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ORIGINAL PAPER

Graphical sustainability analysis using disjoint biplots

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Abstract
The assessment of sustainability is of the utmost importance nowadays. Several approaches exist that measure sustainability at a national level and rank countries accordingly. Comparison of countries could be done numerically or pictorially. This paper introduces a novel clustering disjoint HJ-biplot approach, which is then applied to data from two well-known models: Sustainability Assessment by Fuzzy Evaluation (SAFE) and the United Nations Sustainable Development Goals Index (UN-SDGs). This approach performs a graphical ranking that makes the sustainability standing of countries very transparent. As expected, the pictorial model yielded similar rankings to those of SAFE and UN-SDGs, but it additionally grouped countries according to their most important indicators, thereby yielding a more global picture of sustainability. Our approach thus comprises a useful complement to existing mathematical sustainability ranking models.

Keywords Sustainability assessment · Biplots · Sustainability indicators · Graphical sustainability ranking

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1 Introduction

The term sustainability is often used as a synonym for sustainable development. However, the two terms have different meanings depending on the audience. Sustainable development is used by neoclassical economists as a synonym for economic growth, whereas sustainability is a term that gives priority to environmental and social concerns when used in other disciplines. Additionally, the neoclassical division between developed (rich) and developing (poor) countries focuses only on economic indicators, while a more global vision of sustainable development should include environmental and cultural aspects of a society.

Various authors highlight the major stages or key time periods for the development and formalization of the concept of sustainability (e.g., Giddings et al. 2002; Waas et al. 2011). An initial period of discussion of the term lasted until the end of the 1970s, followed by a period of stagnation in its development from 1980 to 1986. The period from 1987 to 1995 recorded the greatest gains, concluding with a fourth period of modest progress.

Several events in each period marked milestones in the achievement of sustainability. The 1992 Rio Earth Summit declared that sustainability is central to the viability of nations and that we require immediate and concerted action on the concept and also scientific research. Agenda 21, the main outcome of the Summit, stressed the need for developing indicators of sustainability *"to provide solid bases for decision-making at all levels and to contribute to a self-regulating sustainability of integrated environmental and development systems"* (UNSD 1993, Section 40.04). Developing an integrated and widely accepted framework for the measurement of sustainability was a challenging task. The summit of 1992 was followed by the Rio Summit of 2012, which marked some additional progress towards sustainability (Wass et al. 2011). Several important issues were addressed at that meeting, such as climate change and replacement of fossil fuels, transportation, water resources, biodiversity, and desertification.

The 2000 Millennium Summit set the Millennium Development Goals (MDGs) to be achieved by the year 2015. On September 25th of that year, 193 member states of the United Nations adopted a set of 17 SDGs, which will guide the social, economic and environmental actions that all countries will take to achieve a sustainable future by the year 2030 (Sachs et al. 2016).

These 17 SDGs, unlike the MDGs, do not distinguish between 'developed' and 'developing' nations. Instead, the goals apply to all countries. These goals are interconnected and based on the principle of "leaving no one behind." Thus, an awareness of the urgency of conserving our planet is growing among most countries that have made pledges to follow a sustainable path.

The 2017 edition of the Sustainable Development Goals (SDG) Index and Dashboards Report provides a report card for each country regarding performance towards the 2030 Agenda. The annual report shows how leaders can deliver on their promises, and it urges countries not to lose their momentum for important reforms. The Spanish Network for Sustainable Development (REDS) presented the SDG Index and Dashboards Report 2017 in Madrid July 12, 2017, a global report prepared by the Sustainable Development Solutions Network (SDSN) and the Bertelsmann Stiftung Institute (Sachs et al. 2016). This report assesses the degree of compliance of the SDGs in 157 countries of the world, placing them in a global performance ranking, also allowing for regional comparison.

Sustainability is in general a function of precise data, such as concentrations of pollutants or GNI per capita, as well as vague variables, such as human rights or corruption. To handle vagueness and its concomitant uncertainty, fuzzy logic might be used as a way to emulate human thinking in a straightforward manner. Fuzzy logic is well suited to treat qualitative, imprecise, or uncertain information (Zadeh 1971). Sustainability Assessment by Fuzzy Evaluation (SAFE) is a model that assesses sustainability using fuzzy logic. Quantitative and qualitative input variables are converted into linguistic variables through membership functions. A system of fuzzy reasoning evaluates the various composite components of sustainability and sustainability as a whole via "if–then" rules. Finally, the output of the system is converted into a crisp value of sustainability by means of a defuzzification process (Grigoroudis et al. 2014). The model provides country rankings and performs sensitivity analyses that reveal key indices that each country should focus on to improve sustainability (Phillis et al. 2011; Grigoroudis et al. 2014).

It is clear from this brief exposition that sustainability is an issue of global dimensions. It is equally clear that scientific tools should be developed alongside existing ones that assess progress towards sustainability. The objective of this paper is to analyze the sustainability of the world's countries using a new multivariate analysis tool, which is then applied to two global models, the Sustainability Assessment by Fuzzy Evaluation (SAFE) and the United Nations Sustainable Development Goals (SDGs).More specifically, we propose a novel classification approach of countries regarding their sustainability standing. Models that measure sustainability rely on large databases leading to various indicators. It would be desirable to reduce dimensionality and summarize the information captured by a large number of variables in a simple way that can enable a straightforward depiction of the overall sustainability state of the world. Using biplots to depict data associated with many countries and many corresponding sustainability indicators proves to be quite useful in this regard (De Soete and Carroll 1994; Rocci et al. 2011).

Biplot methods are tools widely used to obtain a joint representation of objects or individuals, such as countries and relevant sustainability indicators, structured in a matrix space of reduced dimension to improve visualization and interpretation. An HJ-biplot is a multivariate graphical representation of a matrix $\mathbf{X}$, using markers (vectors) $j_1, j_2, \ldots j_n$ for its rows, and $h_1, h_2, \ldots h_p$ for its columns, chosen so that both markers can overlap in the same reference system with maximum representation quality (Galindo 1986).

The clusters disjoint HJ-biplot (Nieto et al. 2017) is based on ideas from Macedo and Freitas (2015), disjoint biplots (Vichi and Saporta 2009; Vigneau and Qannari 2004; Vines 2000), clustering biplots (De Sarbo and Heiser 1993; Vichi and Kiers 2001; Kiers et al. 2005; Rocci et al. 2011), HJ-biplots (Galindo 1986; Gallego et al. 2015), and biplot methods in Gabriel (1971).

The clustering disjoint HJ-biplot (CD Biplot) algorithm combines the k-means procedure used to form clusters with the HJ-biplot, which improves graphical data representation. The goal is to find the directions that maximize separation between centroids, which represent mean values of a set of point coordinates of P clusters of individuals (e.g., countries) found in the data, and to obtain a representation in an HJ-biplot. In a CD HJ-biplot, the extracted factorial axes are disjoint, that is, each variable (here the sustainability indicators by country) of the starting matrix only contributes to the solution of one axis with zero contributions to the other axes. This disjoint nature is achieved by dividing the total space into disjoint subspaces and extracting from each the direction of maximum variability across all variables. A brief exposition into CD HJ-biplots is given in Sect. 2 below.

