Automated vehicle driving on large lane-free roundabouts*

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***Abstract*— Automated vehicle driving on large, complex, lane-free roundabouts is a major challenge. As a striking example for this challenge, we consider the famous roundabout of Place Charles de Gaulle in Paris, featuring a width of 38 m and comprising a dozen of entering/exiting radial streets. The paper proposes a complete generic methodology to control the lane-free paths of automated vehicles. The developed real-time vehicle movement control strategy relies on appropriate automated offline computation of: (a) wide overlapping movement corridors, one for each Origin-Destination (OD) movement, which delineate the admissible movement zones of corresponding OD vehicles; (b) desired vehicle orientation at each location within each OD corridor. Real-time vehicle movement within the respective corridor is effectuated by a distributed (per vehicle) nonlinear feedback control strategy, such that vehicles can move forward efficiently, accounting, when possible, for the pre-specified desired orientation, while avoiding collisions with other vehicles. Boundary controllers, developed based on linear state-feedback approaches, are used as safety filters defining upper and lower bounds for the vehicle steering angle, such that it is guaranteed that: a vehicle never violates its admissible corridor and roundabout boundaries; and never misses its exit. Microscopic simulation testing results demonstrate the pertinence and effectiveness of the suggested approach.**

I. Introduction

Despite traffic management measures [1, 2], road traffic congestion has been an increasingly serious societal problem that causes excessive delays, substantial environmental pollution and reduced traffic safety. During the last decade, there has been an enormous effort to develop and deploy a variety of Vehicle Automation and Communication Systems (VACS) that are revolutionizing the capabilities of individual vehicles, which may be leveraged towards a new generation of traffic management that may strongly alleviate congestion problems [3, 4]. In fact, numerous companies and research institutions have been developing and testing in real traffic conditions high-automation or virtually driverless autonomous vehicles that monitor their environment and make sensible driving decisions [5].

Recently, the TrafficFluid concept was launched with [6], which is a novel paradigm for vehicular traffic, applicable at high levels of vehicle automation and communication and high penetration rates, as expected to prevail in the not-too-far future. Specifically, vehicles may communicate with each other (V2V) and with the infrastructure (V2I); and drive automatically, based on own sensors, communications, and appropriate movement control strategy. Other than that, vehicles may be of various types and sizes and may have a variety of desired (or allowed) maximum speeds and accelerations. Given these, the TrafficFluid concept is based on the following two combined principles: (a) Lane-free traffic, whereby vehicles are not bound to fixed traffic lanes, as in conventional traffic, but may drive anywhere on the 2-D surface of the road; and (b) Vehicle nudging, whereby vehicles communicate their presence to other vehicles in front of them (or are sensed by them), and this may exert a "nudging" effect on the vehicles in front.

In an excellent recent keynote presentation [7], Luc Julia mentioned two reasons why fully driverless vehicles may never actually exist; one of the two mentioned reasons being that driverless vehicles would hardly be able ever to drive on a road infrastructure as complex as the Place Charles de Gaulle roundabout in Paris (Fig. 1). This famous roundabout has an outer (inner) radius of 84 m (46 m), hence featuring a width of 38 m. The roundabout comprises a dozen of entering/exiting radial streets, i.e., 144 distinct origin-destination (OD) movements for the vehicles. Given this complexity, this road infrastructure has been operating without lanes, hence human drivers must find their way, once on the roundabout, without adhering to traffic lanes. The above remark by Luc Julia was the trigger for us to address the challenge and consider the Place Charles de Gaulle roundabout, which is anyhow a lane-free infrastructure, as a case study for the TrafficFluid concept, i.e., to develop a vehicle movement strategy for automated vehicles that populate and drive on such complex roundabouts, as reported in this paper.

As a matter of fact, several works [8 – 21] in the literature have considered automated vehicles driving on roundabouts, something that is indeed considered challenging due to the special geometric features [8]. Table 1 classifies the reported approaches according to some important features. It appears that all reported works address much simpler roundabout cases that do not even approach the complexity of the case study of this paper. In particular, most works consider single-lane or double-lane roundabouts and a limited number of radial streets.

Over the last couple of years, a number of works have developed movement strategies for automated vehicles on lane-free highways, in accordance with the TrafficFluid paradigm, using different methodologies, such as: ad-hoc strategies [6], optimal model predictive control [22], reinforcement learning [23], nonlinear feedback control [24]; and a generic simulation environment for lane-free traffic has

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also been developed [25]. In particular, [24] proposes, on the basis of the bicycle kinematic model, a two-dimensional (lane-free) nonlinear cruise controller for automated vehicles on straight highways, which guarantees collision and boundary violation avoidance, speeds in the allowable range and their convergence to the desired value, and convergence of accelerations, lateral speeds, and orientations to zero. It is a fully distributed approach, such that each vehicle's controller only needs its own current variables and distances from surrounding vehicles to calculate, in a real-time closed-loop manner, the control inputs, i.e., forward acceleration and steering angle.

