0)$cm for Tx5. The dipole of the receiver is located at $(0, 0)$cm. That is, the line that connects the transmitter and the receiver is 8 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 1 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 1 selected transmit antenna, each time with the other four antennas terminated (nonselected). For the propagation environment, we choose the LOS model.

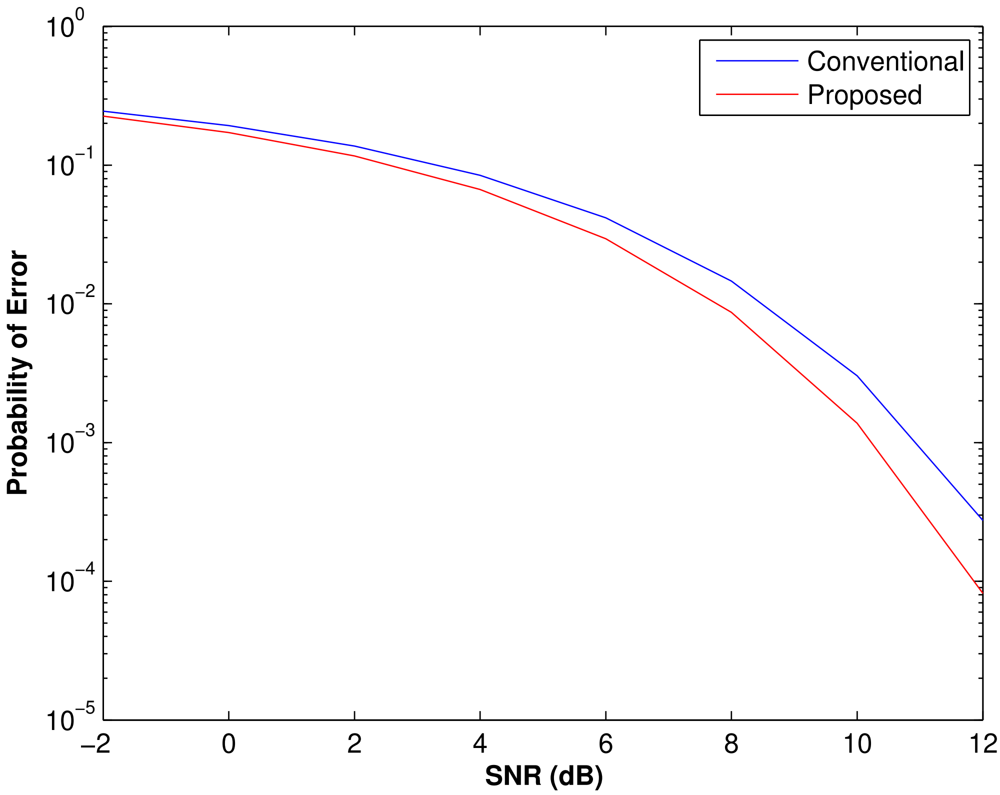

Figure 4.26: Probability of error versus SNR for $K = 1$ selected antenna in a 5×1 MISO system with 8^o angle and 5cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
2	5

.

In the above table, we observe that the conventional method would select Tx2, because, when all transmit antennas operate, the strongest channel coefficient is the one between Tx2 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the antenna that actually provides the strongest channel after it is selected, while the other four are not selected, is Tx5.

The above observation leads to different performance, as illustrated in Fig. 4.26, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 9

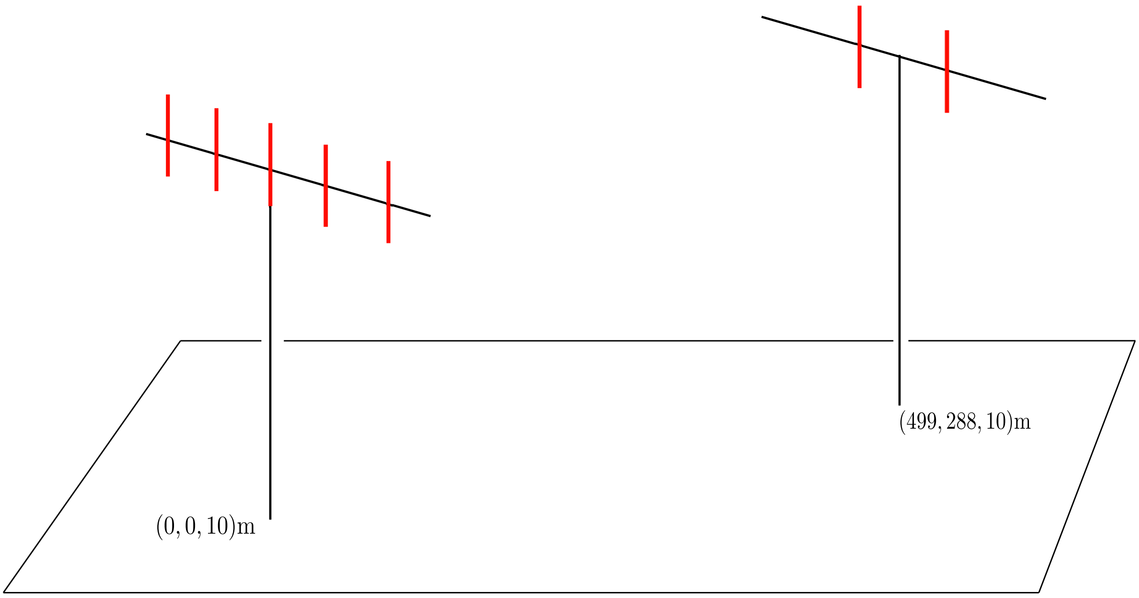

Figure 4.27: MIMO 5×2, 60^o angle, 50cm interelement distance.

In this case, we study a 5×2 MIMO system, illustrated in Fig. 4.27, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipoles of the receiver are located at $(-50, 0)$cm for Tx1 and $(+50, 0)$cm for Tx2. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 3 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 3 selected transmit antennas, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