The main contribution of CD HJ-biplots is their ability to present graphically a large amount of data, containing numerous entities such as countries and indicators, which is particularly useful in sustainability studies. Clustering results provided by CD HJ-biplots may help in sorting similar countries. Furthermore, since CD HJ-biplots reduce dimensionality, it is possible to group and/or select the most important indicators out of many. It should be noted that CD HJ-biplots are exploratory techniques that, unlike other sustainability assessment models, do not require focus on any specific parameter. For example, CD HJ-biplots may be used as a preliminary analysis tool to provide an overall picture of sustainability data, thus becoming useful complements to mathematical assessment models. Compared to dimensionality reduction techniques, as for example principal components analysis, all variables are used in CD HJ-biplots, thus, important information about correlations among these variables is not lost. Other advantages of CD HJ-biplots are outlined in Gallego et al. (2015).

2 CD HJ-biplots

Cluster analysis aims at depicting data according to certain characteristics of similarity. Each cluster, therefore, contains data that share a characteristic at a higher degree than data in other clusters. The mean of the data in a given cluster is called its centroid.

The goal of the CD HJ-biplot algorithm is to specify appropriate matrices for the coordinates of countries, indicators and centroids in the graph. To ensure an appropriate representation of the data, an alternating least squares (ALS) algorithm is used to solve a non-convex optimization problem by reducing it to a linear regression. This is done by fixing one matrix at a time while optimizing the other. The clustering disjoint biplot model results from the application of an HJ-biplot on the transformed data matrix, where each object is replaced by its centroid. The centroids are obtained by applying a k-means algorithm on the original data matrix. Each iteration of the algorithm has two steps: allocation of the objects by using the k-means algorithm followed by a search for a reduced space by using an HJ-biplot on the resulting centroids to obtain the J sustainability indicators that contribute to one of the Q components, or equivalently the axes in the CD HJ-biplot (Nieto et al. 2017).

To start the iterations set $k = 0$. Then consider countries first. A matrix $\mathbf{U}$ contains the I countries in its rows and the P clusters in its columns and allocates these countries into clusters. Then a matrix $\overline{\mathbf{X}}$ of the centroids in the original space is calculated together with a matrix $\mathbf{Z}$ that identifies the countries by cluster centroid.

Next, the J indicators are allocated into Q subsets via a stochastic binary matrix $\mathbf{V}_0$ of order $J \times Q$ where J is the number of sustainability indices and Q the number of components. $\mathbf{V}_0 = |u_{jq}|$, where $u_{jq} = 1$ if indicator j contributes to component q, and $u_{jq} = 0$ otherwise. Then, the coordinates of the indicators in the new space of disjoint components are computed by matrix $\mathbf{B}_0$, and the coordinates of the objects in the same space of the Q disjoint components by matrix $\mathbf{A}$ and the coordinates of the corresponding centroids by matrix $\overline{\mathbf{A}}$.

2.1 Stage 1: cluster of countries

The process starts from an original data matrix $\mathbf{X}$ of order $I \times J$ that contains the information of I countries over which J normalized sustainability indicators have been measured (*step a*).

Define the binary matrix $\mathbf{U}_0$ of order $I \times P$ that contains countries in its rows, and cluster numbers u_{ip} in its columns, such that $u_{ip} = 1$ if country i belongs to cluster p and $u_{ip} = 0$ otherwise (*step b*). This matrix is obviously stochastic.

The object centroid matrix $\overline{\mathbf{X}}_0$ of order $P \times J$ is generated so that the following squared error is minimized with respect to $\overline{\mathbf{X}}_0$ (for details see Nieto et al. 2017).

$$\left\| \mathbf{X} - \mathbf{U}_0 \bar{\mathbf{X}}_0 \right\|^2 \tag{1}$$

Straightforward differentiation yields (*step c*).

$$\overline{\mathbf{X}}_0 = \left(\mathbf{U}_0^T \mathbf{U}_0 \right)^{-1} \mathbf{U}_0^T \mathbf{X} \tag{2}$$

Next compute the matrix $\mathbf{Z}_0 = \mathbf{U}_0 \bar{\mathbf{X}}_0$, which contains the centroid values of the clusters to which each object belongs rather than the original $\mathbf{X}$ values (*step d*).

2.2 Stage 2: indicators

Define $\mathbf{V}_0$, the matrix with only one nonzero element per row, equal to 1 (*step e*). The nonzero elements of the q-th column of $\mathbf{V}_0$ identify the indicators that contribute to component q. Using $\mathbf{Z}_0$ and $\mathbf{V}_0$, the matrix $\mathbf{B}_0$ of the coordinates of the indicators is constructed column by column.

We form a submatrix $\mathbf{W}_{0_q}$ with the nonzero columns q of $\mathbf{V}_0$ (*step f*) These columns signify the indicators that contribute to component q. We then decompose $\mathbf{W}_{0_q}$ as follows

$$\mathbf{W}_{0_q} = \mathbf{R} \overline{\boldsymbol{\Lambda}} \mathbf{T}^T \tag{3}$$

where $\mathbf{R}$ and $\mathbf{T}^T$ are orthonormal matrices (their columns are orthogonal and their norm equals 1), and $\overline{\boldsymbol{\Lambda}}$ the diagonal matrix of eigenvalues of the q-th decomposition. The coordinates of the indicators in the HJ-biplot are $\mathbf{B}_{0_q} = \mathbf{T}\overline{\boldsymbol{\Lambda}}$ (*step g*).

The coordinates of countries and corresponding centroids are (*steps h* and *i*, respectively):

$$\mathbf{A}_0 = \mathbf{X}\mathbf{B}_0\overline{\mathbf{\Lambda}}_0^{-1} \tag{4}$$

$$\overline{\mathbf{A}}_0 = \overline{\mathbf{X}}\mathbf{B}_0\overline{\mathbf{\Lambda}}_0^{-1} \tag{5}$$

where $\overline{\mathbf{\Lambda}}_0$ is the usual diagonal of eigenvalues for the q-th decomposition.

Finally, the value of the objective function $F_0 = \|\mathbf{U}_0\overline{\mathbf{A}}_0\|^2$ is computed.

2.3 *k*-th iteration

Given $\mathbf{U}_{k-1}$, $\overline{\mathbf{X}}_{k-1}$, $\mathbf{Z}_{k-1}$, $\mathbf{A}_{k-1}$, $\overline{\mathbf{A}}_{k-1}$, $\mathbf{V}_{k-1}$, $\mathbf{\Lambda}_{k-1}$, $\mathbf{B}_{k-1}$, and the objective function F_{k-1} at step $k-1$, we proceed as follows:

2.3.1 Cluster of countries

$\mathbf{U}_k$ is updated via the coordinate matrix of countries $\mathbf{A}_{k-1}$ and the matrix of centroid coordinates $\overline{\mathbf{A}}_{k-1}$ through a k-means algorithm in the reduced space. Each country is thus assigned to the closest centroid. Then the following matrices are updated $\overline{\mathbf{X}}_k = \left(\mathbf{U}_k^T\mathbf{U}_k\right)^{-1}\mathbf{U}_k^T\mathbf{X}$ and $\mathbf{Z}_k = \mathbf{U}_k\overline{\mathbf{X}}_k$. If a cluster is empty, the procedure in the initial iteration is repeated.