This paper suggests a comprehensive approach to control vehicles at complex lane-free roundabouts in all involved situations, i.e., entering, navigating, and exiting. Given the urban environment and need for vehicle steering, the vehicle dynamics are modeled by the bicycle model and two appropriate transformations are employed, for circular and skewed motions, which facilitate controller design for those situations. The vehicle movement relies on offline computation of: (a) overlapping movement corridors, one for each OD movement, which delineate the admissible movement zones of corresponding OD vehicles; and (b) desired vehicle orientation at each location within each OD corridor. The nonlinear controller presented in [24] is employed to control vehicles within the defined corridors, after applying some needed modifications, which enable vehicles to follow non-zero orientation, as necessary on roundabouts. Besides, state-feedback boundary controllers guarantee that vehicles will remain within their corridors as well as within the roundabout boundaries.

The structure of the paper is as follows. Section II explains the vehicle dynamics and the transformations for circular and skewed movements. The nonlinear and boundary controllers are expressed in Section III. Section IV describes the offline computation details. Simulation results are presented in Section V. Finally, concluding remarks are given in Section VI.

II. VEHICLE MODELLING AND THE TEST INFRASTRUCTURE

A. Vehicle dynamics

In this study, vehicle dynamics are represented by the kinematic bicycle model, which has been widely used in the literature [24,26]. The model variables are depicted in Fig. 2, and the state-space model is described as below [24]:

$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= \sigma^{-1} v \tan(\delta) \\ \dot{v} &= F \end{aligned} \tag{1}$$

where x and y are the longitudinal and lateral position coordinates of the rear axle midpoint of the vehicle. Also, v and $\theta \in [-\pi/2, \pi/2]$ are speed and orientation of the vehicle. Moreover, the model has two control inputs: acceleration F and steering angle δ. Finally, σ is the length of the vehicle. To simplify the third state equation above, we define [24]

TABLE I. CLASSIFICATION OF REFERENCES ADDRESSING AUTOMATED VEHICLE DRIVING AT ROUNDABOUTS

Reference	Coverage		Model		Traffic		Roundabout Lanes	Number of Entries/ Exits	Merging Lanes	Control Method		
	Merging only	Full Navigation	Double Integrator	Bicycle Model	Automated Vehicles	Mixed Traffic				Optimal/ MPC	Game Theory	Heuristic
[8]		*	*		*		1	4	1	*		
[9]	*		*		*		1	4	1			*
[10]		*	*		*		1	2	1	*		
[11]		*	*		*		1	2	1	*		
[12]		*	2D		*		1	4	1	*		
[13]		*	2D			*	1	4	1	*		
[14]		*	*		*		1	4	1			*
[15]		*	*		*		1	4	1	*		
[16]		*	*		*		1	4	1	*		
[17]		*	real van		*		2	4	2			Fuzzy
[18]		*	*		*		2	4	1	*	*	
[19]		*		*	*		2	4	2	*	*	
[20]		*		*		*	2	4	1	*	*	
[21]		*		*	*		4	5	4	Machine Learning		

$$u = \sigma^{-1} v \tan(\delta) \tag{2}$$

Then, (1) can be written as follows:

$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= u \\ \dot{v} &= F \end{aligned} \tag{3}$$

B. Transformation for circular movement

When a vehicle has a circular movement at a roundabout, transforming its dynamics to polar coordinates is beneficial for easier analysis and controller design. Assuming that the origin of the Euclidean coordinates (x, y) is the centre of the roundabout, the state variables in the new coordinates can be expressed as below:

$$\begin{aligned} x_1 &= r = \sqrt{x^2 + y^2} \\ x_2 &= \theta_r = \begin{cases} \tan^{-1}(y/x) + \pi/2 & x \geq 0 \\ \tan^{-1}(y/x) + 3\pi/2 & x < 0, y > 0 \\ \tan^{-1}(y/x) - \pi/2 & x < 0, y < 0 \end{cases} \\ x_3 &= \theta - \theta_r \\ x_4 &= v \end{aligned} \tag{4}$$

where r is the radius of the vehicle position, measured from the centre of the roundabout; θ_r is the slope of the circle tangent at the current position. For circular vehicle motion, the vehicle orientation would be θ_r; therefore, we define $x_3 = \theta - \theta_r$ as a state variable, which may take positive or negative values, similarly to the vehicle orientation on a straight road in Euclidian coordinates. After calculating the time-derivatives of the new state variables and processing them, the system dynamics for the transformed model are obtained as below:

$$\begin{aligned} \dot{x}_1 &= -x_4 \sin x_3 \\ \dot{x}_2 &= x_4 \cos x_3 / x_1 \\ \dot{x}_3 &= u - x_4 \cos x_3 / x_1 \\ \dot{x}_4 &= F \end{aligned} \tag{5}$$