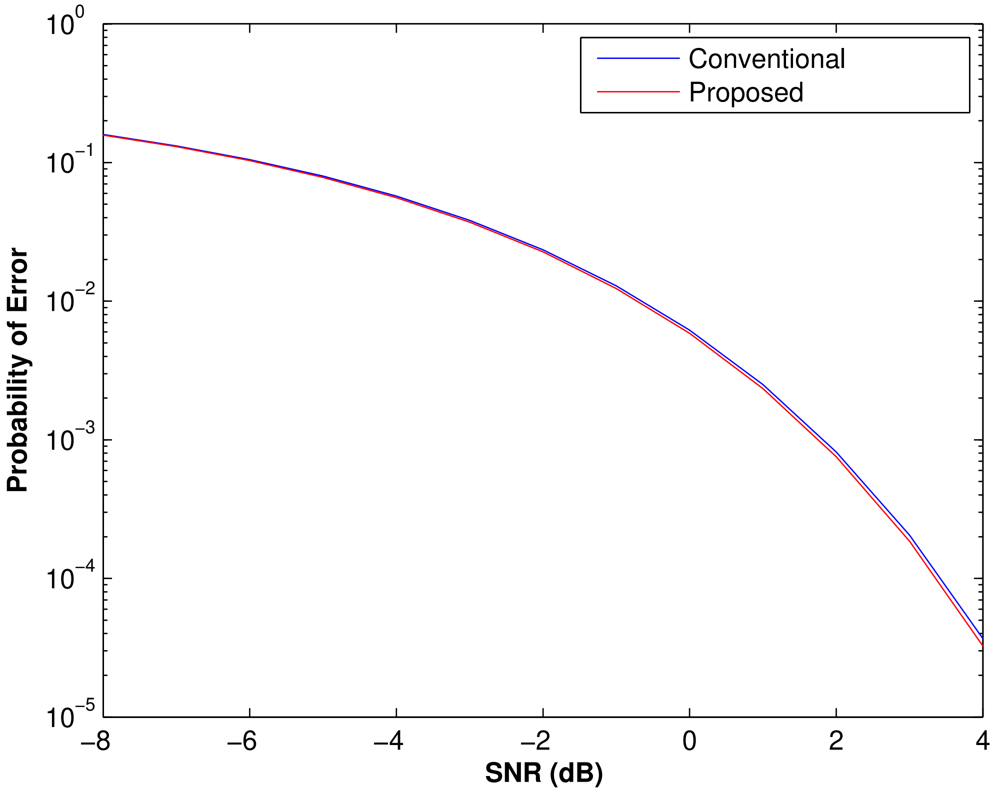

Figure 4.28: Probability of error versus SNR for $K = 3$ selected antennas in a 5×2 MIMO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
Combination 1-2-3	Combination 1-2-4

.

In the above table, we observe that the conventional method would select the combination of Tx1, Tx2 and Tx3, because, when all transmit antennas operate, the strongest $\sigma_{\max}$ is the one of the channel matrix generated between the combination of Tx1, Tx2, and Tx3 and the receiver. However, if we perform the antenna selection process as described in the second way above, based on MIMObit, we conclude that the combination of antennas that actually provides the strongest $\sigma_{\max}$ after they are selected, while the other antennas are not selected, is Tx1, Tx2 and Tx4.

The above observation leads to different performance, as illustrated in Fig. 4.28, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 10

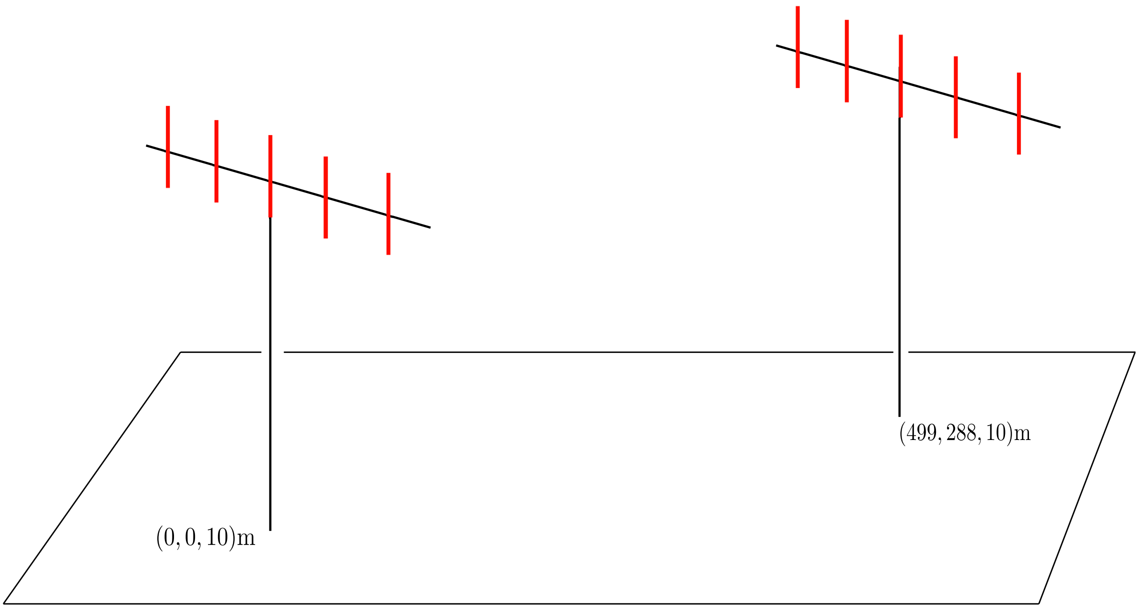

Figure 4.29: MIMO 5×5, 60^o angle, 50cm interelement distance.

In this case, we study a 5×5 MIMO system, illustrated in Fig. 4.29, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipoles of the receiver are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 3 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 3 selected transmit antennas, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

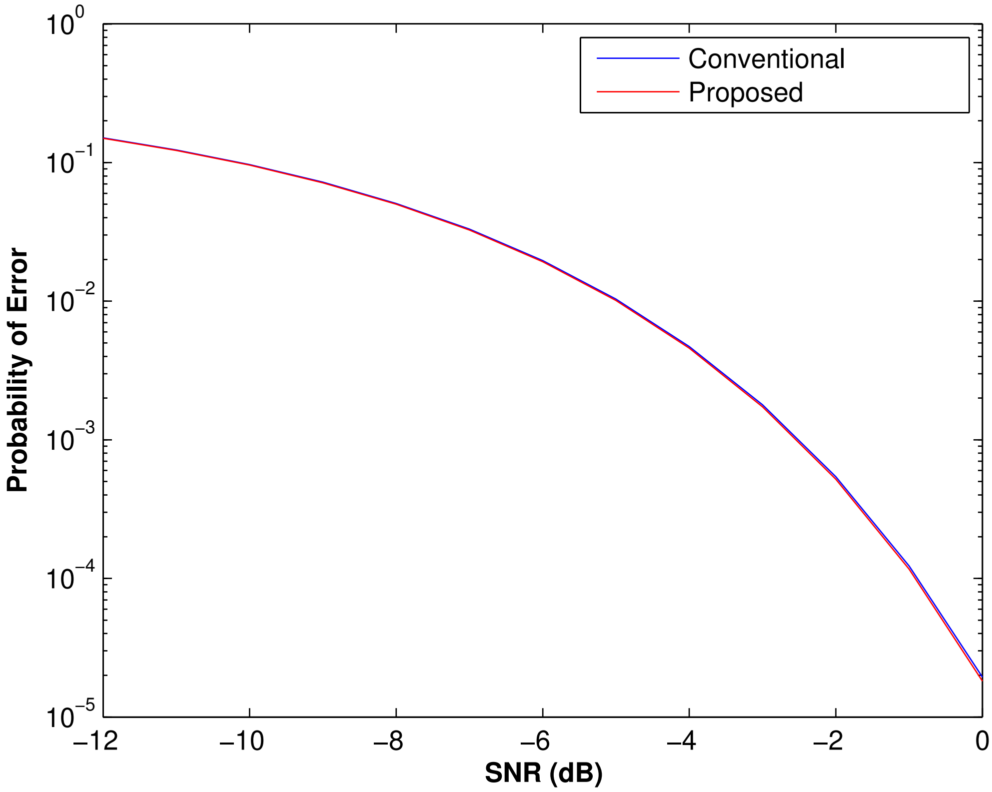

Figure 4.30: Probability of error versus SNR for $K = 3$ selected antennas in a 5×5 MIMO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
Combination 1-2-4	Combination 2-3-4

.