2.3.2 Sustainability indicators

Now we update $\mathbf{V}_k$. Consider row j. Rows $1, \ldots, j-1,\ j+1, \ldots, J$ are fixed, while all elements of row j are set equal to zero. The nonzero element of row j is positioned in all Q positions, thus constructing Q different matrices $\overline{\mathbf{V}}_{kq}$. The matrices $\mathbf{A}_k$, $\overline{\mathbf{A}}_k$, and $\mathbf{B}_k$, as well as F_k are computed using each of the constructed $\overline{\mathbf{V}}_{kq}$. We choose the nonzero element that yields the maximum of the objective function and fix its position in row j. This procedure is repeated for the remaining rows of $\mathbf{V}_k$. Having the updated $\mathbf{V}_k$, we go back to step 2 to update $\mathbf{A}_k$, $\overline{\mathbf{A}}_k$, $\mathbf{\Lambda}_k$, and $\mathbf{B}_k$.

2.4 Stopping

The stopping criterion is set at a difference between F_k and F_{k-1} less than or equal to 10^{-6}. To avoid entrapment, the algorithm is run several times at a minimum of 1000 to find a stable solution (Nieto et al. 2017). Figure 1 shows a graphical representation of the CD HJ-biplot algorithm, including the aforementioned steps.

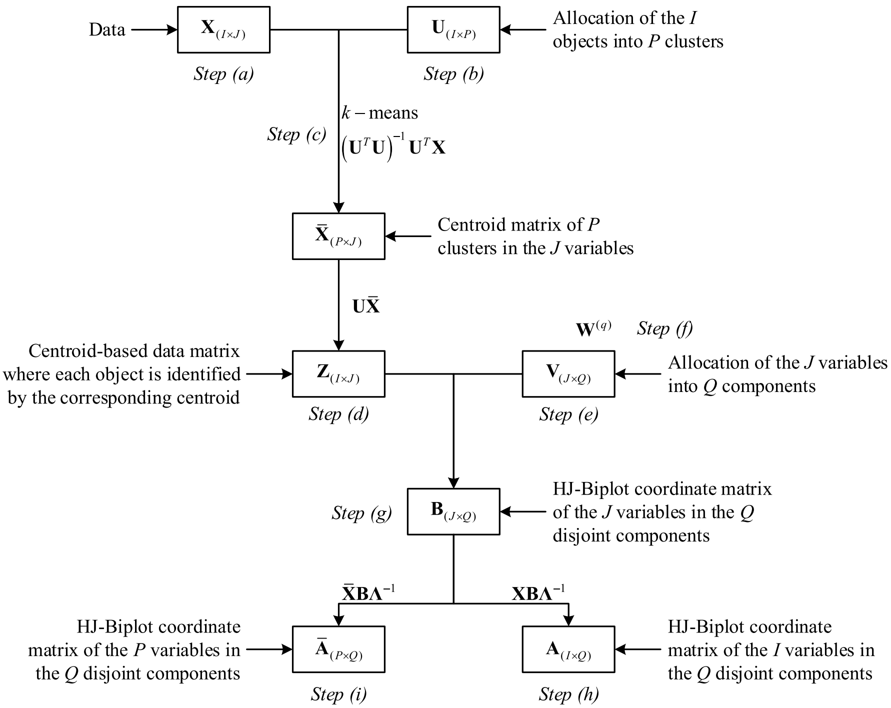

Fig. 1 The CD HJ-biplot algorithm

3 Outline of SAFE

Details of SAFE can be found in Phillis et al. (2011) and Grigoroudis et al. (2014). SAFE uses basic indicators of environmental integrity, economic efficiency and social welfare. Via statistical analysis and fuzzy reasoning, SAFE determines measures of human, ecological and overall sustainability. Data about basic indicators such as emissions are passed through an exponential smoothing filter to account for memory of past performance and then are normalized on [0, 1] according to their sustainability standing, where 0 corresponds to totally unsustainable and 1 to totally sustainable values. Missing data are generated via an imputation procedure. Next a multistage fuzzy inference engine is used together with pertinent rule bases to obtain fuzzy values for composite sustainability variables. A height defuzzification procedure yields crisp sustainability numbers at each stage. The final number of overall sustainability is used to rank countries. Finally, a sensitivity analysis reveals those indicators that have the greatest potential of improving sustainability. It should be stressed that the model is flexible in that its indicators can change in number and importance according to reality.

Several basic indicators are used to compute the four components of the ecosystem dimension, air, land, water, and biodiversity, and the four components of the human system dimension, policies, wealth, health, and knowledge. Finally, an index OSUS of overall sustainability in [0, 1] is derived for each country. The hierarchical structure of indicators in the SAFE model is outlined in Fig. 2.

The database of SAFE 2018 covers 68 basic indicators for a period of 26 years (1990–2016). The initial raw data after exponential smoothing and normalization were used to run the CD HJ-biplot. Data about indicators such as CO_2 emissions, municipal waste, threatened species, immunizations etc., were collected from such sources as Eurostat, World Health Organization, Organization for Economic Co-operation and Development, United Nations, World Bank and similar authoritative entities.

The data are normalized over [0, 1]. The value 0 corresponds to a completely unsustainable indicator while 1 corresponds to a completely sustainable one. Intermediate values are computed via linear interpolation. To avoid relying on only the most recent value for each indicator which might exhibit a sudden misleading temporary variation, time series are used over a period of time and a smoothed value is extracted by a Holt-Winters algorithm. Furthermore, since some indicators lack values a sophisticated statistical imputation procedure generates those missing values. A complete list of indicators and sources as well as the detailed mathematical analysis at all steps of the computation of sustainability are given in Grigoroudis et al. (2014) and the website www.sustainability.tuc.gr.

Additionally, the UN-SDGs index database was used to run the biplot. Details about this database can be found in Sachs et al. (2016). It contains 77 indices for 149 countries for the year 2016, covering the following sustainability goals:

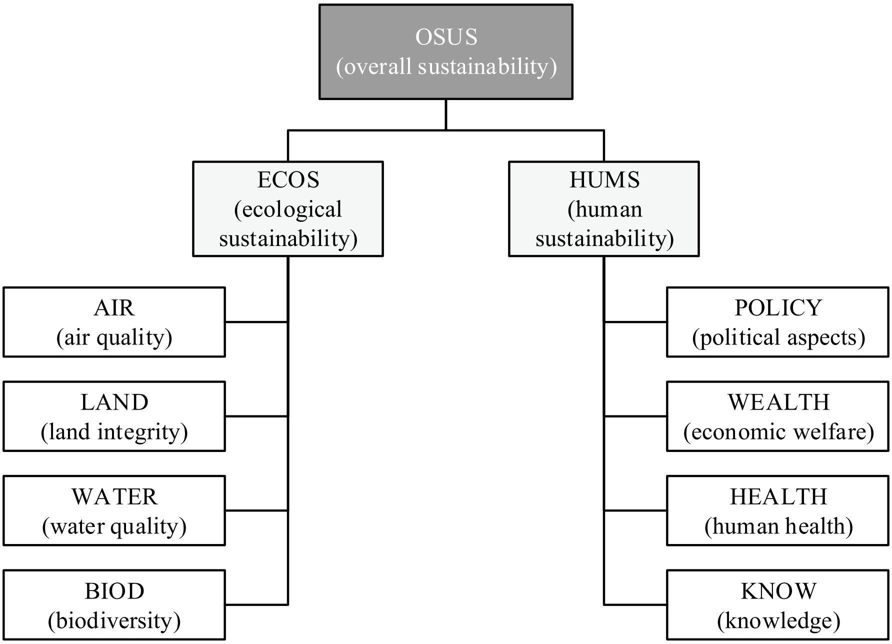

Fig. 2 The SAFE model

1. No poverty
2. Zero hunger
3. Good health and well-being
4. Quality education
5. Gender equality
6. Clean water and sanitation
7. Affordable and clean energy
8. Decent work and economic growth
9. Industry, innovation and infrastructure
10. Reduced inequalities
11. Sustainable cities and communities
12. Responsible consumption and production
13. Climate action
14. Life below water
15. Life on land
16. Peace, justice and strong institution
17. Partnership to achieve goals

As with SAFE, indicator data are normalized over [0, 1] and then aggregated using an arithmetic or geometric mean. The arithmetic mean is used in Sect. 4 since its results differ little from those of the geometric mean.

4 Results

4.1 Clustering disjoint biplot analysis for SAFE

The sustainability rankings of SAFE 2018 are shown in "Appendix 1". Its data base generated the CD HJ-biplot in Fig. 3. The biplot placed the top 35 countries of SAFE in the lower left quadrant of Fig. 3, forming group 1 (Table 1).

Table 2 shows the most influential indicators in the negative X and positive Y axes, all belonging to the human dimension, except for population growth and forest area. The bottom 30 countries were placed in the upper right quadrant of Fig. 3 and form group 3 (Table 1). Again, Table 2 shows the most influential indicators along the positive X and negative Y axes, which now belong to both human and ecosystem dimensions.

The first axis of the CD HJ-biplot is characterized by both human and ecological sustainability indicators (Table 2). More specifically, the positive side of factorial axis 1 is mainly related to environmental sustainability variables (NOx emissions, pesticide consumption, renewables, protected areas, etc.), while the negative side is mainly associated with human sustainability indicators, mostly health and knowledge indicators, such as school enrolment, mean years of schooling, student–teacher ratio, infant or maternal mortality, life expectancy, access to improved water sources and to improved sanitation, etc. Similarly, the positive side of factorial axis 2 is related to human sustainability indicators of health and economy, such as cardiovascular incidences and government debt, as well as environmental indicators, such

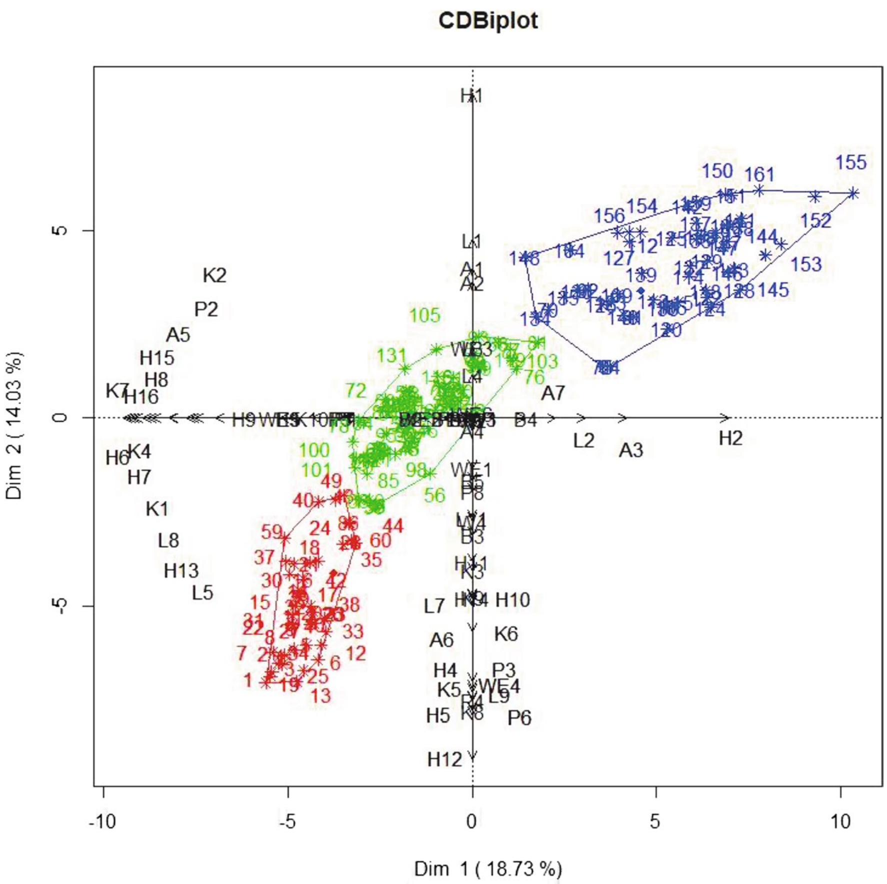

Fig. 3 CD HJ-biplot representation of sustainability indicators and countries 2018

as CO_2 and SO_2 emissions, hazardous wastes, fertilizer consumption, and SLR land impact. The negative side of axis 2 is linked with human sustainability indicators, mainly health, policy and knowledge. These findings, in addition to characterizing the axes of the CD HJ-biplot, may reveal potential correlations between the aforementioned sustainability indicators.

Moreover, the CD HJ-biplot revealed three country groups (Fig. 3). Group 1 is located in the lower left quadrant and consists of the most sustainable countries. These countries have high performance (low values) in major human sustainability indicators, e.g., student–teacher ratio, infant or maternal mortality, malaria or tuberculosis incidences, corruption, civil liberties, political rights and schooling years gender gap. On the other hand, group 3 is located in the upper right quadrant and contains the least sustainable countries, which are characterized by low performance (high values) in specific environmental sustainability indicators, e.g., CO_2, SO_2 and NOx emissions, pesticide consumption, hazardous wastes, fertilizer consumption, and SLR land impact together with the

Table 1 Clustering membership for 161 countries worldwide (SAFE indicators)