C. Transformation for skewed path

In some situations, a vehicle may have to be guided along a skewed path with angle $\theta' \in [0, 2\pi]$. Transformation to corresponding new coordinates can be helpful, in order to employ the controllers originally designed for horizontal roads. The transformation is illustrated in Fig. 3, where the skewed coordinates are (x', y') and can be derived as below:

$$\begin{aligned} x' &= x\cos\theta' + y\sin\theta' \\ y' &= y\cos\theta' - x\sin\theta' \end{aligned} \tag{6}$$

Also, the difference between the vehicle orientation and the skewed angle θ' is considered as a state variable $\xi = \theta - \theta'$. The state equations are obtained by calculating and processing the time-derivatives of the new state variables:

$$\begin{aligned} \dot{x}' &= v\cos\xi \\ \dot{y}' &= v\sin\xi \\ \dot{\xi} &= u \\ \dot{v} &= F \end{aligned} \tag{7}$$

D. The Place Charles de Gaulle roundabout

The designed vehicle movement strategy is generic and suitable for complex roundabouts. Although a specific example is considered in the paper, the methodology does not make any use of its specific topology and characteristics, hence it can be easily applied to other large roundabouts. Our case study, the Place Charles de Gaulle roundabout, has an outer (inner) radius of 84 m (46 m), hence featuring a width of 38 m and comprises 12 radial branches, all of them bidirectional, which results in 144 distinct OD movements. Fig. 4 illustrates the geometry of the roundabout, where "branch1" represents the famous Champs-Élysées Avenue.

III. VEHICLE CONTROLLER DESIGN

A. General Concept

Given the complex nature of large roundabouts with multiple involved OD movements and lane-free structure, it does not seem reasonable to determine rigid space-time paths, on which vehicles should drive from their origin to their destination. Instead, we are pursuing a distributed approach, whereby automated vehicles navigate independently, based on a feedback movement strategy, which is presented in section III.B. More precisely, each vehicle decides in real time about its acceleration and steering angle, taking into account the own position relative to the roundabout geometry, as well as the adjacent vehicles' positions (to avoid collisions). However, to enable pertinent and efficient real-time decisions by the vehicles, it is necessary to provide them with two pre-computed roundabout-related elements: (a) An OD-dependent zone or corridor, within which a vehicle may move; and (b) some guideline regarding the direction of its movement.

With regard to (a), we specify offline, for each OD, a corresponding wide corridor connecting the specific origin and destination, see Section IV.A. The OD corridors are overlapping subsets of the roundabout surface, and a vehicle, having a specific OD, is only allowed to move within the corresponding OD corridor. Appropriate boundary controllers are employed to guarantee that vehicles do not exceed the corridor's (and hence also the roundabout) boundaries; and that vehicles do not miss their destination. With regard to (b), we specify, for each location of each OD corridor, a desired orientation for the vehicle movement, see Section IV.B. While making its real-time movement decisions, a vehicle takes into account the pre-specified desired orientation at its current location, without being forced to rigidly stick to it, as other high-priority objectives (e.g., crash avoidance) may be at stake. The structure of the overall control strategy is described by a block diagram in Fig. 5.

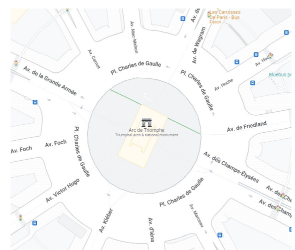


Figure 1. Place Charles de Gaulle roundabout in Paris (GoogleMap photo)

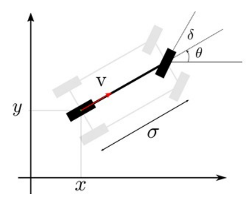


Figure 2. The bicycle model details [24]

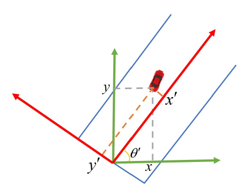


Figure 3. The transformation for the skewed path

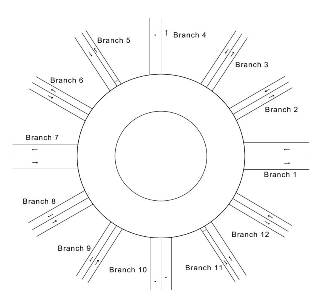


Figure 4. The Place Charles de Gaulle roundabout structure

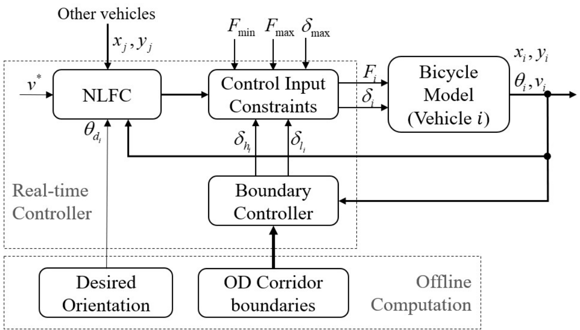


Figure 5. Control strategy structure

B. The Nonlinear feedback controller (NLFC)

A NLFC (Fig. 5) is the kernel for real-time decision making by each vehicle. In [24], a nonlinear controller is presented to control vehicles on a straight lane-free highway using the vehicle's state variables and its distance from other adjacent vehicles. The controller was designed for the continuous-time model (1) and was rigorously shown to have a number of properties. Specifically, the controller avoids collisions, boundary violation, negative speed, and exceeding the allowable maximum speed. Also, if there is sufficient space, vehicles reach the desired longitudinal speeds, while accelerations, orientations, lateral speeds, and steering angles tend to zero (on a straight road). The feedback law reads [24]:

$$\begin{aligned}
&\delta_i = \tan^{-1}\left(\sigma u_i / v_i\right) \\
&u_i = -\left(v^* + \frac{A}{v_i\left(\cos(\theta_i) - \cos(\varphi)\right)^2}\right)^{-1} \begin{pmatrix} \mu_1 v_i \sin(\theta_i) + U'(y_i) \\ + p\sum_{j\neq i} V'(d_{i,j})\left(y_i - y_j\right) / d_{i,j} \\ + \sin(\theta_i) F_i \end{pmatrix} \\
&F_i = -\frac{K_i}{\cos(\theta_i)}\left(v_i \cos(\theta_i) - v^*\right) - \frac{1}{\cos(\theta_i)}\sum_{j\neq i} V'(d_{i,j})\left(x_i - x_j\right) / d_{i,j} \\
&K_i = \mu_2 + \frac{1}{v^*}\sum_{j\neq i} V'(d_{i,j})\left(x_i - x_j\right) / d_{i,j} \\
&\quad + \frac{v_{\max}\cos(\theta_i)}{v^*\left(v_{\max}\cos(\theta_i) - v^*\right)} f\left(-\sum_{j\neq i} V'(d_{i,j})\left(x_i - x_j\right) / d_{i,j}\right)
\end{aligned} \tag{8}$$

where v^* is the desired speed, $v_{\max}$ is the maximum allowable speed, μ_1, μ_2, and A are controller gains. $d_{i,j}$ is the elliptical distance between vehicles, defined as

$$d_{i,j} = \sqrt{\left(x_i - x_j\right)^2 + p\left(y_i - y_j\right)^2} \tag{9}$$

where $p \geq 1$ is a factor that determines the shape (length versus width) of the considered distance ellipses. Moreover, θ_i should remain in $[-\varphi, \varphi]$, where $\varphi \in (0, \pi/2)$ is a constraint. Also, $V(d_{i,j})$ and $U(y_i)$ are repulsive potential functions for the distance and lateral position, respectively; these functions are employed to maintain safe distances from adjacent vehicles and avoid road boundary violation, respectively. In this work, the lateral-position potential function is utterly left out, and its role of avoiding road boundary violation is undertaken by the boundary controllers presented in Section III.C. Furthermore, suggested function for the distance potential function is as below:

$$V(d) = \gamma_1 \gamma_2 \exp(-d/\gamma_2) \tag{10}$$

where γ_1 and γ_2 are design parameters. Moreover, f is defined as in [24]

$$f(x) = \frac{1}{2\varepsilon}\begin{cases} 0 & x \leq -\varepsilon \\ \left(x + \varepsilon\right)^2 & -\varepsilon < x < 0 \\ \varepsilon^2 + 2\varepsilon x & x \geq 0 \end{cases} \tag{11}$$

where $\varepsilon > 0$ is a constant.

The nonlinear controller (8) was designed for straight roads, where the desired orientation is zero (along the road). However, for vehicles driving on a roundabout, it is required to consider non-zero desired orientation θ_{d_i}. In such cases, the nonlinear feedback controller needs to be modified accordingly, by replacing the controller arguments as follows:

- Replacing the orientation θ_i with $\theta_i - \theta_{\mathrm{d}_i}$.
- Replacing the position coordinates x_i and y_i with x_i' and y_i', respectively, according to (6), considering $\theta' = \theta_{\mathrm{d}_i}$.
- Replacing adjacent vehicles' coordinates x_j and y_j with x_j' and y_j', respectively, calculated by (6), considering $\theta' = \theta_{\mathrm{d}_i}$.

C. The boundary controllers

In a lane-free traffic environment, guaranteeing that a vehicle will not violate the road boundaries is more important than in a lane-based environment, where vehicles are typically guided around the middle of each lane (hence also of both outer lanes), and therefore the risk of actually departing from the road is less serious. In addition, in lane-free traffic, there is an interest, e.g., at dense traffic, to let some vehicles drive exactly on a (left or right) road boundary (of course without ever violating it), so as to maximize the exploitation of the available road width.

These two endeavors may be fulfilled via the introduction of two boundary controllers, one for the left and another for the right road boundary, which deliver appropriate upper and lower bounds, respectively, for the steering angle determined by the NLFC of the vehicle. As long as such a bound is not activated, the vehicle drives according to NLFC; when one of the bounds is activated, it is designed to navigate the vehicle asymptotically to the corresponding road boundary and then have it driving exactly on the road boundary for as long as the steering angle produced by NLFC is actually activating that bound. The boundary controller is firstly designed for a straight road and then extended for circular and skewed paths.