In the above table, we observe that the conventional method would select the combination of Tx1, Tx2 and Tx4, because, when all transmit antennas operate, the strongest $\sigma_{\max}$ is the one of the channel matrix generated between the combination of Tx1, Tx2 and Tx4 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the combination of antennas that actually provides the strongest $\sigma_{\max}$ after they are selected, while the other antennas are not selected, is Tx2, Tx3, Tx4.

The above observation leads to different performance, as illustrated in Fig. 4.30, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 11

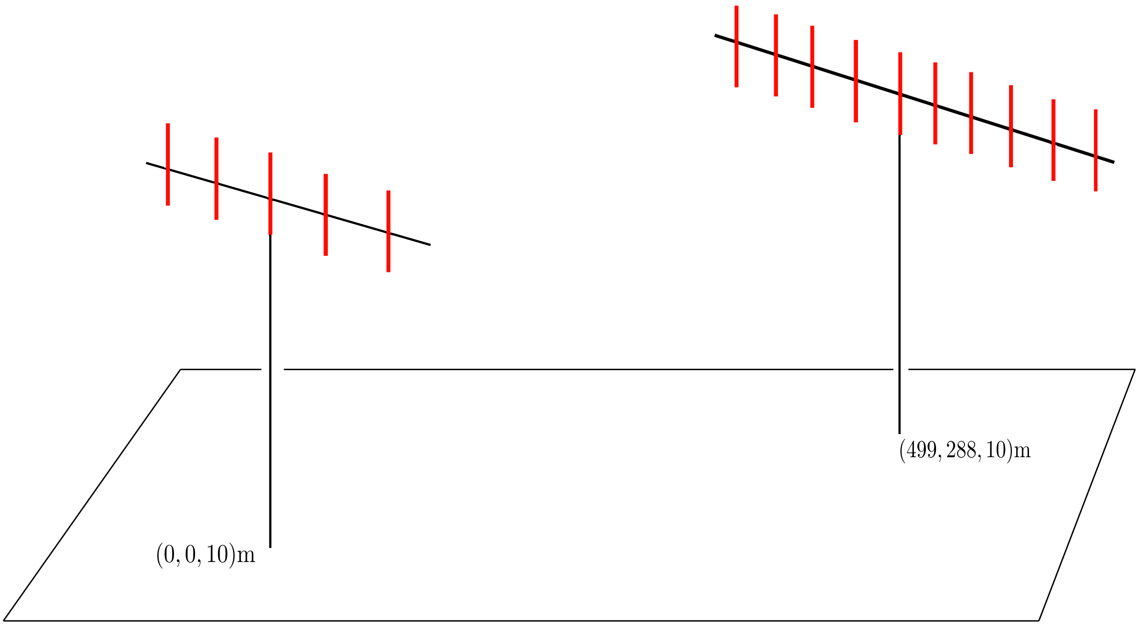

Figure 4.31: MIMO 5×10, 60^o angle, 50cm interelement distance.

In this case, we study a 5×10 MIMO system, illustrated in Fig. 4.31, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipoles of the receiver are located at $(-200, 0)$cm for Tx1, $(-150, 0)$cm for Tx2 , $(-100, 0)$cm for Tx3, $(-50, 0)$cm for Tx4, $(0, 0)$cm for Tx5, $(+50, 0)$cm for Tx6, $(+100, 0)$cm for Tx7, $(+150, 0)$cm for Tx8, $(+200, 0)$cm for Tx9, and $(+250, 0)$cm for Tx10. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 3 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 3 selected transmit antennas, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

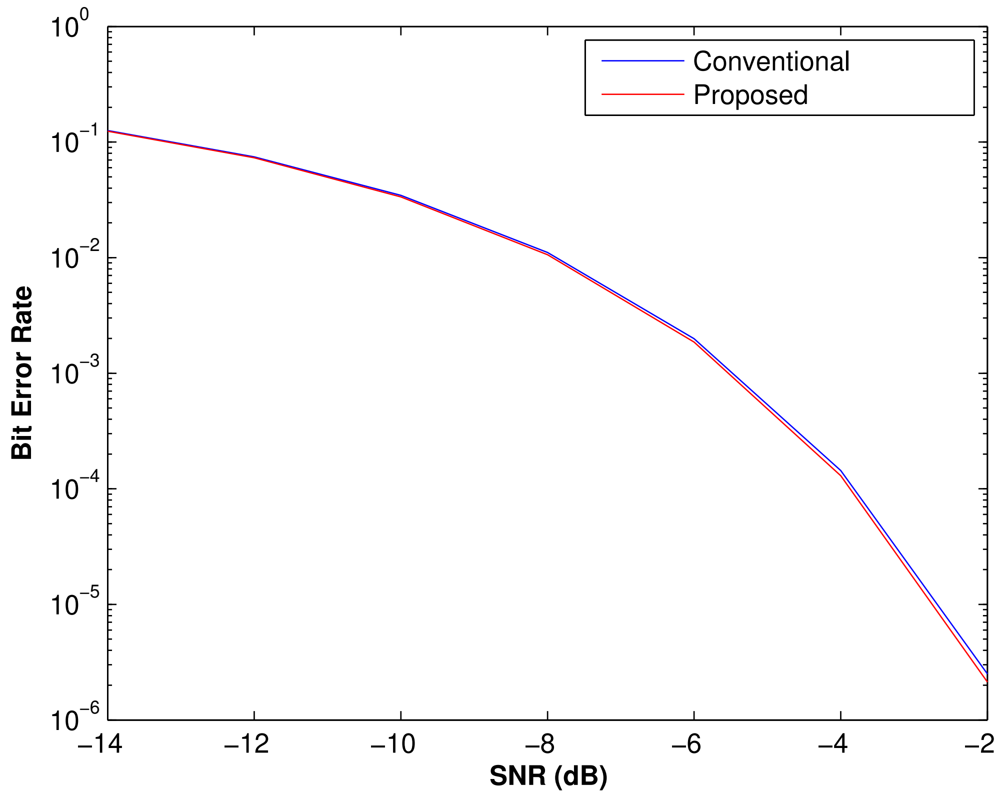

Figure 4.32: Probability of error versus SNR for $K = 3$ selected antennas in a 5×10 MIMO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
Combination 1-2-3	Combination 1-2-4

.

In the above table, we observe that the conventional method would select the combination of Tx1, Tx2 and Tx3, because, when all transmit antennas operate, the strongest $\sigma_{\max}$ is the one of the channel matrix generated between the combination of Tx1, Tx2 and Tx3 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the combination of antennas that actually provides the strongest $\sigma_{\max}$ after they are selected, while the other antennas are not selected, is Tx1, Tx2, Tx4.