	Country	Cl		Country	Cl		Country	Cl		Country	Cl
1	Denmark	1	42	Israel	1	83	Sri Lanka	2	124	Zambia	3
2	Norway	1	43	Brunei	1	84	Armenia	2	125	Cameroon	3
3	Sweden	1	44	Russia	1	85	Trinidad and Tobago	2	126	Cambodia	3
4	Switzerland	1	45	Moldova	2	86	Kuwait	1	127	Djibouti	3
5	United Kingdom	1	46	Brazil	2	87	Bosnia and Herz.	2	128	Sierra Leone	3
6	Austria	1	47	Thailand	2	88	Honduras	2	129	Togo	3
7	Netherlands	1	48	Venezuela	2	89	Colombia	2	130	India	3
8	Finland	1	49	Cuba	1	90	Tajikistan	2	131	Libya	2
9	Slovenia	1	50	Turkey	2	91	Zimbabwe	3	132	Papua N.G.	3
10	Iceland	1	51	Peru	2	92	Lao PDR	3	133	Burkina Faso	3
11	France	1	52	Fiji	2	93	Indonesia	2	134	Guatemala	2
12	Ireland	1	53	Tunisia	2	94	El Salvador	2	135	Bangladesh	3
13	Germany	1	54	Azerbaijan	2	95	China	2	136	Tanzania	3
14	Poland	1	55	Mexico	2	96	Turkmenistan	2	137	Mali	3
15	Czech Rep.	1	56	Mongolia	2	97	Jordan	2	138	Liberia	3
16	Slovakia	1	57	Argentina	2	98	Seychelles	2	139	Gambia	3
17	Lithuania	1	58	Morocco	2	99	Belize	2	140	Myanmar	3
18	Hungary	1	59	South Korea	1	100	Saudi Arabia	2	141	Ethiopia	3
19	Luxembourg	1	60	Singapore	1	101	Qatar	2	142	Nigeria	3
20	Portugal	1	61	Malaysia	2	102	Uzbekistan	2	143	Madagascar	3
21	Latvia	1	62	North Macedonia	2	103	Botswana	2	144	Guinea	3
22	Australia	1	63	Kyrgyzstan	2	104	Nepal	3	145	Mozambique	3
23	Spain	1	64	Ghana	3	105	Algeria	2	146	Burundi	3
24	Croatia	1	65	Paraguay	2	106	Congo Rep.	3	147	Uganda	3

Table 1 (continued)

	Country	Cl		Country	Cl		Country	Cl		Country	Cl
25	Estonia	1	66	Serbia	1	107	Lebanon	2	148	Iraq	3
26	Belgium	1	67	Ecuador	2	108	Iran	2	149	Angola	3
27	Italy	1	68	Cape Verde	2	109	Senegal	3	150	Niger	3
28	Uruguay	1	69	Dominican Rep.	2	110	Un. Arab Emirates	1	151	Congo D.R.	3
29	New Zealand	1	70	Gabon	3	111	Jamaica	2	152	Chad	3
30	Cyprus	1	71	Guyana	2	112	Eq. Guinea	3	153	Guinea-Bissau	3
31	Japan	1	72	Ukraine	2	113	Kenya	3	154	Pakistan	3
32	Malta	1	73	Georgia	2	114	Lesotho	3	155	Central African Rep.	3
33	Canada	1	74	Suriname	2	115	Benin	3	156	Yemen	3
34	United States	1	75	Namibia	2	116	Egypt	2	157	Eritrea	3
35	Chile	1	76	Nicaragua	2	117	Bahrain	2	158	Haiti	3
36	Costa Rica	2	77	Bolivia	2	118	Côte d'Ivoire	3	159	Mauritania	3
37	Greece	1	78	Mauritius	2	119	South Africa	2	160	Sudan	3
38	Bulgaria	1	79	Kazakhstan	2	120	Rwanda	3	161	Afghanistan	3
39	Romania	2	80	Panama	2	121	Oman	2			
40	Belarus	1	81	Philippines	2	122	Malawi	3			
41	Albania	2	82	Vietnam	2	123	Swaziland	3			

Table 2 Contribution of variables to each axis (SAFE indicators)

	Positive	Negative
Factorial axis 1	H2 (+ 8.4) [Neoplastic incidence], A3 (+ 5.063) [NOx emissions], L2 (+ 3.711) [Pesticides], A7 (+ 2.732) [Renewables], B4 (+ 1.768) [Mountain protection KBA]	K7 (− 11.418) [Secondary school enrollment], H6 (− 11.308) [Infant mortality], H16 (− 11.288) [Access sanitation], K4 (− 11.163) [Mean years schooling], H8 (− 11.069) [Life expectancy], H7 (− 10.782) [Maternal mortality], H15 (− 10.616) [Access safe water], K1 (− 10.477) [Primary student–teacher], A5 (− 9.934) [Mortality household], L8 (− 9.906) [Municipal waste collected], P2 (− 9.356) [Undernourishment], H13 (− 9.212) [Hospital beds], K2 (− 9.077) [Secondary student–teacher], L5 (− 8.4) [Pop growth], H9 (− 7.438) [Immunization DPT]
Factorial axis 2	H1 (+ 10.412) [Cardiovascular incidence], L1 (+ 5.728) [Hazardous w], A1 (+ 4.815) [CO_2_emissions], A2 (+ 4.361) [SO_2_emissions], L3 (+ 2.27) [Fertilizers], E3 (+ 2.269) [Government debt], L4 (+ 1.417) [SLR land impact]	H12 (− 10.916) [Physicians], P6 (− 9.525) [Corruption], K8 (− 9.387) [Literacy rate], H5 (− 9.346) [Malaria incidence], L9 (− 9.056) [Municipal waste recycled], P4 (− 9.042) [Civil liberties], K5 (− 9.005) [Schooling years gender gap], E4(− 8.788) [GNI], P3 (− 8.674) [Political rights], H4 (− 8.425) [Tuberculosis incidence], K6 (− 6.834) [Primary school enrollment], A6 (− 6.822) [PM2.5], L7 (− 5.939) [Forest area], H10 (− 5.789) [Immunization measles], H14 (− 5.764) [Health expenditure], K9 (− 5.7.16) [RD expenditure]

human sustainability indicators cardiovascular incidences and government debt. Finally, group 2 consists of countries having a moderate performance in the previous sustainability indicators.

According to Table 1, the clusters of the CD HJ-biplot agree with the overall ranking of the SAFE model. More specifically, the SAFE ranking and the CD HJ-biplot clusters are consistent for 85.7% of the countries. Additionally, the average SAFE scores of group 1, 2 and 3 are 74.60, 59.94, and 46.65, respectively. Some inconsistencies may be justified by the overall variance explained by the generated CD HJ-biplot which is approximately 33%.

4.2 Clustering disjoint HJ-biplot analysis for UN-SDGs index database

The sustainability rankings of UN-SDGs are shown in "Appendix 2" (see also Sachs et al. 2016). The corresponding data base generated the CD HJ-biplot in Fig. 4. The biplot placed in the lower left quadrant the top 18 countries called group 1 in Table 3. Similarly, the bottom 41 countries located in the upper right quadrant were placed into group 3. Table 4 extracts the most influential indicators in the negative *X* and *Y* axes.