Boundary controller for straight roads: The boundary controllers are designed such that, if activated, vehicles asymptotically reach to a road boundary, whereby overshooting in the responses is not allowed, as it might lead to road boundary violation. For boundary control on a straight road, we consider the bicycle model (1) and have two main goals: convergence of the vehicle's lateral position $y(t)$ to the road boundary; and convergence of the vehicle orientation $\theta(t)$ to zero. Since longitudinal position and speed have a minor role in these endeavors, we focus on a subsystem of the bicycle model, which includes only the two state variables of interest, $y(t)$ and $\theta(t)$, and one control variable, $\delta(t)$, while the speed v, which is also involved in the subsystem equations, may be considered as a measurable exogenous quantity (disturbance), if necessary.

A simple but efficient approach for designing a boundary controller is to linearize the subsystem around the desired equilibrium point ($y = y_d, \theta = 0, \delta = 0$) and use a linear state-feedback controller; where y_d is the desired lateral position, i.e., the road boundary. Then, the linearized system can be derived as

$$\begin{bmatrix} \dot{y}(t) & \dot{\theta}(t) \end{bmatrix}^T = \mathbf{A}\begin{bmatrix} y(t) - y_d & \theta(t) \end{bmatrix}^T + \mathbf{B}u(t) \quad (12)$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & v^* \\ 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (13)$$

Now, a linear state-feedback controller can be designed for the subsystem as below:

$$u(t) = -\mathbf{K}\begin{bmatrix} y(t) - y_d & \theta(t) \end{bmatrix}^T \quad (14)$$

where $\mathbf{K}$ is the feedback gain, which can be readily computed such that the closed-loop eigenvalues are real and negative, to provide asymptotical convergence of the state variables to their desired values.

Boundary controller for circular paths: When a vehicle has a circular movement at a roundabout, inner and outer boundaries of the roundabout must be taken into account; hence, the transformed model for the circular motion, presented in Section II.B, should be considered. Following a similar design procedure as for the straight road, a subsystem containing $x_1 = r$ and $x_3 = \theta - \theta_r$ is considered for the design of the boundary controllers. The system is linearized at the equilibrium point ($x_1 = r_d$, $x_3 = 0$, $\delta = \delta_d$), while the external variables for the subsystem are set $x_4 = v^*$ and $F = 0$; where r_d is the desired radius, i.e., the inner or outer radius of the roundabout, and it is easy to show that the stationary value of the steering angle δ_d in circular motion with radius r_d is given by

$$\delta_d(r_d) = \tan^{-1}(\sigma / r_d) \quad (15)$$

Given (5), the linearized subsystem reads

$$\begin{bmatrix} \dot{x}_1(t) & \dot{x}_3(t) \end{bmatrix}^T = \mathbf{A}\begin{bmatrix} x_1(t) - r_d & x_3(t) \end{bmatrix}^T + \mathbf{B}\left(u(t) - u_d(r_d)\right) \quad (16)$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & -v^* \\ v^*/r_d^2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, u_d(r_d) = \sigma^{-1} v \tan(\delta_d(r_d)) \quad (17)$$

Also, considering (2), (15) and (16), the controller is

$$u(t) = -\mathbf{K}_c\begin{bmatrix} r(t) - r_d & \theta(t) - \theta_r(t) \end{bmatrix}^T + v(t)/r_d \quad (18)$$

where $\mathbf{K}_c$ is the feedback gain for the circular boundary controllers.

Boundary controller for skewed paths: A suitable boundary controller is also needed for skewed boundaries, and the corresponding model was presented in Section II.C. Since the transformed model for a skewed path (7) is same as the original bicycle model (3), we may use the designed state-feedback controller for the straight road by only replacing y and θ with y' and ξ in the feedback law. So, the feedback law for a skewed boundary controller is given by

$$u(t) = -\mathbf{K}\begin{bmatrix} y'(t) - y'_d & \xi(t) \end{bmatrix}^T \quad (19)$$

where y'_d is the desired lateral position for the vehicle in the transformed coordinates. Moreover, $\mathbf{K}$ is the same feedback gain determined for the straight boundary controller.

IV. OD CORRIDORS AND DESIRED ORIENTATIONS

A. *Defining OD corridors*

Given the high number of OD couples, it is reasonable to have the corresponding corridors being defined automatically, following specific rules. The outer boundary of all OD corridors coincides with the roundabout's outer boundary between the specific origin and destination (see Fig. 6). On the other hand, for better infrastructure utilization, a more pertinent definition of the corridors' inner boundary is required. To this end, OD corridors are distinguished in two types: (1) The destination is visible from the origin (Fig. 6(a)); and (2) The destination is not visible from the origin (Fig. 6(b)).