The above observation leads to different performance, as illustrated in Fig. 4.32, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 12

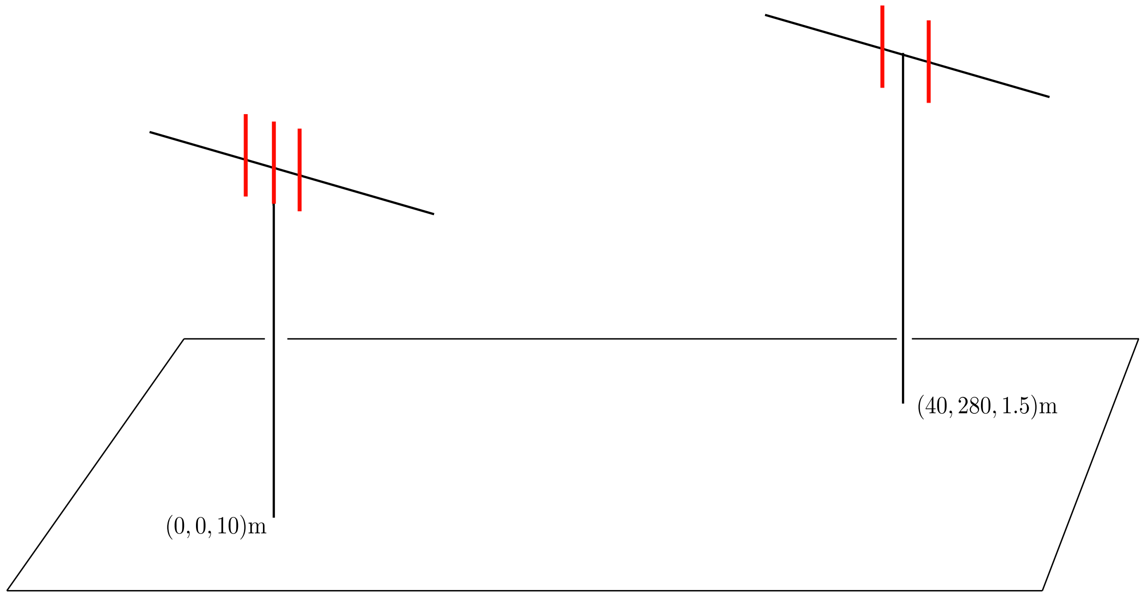

Figure 4.33: MIMO 3×2, 8^o angle, 5cm interelement distance.

In this case, we study a 3×2 MIMO system, illustrated in Fig. 4.33, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(40, 280, 1.5)$m. The dipoles of the transmitter are located at $(-5, 0)$cm for Tx1, $(0, 0)$cm for Tx2, and $(+5, 0)$cm for Tx3. The dipoles of the receiver are located at $(-5, 0)$cm for Tx1 and $(+5, 0)$cm for Tx2. That is, the line that connects the transmitter and the receiver is 8 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 2 out of 3 transmit antennas. First, we transmit from all 3 antennas and then we repeat the transmission but only from 2 selected transmit antennas, each time with the other one antenna terminated (nonselected). In this case, the termination of the nonselected antennas was implemented by changing the impedance from 50 0hms to 50000 Ohms. For the propagation environment, we choose the LOS model.

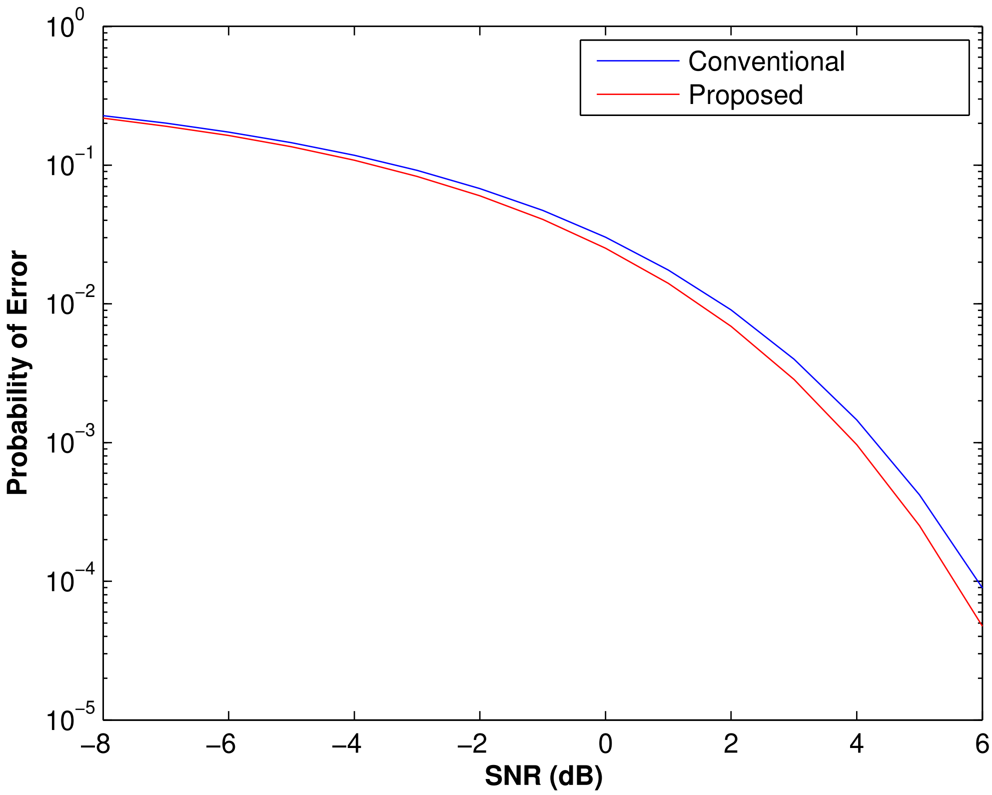

Figure 4.34: Probability of error versus SNR for $K = 2$ selected antennas in a 3×2 MIMO system with 8^o angle and 5cm interelement distance.

Selected Antenna Index	
Conventional	Proposed
Combination 2-3	Combination 1-3

.

In the above table, we observe that the conventional method would select the combination of Tx2 and Tx3, because, when all transmit antennas operate, the strongest $\sigma_{\max}$ is the one of the channel matrix generated between the combination of Tx2 and Tx3 and the receiver. However, if we perform the antenna selection process as described in the second way above based on MIMObit, we conclude that the combination of antennas that actually provides the strongest $\sigma_{\max}$ after they are selected, while the other antennas are not selected, is Tx1 and Tx3.

The above observation leads to different performance, as illustrated in Fig. 4.34, where we plot the probability of error versus SNR per path for the two selection methods. The BER decreases when we take into consideration the channel changes which occur due to the antenna selection process.