As in the previous biplot (Fig. 3), the positive side of the first axis in Fig. 4 is characterized mainly by high values of sustainability indicators related to zero hunger, such as prevalence of stunting or wasting in children under 5 years of age, as well as good health and well-being: neonatal mortality rate, tuberculosis incidences, rate of traffic-related deaths, etc. The negative side of axis 2 is mainly related to economic growth indicators, such as employment, quality education (PISA score, expected years of schooling) and well-being (healthy life expectancy at birth, daily smokers). The

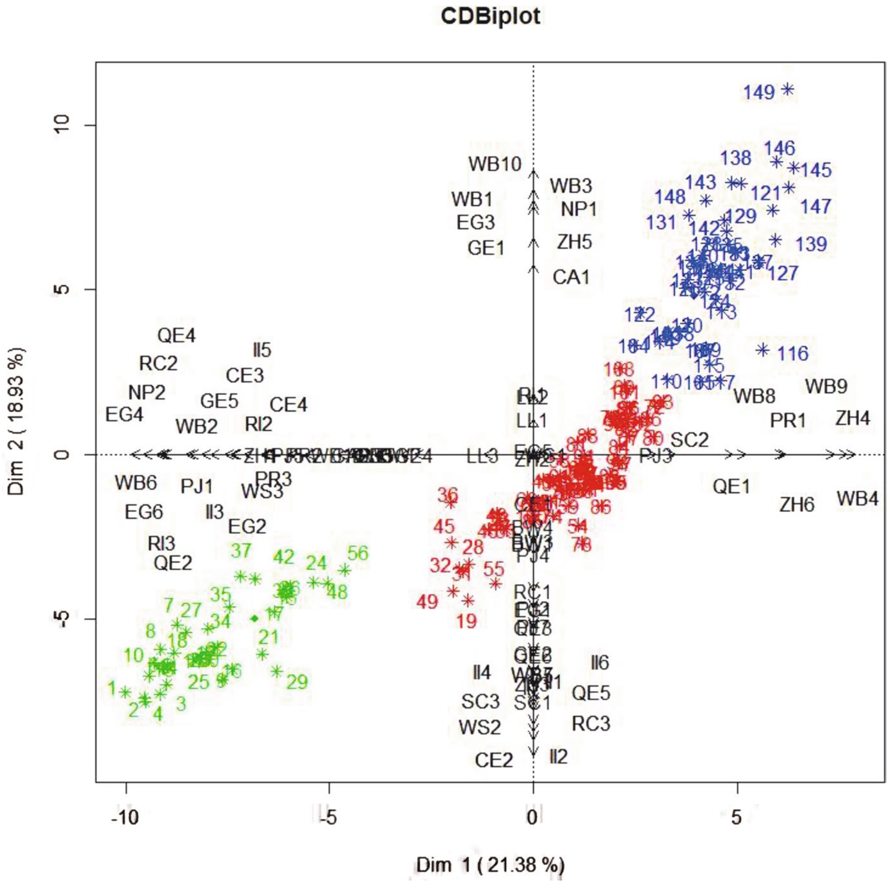

Fig. 4 CD HJ-biplot representation of UN-SDGs index

Table 3 Clustering membership for 149 countries worldwide (UN-SDGs)

	Country	Cl		Country	Cl		Country	Cl		Country	Cl
1	Sweden	1	39	Serbia	1	77	Jamaica	2	115	Pakistan	3
2	Denmark	1	40	Uruguay	2	78	Trinidad and Tobago	2	116	Swaziland	3
3	Norway	1	41	Romania	2	79	Iran	2	117	Myanmar	3
4	Finland	1	42	Chile	2	80	Botswana	2	118	Bangladesh	3
5	Switzerland	1	43	Argentina	2	81	Peru	2	119	Cambodia	3
6	Germany	1	44	Moldova	2	82	Bhutan	2	120	Kenya	3
7	Austria	1	45	Cyprus	2	83	Algeria	2	121	Angola	3
8	Netherlands	1	46	Ukraine	1	84	Mongolia	2	122	Rwanda	3
9	Iceland	1	47	Russia	2	85	Saudi Arabia	2	123	Uganda	3
10	United Kingdom	1	48	Turkey	2	86	Lebanon	2	124	Côte d'Ivoire	3
11	France	1	49	Qatar	2	87	Suriname	2	125	Ethiopia	3
12	Belgium	1	50	Armenia	2	88	Vietnam	2	126	Tanzania	3
13	Canada	1	51	Tunisia	2	89	Bolivia	2	127	Sudan	3
14	Ireland	1	52	Brazil	2	90	Nicaragua	2	128	Burundi	3
15	Czech Rep.	1	53	Costa Rica	2	91	Colombia	2	129	Togo	3
16	Luxembourg	1	54	Kazakhstan	2	92	Dominican Rep.	2	130	Benin	3
17	Slovenia	1	55	United Arab Emirates	1	93	Gabon	3	131	Malawi	3
18	Japan	1	56	Mexico	1	94	El Salvador	2	132	Mauritania	3
19	Singapore	1	57	Georgia	2	95	Philippines	2	133	Mozambique	3
20	Australia	2	58	North Macedonia	2	96	Cape Verde	2	134	Zambia	3
21	Estonia	1	59	Jordan	2	97	Sri Lanka	2	135	Mali	3
22	New Zealand	1	60	Montenegro	2	98	Indonesia	2	136	Gambia	3
23	Belarus	2	61	Thailand	2	99	South Africa	2	137	Yemen	3
24	Hungary	1	62	Venezuela	2	100	Kuwait	2	138	Sierra Leone	3

Table 3 (continued)

	Country	Cl		Country	Cl		Country	Cl		Country	Cl
25	United States	2	63	Malaysia	2	101	Guyana	2	139	Afghanistan	3
26	Slovakia	1	64	Morocco	2	102	Honduras	2	140	Madagascar	3
27	South Korea	1	65	Azerbaijan	2	103	Nepal	3	141	Nigeria	3
28	Latvia	1	66	Egypt	2	104	Ghana	3	142	Guinea	3
29	Israel	1	67	Kyrgyzstan	2	105	Iraq	2	143	Burkina Faso	3
30	Spain	1	68	Albania	2	106	Guatemala	2	144	Haiti	3
31	Lithuania	1	69	Mauritius	2	107	Lao PDR	3	145	Chad	3
32	Malta	2	70	Panama	2	108	Namibia	2	146	Niger	3
33	Bulgaria	2	71	Ecuador	2	109	Zimbabwe	3	147	Congo D.R.	3
34	Portugal	2	72	Tajikistan	2	110	India	3	148	Liberia	3
35	Italy	1	73	Bosnia and Herzegovina	2	111	Congo Rep.	3	149	Central African Rep.	3
36	Croatia	2	74	Oman	2	112	Cameroon	3			
37	Greece	1	75	Paraguay	2	113	Lesotho	3			
38	Poland	2	76	China	2	114	Senegal	3			

Table 4 Contribution of variables to each axis (UN-SDGs)