For Type-1 ODs, it appears reasonable to define corridors that do not extend up to the inner roundabout boundary, as vehicles are bound to exit soon after entering and should therefore not navigate through the inner part of the roundabout, which may be reserved for vehicles with longer trips. In contrast, vehicles belonging to Type-2 ODs must drive partly on circular paths (around a part of the roundabout), and therefore the corresponding corridors may extend up to the inner roundabout boundary. Below, we describe the details for each corridor type.

Visible destination: For OD couples of a roundabout where the origin and destination are relatively close to each other, e.g., when the destination is visible from the origin, the shortest and simplest way to get there is to take a direct path, which is close to or on the outer roundabout boundary, without passages through the inner part of the roundabout. The selection of ODs belonging to this type depends on the specific roundabout geometry. For the case of Place Charles de Gaulle roundabout, the destination is visible from the origin if is up to 3 branches away from the origin. The considered corridor for Type-1 ODs is illustrated in Fig. 6(a). The corridor's right-hand side coincides with the outer roundabout boundary; while its left-hand boundary is a straight line connecting the left-most point of the origin branch with the right-most point of the destination branch.

Invisible destination: For the Place Charles de Gaulle roundabout, ODs having more than 2 branches between origin and destination belong to Type 2. For such ODs, vehicles cannot move on a direct path toward the destination, and a circular motion must be partially pursued. In this condition, a vehicle is allowed to use the full width of the roundabout, till it gets close to the destination; when the destination becomes visible from the inner roundabout boundary, the left-hand delineation of the corridor is again a straight line that leads to the right-most point of the destination branch. In summary, the corridor for Type-2 ODs consists of two sections. The first section starts at the left-most point of the origin branch and is bounded by two roundabout boundaries; while the second section is similar to Type 1, having a circular right boundary and a straight right boundary, which coincides with the tangent line from the right-most point of the destination branch to the inner roundabout boundary. The mentioned corridor type is illustrated in Fig. 6(b).

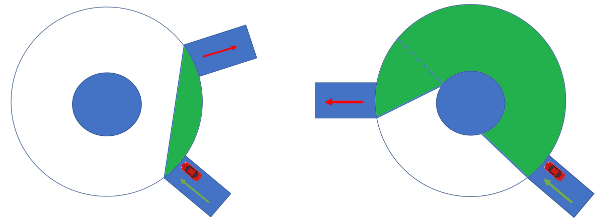
B. *Specifying Desired Orientations*

While moving within its OD corridor, a vehicle should have some guideline regarding its direction of movement, so that it merges in the roundabout traffic, advances towards its destination and eventually exits. This guideline is provided in form of desired orientations for the vehicle that are computed for every vehicle position within the OD corridor and are fed to the NLFC for consideration in the vehicle movement decisions. The desired orientations are computed based on some rules, which differ according to the corridor type. To specify these rules, it is easier to work with polar coordinates, where position is represented by radius and angle. Since the radius is constant for all entrances and exits (R_{out}), we may use only the angle to refer to their positions on the outer boundary. In particular, an entrance is introduced by an angular interval $[\theta_{\text{en}}, \theta_{\text{en}} + \Delta\theta_{\text{en}}]$ where $\theta_{\text{en}} \in [0, 2\pi)$, and $\Delta\theta_{\text{en}} > 0$ is the angular width of the entrance. Similarly, an exit branch is characterised by $[\theta_{\text{ex}}, \theta_{\text{ex}} + \Delta\theta_{\text{ex}}]$, where $\theta_{\text{ex}} \in [0, 2\pi)$ and $\Delta\theta_{\text{ex}} > 0$.

Visible destinations: For Type-1 corridors, a direct path appears reasonable. In order to exploit the whole width of the exit branch and avoid unnecessary path crossing of entering vehicles with the same destination, an exit angle θ_c is determined for each entering vehicle through following linear mapping:

$$\theta_{\text{c}} = \theta_{\text{ex}} + \left(1 - (\psi_{\text{en}} - \theta_{\text{en}})/\Delta\theta_{\text{en}}\right)\Delta\theta_{\text{ex}} \tag{20}$$

Invisible destinations: For Type-2 corridors, where the destination is not visible from the origin, the vehicle is navigated to a circular movement, till it gets close enough to its exit. If vehicles would start a circular motion as soon as they enter the roundabout, most vehicles would tend to move along the outer boundary. As a result, the capacity of the roundabout would not be fully used, and the outer area would be more crowded. Therefore, vehicles should be allowed to move towards the roundabout interior area; on the other hand, perpendicular movements should be avoided, as they may cause strong conflicts with other vehicles driving on circular paths. To account for these issues, an angular entrance transition zone is considered, within which the desired orientation of the vehicle gradually tends to the slope of circle tangent ($\theta_r(t)$ in (4)) that is the desired heading for a circular motion. The zone size is determined from



(a) (b)