Case 13

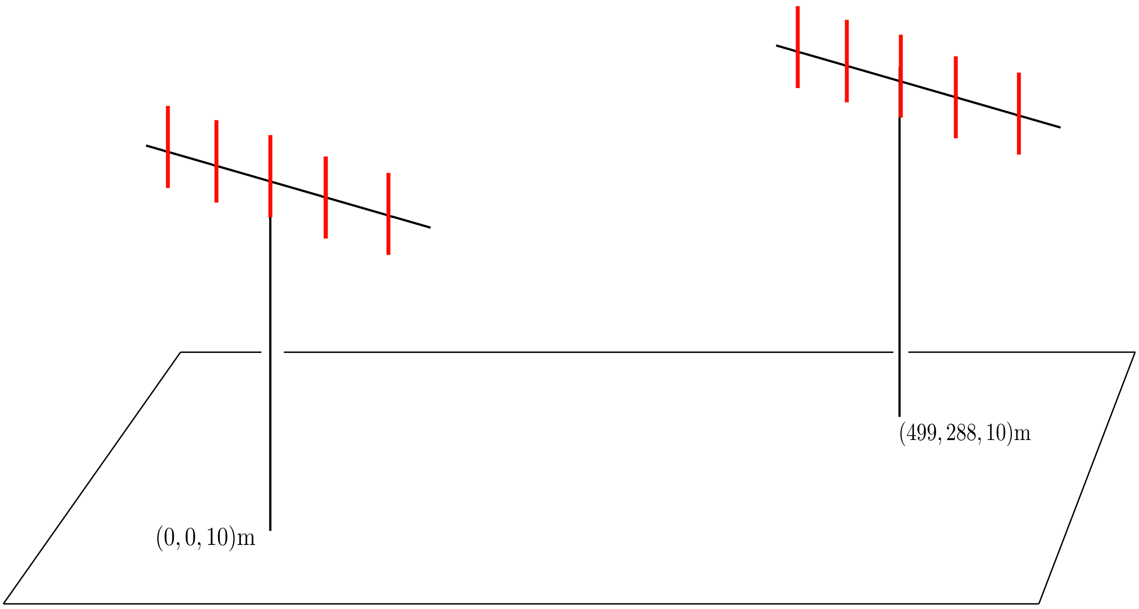

Figure 4.35: MIMO 5×5, 60^o angle, 50cm interelement distance.

In this case, we study a 5×5 MIMO system, illustrated in Fig. 4.35, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipoles of the receiver are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 3 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 3 selected transmit antennas, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

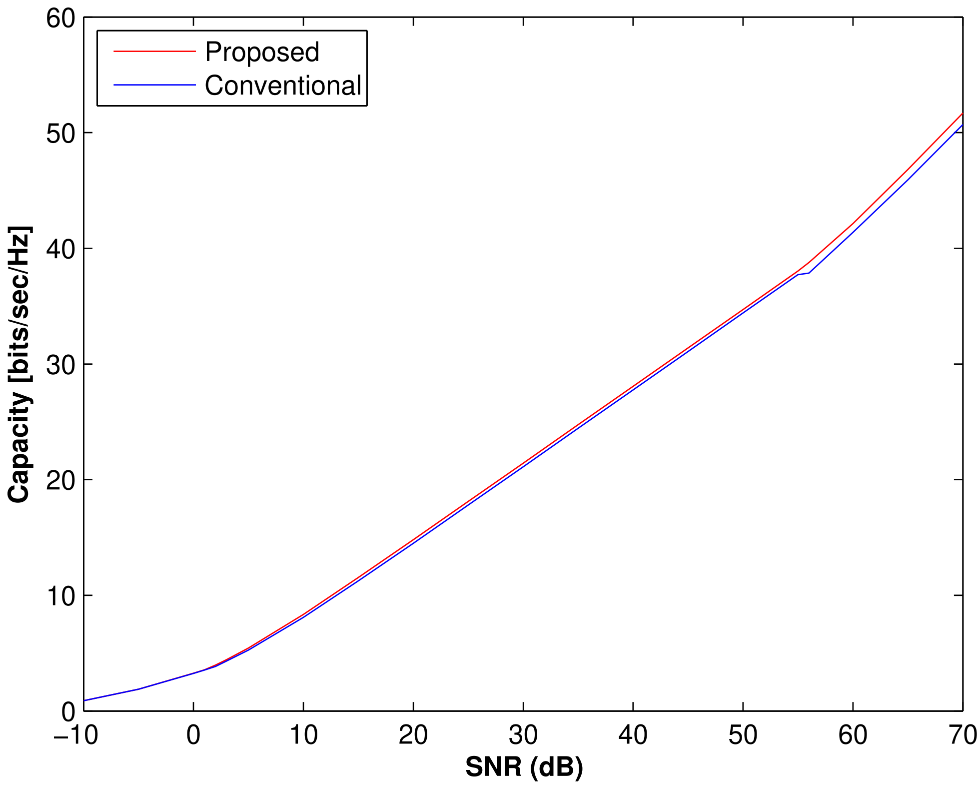

Figure 4.36: Capacity versus SNR for $K = 3$ selected antennas in a 5×5 MIMO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index		
SNR (db)	Proposed	Conventional
-10...1	Combination 2-3-4	Combination 1-2-4
2	Combination 1-3-5	Combination 1-2-4
3...55	Combination 1-3-5	Combination 1-2-3
56...70	Combination 1-2-5	Combination 1-3-5

In the above table, we observe that the conventional method would select different combination of antennas, in different SNR ranges, compared with the method based on MIMObit. More specifically between $(-10, 1)$ db the conventional method leads to the selection of the combination of Tx1, Tx2 and Tx4 while the second method leads to the selection of the combination of Tx2, Tx3 and Tx4. At 2 db, the conventional method selects the combination of Tx1, Tx2 and Tx4, but the method based on MIMObit selects the combination of Tx1, Tx3 and Tx5 while between $(3, 55)$ db the conventional method selects the combination of Tx1, Tx2 and Tx3, but the method based on MIMObit selects the combination of Tx1, Tx3 and Tx5. Finally, after 56 db the conventional way selects the conventional method selects the combination of Tx1, Tx3 and Tx5, but the method based on MIMObit selects the combination of Tx1, Tx2 and Tx5.