	Positive	Negative
Factorial axis 1	ZH4 (+ 9.027) [Prevalence of stunting (low height-for-age) in children under 5 years of age (%)], WB4 (+ 8.8) [Neonatal mortality rate (per 1000 live births)], WB9 (+ 8.442) [Traffic deaths rate (per 100,000 people)], ZH6 (+ 7.093) [Prevalence of wasting in children under 5 years of age (%)], PR1 (+ 7.014) [For all other countries: Tax revenue (% of GDP)], WB8 (+ 5.994) [Incidence of tuberculosis (per 100,000 people)], QE1 (+ 5.595) [Literacy rate of 15–24 year olds, both sexes (%)], SC2 (+ 3.987) [Annual mean concentration of particulate matter of less than 2.5 microns of diameter (PM2.5) (μg/m3) in urban areas],PJ3 (+ 3.438) [Homicides (per 100,000 people)]	EG4 (− 11.261) [Employment-to-population ratio (%)], WB6 (− 10.914) [Daily smokers (% of population aged 15 +)], NP2 (− 10.564) [Poverty rate after taxes and transfers, poverty line 50% (% of population)], EG6 (− 10.474) [Youth not in employment, education or training (NEET) (%)], RC2 (− 10.432) [Non-recycled municipal solid waste (kg/person/year)], RI3 (− 10.36) [Palma ratio], QE4 (− 9.73) [Expected years of schooling (years)], QE2 (− 9.616) [PISA score (0–600)], WB2 (− 9.29) [Healthy life expectancy at birth (years)], PJ1 (− 9.036) [Corruption perception index (0–100)], GE5 (− 8.644) [Gender wage gap (% of male median wage)], II3 (− 8.488) [Logistics Performance index: Quality of trade and transport-related infrastructure], CE3 (− 8.446) [CO_2 emissions from fuel combustion and electricity output (MtCO2/TWh)]
Factorial axis 2	WB10 (+ 9.877) [Mortality rate, under-5 (per 1000 live births)], WB3 (+ 9.186) [Maternal mortality rate (per 100,000 live births)], WB1 (+ 8.798) [Adolescent fertility rate (births per 1000 women ages 15–19)], NP1 (+ 8.789) [Poverty headcount ratio at $1.90 a day (2011 PPP) (% of population)], EG3 (+ 8.605) [Percentage of children 5–14 years old involved in child labor (%)], ZH5 (+ 7.479) [Prevalence of undernourishment (% of population)], GE1 (+ 7.478) [Estimated demand for contraception that is unmet (% of women married or in union, ages 15–49)], CA1 (+ 6.588) [Climate change vulnerability monitor]	II2 (− 10.433) [Proportion of the population using the internet (%)], CE2 (− 9.889) [Access to non-solid fuels (% of population)], WS2 (− 9.618) [Access to improved sanitation facilities (% of population)], RC3 (− 9.34) [Municipal solid waste (kg/year/capita)], SC3 (− 8.877) [Rooms per person], QE5 (− 8.706) [Population aged 25–64 with tertiary education (%)], SC1 (− 8.582) [Improved water source, piped (% of urban population with access)], II6 (− 8.356) [Research and development researchers (per 1000 employed)], II4 (− 8.335) [Mobile broadband subscriptions (per 100 inhabitants)], II1 (− 8.313) [Quality of overall infrastructure], II7 (− 8.082) [Research and development expenditure (% of GDP)]

positive side of factorial axis 2 is related to well-being indicators, such as adolescent fertility rate, maternal mortality rate and prevalence of undernourishment, while the negative side is mainly associated with industry, innovation, and infrastructure indicators, as for example quality of overall infrastructure, proportion of the population using the internet, mobile broadband subscriptions, number of R&D researchers and R&D expenditure.

The country grouping in Fig. 4 is quite similar to that in Fig. 3: 90% of the countries that are categorized into a specific group according to SAFE are categorized into the same group based on UN-SDGs variables. More specifically, group 1 (in the lower left quadrant) consists of the most sustainable countries, group 3 (in the upper right quadrant) the least sustainable countries, and group 2 (close to the origin of axes) consists of countries with a moderate sustainability performance.

The UN-SDGs ranking and the CD HJ-biplot clustering are consistent for 91.3% of the countries. The average UN-SDGs scores of group 1, 2 and 3 are 75.14, 61.27, and 41.31, respectively, showing that the CD HJ-biplot reproduces the UN-SDGs rankings satisfactorily. Any inconsistencies of Table 3 are justified by the fact that the CD HJ-biplot explains approximately 40% of the variance of the initial dataset.

5 Conclusions

CD HJ-biplots to portray graphically the sustainability position of a large number of countries are a useful complement to mathematical models of sustainability, just as graphs complement equations. Graphical information could be useful to planners it shows directly how countries are grouped according to sustainability together with the most related indicators. Thus, planners can prioritize social, environmental and economic policies and make the most effective decisions.

In a sense, CD HJ-biplots verify statistically the findings of SAFE and UN-SDGs. However, one could go one step further and investigate relationships of indicators that could be excluded from these indices in order to reduce the dimensionality of the original models. Additionally, deviations of rankings between biplots and SAFE or UN-SDGs could serve as venues for possible improvements of these models. All these are subjects for future research.

As a general conclusion, a large number of countries still remain in the areas of moderate or low sustainability. One could graphically observe the dynamic evolution of sustainability worldwide over time with a graphical approach used to draw relevant conclusions. In an era of climate change, species extinction, poverty, and environmental migration, such observations could aid political decision making regarding the future of our planet.

Appendix 1: SAFE 2018 sustainability ranking of countries: data for 1990–2016 (Grigoroudis et al. 2019)

	Country	SAFE		Country	SAFE		Country	SAFE
1	Denmark	0.8734	55	Mexico	0.6339	109	Senegal	0.5444
2	Norway	0.8686	56	Mongolia	0.6328	110	Un. Arab Emirates	0.5438
3	Sweden	0.8630	57	Argentina	0.6309	111	Jamaica	0.5387
4	Switzerland	0.8615	58	Morocco	0.6308	112	Eq. Guinea	0.5315
5	United Kingdom	0.8384	59	South Korea	0.6296	113	Kenya	0.5295
6	Austria	0.8296	60	Singapore	0.6278	114	Lesotho	0.5289
7	Netherlands	0.8259	61	Malaysia	0.6274	115	Benin	0.5217
8	Finland	0.8231	62	North Macedonia	0.6266	116	Egypt	0.5190
9	Slovenia	0.8192	63	Kyrgyzstan	0.6265	117	Bahrain	0.5187
10	Iceland	0.8141	64	Ghana	0.6259	118	Côte d'Ivoire	0.5147
11	France	0.8078	65	Paraguay	0.6259	119	South Africa	0.5104
12	Ireland	0.8032	66	Serbia	0.6250	120	Rwanda	0.5085
13	Germany	0.7975	67	Ecuador	0.6240	121	Oman	0.5041
14	Poland	0.7835	68	Cape Verde	0.6230	122	Malawi	0.5021
15	Czech Rep.	0.7834	69	Dominican Rep.	0.6226	123	Swaziland	0.5011
16	Slovakia	0.7790	70	Gabon	0.6224	124	Zambia	0.5007
17	Lithuania	0.7743	71	Guyana	0.6224	125	Cameroon	0.5006
18	Hungary	0.7731	72	Ukraine	0.6213	126	Cambodia	0.5002
19	Luxembourg	0.7681	73	Georgia	0.6204	127	Djibouti	0.4984
20	Portugal	0.7565	74	Suriname	0.6203	128	Sierra Leone	0.4918
21	Latvia	0.7546	75	Namibia	0.6184	129	Togo	0.4893
22	Australia	0.7525	76	Nicaragua	0.6164	130	India	0.4884
23	Spain	0.7518	77	Bolivia	0.6158	131	Libya	0.4852
24	Croatia	0.7517	78	Mauritius	0.6144	132	Papua N.G.	0.4803
25	Estonia	0.7517	79	Kazakhstan	0.6113	133	Burkina Faso	0.4711
26	Belgium	0.7496	80	Panama	0.6109	134	Guatemala	0.4709
27	Italy	0.7457	81	Philippines	0.6096	135	Bangladesh	0.4688
28	Uruguay	0.7451	82	Vietnam	0.6058	136	Tanzania	0.4610
29	New Zealand	0.7431	83	Sri Lanka	0.6026	137	Mali	0.4586
30	Cyprus	0.7355	84	Armenia	0.5949	138	Liberia	0.4543
31	Japan	0.7280	85	Trinidad and Tobago	0.5940	139	Gambia	0.4463
32	Malta	0.7270	86	Kuwait	0.5934	140	Myanmar	0.4457
33	Canada	0.7217	87	Bosnia and Herz.	0.5910	141	Ethiopia	0.4418
34	United States	0.7157	88	Honduras	0.5906	142	Nigeria	0.4364
35	Chile	0.7150	89	Colombia	0.5891	143	Madagascar	0.4343
36	Costa Rica	0.7150	90	Tajikistan	0.5828	144	Guinea	0.4289
37	Greece	0.7138	91	Zimbabwe	0.5818	145	Mozambique	0.4087
38	Bulgaria	0.7067	92	Lao PDR	0.5794	146	Burundi	0.4072
39	Romania	0.7000	93	Indonesia	0.5765	147	Uganda	0.4030