Figure 6. Defined corridor: (a) for visible ODs (Type 1), (b) for invisible ODs (Type 2)

$$A_{\text{t}} = (\Delta b/12)\, A_{\text{t}_{\max}} \tag{21}$$

where Δb is the number of branches that the vehicle must pass to reach its exit; and $A_{\text{t}_{\max}}$ is a design parameter. Within the transition zone, i.e., for vehicular angles $\psi(t) \in [\theta_{\text{en}}, \theta_{\text{en}} + A_{\text{t}}]$, the desired orientation is calculated as follows:

$$\theta_{\text{d}}(t) = \frac{A_{\text{t}} - (\psi(t) - \theta_{\text{en}})}{A_{\text{t}}}\theta_{\text{d}}(0) + \frac{\psi(t) - \theta_{\text{en}}}{A_{\text{t}}}\theta_{\text{r}}(t) \tag{22}$$

where $\theta_{\text{d}}(0) = \theta_{\text{en}} + \pi - \theta_0$ is an initial angle to avoid perpendicular entrance of the vehicle, and θ_0 is a design parameter.

After passing the transition zone, the vehicle's desired orientation is $\theta_{\text{r}}(t)$, which is permanently changing as the vehicle advances. The circular motion continues till the vehicle gets close to the exit branch. To avoid perpendicular motion, an exit transition zone is then defined with a length of $2A_{\text{sk}}$, where:

$$A_{\text{sk}} = \left((R_{\text{out}} - r(t))/(R_{\text{out}} - R_{\text{in}})\right) A_{\text{sk}_{\max}} \tag{23}$$

where $A_{\text{sk}_{\max}}$ is a design parameter, R_{in} is the radius of inner circle of roundabout, and $r(t)$ is the radius of the vehicle position in polar coordinate. With (23), vehicles which are closer to the interior boundary of the roundabout start their exit procedure sooner. The exit transition zone is divided into two parts. In the first part, the circular motion is gradually changing to a skewed motion; while in the second part, the desired orientation points directly toward the exit point which can be determined based on its distance from the centre as below:

$$\theta_{\text{c}} = \theta_{\text{ex}} + \left(1 - (r_{\text{sk}} - R_{\text{in}})/(R_{\text{out}} - R_{\text{in}})\right)\Delta\theta_{\text{ex}} \tag{24}$$

where r_{sk} is the radius of the vehicle position in polar coordinates at the beginning of the exit transition zone. In the first part, i.e., for $\psi(k) \in [\theta_{\text{ex}} - 2A_{\text{sk}}, \theta_{\text{ex}} - A_{\text{sk}}]$, the desired orientation is calculated by a weighted average, based on the vehicle's angle in polar coordinate as below:

$$\theta_{\text{d}}(k) = \frac{(\theta_{\text{ex}} - \psi(t)) - A_{\text{sk}}}{A_{\text{sk}}}\theta_{\text{r}_{\text{sk}}} + \frac{2A_{\text{sk}} - (\theta_{\text{ex}} - \psi(t))}{A_{\text{sk}}}\theta_{\text{d}_{\text{sk}}}(t) \tag{25}$$

where $\theta_{\text{r}_{\text{sk}}}$ is the value of $\theta_{\text{r}}(t)$ at the beginning of this zone. Also, $\theta_{\text{d}_{\text{sk}}}(t)$ is the slope of a direct line connecting the current position to the exit point, which can be obtained by (22). Also, in the second part of the exit transition zone, i.e., for $\psi(t) \in [\theta_{ex} - A_{sk}, \theta_c]$, the desired orientation is obtained by (22).

Based on the above explanations, desired orientation can be specified and recorded at all positions for a given OD. Then, in real-time, a vehicle only needs to retrieve the desired orientation corresponding to its current position. Fig. 7 depicts the desired directions for an OD, where branches 1 and 9 are considered as the entrance and exit, respectively. Despite some complexity, the above rules are generally applicable and very fast to execute. Alternatively, more methodologically transparent approaches for determining the desired orientations are currently in development.

V. Simulation Results

To investigate the effectiveness of the proposed methodology, a preliminary simulation with 10 vehicles moving on Place Charles de Gaulle roundabout is carried out. The simulation is done in Matlab and will be implemented in TrafficFluid-Sim [27] later. Along with the presented approach, some practical issues are considered in the simulation. System dynamics and controllers are implemented in discrete-time domain with a reasonable sample period of 0.1 s that facilitates real-time implementation of the suggested approach. Also, practical bounds are applied to the control inputs, such that acceleration and steering angle are truncated if they exceed the convenience ranges $[F_{\min}, F_{\max}]$ and $[-\delta_{\max}, \delta_{\max}]$, respectively. Furthermore, following parameters are used:

$$\begin{aligned}
&\sigma = 5, \varepsilon = 0.2, \varphi = 1, \mathrm{p} = 5.11, \mathrm{A} = 0.1, L = 5.69, \mathrm{v}^* = 12,\\
&\mathrm{v}_{\max} = 30.1, \mu_1 = 4.2, \mu_2 = 0.1, A_{t_{\max}} = \pi/4, A_{sk_{\max}} = \pi/4,\\
&\theta_0 = \pi/6, \gamma_1 = 25, \gamma_2 = 6, F_{\min} = -2, F_{\max} = 0.6, \delta_{\max} = 50^\circ,\\
&\mathbf{K} = \begin{bmatrix} 25 & 35 \end{bmatrix}, \mathbf{K}_c = \begin{bmatrix} -12.5 & 25 \end{bmatrix}
\end{aligned} \tag{26}$$