To examine the effect of wrong antenna selection by the conventional method, we proceed as follows. Let $\widetilde{\mathbf{H}}$ be the submatrix of the initial channel matrix that consists of the columns that correspond to the selected antennas. Then,

$$\widetilde{\mathbf{H}} = \widetilde{\mathbf{U}}\widetilde{\mathbf{\Sigma}}\widetilde{\mathbf{V}}^H. \tag{4.3}$$

For the same wrong antenna selection, let $\mathbf{H}$ be the true channel matrix after these antennas have been selected where

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H. \tag{4.4}$$

Apparently, $\widetilde{\mathbf{H}}$ is not necessarily equal to $\mathbf{H}$ due to the channel changes inducted by antenna selection/termination. Then, the transmitter and the receiver use the wrong version of $\mathbf{H}$ (i.e., $\widetilde{\mathbf{H}}$) for pre- and post-processing while the true propagation channel is now $\mathbf{H}$. Hence, the transmitter pre-processes $\mathbf{Ab}$ by $\widetilde{\mathbf{V}}$ while the receiver post-processes $\mathbf{y}$ by $\widetilde{\mathbf{U}}^H$, i.e., the filtered received vector is

$$\widetilde{\mathbf{y}} = \widetilde{\mathbf{U}}^H\mathbf{y} = \widetilde{\mathbf{U}}^H(\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H\widetilde{\mathbf{V}}\mathbf{Ab}) + \widetilde{\mathbf{U}}^H\mathbf{n} = \widetilde{\mathbf{U}}^H\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H\widetilde{\mathbf{V}}\mathbf{Ab} + \widetilde{\mathbf{U}}^H\mathbf{n} = \mathbf{Sb} + \mathbf{n}' \tag{4.5}$$

where

$$\mathbf{S} = \widetilde{\mathbf{U}}^H\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H\widetilde{\mathbf{V}}\mathbf{A} \tag{4.6}$$

and $\mathbf{n}' \sim \mathbb{CN}(\mathbf{0}, \sigma^2\mathbf{I}_{N_\mathrm{r}})$.

Note that $\mathbf{A}$ is a diagonal $\widetilde{r} \times \widetilde{r}$ matrix that contains the square roots of the powers provided by the waterfilling algorithm run over the "fake" channel matrix $\widetilde{\mathbf{H}}$ assumed by the conventional approach, when $\widetilde{r}$ is the rank of $\widetilde{\mathbf{H}}$.

Then, based on the above, the capacity of the MIMO system after antenna selection using the conventional method can be expressed as

$$C = \log_2\left|\mathbf{I}_{N_\mathrm{r}} + \frac{1}{\sigma^2}\mathbf{S}\mathbf{S}^H\right| \qquad \text{bits/symbol.} \tag{4.7}$$

The above observation leads to different performance, as illustrated in Fig. 4.36, where we plot the capacity versus SNR per path for the two selection methods. Capacity is improved when we take into consideration the channel changes which will occur due to the antenna selection process.

Case 14

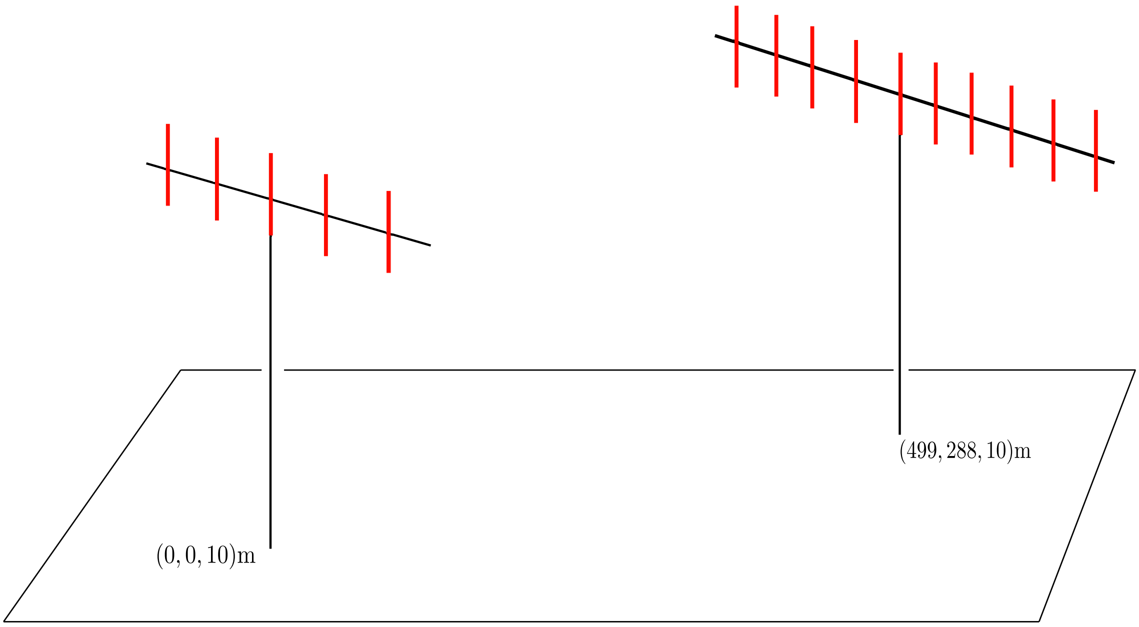

Figure 4.37: MIMO 5×10, 60^o angle, 50cm interelement distance.

In this case, we study a 5×10 MIMO system, illustrated in Fig. 4.37, where the transmitter is located at $(0, 0, 10)$m and the receiver is located at $(499, 288, 10)$m. The dipoles of the transmitter are located at $(-100, 0)$cm for Tx1, $(-50, 0)$ for Tx2, $(0, 0)$cm for Tx3, $(+50, 0)$cm for Tx4, and $(+100, 0)$cm for Tx5. The dipoles on the receiver are located at $(-200, 0)$cm for Tx1, $(-150, 0)$cm for Tx2, $(-100, 0)$cm for Tx3, $(-50, 0)$cm for Tx4, $(0, 0)$cm for Tx5, $(+50, 0)$cm for Tx6, $(+100, 0)$cm for Tx7, $(+150, 0)$cm for Tx8, $(+200, 0)$cm for Tx9, and $(+250, 0)$cm for Tx10. That is, the line that connects the transmitter and the receiver is 60 degrees off the perpendicular bisector of the line segment created by the dipoles of the transmitter.

Our objective is to select 3 out of 5 transmit antennas. First, we transmit from all 5 antennas and then we repeat the transmission but only from 3 selected transmit antennas, each time with the other two antennas terminated (nonselected). For the propagation environment, we choose the 2ray model.