	Country	SAFE		Country	SAFE		Country	SAFE
40	Belarus	0.6724	94	El Salvador	0.5762	148	Iraq	0.3924
41	Albania	0.6685	95	China	0.5750	149	Angola	0.3909
42	Israel	0.6630	96	Turkmenistan	0.5749	150	Niger	0.3881
43	Brunei	0.6585	97	Jordan	0.5748	151	Congo D.R.	0.3856
44	Russia	0.6542	98	Seychelles	0.5723	152	Chad	0.3817
45	Moldova	0.6459	99	Belize	0.5719	153	Guinea-Bissau	0.3783
46	Brazil	0.6456	100	Saudi Arabia	0.5660	154	Pakistan	0.3773
47	Thailand	0.6454	101	Qatar	0.5659	155	Central African Rep.	0.3767
48	Venezuela	0.6440	102	Uzbekistan	0.5659	156	Yemen	0.3766
49	Cuba	0.6439	103	Botswana	0.5635	157	Eritrea	0.3713
50	Turkey	0.6426	104	Nepal	0.5625	158	Haiti	0.3648
51	Peru	0.6418	105	Algeria	0.5588	159	Mauritania	0.3563
52	Fiji	0.6381	106	Congo Rep.	0.5507	160	Sudan	0.3500
53	Tunisia	0.6373	107	Lebanon	0.5506	161	Afghanistan	0.3267
54	Azerbaijan	0.6343	108	Iran	0.5464			

Appendix 2: UN-SDGs index ranking of countries: data for 2016 (Sachs et al. 2016)

	Country	Score		Country	Score		Country	Score
1	Sweden	84.5	51	Tunisia	65.1	101	Guyana	52.4
2	Denmark	83.9	52	Brazil	64.4	102	Honduras	51.8
3	Norway	82.3	53	Costa Rica	64.2	103	Nepal	51.5
4	Finland	81.0	54	Kazakhstan	63.9	104	Ghana	51.4
5	Switzerland	80.9	55	Un. Arab Emirates	63.6	105	Iraq	50.9
6	Germany	80.5	56	Mexico	63.4	106	Guatemala	50.0
7	Austria	79.1	57	Georgia	63.3	107	Lao PDR	49.9
8	Netherlands	78.9	58	North Macedonia	62.8	108	Namibia	49.9
9	Iceland	78.4	59	Jordan	62.7	109	Zimbabwe	48.6
10	United Kingdom	78.1	60	Montenegro	62.5	110	India	48.4
11	France	77.9	61	Thailand	62.2	111	Congo Rep.	47.2
12	Belgium	77.4	62	Venezuela	61.8	112	Cameroon	46.3
13	Canada	76.8	63	Malaysia	61.7	113	Lesotho	45.9
14	Ireland	76.7	64	Morocco	61.6	114	Senegal	45.8
15	Czech Rep.	76.7	65	Azerbaijan	61.3	115	Pakistan	45.7
16	Luxembourg	76.7	66	Egypt	60.9	116	Swaziland	45.1
17	Slovenia	76.6	67	Kyrgyzstan	60.9	117	Myanmar	44.5
18	Japan	75.0	68	Albania	60.8	118	Bangladesh	44.4
19	Singapore	74.6	69	Mauritius	60.7	119	Cambodia	44.4

	Country	Score		Country	Score		Country	Score
20	Australia	74.5	70	Panama	60.7	120	Kenya	44.0
21	Estonia	74.5	71	Ecuador	60.7	121	Angola	44.0
22	New Zealand	74.0	72	Tajikistan	60.2	122	Rwanda	44.0
23	Belarus	73.5	73	Bosnia and Herz.	59.9	123	Uganda	43.6
24	Hungary	73.4	74	Oman	59.9	124	Côte d'Ivoire	43.5
25	United States	72.7	75	Paraguay	59.3	125	Ethiopia	43.1
26	Slovakia	72.7	76	China	59.1	126	Tanzania	43.0
27	South Korea	72.7	77	Jamaica	59.1	127	Sudan	42.2
28	Latvia	72.5	78	Trinidad and Tobago	59.1	128	Burundi	42.0
29	Israel	72.3	79	Iran	58.5	129	Togo	40.9
30	Spain	72.2	80	Botswana	58.4	130	Benin	40.0
31	Lithuania	72.1	81	Peru	58.4	131	Malawi	39.8
32	Malta	72.0	82	Bhutan	58.2	132	Mauritania	39.6
33	Bulgaria	71.8	83	Algeria	58.1	133	Mozambique	39.5
34	Portugal	71.5	84	Mongolia	58.1	134	Zambia	38.4
35	Italy	70.9	85	Saudi Arabia	58.0	135	Mali	38.2
36	Croatia	70.7	86	Lebanon	58.0	136	Gambia	37.8
37	Greece	69.9	87	Suriname	58.0	137	Yemen	37.3
38	Poland	69.8	88	Vietnam	57.6	138	Sierra Leone	36.9
39	Serbia	68.3	89	Bolivia	57.5	139	Afghanistan	36.5
40	Uruguay	68.0	90	Nicaragua	57.4	140	Madagascar	36.2
41	Romania	67.5	91	Colombia	57.2	141	Nigeria	36.1
42	Chile	67.2	92	Dominican Rep.	57.1	142	Guinea	35.9
43	Argentina	66.8	93	Gabon	56.2	143	Burkina Faso	35.6
44	Moldova	66.6	94	El Salvador	55.6	144	Haiti	34.4
45	Cyprus	66.5	95	Philippines	55.5	145	Chad	31.8
46	Ukraine	66.4	96	Cape Verde	55.5	146	Niger	31.4
47	Russia	66.4	97	Sri Lanka	54.8	147	Congo D.R.	31.3
48	Turkey	66.1	98	Indonesia	54.4	148	Liberia	30.5
49	Qatar	65.8	99	South Africa	53.8	149	Central African Rep.	26.1
50	Armenia	65.4	100	Kuwait	52.5			

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