The simulation results are presented in Fig. 8 to Fig. 12. Vehicles, depending on their origins and destinations, navigate such that they are properly distributed on the roundabout, and perpendicular movements are avoided, as can be observed in Fig. 8. In Fig. 9, there are some few large accelerations and decelerations, which are generated by the nonlinear controller to avoid crashes. Such peaks can be mitigated with better tuning of the NLFC parameters responsible for repulsion (ongoing work). For passenger convenience, the gain μ_2, responsible for desired speed tracking was given a low value, leading to conveniently slow speed convergence (see Fig. 10). Also, Fig. 11 shows that vehicles may exhibit large steering angles at the entrances and exits, which is deemed natural, but they do not have sharp steering within the roundabout. Finally, the minimum inter-vehicle distance is used as a simple index to detect if any collisions have occurred. As shown in Fig. 12, all inter-vehicle distances are always higher than the safety distance L, which means no collision has occurred. It should be mentioned that an exact collision detection approach is utilized in the simulation that considers length, width, and orientation of vehicles. A video with 40 vehicles involved may be viewed at https://bit.ly/36exR42.

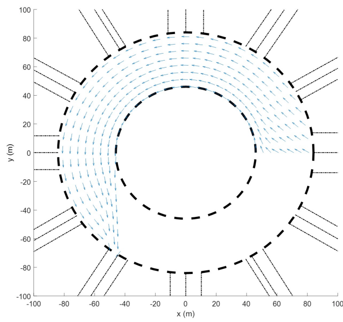


Figure 7. Desired orientations for a specific OD

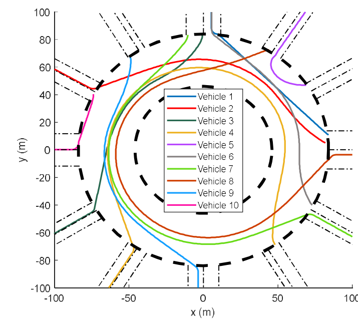


Figure 8. Vehicles' trajectories

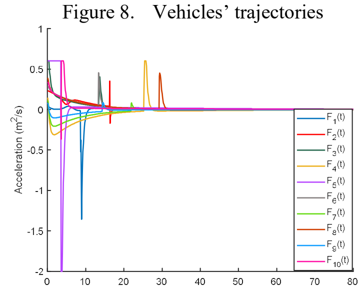


Figure 9. Vehicles' accelerations

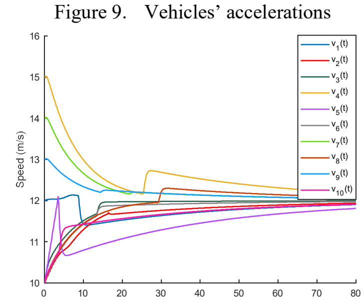


Figure 10. Vehicles' speeds

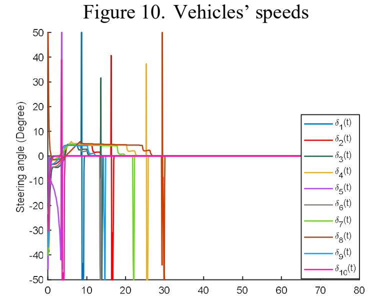


Figure 11. Vehicles' steering angles

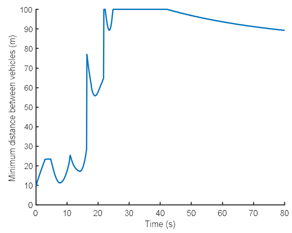


Figure 12. Minimum distance between vehicles

VI. Conclusion

The present work develops an effective strategy to control automated vehicles at large lane-free roundabouts. Vehicle dynamics are considered by the bicycle model, which is also transformed in suitable coordinates for circular and skewed paths to facilitate the controller design. For each OD, a corridor is defined, in which vehicles can move based on the developed movement strategy. To this end, at every position, a desired orientation is fed to the nonlinear controller, which incorporates the ideal path. Corridors and desired orientations are computed offline according to developed general rules. Then, in real-time, vehicles only need to retrieve the desired direction corresponding to their current position. Moreover, state-feedback boundary controllers are designed for straight, circular, and skewed motions to guarantee that vehicles do not violate the defined corridors, nor the roundabout boundaries. Preliminary simulation results confirm the pertinence and effectiveness in vehicle driving tasks of the controller, such that vehicles properly navigate towards their destinations and remain within the roundabout boundaries, while control signals meet the practical limits, and no collision occurs. Our ongoing work focuses on investigating traffic-level efficiency by considering a higher number of vehicles.

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