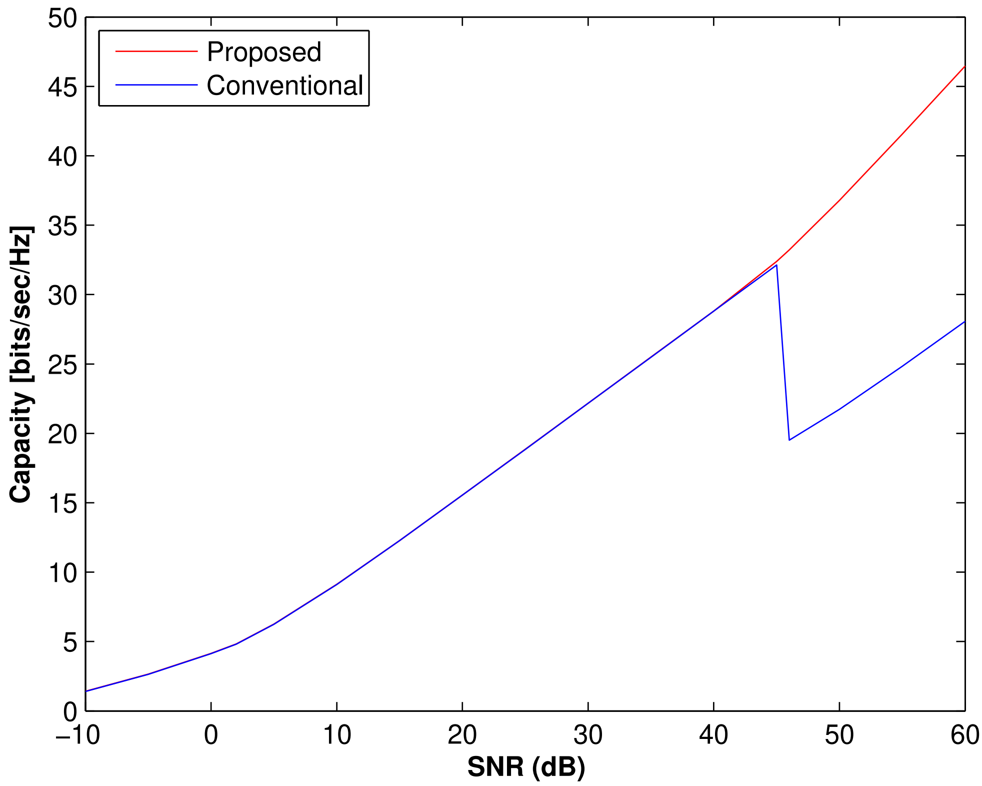

Figure 4.38: Capacity versus SNR for $K = 3$ selected antennas in a 5×10 MIMO system with 60^o angle and 50cm interelement distance.

Selected Antenna Index		
SNR (db)	Proposed	Conventional
-10...1	Combination 1-2-4	Combination 1-2-3
2...45	Combination 1-3-5	Combination 1-2-5
46...60	Combination 1-3-5	Combination 1-4-5

.

In the above table, we observe that the conventional method would select different combination of antennas, in different SNR ranges, compared with the method based on MIMObit. More specifically between $(-10, 1)$ db the conventional method leads to the selection of the combination of Tx1, Tx2 and Tx3 while the second method leads to the selection of the combination of Tx1, Tx2 and Tx4. Between $(2, 45)$ db, the conventional method selects the combination of Tx1, Tx2 and Tx5, but the method based on MIMObit selects the combination of Tx1, Tx3 and Tx5. Finally, after 46 db the conventional way selects the conventional method selects the combination of Tx1, Tx4 and Tx5, but the method based on MIMObit selects the combination of Tx1, Tx3 and Tx5.

The above observation leads to different performance, as illustrated in Fig. 4.38, where we plot the capacity versus SNR per path for the two selection methods. Capacity is improved when we take into consideration the channel changes which will occur due to the antenna selection process.

Chapter 5

Conclusions and Future Work

In this work, we studied antenna selection criteria and algorithms for multiple-input multiple-output (MIMO) wireless systems using channel data produced by the specialized software MIMObit. We compared the conventional method used in antenna selection, where we assume that the channel coefficients of the selected antennas do not change after the antenna selection, to the proposed method that relies on the channel data generated by MIMObit after the antenna selection process. We concluded that the proposed method always outperforms the conventional one. In some cases, (i.e., when the distance between the antennas on the transmitter is small and the termination load is very high), the performance gain is noticeable. We also observed that the coupling effect is stronger when the interelement distance is small (i.e., equal to 0.5λ of the dipole antenna).

However, there is still work to be done. Based on our observations, we can experiment on antennas with actual data, such as the antennas used in cell phones. We can design such antennas using the CST platform and import the corresponding files into MIMObit. Therefore, we can study if the proposed method still outperforms the conventional one in terms of antennas used in everyday life/reality.

Chapter 6

Appendix

Proof of (1.2)

We decide in favor of bit $b = 1$ iff

$$f\left(\mathbf{y}|b=1\right) > f\left(\mathbf{y}|b=-1\right)$$

$$\frac{1}{\pi^2|\sigma^2\mathbf{I}|}\left(e^{-(\mathbf{y}-A\mathbf{h})^H\left(\sigma^2\mathbf{I}\right)^{-1}(\mathbf{y}-A\mathbf{h})}\right) > \frac{1}{\pi^2|\sigma^2\mathbf{I}|}\left(e^{-(\mathbf{y}+A\mathbf{h})^H\left(\sigma^2\mathbf{I}\right)^{-1}(\mathbf{y}+A\mathbf{h})}\right)$$

$$e^{-(\mathbf{y}-A\mathbf{h})^H\frac{1}{\sigma^2}\mathbf{I}(\mathbf{y}-A\mathbf{h})} > e^{-(\mathbf{y}+A\mathbf{h})^H\frac{1}{\sigma^2}\mathbf{I}(\mathbf{y}+A\mathbf{h})}$$

$$\ln\left(e^{-\frac{1}{\sigma^2}(\mathbf{y}-A\mathbf{h})^H(\mathbf{y}-A\mathbf{h})}\right) > \ln\left(e^{-\frac{1}{\sigma^2}(\mathbf{y}+A\mathbf{h})^H(\mathbf{y}+A\mathbf{h})}\right)$$

$$-\left(\mathbf{y}-A\mathbf{h}\right)^H\left(\mathbf{y}-A\mathbf{h}\right) > -\left(\mathbf{y}+A\mathbf{h}\right)^H\left(\mathbf{y}+A\mathbf{h}\right)$$

$$\left(\mathbf{y}+A\mathbf{h}\right)^H\left(\mathbf{y}+A\mathbf{h}\right) > \left(\mathbf{y}-A\mathbf{h}\right)^H\left(\mathbf{y}-A\mathbf{h}\right)$$

$$\mathbf{y}^H\mathbf{y}+\mathbf{y}^HA\mathbf{h}+A\mathbf{h}^H\mathbf{y}+A^2\mathbf{h}^H\mathbf{h} > \mathbf{y}^H\mathbf{y}-\mathbf{y}^HA\mathbf{h}-A\mathbf{h}^H\mathbf{y}+A^2\mathbf{h}^H\mathbf{h}$$

$$2A\left(\mathbf{y}^H\mathbf{h}+\mathbf{h}^H\mathbf{y}\right) > 0$$

$$\mathrm{Re}(\mathbf{h}^H\mathbf{y}) > 0. \tag{6.1}$$

Proof of (1.3)

The probability of error of the optimal detector of (1.2) is

$$
\begin{aligned}
P_\mathrm{e} &= P\left(\hat{b} \neq b\right) = \frac{1}{2}P\left(\hat{b} = 1|b=-1\right) + \frac{1}{2}P\left(\hat{b}=-1|b=+1\right) = P\left(\mathrm{Re}\big(\mathbf{h}^H\mathbf{y}\big) > 0|b=-1\right) \\
&= P\left(\mathrm{Re}\left(\mathbf{h}^H\left(A\mathbf{h}b+\mathbf{n}\right)\right) > 0|b=-1\right) = P\left(\mathrm{Re}\left(\mathbf{h}^H\left(-A\mathbf{h}+\mathbf{n}\right)\right) > 0|b=-1\right) \\
&= P\left(\mathrm{Re}\left(\mathbf{h}^H\mathbf{n}\right) > A\|\mathbf{h}\|^2\right). \qquad (6.2)
\end{aligned}
$$

Since $\mathbf{n}$ is a complex Gaussian vector, $\mathbf{h}^H\mathbf{n}$ is a complex Gaussian variable with mean

$$
E[\mathbf{h}^H\mathbf{n}] = \mathbf{h}^H E[\mathbf{n}] = \mathbf{h}^H\mathbf{0} = 0 \qquad (6.3)
$$

and variance

$$
\begin{aligned}
\mathrm{var}\left(\mathbf{h}^H\mathbf{n}\right) &= E[|\mathbf{h}^H\mathbf{n}|^2] - |E[\mathbf{h}^H\mathbf{n}]|^2 = E[|\mathbf{h}^H\mathbf{n}|^2] = E[(\mathbf{h}^H\mathbf{n})(\mathbf{n}^H\mathbf{h})] = \mathbf{h}^H E[\mathbf{n}\mathbf{n}^H]\mathbf{h} \\
&= \mathbf{h}^H(\sigma^2\mathbf{I})\mathbf{h} = \|\mathbf{h}\|^2\sigma^2. \qquad (6.4)
\end{aligned}
$$

Hence, $\mathrm{Re}\left(\mathbf{h}^H\mathbf{n}\right)$ is a real Gaussian variable with mean

$$
E[\mathrm{Re}\left(\mathbf{h}^H\mathbf{n}\right)] = \mathrm{Re}\left(E[\mathbf{h}^H\mathbf{n}]\right) = \mathrm{Re}\left(0\right) = 0 \qquad (6.5)
$$

and variance

$$
\mathrm{var}\left(\mathrm{Re}\left(\mathbf{h}^H\mathbf{n}\right)\right) = \frac{1}{2}\mathrm{var}\left(\mathbf{h}^H\mathbf{n}\right) = \frac{\|\mathbf{h}\|^2\sigma^2}{2}. \qquad (6.6)
$$

Then, (6.2) becomes

$$
\begin{aligned}
P_\mathrm{e} &= P\left(\frac{\mathrm{Re}(\mathbf{h}^H\mathbf{n}) - 0}{\sqrt{\mathrm{var}\left(\mathrm{Re}(\mathbf{h}^H\mathbf{n})\right)}} > \frac{A\|\mathbf{h}\|^2}{\sqrt{\mathrm{var}\left(\mathrm{Re}\left(\mathbf{h}^H\mathbf{n}\right)\right)}}\right) = P\left(\frac{\mathrm{Re}(\mathbf{h}^H\mathbf{n}) - 0}{\sqrt{\mathrm{var}\left(\mathrm{Re}(\mathbf{h}^H\mathbf{n})\right)}} > \frac{A\|\mathbf{h}\|^2}{\sqrt{\frac{\|\mathbf{h}\|^2\sigma^2}{2}}}\right) \\
&= P\left(\frac{\mathrm{Re}(\mathbf{h}^H\mathbf{n}) - 0}{\sqrt{\mathrm{var}\left(\mathrm{Re}(\mathbf{h}^H\mathbf{n})\right)}} > \sqrt{\frac{2\|\mathbf{h}\|^2A^2}{\sigma^2}}\right) = Q\left(\sqrt{2\|\mathbf{h}\|^2\frac{A^2}{\sigma^2}}\right). \qquad (6.7)
\end{aligned}
$$

Proof of Average receive SNR per path

We consider the MIMO case which we present in Chapter 2. The received signal is

$$\mathbf{y} = A\mathbf{H}\mathbf{w}b + \mathbf{n}, \tag{6.8}$$

After filtering the received signal $(\mathbf{H}\mathbf{w})^H y$, we proved that the optimal beamformer is

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{v}_1. \tag{6.9}$$

By substituting (6.9) into (6.8), the received signal at the predetection stage is

$$y = \sigma_1 A b + \mathrm{Re}(n) \tag{6.10}$$

where $\mathrm{Re}(n) \sim \mathbb{CN}(0, \frac{\sigma^2}{2})$ represents additive white (in time) complex Gaussian noise with mean zero and half the variance of σ^2.

Thus, for a fixed $\mathbf{H}$

$$\mathrm{SNR} = \frac{{\sigma_1}^2 A^2}{\frac{\sigma^2}{2}} = \frac{2A^2\sigma_1^2}{\sigma^2}. \tag{6.11}$$

For random $\mathbf{H}$, we get the expected SNR, which is

$$\mathrm{SNR} = \frac{2A^2 E\{\sigma_1^2\}}{\sigma^2}. \tag{6.12}$$

which is the SNR for a MIMO with $N_{\mathrm{t}} \times N_{\mathrm{r}}$ paths. So the average SNR per path is

$$\text{Average SNR per path} = \frac{2A^2}{\sigma^2}\frac{E\{\sigma_1^2\}}{N_{\mathrm{t}}N_{\mathrm{r}}}. \tag{6.13}$$

Average receive SNR per path for MISO

In a MISO system, in contrast with a MIMO system, we have $N_{\mathrm{r}} = 1$ receive antenna, vector $\mathbf{h}$ instead of matrix $\mathbf{H}$ and $\|\mathbf{h}\|$ instead of σ_1. Thus, $E\{\sigma_1^2\} = E\{\|\mathbf{h}\|^2\} = N_{\mathrm{t}}$. Therefore, by substituting into (6.13), the average SNR per path is

$$\text{Average SNR per path} = \frac{2A^2}{\sigma^2}\frac{N_{\mathrm{t}}}{N_{\mathrm{t}}1} = \frac{2A^2}{\sigma^2}. \tag{6.14}$$

Average receive SNR per path for SIMO

In a SIMO system, in contrast with a MIMO system, we have $N_{\mathrm{t}} = 1$ receive antenna, vector $\mathbf{h}$ instead of matrix $\mathbf{H}$ and $\|\mathbf{h}\|$ instead of σ_1. Thus, $E\{\sigma_1^2\} = E\{\|\mathbf{h}\|^2\} = N_{\mathrm{r}}$. Therefore, by substituting into (6.13), the average SNR per path is

$$\text{Average SNR per path} = \frac{2A^2}{\sigma^2}\frac{N_{\mathrm{r}}}{1N_{\mathrm{r}}} = \frac{2A^2}{\sigma^2}. \tag{6.15}$$

Average receive SNR per path for SISO

In a SISO system, in contrast with a MIMO system, we have $N_\mathrm{t} = 1$ transmit antenna and $N_\mathrm{r} = 1$ receive antenna and scalar h instead of matrix $\mathbf{H}$. Therefore, by substituting into (6.13), the average SNR per path is

$$\text{Average SNR per path} = \frac{2A^2}{\sigma^2}. \tag{6.16}$$

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